Tuned liquid dampers for multi-storey structure: numerical simulation using a partitioned FSI algorithm and experimental validation

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Abstract. Wind-induced and earthquake-induced vibrations of structures such as super-tall towers and bridges can be efficaciously controlled by tuned liquid dampers (TLDs). This work presents a numerical simulation procedure to study the performance of TLDs–structure system through sigma (σ)-transformation-based fluid–structure coupled solver. For this, a ‘C’-based computational code has been developed. The structural equations, which are coupled with the fluid equations in order to achieve the transfer of sloshing forces to structure for damping, are solved by the fourth-order Runge–Kutta method, while the fluid equations are solved using finite-difference-based sigma-transformed algorithm. Different iterative and error schemes are used to optimize the code for larger convergence rate and higher accuracy. For validation, a few experiments are conducted with a three-storey structure using TLDs arrangement. The present numerical results of response of TLD-installed structures match well with the experimental results. The minimum displacement of structure is observed when the resonance condition of the coupled system is achieved through proper tuning of TLDs. Since real-time excitations are random in nature, the performance study of TLDs under random excitation has also been carried out in which the Bretschneider spectrum is used to generate the random input wave.

Keywords. Tuned liquid dampers; sloshing; σ -transformation; fluid–structure interaction; experiments.

1. Introduction

A partially filled water tank can be used as an effective damper for suppressing horizontal vibration of structures. This damper is called tuned liquid damper (TLD) because it utilizes the dynamic effect of liquid sloshing. A TLD is liquid confined in a container that uses the sloshing energy of the water to diminish the vibration of the system when the system is subjected to external excitation such as wind load and earthquake load. Several TLD systems can be kept in a structure to effectively control the structural oscillations with distributed sloshing frequencies over a range near the structural frequency and this is known as MTLT. MTLT has also been found to be very effective in revoking vibrations caused due to wind. However, in this work, several TLDs are used with same sloshing frequencies for mitigation of structural vibration. This damper is extremely practical and works on the principle of energy dissipation through liquid sloshing and wave breaking on the free surface. Fundamental mode frequency of liquid sloshing is

tuned to the natural frequency of the structure, and the damping ratio of the sloshing mode is set to an optimal value. As depicted by Tamura [1], the first TLD installation of an actual ground structure was performed successfully in Nagasaki Airport Tower (NAT), Nagasaki, Japan, in 1987. Now these dampers are used in many super-tall structures and bridges of developed countries, for example, in the John Hancock Tower in Boston, Citicorp Building in the New York and Ikuchi bridge and Sakitama bridge in Japan [2, 3].

When building a structure or component, it is important to calculate wind load to ensure that it can withstand high winds [4]. Mass and liquid dampers and base isolators are used to suppress the vibrations of the structures among the various alternatives. This paper will emphasize on one of these devices, TLD. A TLD consists of, partially filled liquid (usually water) tanks, which is typically situated on the top of a building. As the building moves in the extreme wind or earthquake attack, the fluid contained within the tank starts sloshing. Vibrational energy from the structure will be engrossed by the fluid, which transforms it into kinetic and potential energy of the sloshing fluid. Sloshing

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forces from the damper are used to mitigate structural responses. Since its inherent benefits include easy installation and handling, easy maintenance and lower cost, TLDs are gradually being used as vibration absorbers and have attracted increasing research interest in recent years [5]. Practical analysis of such dampers has been reported mainly for controlling structures due to wind loads [1, 6].

In this paper, a 'C'-based computational code is developed using finite-difference approach to capture the liquid free surface and liquid load estimation. Here the liquid free surface is captured using σ transformation as it eliminates the requirement for free surface smoothing and re-meshing. The calculated liquid loads are effectively coupled with structural equations to estimate the displacement of a TLD-coupled structure.

1.1 σ -Transformation technique

Along with appropriate fluid equations, an additional algorithm is required to capture the moving free surface. For this, algorithms such as marker and cell (MAC) method, level set method (LSM), volume of fluid (VOF), etc. are commonly used during the simulation process [7–14]. The coordinate transformation technique can also be used to track the liquid free surface [15, 16]. The particular type of coordinate transformation that features stretching in vertical direction is known as σ transformation. This approach has earned reputation because of its simplicity, since all others requires a complex computer programming in order to treat the time-varying free surface boundary and update the computational mesh.

Initially, Phillips [17] proposed the σ -transformation technique for meteorological forecasting. Later, Blumberg and Mellor [18] applied this to oceanic and coastal flows. The σ transformation was applied to nonlinear steep waves in fixed and base-excited tanks by Chern *et al* [19] and Turnbull *et al* [20]. Frandsen [21] studied steep free surface sloshing in fixed and base-excited rectangular tanks with a focus on moving liquid tank with horizontal and vertical excitations. Chen and Nokes [22], Dai and Xu [23], Eswaran *et al* [16], Wu *et al* [24] and Eswaran and Reddy [25] applied σ transformation to appraise the sloshing effects on simple 2-D and 3-D tanks such as a horizontal cylindrical container and rectangular tank, etc. In this work, the σ transformation is used to study the effectiveness of TLDs to track the free surface in fluid–structure coupled finite-difference solver.

1.2 Problem definition and objective

In order to find the structural displacement in TLD-coupled structure, many works approximate the behaviour of the liquid through the spring–mass model, as the actual liquid motion is complex and not amenable to exact mathematical

modelling. This spring–mass model is available for only simple geometries like rectangular and cylindrical tanks and procedures can be found in ACI [26] and Housner [27]. This model does not consider the nonlinear motion of the wave. The response of a TLD, even for small amplitude motion, is highly nonlinear due to liquid sloshing. For accurately capturing the motion of sloshing water in a vibrating tank, nonlinear functions must be employed. Hence, a dynamic fluid–structure coupled analysis is necessary. Therefore, it is necessary to develop a fluid–structure partitioned code for the accurate predictions of the structural motion, which is coupled with TLDs. Here, the fluid and structure equations are solved separately and effectively coupled in each iteration to study the performance of TLDs. The objectives of the present work are as follows.

- (i) To develop a computational fluid–structure coupled algorithm for the TLDs with a three-storey structure (as shown in figures 1 and 2) under base excitation. The code is used to analyse the effectiveness of the TLDs in reducing the structural oscillations under regular and random excitation through a fluid–structure coupled solver.
- (ii) To optimize the code performance using various iterative schemes, error norms, stopping criteria for larger convergence rate and accurate solutions.
- (iii) To validate the numerical solution with experimental results for various cases to increase the code credibility.
- (iv) To study the floor-wise performance of TLDs.

In order to accomplish this, a modified σ -transformation technique is used to map the asymmetric physical domain onto a rectangle computational domain, such that the moving free surface in the physical plane becomes a fixed plane surface in the computational domain. A finite-

Figure 1. Simple TLD and TLDs arrangements in three-storey skeleton structure.

Figure 2. Schematic of a TLD-structure system with notations.

difference scheme is used to solve the discretized fluid governing equations in the transformed computational domain to obtain the free surface elevation time histories, phase plane diagram, spectrum analysis and free surface profiles in the physical domain. Structural equations are solved by the fourth-order Runge–Kutta method. Section 2 elaborates on the mathematical formulation of fluid, its mapping procedure and the structural formulations. In section 2.2, the finite-difference discretizations of the fluid governing equation and boundary conditions in the computational domain have been described. Experimental procedure is explained in section 3. After a performance analysis of various iteration schemes, error schemes and independence study, a detailed validation is performed with experiments. Experimental results are compared with numerical results qualitatively and quantitatively in section 4. The concluding remarks are in section 5.

2. Mathematical formulation

2.1 Fluid formulations

Formulation for three-story structure-coupled TLDs is developed in this section. A rectangular Cartesian coordinate system is initially engaged with origin on the mean liquid free surface at the left wall of the container. Primarily, a nonlinear wave problem is considered, as depicted in figures 3 and 4, where ζ is the free-surface elevation above still water level, l is the length of the container and h_s is the still water depth. Here, the fluid in the container is assumed to be inviscid and irrotational. Due to the

Figure 3. Physical domain with initial wave profiles 1 and 2.

Figure 4. Sectional view of liquid tank with notations.

assumption that the fluid is governed by the potential flow theory, the velocity potential ϕ satisfies the Laplace equation. The velocity components normal to the fixed boundaries are zero by boundary condition. The left, right, front, back and bottom boundaries are indicated by L_W , R_W , F_W , B_W and BM_W , respectively (figure 4). Two boundary conditions are to be applied at the free surface (i.e., the free surface occurs at the interface between two fluids). The first boundary condition is a kinematic condition that relates the

motion of the free interface to the fluid velocities at the free surface (i.e., T_{W1}), while the second one is a dynamic condition, which is concerned with the force balance at the free surface (i.e., T_{W2}). Liquid velocity components normal to the walls and bottom of the container are zero. Zero pressure at the free surface of the fluid is also assumed in the analysis. In view of the afore-mentioned fact, kinematic and dynamic conditions must be satisfied on the free surface. The following quantities are introduced for generating dimensionless governing equations for the present study [16, 28]:

$$\begin{aligned} x' &= \frac{x}{l}; & y' &= \frac{y}{l}; & w' &= \frac{w}{l}; & \zeta' &= \frac{\zeta}{A} & \ddot{X}_t' &= \frac{\ddot{X}_t}{g}; \\ \ddot{Y}_t' &= \frac{\ddot{Y}_t}{g}; & \ddot{Z}_t' &= \frac{\ddot{Z}_t}{g}; & t' &= \sqrt{\frac{g}{l}}t; & \phi' &= \frac{1}{A\sqrt{lg}}\phi, \end{aligned} \quad (1)$$

where g is the acceleration due to gravity; l , w are, respectively, the length and width of the rectangular liquid damper. Here, $\ddot{X}_t$, $\ddot{Y}_t$ and $\ddot{Z}_t$ are the acceleration of the containers in the horizontal, vertical and lateral directions, respectively, which can be neglected from the free surface dynamic boundary condition for fixed container analysis. A is the wave amplitude, and t is the time.

The forcing characteristics amplitude is calculated as $A_f = F/A$, where F is the total external force applied to the system. The f_f is the force parameter ($=A_f/\rho gh^2$) and it is a measure of nonlinearity. Hereafter, primes are omitted for simplification; the non-dimensional governing equation and boundary conditions can be written as follows:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (2)$$

$$L_W \text{ and } R_W: \frac{\partial \phi}{\partial x} = 0 \text{ on } x = 0, l \quad (3)$$

$$F_W \text{ and } B_W: \frac{\partial \phi}{\partial z} = 0 \text{ on } z = 0, w \quad (4)$$

$$B_{M_W}: \frac{\partial \phi}{\partial y} = 0 \text{ on } y = -h_s \quad (5)$$

$$T_{W1}: \frac{\partial \zeta}{\partial t} + f_f \frac{\partial \phi}{\partial x} \frac{\partial \zeta}{\partial x} + f_f \frac{\partial \phi}{\partial z} \frac{\partial \zeta}{\partial z} = \frac{\partial \phi}{\partial y} \text{ on } y = \zeta \quad (6)$$

$$\begin{aligned} T_{W2}: & \frac{\partial \phi}{\partial t} + f_f \left(\frac{1}{2} \nabla \phi \nabla \phi \right) + (1 + \ddot{Y}_t) \zeta + x \ddot{X}_t + z \ddot{Z}_t \\ & = 0 \text{ on } y = \zeta \end{aligned} \quad (7)$$

Eqs. (2)–(7) form an initial boundary value problem, which is the Laplace equation with nonlinear boundary conditions imposed on the free surface. Here, the nonlinearity of the free surface is significant for two reasons: (i) the elevation

of the moving free surface is not known *a priori* and (ii) the boundary conditions on the free surface (i.e., Eqs. (6) and (7)) contain second-order differential terms or products of unknown parameters.

2.1a Mapping procedure: Through proper coordinate transformations, the time-varying liquid free surface can be mapped onto a fixed plane surface by the σ transformation, which prevents the need for free surface smoothing for the cases considered herein. In this paper, σ transformation is applied in the horizontal direction, which stretches between the left and the right wall and in the vertical direction it stretches between the moving liquid free surface and the bottom of the liquid container, to convert the moving free-surface physical domain onto a fixed square computational domain. The following section discusses the transformation technique in a 3-D container in detail. In this formulation portion, the longitudinal and lateral excitation terms are removed from free surface boundary equations. During transformation, the corresponding governing equation and boundary conditions will change appropriately.

2.1b Coordinate transformation: This transformation adopted the σ -transformation technique to map the liquid domain onto a rectangle, such that the moving free surface in the physical plane (figures 4 and 5) becomes a fixed horizontal line in the σ -transformed domain (figure 6). The mapping function $\sigma(x, z, t)$ is defined as

$$\sigma = \frac{y + h_s}{h}, \quad (8)$$

where

$$h(x, z, t) = I_b \zeta(x, z, t) + h_s. \quad (9)$$

Here, $\sigma(x, t)$ is the stretching factor, which varies from 0 to 1. The value of σ at the bottom of the container is 0, while at the free surface it is 1. The first-order derivatives of σ can be calculated as follows:

$$\frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial t} \left(\frac{y + h_s}{h} \right) = -I_b \frac{\sigma}{h} \frac{\partial \zeta}{\partial t} \text{ and } \frac{\partial \sigma}{\partial x} = -I_b \frac{\sigma}{h} \frac{\partial \zeta}{\partial x}. \quad (10)$$

The potential function $\phi(x, y, z, t)$ in the physical domain is transformed to the potential function $\Phi(X, \sigma, Z, T)$ in the σ -transformed domain. The mapping relations for first transformation are as follows:

$$X = 2x - 1; Y = 2\sigma - 1; Z = 2z - 1 \text{ and } T = t. \quad (11)$$

Hence the transformed governing equation becomes

$$\frac{\partial^2 \Phi}{\partial X^2} + K_1 \frac{\partial \Phi}{\partial Y} - K_2 \frac{\partial^2 \Phi}{\partial X \partial Y} - K_3 \frac{\partial^2 \Phi}{\partial Z \partial Y} + K_4 \frac{\partial^2 \Phi}{\partial Y^2} + \frac{\partial^2 \Phi}{\partial Z^2} = 0 \quad (12)$$

where

Figure 5. Physical domain with grid.

Figure 6. The 3-D intermediate domain after coordinate transformation.

$$\begin{aligned}
 K_1 &= \left[\frac{4\sigma}{h^2} \left(\frac{\partial h}{\partial X} \right)^2 - \frac{2\sigma}{h} \frac{\partial^2 h}{\partial X^2} + \frac{4\sigma}{h^2} \left(\frac{\partial h}{\partial Z} \right)^2 - \frac{2\sigma}{h} \frac{\partial^2 h}{\partial Z^2} \right]; \\
 K_2 &= \frac{2\sigma}{h} \frac{\partial h}{\partial X}; \quad K_3 = \frac{2\sigma}{h} \frac{\partial h}{\partial Z}; \quad K_4 = \left[\frac{1}{h^2} + K_2^2 + K_3^2 \right].
 \end{aligned} \quad (13)$$

The boundary conditions are given as

$$L_W \text{ and } R_W: \frac{\partial \Phi}{\partial X} - \frac{2\sigma}{h} \frac{\partial h}{\partial X} \frac{\partial \Phi}{\partial Y} = 0 \text{ on } X = -1, +1 \quad (14)$$

$$F_W \text{ and } B_W: \frac{\partial \Phi}{\partial Z} - \frac{2\sigma}{h} \frac{\partial h}{\partial Z} \frac{\partial \Phi}{\partial Y} = 0 \text{ on } Z = -1, +1 \quad (15)$$

$$B_M W: \frac{\partial \Phi}{\partial Y} = 0 \text{ on } Y = -1. \quad (16)$$

On the free surface

$$T_{W1}: \frac{\partial \zeta}{\partial T} = K_5 \frac{\partial \Phi}{\partial Y} - 4I_b \frac{\partial \Phi}{\partial X} \frac{\partial \zeta}{\partial X} + -4I_b \frac{\partial \Phi}{\partial Z} \frac{\partial \zeta}{\partial Z} \text{ on } Y = +1 \quad (17)$$

$$\begin{aligned}
 T_{W2}: \frac{\partial \Phi}{\partial T} &= +K_6 \frac{\partial \Phi}{\partial Y} - (1 + \ddot{Y}_t) \zeta + 2I_b \left[\left(\frac{\partial \Phi}{\partial X} - K_7 \frac{\partial \Phi}{\partial Y} \right)^2 \right. \\
 &\quad \left. + \left(\frac{1}{h} \frac{\partial \Phi}{\partial Y} \right)^2 + \left(\frac{\partial \Phi}{\partial Z} - K_8 \frac{\partial \Phi}{\partial Y} \right)^2 \right] \text{ on } Y = +1
 \end{aligned} \quad (18)$$

where

$$\begin{aligned}
 K_5 &= \left[\frac{2}{h} + \frac{8I_b \sigma}{h} \left(\frac{\partial \zeta}{\partial X} \right)^2 + \frac{8I_b \sigma}{h} \left(\frac{\partial \zeta}{\partial Z} \right)^2 \right]; \quad K_6 = \frac{2\sigma}{h} \frac{\partial h}{\partial T}; \\
 K_7 &= \frac{2\sigma}{h} \frac{\partial h}{\partial X}; \quad K_8 = \frac{2\sigma}{h} \frac{\partial h}{\partial Z}.
 \end{aligned} \quad (19)$$

2.1c *Grid stretching near free surface:* The mapping relations for second transformation are given as follows:

$X \leftrightarrow \xi, \quad \xi = X; \quad Z \leftrightarrow \kappa, \quad \kappa = Z; \quad T \leftrightarrow \tau, \quad \tau = T; \quad \text{and} \quad Y \leftrightarrow \eta,$

$$\eta = \alpha_1 + (1 - \alpha_1) \frac{\ln \left(\left(\alpha_2 + \left[\frac{Y+1}{h_{cd}} \right] \right) / \left(\alpha_2 - \left[\frac{Y+1}{h_{cd}} \right] \right) \right)}{\ln[(\alpha_2 + 1)/(\alpha_2 - 1)]}. \quad (20)$$

The mapping is performed from the σ -transformed $\Phi(X, Y, Z, T)$ domain to the computational domain $\varphi(\xi, \eta, \kappa, \tau)$. The height of the liquid, $h(X, Z, T)$ surface, is transformed to the height of the liquid in the transformed computational domain $h_*(\xi, \kappa, \tau)$. The h_{cd} is the height between two successive nodal points along the Y axis of the grid after the first coordinate transformation. The derivatives of the function $\Phi(X, Y, Z, T)$ with respect to X, Y, Z and T are transformed into derivatives of $\varphi(\xi, \eta, \kappa, \tau)$ with respect to ξ, η, κ and τ . After the second coordinate transformation the governing equation in the fluid domain becomes

$$\begin{aligned} \frac{\partial^2 \varphi}{\partial \xi^2} + C_4 \frac{\partial \varphi}{\partial \eta} - C_3 U_1 \frac{\partial^2 \varphi}{\partial \xi \partial \eta} - C_3 U_2 \frac{\partial^2 \varphi}{\partial \kappa \partial \eta} + C_5 \frac{\partial^2 \varphi}{\partial \eta^2} \\ + C_6 \frac{\partial^2 \varphi}{\partial \kappa^2} = 0 \end{aligned} \quad (21)$$

$$\begin{aligned} U_9 = \frac{8I_b \sigma^*}{h_*} \left(\left(\frac{\partial h_*}{\partial \xi} \right)^2 + \left(\frac{\partial h_*}{\partial \kappa} \right)^2 \right); \quad U_7 = C_3 \frac{2\sigma^*}{h_*} \frac{\partial h_*}{\partial \xi}; \\ U_8 = C_3 \frac{2\sigma^*}{h_*} \frac{\partial h_*}{\partial \kappa}; \quad U_6 = C_3 \frac{2\sigma^*}{h_*} \frac{\partial h_*}{\partial \tau} \end{aligned} \quad (28)$$

where

$$\begin{aligned} U_1 = \left[\frac{2\sigma^*}{h_*} \frac{\partial h_*}{\partial \xi} \right]; \quad U_2 = \left[\frac{2\sigma^*}{h_*} \frac{\partial h_*}{\partial \kappa} \right]; \quad U_3 = \left(\frac{\log(\alpha_2 - 1)(\alpha_2 - 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}}}{(\alpha_1 - 1)} - \frac{\log(\alpha_2 + 1)(\alpha_2 + 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}}}{(\alpha_1 - 1)} \right); \\ U_4 = \left(\frac{\log(\alpha_2 - 1)(\alpha_2 - 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}}}{(\alpha_1 - 1)} + \frac{\log(\alpha_2 + 1)(\alpha_2 + 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}}}{(\alpha_1 - 1)} \right) \\ U_5 = \left[\frac{1}{h_*^2} + \left(\frac{2\sigma^*}{h_*} \frac{\partial h_*}{\partial \xi} \right)^2 + \left(\frac{2\sigma^*}{h_*} \frac{\partial h_*}{\partial \kappa} \right)^2 \right]; \quad C_1 = \frac{(1 - \alpha_1)}{\ln \left(\frac{\alpha_2 + 1}{\alpha_2 - 1} \right)} \times \frac{4\alpha_2 \times \sigma^* \times C_2}{\left(h_{cd} \times [\alpha_2^2 - \sigma^{*2}]^2 \right)}; \\ C_3 = \frac{\partial \eta}{\partial Y}; \quad C_5 = C_3^2 \times \left[\frac{1}{h_*^2} + \left(\frac{2\sigma^*}{h_*} \frac{\partial h_*}{\partial \xi} \right)^2 + \left(\frac{2\sigma^*}{h_*} \frac{\partial h_*}{\partial \kappa} \right)^2 \right]; \quad C_7 = \frac{\partial \kappa}{\partial Z} \\ C_2 = \frac{\alpha_2 \times U_3}{(\alpha_2 + 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}} + (\alpha_2 - 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}}} + \frac{\beta \times \left[(\alpha_2 - 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}} - (\alpha_2 + 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}} \right] \times U_4}{\left[(\alpha_2 + 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}} + (\alpha_2 - 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}} \right]^2}; \quad \sigma^* = \left(\alpha_2 \left[\frac{(\alpha_2 + 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}} - (\alpha_2 - 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}}}{(\alpha_2 + 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}} + (\alpha_2 - 1)^{\frac{\eta - \alpha_1}{1 - \alpha_1}}} \right] \right) \\ C_4 = \left(\frac{4\sigma^*}{h_*^2} \left(\left[\frac{\partial h_*}{\partial \xi} \right]^2 + \left[\frac{\partial h_*}{\partial \kappa} \right]^2 \right) \right) - \frac{2\sigma^*}{h_*} \left(\frac{\partial^2 h_*}{\partial \xi^2} + \frac{\partial^2 h_*}{\partial \kappa^2} \right) + C_1 \times U_5; \quad C_6 = C_7^2 \times \left[\frac{1}{h_*^2} + \left(\frac{2\sigma^*}{h_*} \frac{\partial h_*}{\partial \xi} \right)^2 + \left(\frac{2\sigma^*}{h_*} \frac{\partial h_*}{\partial \eta} \right)^2 \right]. \end{aligned} \quad (22)$$

Boundary conditions after the second transformation are given as follows:

$$L_W \text{ and } R_W: \frac{\partial \varphi}{\partial \xi} - \frac{2\sigma^*}{h_*} C_3 \frac{\partial h_*}{\partial \xi} \frac{\partial \varphi}{\partial \eta} = 0 \text{ on } x = -1, 1 \quad (23)$$

$$F_W \text{ and } B_W: \frac{\partial \varphi}{\partial \kappa} - \frac{2\sigma^*}{h_*} C_3 \frac{\partial h_*}{\partial \kappa} \frac{\partial \varphi}{\partial \eta} = 0 \text{ on } \kappa = -1, 1 \quad (24)$$

$$BM_W: \frac{\partial \varphi}{\partial \eta} = 0 \text{ on } \eta = -1 \quad (25)$$

On the free surface,

$$\begin{aligned} T_{W1}: \frac{\partial \zeta^*}{\partial \tau} = -4I_b \left(\frac{\partial \varphi}{\partial \xi} \frac{\partial \zeta}{\partial \xi} + \frac{\partial \varphi}{\partial \kappa} \frac{\partial \zeta}{\partial \kappa} \right) \\ + C_3 \left[\frac{2}{h_*} + U_9 \right] \frac{\partial \varphi}{\partial \eta} \text{ on } \eta \\ = 1 \end{aligned} \quad (26)$$

$$\begin{aligned} T_{W2}: \frac{\partial \varphi}{\partial \tau} = -2I_b \left[\left(\frac{\partial \varphi}{\partial \xi} - U_7 \frac{\partial \varphi}{\partial \eta} \right)^2 + \left(\frac{\partial \varphi}{\partial \kappa} - U_8 \frac{\partial \varphi}{\partial \eta} \right)^2 \right]^* \\ - 2I_b \left(\frac{C_3}{h_*} \frac{\partial \varphi}{\partial \eta} \right)^2 + U_6 \frac{\partial \varphi}{\partial \eta} - (1 + Y_\tau^*) \zeta \text{ on } \eta = 1 \end{aligned} \quad (27)$$

where

The stretched grid system will exactly match the time-dependent free-surface wave profile due to the σ transformation. Since the mean water level in the container remains constant, the sum of the wave height along the x direction is kept at zero to maintain the volume conservation. Since sloshing is a highly nonlinear phenomenon, the amplitude of excitation, frequency of excitation and container dimensions play a vital role in determining the shape of the nonlinear free surface. An amplitude–frequency relationship for the fluid response in a two-dimensional rectangular container is presented by Faltinsen *et al* [29].

2.2 Finite-difference discretization in the computational domain

In this work, the finite-difference scheme is used for liquid sloshing simulation. The governing equation and boundary conditions in the computational domain (Eqs. (21)–(28)) are discretized using the finite-difference method. The Adams–Moulton scheme is utilized for the computation of nonlinear solutions. As the Adams–Moulton scheme requires results from previous time steps to calculate the current time step result, a semi-implicit scheme is used to generate the initial time steps.

2.2a *Semi-implicit scheme at top boundary (free surface)*: A semi-implicit scheme has been applied at the top boundary of the computational domain. As discussed, the top boundary consists of two conditions, viz., kinematic condition and dynamic condition. In computational domain, the kinematic condition (Eq. (26)) is discretized by a forward scheme at $\eta = 1$ as follows:

$$\left. \frac{\partial \zeta^*}{\partial \tau} \right|^n = L_4 C_3 \left. \frac{\partial \varphi}{\partial \eta} \right|^n - 4 I_b \left. \frac{\partial \zeta}{\partial \xi} \right|^n \left. \frac{\partial \varphi}{\partial \xi} \right|^n \quad (29)$$

where

$$\left[\frac{\partial \zeta}{\partial T} \right]^n = \frac{\zeta^{n+1} - \zeta^n}{\Delta T} \quad (30)$$

At $\sigma = 1$, the dynamic condition (Eq. (27)) is discretized as follows:

$$\begin{aligned} \left. \frac{\partial \varphi}{\partial \tau} \right|^n &= C_3 L_5 \left. \frac{\partial \varphi}{\partial \eta} \right|^n - (1 + Y_\tau^*) \zeta^{*n+1} \\ &- 2 E_b \left[\left(\left. \frac{\partial \varphi}{\partial \xi} \right|^n - C_3 L_6 \left. \frac{\partial \varphi}{\partial \eta} \right|^n \right)^2 + \left(\frac{C_3}{h_*^2} \left. \frac{\partial \varphi}{\partial \eta} \right|^n \right)^2 \right] \end{aligned} \quad (31)$$

where

$$\left[\frac{\partial \varphi}{\partial T} \right]^n = \frac{\varphi^{n+1} - \varphi^n}{\Delta T}. \quad (32)$$

The value of ζ^{n+1} is computed using Eq. (29) and is substituted in Eq. (30). Then, the new φ^{n+1} is found at the top boundary of the computational domain using Eq. (32). The first- and second-order derivatives in these equations are discretized by means of a second-order central difference scheme within the liquid domain, and second-order forward and backward differences are employed at the boundaries.

2.2b *Adams–Moulton scheme*: Generally, Adams methods are known as explicit schemes because current and previous time step values are used to obtain the values for future time steps. However, the Adams–Moulton scheme is a set of multi-step methods, which are known as implicit methods. This method is initiated using either a set of known results or from the results of a Runge–Kutta method to start the initial value problem. It uses the explicit scheme to estimate the initial guess and uses the value to estimate the future time steps. Here, a four-point Adams–Moulton scheme is used as follows.

Predictor step: Adam–Bashforth four-point explicit scheme for the initial guess.

1. Camera setup to record slosh height; 2. Weight for fixed support; 3. Motor with eccentric weight for external vibration; 4. Extra weight; 5. Skeletal structure; 6. TLDs; 7. Acceleration probe; 8. Data acquisition system; 9. Auto-transformer for voltage variation.

Figure 7. Experimental set-up of three-storey structure with TLDs.

due to the action of liquid sloshing at 3rd floor is given as follows:

$$\text{internal force } (F_{i3}) \text{ at 3rd floor} = B_{SF3} + P_{F3} \quad (40)$$

where B_{SF3} and P_{F3} are the total liquid shear force from 3rd floor dampers and total force due to pressure on tank walls from 3rd floor dampers, respectively. Base shear of a damper (B_{SF}) in physical domain is calculated as follows:

$$B_{SF} = \frac{1}{2} \rho g \left(\bar{\zeta}(0, z, t)^2 - \bar{\zeta}(b, z, t)^2 \right). \quad (41)$$

Liquid slosh force for a damper (P_F) can be obtained by integrating the pressure values over the right and left tank walls:

$$F_i(t) = \int_{-h_s}^{\zeta} p(x, y, t) dy; \text{ over } x = 0 \text{ and } b, \quad (42)$$

where pressure (p) is obtained from the unsteady Bernoulli equation in the physical domain:

$$\frac{p}{\rho} = -\frac{\partial \phi}{\partial t} - gy - xX_t'' - \frac{1}{2}(\nabla \phi)^2 \quad (43)$$

over the left and right tank walls. The initial conditions are such that the structure is assumed to be at rest and the water surface is still and undisturbed:

$$\begin{aligned} \bar{\phi}(t=0) = 0; \bar{\zeta}(t=0) = 0; X_t''(0) = F_e(0)/m; X_t(0) \\ = 0 \text{ and } X_t'(0) = 0. \end{aligned} \quad (44)$$

Many schemes are available to solve the structural nonlinear time integration equations. In this work, structural equations are solved by the fourth-order Runge–Kutta method, since the present system has moderate vibrations.

2.4 Fluid–structure coupled solver

Structural equations are coupled with fluid equations in order to achieve the transfer of sloshing forces to structure for damping. A numerical algorithm for the three-storey structure is developed and a flowchart of the algorithm is illustrated in figure 8. In order to incorporate the damping action of TLDs into the structure, the resisting internal forces due to sloshing are calculated floor-wise by adopting the acceleration developed in the previous or bottom floor, which is included in the equation of the respective floor. In case of random excitation, the present external force equation is replaced by the force developed due to the acceleration generated by the Bretschneider spectrum, which serves to be the source for random input waves.

2.5 Convergence criteria used for the iterative solver

Each and every numerical algorithm necessarily solves a linear equation $A\phi = B$ in the computational domain at every time step. To solve this equation, various methods are available such as the simply implicit procedure (SIP) [30–32], bi-conjugate gradient stabilized method (BiCGSTAB) [33], preconditioned bi-conjugate gradient stabilized method (PBiCGSTAB) [32], etc. De [32] found that the rate of convergence improves significantly from the SIP to the PBiCGSTAB technique while BiCGSTAB lies between them with non-monotonic convergence. Figure 9 shows the number of iterations required for Gauss–Seidel successive over-relaxation (GS-SOR), SIP, BiCGSTAB and PreBiCGSTAB schemes. The PreBiCGSTAB iterative method has been chosen for this work to find the solution to the velocity potential ϕ at every time step since these methods have the advantages of lesser discretization error and better speed of computation as compared with other direct methods as shown in figure 10. The volume error is computed for $n = 1$; $\Omega_x = 1.253$; $f_f = 0.0033$ and $K_x = 0.015$ over non-dimensional time, where Ω_x and K_x are liquid frequency ratio (= excitation frequency to natural frequency) and wave number in x direction, respectively.

While using an iterative solver it is very important to stop the iterations at the right time, i.e., when the iteration error has decreased by an acceptable amount. The volume errors for all the cases studied are found to be within the range of $[-10^{-8}, 10^{-8}]$ as shown in figure 11.

Various iterative solver stopping criteria are used for optimization of the present code using the simple tank problem as shown in figure 11. It can be noted that the difference norm falls well below 0.1×10^{-8} by 50

Figure 9. Comparison of iteration schemes.

Figure 10. Volume error (for $n = 1$; $\Omega_v = 1.253$; $K_v = 0.4$; $E_b = 0.0033$ and $K_x = 0.015$) versus non-dimensional time (ωt).

iterations, whereas the exact iteration error is still above 0.3×10^{-5} and hence this stopping criterion is not suited for this case. Taking into account the residual norm, it is clear that it has quite a higher value than the exact iteration error for all iterations. The estimated iteration error is almost the same as the exact iteration error in this case (except for the initial iterations, where it shows slight oscillations). The residual stopping criterion ensures that the exact iteration error has fallen well below the tolerance limit, but it results in higher computational time for the solver without achieving higher accuracy than the estimated iteration error stopping criterion. Comparisons made for later time steps also showed a similar behaviour of the norms of the stopping criteria with iterations. Hence, the error estimation stopping criterion with the PreBiCGSTAB iterative scheme is the most suited for the current problem. As shown in figure 12, grid independence study is performed and $61 \times 61 \times 61$ grid size is taken for the present study.

3. Experimental procedure

An experimental study is conducted to evaluate the performance of TLDs and their effectiveness in reducing the structural response. TLD tanks made of acrylic sheets of thickness 4 mm are fabricated. The tanks are made leak proof by applying a combination of resin powder and chloroform at the edges. The length, width and height of the tanks are taken as 50 mm $\times$ 50 mm $\times$ 60 mm. A three-storey skeletal structure is taken for study as shown in figure 7. The height of each floor is 300 mm, which

Figure 11. Comparison of error norms.

Figure 12. Grid independence study.

occupies a flat plate of dimensions 300 mm $\times$ 300 mm and a thickness of 10 mm. The ground floor movement is constrained by the addition of heavy weights. An exciter (motor connected with an eccentric weight and dimmerstat) is attached to the first floor, which acts as the source of external vibration. The frequency of excitation can be altered by the use of dimmerstat, by virtue of which the voltage is regulated to obtain the desired frequency of excitation. The limitation of this exciter is its ability to excite only in one direction. The top two floors are utilized for placing liquid dampers as shown in figure 7. Acceleration probes are attached to the floors, which are connected to the data acquisition system for recording the data. A camera is fixed on a stand in order to capture the slosh height.

4. Results and discussion

The effect of the liquid sloshing on the structural motion is established by the fluid-to-structure mass ratio M_r and tuning ratio relative to the structure $\Omega_t = \omega_n/\omega_0$ where ω_n and ω_0 are the sloshing frequency of mode n and first mode structural frequency, respectively. The free-surface behaviour is investigated by varying the external forcing amplitude through the forcing parameter ($f_f = A_f/\rho gh^2$).

4.1 Code validation with experiments

To prove the credibility of the present numerical code, results need to be compared with the experimental results. Experiments are conducted as explained in section 3. The natural frequency of TLDs is tuned with a structural system by adjusting the water fill level and is kept on the three-storey structure as shown in figure 7. This case involves qualitative comparison between numerical and

experimental results by showing the free surface profile at different time instants with five TLDs at the top floor as shown in figure 13. Snapshots of the experimental and numerical free surface profile are shown and it is observed that the numerical results are in good agreement with the experiment.

4.2 Effect of forcing frequency ratio (β)

$\beta (= \omega/\omega_0)$ is defined as the ratio of excitation frequency to first mode structural frequency. This case study features the effect of frequency ratio at $\Omega_t = 1$, $M_r = 0.01$ and $f_f = 0.0001$. Figure 14 shows the non-dimensional displacement variation over non-dimensional time with and without five-TLDs at the top floor. It is seen that the structural displacement significantly reduces with the application of the TLD when it is tuned to the structural natural frequency. Figure 15 illustrates the fast Fourier transformation (FFT) plot of displacement with the TLD

Figure 13. Qualitative comparison of free surface profile for experimental and numerical results at different time instants.

Figure 14. Displacement plot with and without TLD.

Figure 16. Displacement plot at various β values.

Figure 15. FFT plot of displacement curve with TLD.

case in which two dominant frequencies are obtained. Frandsen [28] discussed about the existence of two distinct frequencies for an effective tank–structure coupled system and also described the two values as eigen-frequencies of the system, obtained by solving the equations of motion through an analytical approach. Here, it is observed that the maximum response is found at $\beta = 0.974$ and 1.04 as presented in figure 15. There will be a rise in amplitude of the corresponding sloshing mode due to the structural oscillations, when one of the sloshing frequencies is close to the structural natural frequency. This increase in sloshing mode begins to interact with the structure, thereby resulting in a change in the natural frequency of the coupled system.

To lend credibility to the numerical code, the results are further compared with the experimental results quantitatively for different β . Quantitative validation with the numerical code for selected cases has been performed and the structural displacement is illustrated in figures 16, 17, 18, 19, 20 and 21. For this purpose, the required parameters are taken as $h = 1$ m, $b/h = 2$, $M_r = 1\%$, $\Omega_t = 1$, $f_f = 1E-4$, $\xi = 0.0013$, $M_t = 202$, $k = 291.013$, $c = 0.63$ and nine TLDs kept at the top floor. Figure 16 shows the plot of non-dimensional displacement against non-dimensional time for different frequency ratios. For the lower frequency ratio, the numerical results exhibit an excellent match with the experiments as shown in figure 17, whereas figure 18 shows the numerical and experimental structural displacement at $\beta = 1.2$.

Due to violent motion of the structure, small deviation is found at $\beta = 0.974$ (figure 19). The violent motion is caused because of the TLD frequency matching with a structural frequency, which leads the displacement sensor to over-prediction. The eccentric arrangement used creates a few additional frequencies in the experimental data. During the violent motion of a structure the difference becomes more due to the above issues. Numerical dissipation can be controlled by higher-order upwind schemes. However, dominant frequencies are the same in both cases. The repeatable patterns in the phase plane diagram in figure 20 show the nonlinear cyclic behaviour of the displacement curve. The effectiveness of TLDs in a single floor is shown in table 1. The nine MLTDs are placed at the top floor and reduction of structural displacement is noted for various M_r and β . The maximum reduction is found at $M_r = 1.4\%$ and $\beta = 0.974$. As stated before, the TLD performance is more for M_r between 1% and 5%.

Figure 17. Experimental and numerical comparison at $\beta = 0.6$.

Figure 19. Experimental and numerical comparison at $\beta = 0.974$.

Figure 18. Experimental and numerical comparison at $\beta = 1.2$.

Figure 20. Numerical phase plane diagram at $\beta = 0.974$.

4.3 Effect of TLDs in floor-wise numerical investigations

The effect of introducing TLDs on structure floors is discussed in this section. The numerical predictions are put forward for different TLDs system configurations. Case studies of without TLD, three TLDs on the top floor and single TLD on each floor are conducted as shown in figure 21a, b and c, respectively. The results of the aforementioned cases under normal excitation and random excitation are illustrated in figures 22 and 23, respectively. In most practical situations, the excitation or the time variation of the system parameters is random in nature.

Hence, random motion has also been taken into consideration in this section. Linear superposition of a number of monochromatic waves can generate a random input wave. Vibration control dampers are usually designed based on random acceleration input such as real earthquake time history [34]. In this work, the Bretschneider spectrum has been selected to serve as the input excitation spectrum to generate the random input wave. A floating random number generating algorithm is used for the selection of linear angular frequencies and phase angle within the prescribed range. More details on the generation of random waves can be found in Sriram *et al* [35] and Eswaran *et al* [16].

Figure 21. Numerical case studies on multiple TLDs.

Table 1. Effectiveness of multiple TLDs in a single floor.

Method	β	No. of TLDs	Mass ratio M_r (%)	Max. displacement in m		
				Without TLD	With TLD	Reduction (%)
Experiment	0.6	9	0.56	4.5E-4	2.5E-4	44.44
Numerical	0.6	9	0.56	4.5E-4	2.5E-4	44.44
Experiment	0.974	9	0.56	2.1E-3	3.3E-4	84.2
Numerical	0.974	9	0.56	2.2E-3	3.2E-4	85.45
Experiment	0.974	9	1.4	5.3E-3	3.5E-4	93.40
Numerical	0.974	9	1.4	5.3E-3	3.7E-4	93.02

Displacement from top floor of the structure and $f_{TLD} = 3.86$, $M_I = 0.56$ kg.

Figure 22. Non-dimensional displacement plot under normal excitation.

Figure 23. Non-dimensional displacement plot under random excitation.

From figures 22 and 23, it can be observed that there is minimum structural displacement when the TLDs are placed on the top floor of the structure when compared with that arranged in individual floors. It can also be stated that the multiple TLDs arranged on a single floor provide more resisting sloshing forces required for damping the structural vibration. In the random case, the TLDs on top floor provided more damping initially but as time proceeds, both the cases show similar effects in mitigating structural response.

5. Summary

The international guidelines for liquid tank designs are based on the spring–mass model system. Since they are based on linear theory and also provide static liquid load, the estimation of a liquid load leads to imprecision. For accurate prediction, a dynamical fluid–structure coupled analysis is necessary. Therefore, it is also essential to develop a fluid–structure coupled code for the accurate predictions of the structural motion, which is coupled with TLDs.

In this work, the fluid and structure equations are solved separately and coupled effectively in each iteration to study the performance of TLDs. Structural equations are coupled with fluid equations in order to achieve the transfer of sloshing forces into the structure for damping using a partitioned FSI solver. Structural equations are solved by the fourth-order Runge–Kutta method while fluid equations are solved using a finite-difference-based sigma-transformed algorithm. Different iterative schemes and error schemes are used to optimize the code for larger convergence rate with higher accuracy. The error estimation stopping criterion with PreBiCGSTAB iterative scheme suits well for the current problem. Grid independence study has also been performed. To lend credibility to the numerical code, the computed results are compared with experimental data. For this, a detailed experiment on TLDs is conducted. The numerical results match well with the experimental results. The present numerical approach is easy to implement, computationally accurate and efficient. The cases presented herein eliminate the requirement for free surface smoothing and re-meshing. This model offers a simple way of simulating steep non-breaking waves.

It is clear from the above study that as the mass ratio is increased, the TLDs can effectively damp the structural vibration. In this case, the mass ratio is increased by increasing the number of TLDs actively participating in mitigating the structural vibration. Although the arrangement of TLDs in single floor seems to provide greater damping than that of dividing the same number of TLDs among the top two floors, the latter configuration proved effective in mitigating structural response in both of the perpendicular directions.

The numerical model is simple, computationally inexpensive and accurate and there is no necessity for free

surface smoothing. The present potential flow model may be readily extended to the prediction of magnetorheological (MR) fluid sloshing effects.

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List of symbols

A_i	wave amplitude of i th linear wave
A_d	non-dimensional constant
A_F	forcing characteristics amplitude
Amp	external force characteristic amplitude
α_1, α_2	grid stretching factors
b	length of tank
$d_i, \dot{d}_i, \ddot{d}_i$	structural displacement, velocity and acceleration, respectively, of floor i
h	instant water height from tank bottom
h_s	still water depth
H_s	significant wave height
N_W	number of linear monochromatic waves
ρ	density of water
g	acceleration due to gravity
ζ	non-dimensional free surface elevation
$\bar{m}, \bar{c}, \bar{k}$	non-dimensional structure mass, damping and stiffness, respectively
D	non-dimensional structural displacement
τ	non-dimensional time
$\bar{\phi}$	non-dimensional velocity potential function
$\bar{F}_e(t)$	non-dimensional external force of excitation
$\bar{F}_i(t)$	non-dimensional internal force of sloshing
n	mode number of oscillation
p	pressure
ω_n	the n th mode sloshing angular frequency
ω_1	first mode sloshing angular frequency
ω_0	structural natural frequency
ω	excitation angular frequency
ω_i	frequency of i th linear wave
ω_p	peak frequency of wave spectrum
k_n	wave number of n th mode
M_r	mass ratio
M_t	non-dimensional total mass
ξ	damping ratio
f_f	forcing factor
Ω_t	tuning ratio
Ω_x	liquid frequency ratio in x direction
β	forcing frequency ratio
$\phi(x, y, z, t)$	velocity potential function in physical domain (x, y, z, t)

$\Phi(X, Y, Z, T)$	velocity potential function in computational domain (X, Y, Z, T)
m_1, m_2, n_1, n_2	coordinate points on computational domain axes
S_ζ	Bretschneider spectrum wave energy
ψ_i	phase angle of i th linear wave
BSF	base shear force
c	damping
CFD	computational fluid dynamics
F_{ext}	external force
FFT	fast Fourier transform
k	stiffness
M	mass of the floor
PF	force due to pressure on tank walls
t	time (s)
TLCD	tuned liquid column damper
TLD	tuned liquid damper
TSD	tuned sloshing damper
x	structural floor displacement

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