On high-resolution manoeuvres control via trajectory optimization

A H MAZINAN^{1,*} and M SHAHI²

¹Department of Control Engineering, Faculty of Electrical Engineering, South Tehran Branch, Islamic Azad University (IAU), P.O. Box 11365/4435, Tehran, Iran

²Department of Electrical Engineering, Birjand Branch, Islamic Azad University (IAU), Birjand, Iran
e-mail: mazinan@azad.ac.ir; m.shahi@iaubir.ac.ir

MS received 22 September 2015; revised 17 July 2016; accepted 14 September 2016

Abstract. This research is on a realization of control approach in line with the trajectory optimization for the purpose of dealing with overactuated spacecraft in the process of the high-resolution manoeuvres. The idea behind the research is to realize closed control loops to cope with the rotational angles and the corresponding angular rates, synchronously, to handle the spacecraft manoeuvres. It is to be noted that the traditional techniques may not have sufficient merit to deal with such a complicated process, suitably. The proposed trajectory optimization is designed to provide the three-axis referenced commands, in finite burn, for transferring the aforementioned overactuated spacecraft from the initial orbit to its final outcomes in the orbital transfer process. The outcomes are realized through the variations of the orbital parameters, including the inclination, the eccentricity, the angular momentum, the semi-major axis and so on, in the high-resolution manoeuvres. It aims to get the system under control to guarantee the performance of the three-dimensional rotational angles tracking to be desirable, instantly. The contribution of the research is to make the high-thrust optimization trajectory, which is organized in association with the new configuration of the three-axis attitude control approach, to be applicable to manage the present overactuated spacecraft in the procedure of high-resolution orbital transfer process. The investigated outcomes of the research are efficient and competitive along with the potential materials through a series of experiments, as long as the desirable tracking performance in the three-dimensional space manoeuvres is apparently guaranteed.

Keywords. Trajectory optimization; high-resolution manoeuvres control; tracking performance; a set of experiments; process control systems.

1. Introduction

One of the responsibilities of a number of the spacecrafts is to handle the orbital transfer process, in an efficient manner, to achieve the required space mission operation plans. There is consensus among the experts of this area that a new insight to outperform the aforementioned plans is worthy of consideration, because of the fact that it is applicable to cope with three-dimensional spacecraft in both academic and real environments, usually. Regarding this point, the control of high-resolution manoeuvres is targeted for investigation in the present research, while the idea of the high-thrust trajectory optimization is taken into real consideration. In fact, the aim is to be able to guarantee desirable performance of the three-dimensional manoeuvres through a realization of the control approach, which is organized upon the double closed control loops involving the inner and the outer ones, as well. The aforementioned inner closed loop is designed to adjust the three-axis angular rates of the present spacecraft to be

about zero in steady state, as long as the corresponding rotational angles of such a system are desirable based upon its three-axis referenced commands, correspondingly. It is to be noted that the proposed high-thrust trajectory optimization plays a significant role in the process of orbital transfer in providing the present three-axis referenced commands, all the required time. The investigated outcomes are to transfer the present overactuated spacecraft from initial orbit to its final ones through updating the orbital parameters, including the inclination, the eccentricity, the angular rate, the semi-major axis and so on. Subsequently, in order to describe the contribution of the proposed research, the high-thrust trajectory optimization in connection with the space manoeuvres control are not efficiently integrated in the related referenced materials to deal with a set of unique orbital transfer processes, while the intelligence-based technique in the form of the genetic algorithm (GA) is designed to outperform the outcomes. In general, it is easy to note that the number of objects in high-thrust trajectory optimization is very rare, although the number of outcomes to cope with low-thrust trajectory optimization is somehow manageable.

*For correspondence

In case of the high-thrust trajectory optimization, Olds *et al* [1] designed the procedure of the interplanetary mission through differential evolution. A promising global approach is in fact identified as being especially well suited to the high-thrust optimization problem. The qualities of the results, acquired with the differential evolution, are identified to be sensitive to the selection of the tuning parameters that are efficient in the rapid global optimization [1]. Balkanyi and Spurlock [2] present an applicable high-thrust trajectory optimization code to design the missions on expendable launch vehicles through a mathematical model that is addressed through an objective function to be optimized subject to an assortment of algebraic and dynamic constraints. In case of the low-thrust trajectory optimization, Xincheng *et al* [3] reveal trajectory optimization regarding the low-thrust spacecraft via the pseudo spectral method. The movement of the spacecraft is discussed by the Gauss equation with a focus on the modified equinoctial elements in the Gauss orbital coordinates [3]. Yam *et al* [4] present constrained global optimization of the low-thrust interplanetary trajectories, where the Sims–Flanagan transcription idea is realized, to provide a nonlinear programming problem. The use of two global optimization algorithms including the basin hopping and the simulated annealing in an adaptive neighborhood is made to explore the solution space [4]. Xiao-yong *et al* [5] propose an approach based on a virtual gravitational body for low-thrust trajectory optimization, while the original optimal control problem is approached as a nonlinear programming problem that could be solved using the analytical nature of conic arcs [5].

In another attempt, Xiao-yong *et al* [6] present a multi-impulse extended method for low-thrust trajectory optimization. In fact, there is a multi-impulse trajectory that can be converted into the low-thrust one by applying all the other segments [6]. Bando *et al* [7] suggest low-thrust trajectory optimization via second-order generating functions. It is noted that a robust control approach is realized for the spacecraft rendezvous problem. Hereinafter, the optimal rendezvous problem via continuous low thrust is realized, as the optimal and robust control. It leads to the formulation of a linear–quadratic control with the worst-case-disturbance rejection and also the worst-case initial condition, which are related to generating functions [7]. Dai *et al* [8] propose optimal trajectory generation to establish connectivity in proximity networks. In fact, the problem of designing the optimal trajectories is examined to establish connectivity in a network of initially scattered dynamic agents, where minimizing the squared integral of the total control effort is emphasized. An optimal control problem with specified initial states, linear dynamics and a connectivity constraint is realized in the final induced topology [8]. Tian *et al* [9] suggest real-time trajectory and attitude coordination control of a reusable launch vehicle in reentry phase. The pseudo-spectral-based optimal feedback reentry guidance is realized using successive real-time optimal

open loop control of the guidance system, which has sufficient robustness against initial reentry perturbations [9]. In case of the space manoeuvres, Curtis [10] considers the fundamental theories in the orbital mechanics, which can be efficient in the field of realizing orbital transfer processes.

Concerning the recent potential research in the area of spacecraft control, Sidi [11] presents the fundamental potential outcomes in the area of the spacecraft dynamics and control. He *et al* [12] discuss robust output feedback control design in the area of autonomous spacecraft rendezvous with actuator faults, while Huang *et al* [13] address output feedback sliding mode control regarding the Lorentz-augmented spacecraft hovering through neural networks as well. Song *et al* [14] suggest finite-time control for nonlinear spacecraft attitude through the terminal sliding mode strategy. In their research, the terminal sliding mode control strategy with double closed loops is suggested to deal with the attitude control problem. They include both the inner and the outer control loops. The strategy is considered using Lyapunov's concept to guarantee the occurrence of finite-time sliding motion [14]. Lu *et al* explore the adaptive attitude tracking control of rigid spacecraft with convergence. In this research work, the attitude tracking control problem in finite time is addressed under external disturbances and inertia uncertainties. As reported there, a fast nonsingular terminal sliding mode surface is realized without the constraint. The proposed chattering-free control laws are organized in line with the adaptive control architecture [15].

Yang [16] reviews the spacecraft attitude determination and control by using the quaternion based method. In this review, the quaternion-based proportional-derivative method is first discussed for the attitude determination and control. In this quaternion-based proportional-derivative control approach, a number of reduced quaternion models using vector component of the quaternion in the state space models are presented. Subsequently, a number of methods, including the analytical solution of the linear-quadratic regulator and also the reduced disturbance effect with stable nonlinear spacecraft, are presented [16]. Mazinan *et al* [17] investigate the three-axis detumbling mode control approach and also the full quaternion-based finite-time cascade attitude control approach via pulse modulation synthesis, whereas the work of Zou and Kumar [18] is to realize the adaptive fuzzy fault-tolerant attitude control, where unknown mass moments of inertia matrix, external disturbances, actuator failures and input constraints are all taken into consideration. The work of Hu *et al* [19] is to design robust attitude control under assigned velocity and control constraints, while Cai and Huang [20] describe the leader–follower attitude control of multiple rigid spacecraft. The work of Kuo and Wu [21] introduces the attitude dynamics and control of miniature spacecraft through the pseudo-wheels and finally the research of Zhang *et al* [22] presents the attitude control of a rigid spacecraft with disturbance, which is provided through time varying exosystems.

With a focus on the contribution made in the research, it is to be noted that the innovations points can be listed as the integration of the high-thrust trajectory optimization with regard to realization of a manoeuvres control approach, which is novel and not reported so far. Moreover, the configuration of the high-resolution three-dimensional control approach is efficient to deal with the overactuated spacecraft in the process of orbital transfer. Subsequently, the desirable tracking performance of the proposed control approach can be guaranteed through a series of experiments, carried out in the simulation study. Therefore, the process of transferring the spacecraft under control in some real situations is efficient in choosing a number of separated initial and final orbits, in finite burn, as long as the abilities of the high-thrust trajectory optimization in connection with the intelligence-based technique in the form of the *GA* are taken into consideration.

The rest of the manuscript is organized as follows. The proposed control approach is first presented in section 2, while the simulation results are then given in section 3. The research is finally concluded with outcomes in section 4.

2. The proposed control approach

The proposed control approach is organized in association with a number of sub-systems, including the referenced commands generator, the three-axis control approach, the rate feedback, the modulator module, the control allocator scheme, the system attitude–position, the orbital module and finally the optimization module, as illustrated in figure 1 [11–17]. There are double closed control loops including the inner loop and the outer loop to deal with the angular rates and rotational angles through dynamics and kinematics of the spacecraft under control, as long as the position relation of the aforementioned system is realized to handle the orbital module.

Also there is the optimization module to be able to provide the referenced steering angles commands. The referenced command generator is designed to provide three-axis rotational angles errors to be applied to the control approach, while the rotational angles of the system and also the same referenced steering angles commands from the optimization modules could be acquired. The modulator module that is realized based upon the pulse-width pulse-frequency technique is employed to provide on–off thrusts to enable reaction thrusters, which are designed, along with the control allocator, as an important part of overactuated systems. It is to be realized that the number of control effort signals in the procedure of realization of the three-axis control approach should be three, while the number of reaction thrusters in the overactuated systems under control should be more than three to be enabled.

The rate feedback is designed to complete the inner control loop to deal with desirable angular rates in steady state. The aforementioned sub-systems are discussed in the subsequent sections.

2.1 The referenced commands generator

The referenced commands generator is realized to provide the three-axis referenced commands, i.e., $\varphi_c(t)$, $\theta_c(t)$ and $\psi_c(t)$, to be able to apply to the control approach through the referenced steering angles commands, i.e., $\alpha_c(t)$ and $\beta_c(t)$. The aforementioned generator is designed, as long as the corresponding referenced rotational angles errors, i.e., $\varphi_e(t)$, $\theta_e(t)$ and $\psi_e(t)$ and the system rotational angles, i.e., $\varphi(t)$, $\theta(t)$ and $\psi(t)$ are taken into consideration. The present sub-system is given by

$$\begin{cases} \varphi_c(t) = 0; \varphi_e(t) = \varphi(t) \\ \theta_c(t) = -\alpha_c(t); \theta_e(t) = \theta(t) + \alpha_c(t) \\ \psi_c(t) = \beta_c(t); \psi_e(t) = \psi(t) - \beta_c(t) \end{cases} \quad (1)$$

Figure 1. Schematic diagram of the control approach proposed here.

2.2 The three-axis control approach and the rate feedback

The three-axis control approach is designed to cope with the angular rates and the corresponding rotational angles of the system under control to be desirable as the rotational angles need to be adjusted through the referenced commands generator and the angular rates need to be about zero in steady state, as well. In fact, the three-axis control approach, which is organized based upon the proportional-derivative gains, plays an important role in the inner closed control loop, which is related to the angular rates, and also the outer closed control loop, which is related to the rotational angles. It is to be noted that the angular rates can be dealt with through the mentioned inner closed control loop, where the rate feedback is suitably designed based upon the proportional gains to guarantee the stability of the aforementioned closed control loop. There is a theorem to analyse the stability of the inner control loop to guarantee that the same loop is stable provided that the coefficients of the three-axis control approach are appropriately chosen [11].

Theorem 1 *The coverage of the angular rates of the system under control in the inner closed control loop, i.e., ω_λ ; $\lambda = x, y, z$ is to provide a state of the equilibrium, where the moments of inertia of the same system, i.e., I_λ ; $\lambda = x, y, z$ are taken into consideration. The present theorem indicates that the closed control loop can be stable if the relations of the present angular rates are appropriately organized. Now, the Lyapunov function is taken as*

$$V = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2). \quad (2)$$

The derivative of the aforementioned Lyapunov function is easily given as

$$\dot{V} = I_x \omega_x \dot{\omega}_x + I_y \omega_y \dot{\omega}_y + I_z \omega_z \dot{\omega}_z. \quad (3)$$

The relations of the angular rates with regard to their derivatives are

$$\begin{cases} \dot{\omega}_x + k_{\omega x} \omega_x = 0 \\ \dot{\omega}_y + k_{\omega y} \omega_y = 0 \\ \dot{\omega}_z + k_{\omega z} \omega_z = 0 \end{cases} \quad (4)$$

Now, by substituting Eq. (4) into Eq. (3), the following outcomes are easily investigated:

$$\dot{V} = -(k_{\omega x} I_x \omega_x^2 + k_{\omega y} I_y \omega_y^2 + k_{\omega z} I_z \omega_z^2) < 0. \quad (5)$$

Here, $k_{\omega \mu} I_\mu > 0$; $\mu = x, y, z$ have to be assumed.

2.3 The modulator module

The modulator module is realized in the proposed approach along with the pulse-width pulse-frequency technique to

Figure 2. Schematic diagram of the modulator module used here.

handle a number of the reaction thrusters through the on-off thrusts, as illustrated in figure 2. In fact, the thrusts to be provided and processed via the present modulator have two separated levels including on and off values. To design the aforementioned modulator in a case study, the parameters including A , T , U , U_{on} and U_{off} need to be initialized [11].

The parameter U is related to the thrust level that appears in $c_{pw}(t)$, where U_{on} and U_{off} are related to the modulator hysteresis, as well. It is first supposed that the input of the modulator is restricted by

$$\frac{U_{on}}{A} \leq r_{pw}(t) \leq \frac{U_{off}}{A+1}. \quad (6)$$

The t_{on} and t_{off} , which are the duration of thrust levels in on or off, are presented by

$$\begin{cases} t_{on} = T \frac{U_{on} - U_{off}}{A - Ar_{pw} + U_{on}} \\ t_{off} = \frac{U_{on} - U_{off}}{Ar_{pw} - U_{off}} \end{cases} \quad (7)$$

The frequency of the modulator f and the corresponding duty cycle d_u are, respectively, calculated by

$$\begin{cases} f = \frac{1}{t_{on} + t_{off}} \\ d_u = \frac{t_{on}}{t_{on} + t_{off}} \end{cases} \quad (8)$$

Finally, the band width of the modulator is given by

$$b_w = \frac{T}{A} (U_{on} - U_{off}). \quad (9)$$

2.4 The control allocator scheme

The control allocator scheme is briefly considered to handle a set of more than three on-off reaction thrusters in the proposed approach. In one such case, the number of torques in the procedure of designing the three-axis control approach is less than the number of employed reaction thrusters [11]. Therefore, the pseudo-inverse matrix, i.e., $Conf^+ = Conf^T (Conf Conf^T)^{-1}$, which is directly related

to the physical configuration of the reaction thrusters, needs to be calculated, while the *Conf* is parametrically given by

$$\text{Conf} = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1N} \\ c_{21} & c_{22} & \dots & c_{2N} \\ c_{31} & c_{32} & \dots & c_{3N} \end{bmatrix}. \quad (10)$$

Here, $c_{\eta\kappa}$; $\eta = 1, 2, 3$; $\kappa = 1, 2, \dots, N$ are directly related to p and q , taken as the physical thruster cylinder radius and the corresponding arm, respectively.

With this purpose, the N number of the thruster's enabling signals, i.e. te_κ ; $\kappa = 1, 2, \dots, N$ can be provided along with the control torques about the three axes, i.e., M_x , M_y and M_z , by the following relations:

$$\begin{bmatrix} te_1 \\ te_2 \\ \vdots \\ te_N \end{bmatrix} = T_{level} \text{Conf}^+ \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} \quad (11)$$

where T_{level} indicates the level of the thrusters, while the parameter $U = 1$ is taken in the modulator module.

2.5 The system attitude–position

2.5a The system dynamics and kinematics: From Newton's second law, the external moments summation acting on the spacecraft is equal to the time rate of angular momentum variation in the inertial frame, given by [11]

$$D^I(h_B^{BI}) = \sum m_B. \quad (12)$$

The rotational time derivative based upon Eq. (12) with respect to the body frame B is rewritten as

$$D^I(I_B^B \omega^{BI}) + \Omega^{BI} I_B^B \omega^{BI} = \sum m_B \quad (13)$$

where I_B^B is taken as the moment of inertia of the spacecraft, ω^{BI} is the system angular rate in terms of the inertial coordinate and Ω^{BI} is taken as a skew-symmetric matrix as well. Picking the body coordinate frame $[\cdot]^B$, the outcomes of Eq. (13) result in the following form:

$$[I_B^B]^B \left[\frac{d\omega^{BI}}{dt} \right]^B + [\Omega^{BI}]^B [I_B^B]^B [\omega^{BI}]^B = \left[\sum m_B \right]^B. \quad (14)$$

Focusing on the M_λ , ω_λ , I_λ ; $\lambda = x, y, z$ and φ , θ , ψ as the three-axis torques, three-axis angular rates, three-axis moments of inertia and the three-axis rotational angles, respectively, the dynamics and the kinematics of the spacecraft under control are presented as follows:

$$\begin{cases} \dot{\omega}_x(t) = \frac{M_x(t)}{I_x} - \frac{(I_z - I_y)}{I_x} \omega_y(t) \omega_z(t) \\ \dot{\omega}_y(t) = \frac{M_y(t)}{I_y} - \frac{(I_x - I_z)}{I_y} \omega_x(t) \omega_z(t) \\ \dot{\omega}_z(t) = \frac{M_z(t)}{I_z} - \frac{(I_y - I_x)}{I_z} \omega_x(t) \omega_y(t) \\ \dot{\varphi}(t) = \omega_x(t) + (\tan \theta(t) \sin \varphi(t)) \omega_y(t) + (\tan \theta(t) \cos \varphi(t)) \omega_z(t) \\ \dot{\theta}(t) = (\cos \varphi(t)) \omega_y(t) - (\sin \varphi(t)) \omega_z(t) \\ \dot{\psi}(t) = \left(\frac{\sin \varphi(t)}{\cos \theta(t)} \right) \omega_y(t) + \left(\frac{\cos \varphi(t)}{\cos \theta(t)} \right) \omega_z(t). \end{cases} \quad (15)$$

2.5b The system position: The spacecraft position satisfies Kepler's equation [10]:

$$\ddot{\mathbf{r}}(t) = -\frac{\mu}{r^3(t)} \mathbf{r}(t) + \frac{\mathbf{F}(t)}{m(t)} \quad (16)$$

where $\mathbf{r}(t)$ is the radius vector with respect to the inertial reference frame, $r(t)$ is the scalar value of $\mathbf{r}(t)$, $\mu = 398,600$ is the constant gravitational amount of earth and finally $m(t)$ denotes the mass of the spacecraft. Now, the thrust vector, relative to the inertial coordinate system, is taken as

$$\mathbf{F}(t) = F(t) \begin{bmatrix} \cos(\alpha_c(t)) \cos(\beta_c(t)) \\ \cos(\alpha_c(t)) \sin(\beta_c(t)) \\ \sin(\alpha_c(t)) \end{bmatrix} \quad (17)$$

where $F(t)$ is the magnitude of the thrust vector. The mass is decreasing at the rate of $\dot{m}(t)$, presented by the following relation:

$$\dot{m}(t) = \frac{T(t)}{I_{sp} g_0} \quad (18)$$

where I_{sp} is the specific impulse of the propulsion engine and g_0 is the sea-level acceleration due to the gravity. Therefore, the mass can be rewritten as follows:

$$m(t) = m_o - \dot{m}(t)t. \quad (19)$$

Here, m_o is taken as the initial mass of the spacecraft.

2.6 The orbital module

The orbital module involves a number of independent parameters related to the Keplerian orbit, including the eccentricity $e(t)$, the semi-major axis $a(t)$, the inclination $i(t)$, and so on. The angular momentum vector is acquired through the cross product of the distance and the corresponding rate [10]:

$$\mathbf{h}(t) = \mathbf{r}(t) \times \mathbf{v}(t) \quad (20)$$

where its scalar value is calculated through the dot product of the angular momentum vectors, i.e., $h^2(t) = \mathbf{h}(t) \cdot \mathbf{h}(t)$. The inclination is given as

$$i(t) = \cos^{-1} \left(\frac{h_z(t)}{h(t)} \right). \quad (21)$$

Here, $h_z(t)$ indicates z -axis angular momentum. The eccentricity is presented as

$$\mathbf{e}(t) = \frac{1}{\mu} \left(v^2(t) - \frac{\mu}{r(t)} \right) \mathbf{r}(t) - \frac{1}{\mu} r(t) v_r(t) \mathbf{v}(t) \quad (22)$$

where $v_r(t) = \frac{\mathbf{v}(t) \cdot \mathbf{r}(t)}{r(t)}$. The eccentricity, in its scalar form, is given as

$$e^2(t) = \mathbf{e}(t) \cdot \mathbf{e}(t) = 1 + \frac{h^2(t)}{\mu^2} \left(v^2(t) - \frac{2\mu}{r(t)} \right)^2. \quad (23)$$

The perigee and apogee, respectively, are

$$\begin{cases} r_p(t) = \frac{h^2(t)}{\mu(1+e(t))} \\ r_a(t) = \frac{h^2(t)}{\mu(1-e(t))}. \end{cases} \quad (24)$$

The semi-major-axis is

$$a(t) = \frac{r_a(t) + r_p(t)}{2}. \quad (25)$$

2.7 The optimization module

The optimization module is organized to acquire the coefficients of the referenced steering angles commands, i.e., $A_c^\eta, B_c^\eta; \eta = 1, 2, \dots, n$, which are presented through the n th order polynomials, in its off-line manner, through the GA [1–9]. The GA parameters, including the maximum generations, the crossover points, the population size, the mutation probability and the crossover probability, need to be appropriately chosen to acquire the optimum values for these coefficients of the referenced steering angles commands, as long as the present GA can successfully be terminated at the limited number of the iterations. Now, the aforementioned n th order polynomials are given by

$$\begin{cases} \alpha_c(t) = \sum_{\eta=0}^n A_c^\eta t^\eta \\ \beta_c(t) = \sum_{\eta=0}^n B_c^\eta t^\eta. \end{cases} \quad (26)$$

It should be noted that the performance index as the fitness function can be organized based upon the orbital parameters to be optimized. In one such case, the present fitness function needs to satisfy the best value and the

corresponding mean value of the coefficients of the referenced steering angles commands to be covered in the limited number of iterations. Moreover, the behaviour of the fitness function versus the variation of the mentioned coefficients of the referenced steering angles commands may synchronously be considered to indicate that the GA is able to converge to reach reliable information with respect to time. There are two orbital parameters including the inclination and the eccentricity to be chosen to realize the present function by the following, while the scaling factors including $SF_\chi; \chi = 1, 2$ are used to weight the effectiveness of the items to be considered:

$$\Gamma(t) = SF_1(i(t) - i_d)^2 + SF_2(e(t) - e_d)^2 \quad (27)$$

where i_d and e_d denote the desirable values for the inclination and the eccentricity, respectively.

3. The simulation results

The control approach proposed here has been implemented first on a computer, whose main specifications are presented in table 1.

Now, to start carrying out the control approach, the distance and its rate for the initial orbit are first taken as

$$\begin{cases} r = [1.58 \ 4.02 \ 5.84]^T \text{ km} \\ \dot{r} = [6.75 \ -1.05 \ -1.42]^T \text{ km/s}. \end{cases} \quad (28)$$

In this case, the mission operation plan in association with the variations of the orbital parameters including the inclination, the eccentricity, the angular momentum and the semi-major axis are organized to deal with the orbital transfer process, as tabulated in table 2. It is to be noted that

Table 1. The specifications of the computer that is used in carrying out the control approach proposed.

	Items	Values
1	Processor	AMD Phenom9950 quad-core 2.6 GHZ
2	Installed memory	4.00 GB
3	System type	x-64-based processor, 32-bit operating system

Table 2. Orbital transfer mission.

	Parameters	Initial values	Final values
1	$i(t)$	55.35 deg.	5230 deg.
2	$e(t)$	0.015	0.14
3	$h(t)$	5.08e4 km ² /s	5.53e4 km ² /s
4	$a(t)$	6480 km	7820 km

Figure 3. The variation of the inclination in the orbital transfer process.

the outcomes are presented along with the initial and final values of the orbits.

The variations of the aforementioned orbital parameters with respect to time based upon the predefined mission are illustrated in figures 3, 4, 5 and 6, while the average values of the three-axis high-thrust vector profile are approximately taken as

$$\mathbf{F} = [1e4 \ 0 \ 0]^T \text{N}. \quad (29)$$

To calculate the coefficients of the referenced steering angles commands, the GA is realized to minimize the fitness function, while $SF_1 = \frac{5}{|i_i - i_d|} = 1.640$ and $SF_2 = \frac{0.05}{|e_i - e_d|} = 0.905$ are considered. It is to be noted that i_i and e_i denote, respectively, the values for the inclination and eccentricity in the initial orbit. Subsequently, the GA parameters are tabulated in table 3.

The GA is terminated at the 75th generation, where the fitness function is appropriately 0.00180 and the corresponding mean value is 0.00195. The variation of the performance index is illustrated in figure 7 in the orbital transfer process. The optimum coefficients of the referenced steering angles commands, presented in Eq. (26), are satisfied at the 2250th iteration by the following:

$$\begin{cases} a_c^0 = 15, a_c^1 = -3.10, a_c^2 = 0.46, a_c^3 = -0.025, a_c^4 = 0.00062, a_c^5 = -5.1e-6 \\ b_c^0 = 110, b_c^1 = 3.31, b_c^2 = -0.47, b_c^3 = 0.025, b_c^4 = -0.00060, b_c^5 = 5.0e-6 \end{cases} \quad (30)$$

The referenced steering angles commands are illustrated in figure 8 based upon the outcomes investigated in Eq. (30).

Figure 4. The eccentricity variation.

Figure 5. The angular momentum variation.

Now, to consider the efficiency of the control approach, a set of experiments are considered. It is to be noted that the

control approach is designed to deal with the three-axis rotational angles along with the referenced commands, acquired through Eqs. (1), (26) and (30), as long as the three-axis angular rates approach zero in the steady state for

Figure 6. The semi-major axis variation.

Figure 8. The steering angles commands.

Table 3. GA parameters.

	Parameters	Values
1	Crossover probability	0.80
2	Mutation probability	0.01
3	Crossover points	2.0
4	Population size	30
5	Maximum generations	80

Figure 7. The variation of the performance index in the orbital transfer process.

the purpose of guaranteeing system stability. The values of the parameters of the sub-systems, realized in carrying out the control approach, are tabulated in table 4.

Moreover, it is to be noted that in case of control allocator scheme, the configuration matrix is taken as

Table 4. The values of the parameters of the sub-systems realized in the procedure of carrying out the proposed control approach.

	Sub-systems	Values of the parameters
1	Three-axis control	$k_{pd} = [1020]$
2	Modulator	$A = 55, T = 0.15 \text{ s}, U_{on} = 0.8, U_{off} = 0.1$
3	Rate feedback	$k_{\omega xyz} = [153030]$
4	Control allocator	$p = 0.5, q = 0.25 \text{ m}$
5	Moments of inertia	$I_x = 20, I_y = 40, I_z = 45 \text{ kg m}^2$
6	T_{level}	6N
7	N	8

$$\text{Conf} = \begin{bmatrix} 0 & 0 & 0 & 0 & -p & -p & p & p \\ -p & 0 & p & 0 & 0 & q & 0 & -q \\ 0 & p & 0 & -p & q & 0 & -q & 0 \end{bmatrix}. \quad (31)$$

The rotational angles commands are considered versus the outcomes investigated in figure 8 through Eq. (1), as the three-axis tracking results are shown in figure 9, correspondingly, while the initial values of the same three-axis rotational angles of the system are given by

$$\begin{cases} \varphi_0 = \varphi(0) = 0^\circ \\ \theta_0 = \theta(0) = -10^\circ \\ \psi_0 = \psi(0) = 115^\circ. \end{cases} \quad (32)$$

Moreover, the initial values of the three-axis referenced commands based upon Eq. (1) and figure 8 are

$$\begin{cases} \varphi_{c0} = \varphi_c(0) = 0^\circ \\ \theta_{c0} = \theta_c(0) = -15^\circ \\ \psi_{c0} = \psi_c(0) = 110^\circ. \end{cases} \quad (33)$$

Figure 9. The tracking of the three-axis rotational angle.

Figure 10. The three-axis rotational angle tracking errors.

The errors of the corresponding three-axis rotational angles tracking are synchronously shown in figure 10.

3.1 The verification of the proposed control approach

The overall performance of the control approach is presented with respect to the state-dependent Riccati equation (SDRE). A number of cases are considered, including

$$\mathbf{x}(t) = \begin{bmatrix} \omega_{x,y,z}(t) \\ \varphi(t) \\ \theta(t) \\ \psi(t) \end{bmatrix} \quad (35)$$

$$\mathbf{u}_{SDRE}(t) = \begin{bmatrix} \mathbf{M}_{x,y,z}(t) \\ \mathbf{0}_{3 \times 1} \end{bmatrix}. \quad (36)$$

The $\mathbf{A}(\mathbf{x})$ and the $\mathbf{B}(\mathbf{x})$ are as follows:

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} 0 & \frac{-\omega_{z0}(I_x(t) - I_y(t))}{I_x(t)} & \frac{-\omega_{y0}(I_x(t) - I_y(t))}{I_x(t)} \\ \frac{-\omega_{z0}(I_x(t) - I_y(t))}{I_x(t)} & 0 & \frac{-\omega_{x0}(I_x(t) - I_z(t))}{I_y(t)} \\ \frac{-\omega_{z0}(I_x(t) - I_y(t))}{I_x(t)} & \frac{-\omega_{x0}(I_x(t) - I_z(t))}{I_y(t)} & 0 \end{bmatrix} \quad \mathbf{0}_{3 \times 3}$$

$$\mathbf{B}(\mathbf{x}(t)) = \begin{bmatrix} I_{x,y,z}^{-1} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & \sin(\varphi_0) \tan(\theta_0) & \cos(\varphi_0) \tan(\theta_0) & \omega_{y0} \cos(\varphi_0) \tan(\theta_0) - \omega_{z0} \sin(\varphi_0) \tan(\theta_0) & \frac{\omega_{z0} \cos(\varphi_0) + \omega_{y0} \sin(\varphi_0)}{\cos^2(\theta_0)} & 0 \\ 0 & \cos(\varphi_0) & -\sin(\varphi_0) & -\omega_{y0} \sin(\varphi_0) - \omega_{z0} \cos(\varphi_0) & 0 & 0 \\ 0 & \frac{\sin(\varphi_0)}{\cos(\theta_0)} & \frac{\cos(\varphi_0)}{\cos(\theta_0)} & \frac{\omega_{y0} \cos(\varphi_0) - \omega_{z0} \sin(\varphi_0)}{\cos(\theta_0)} & \frac{\omega_{y0} \sin(\varphi_0) \sin(\theta_0) + \omega_{z0} \cos(\varphi_0) \sin(\theta_0)}{\cos^2(\theta_0)} & 0 \end{bmatrix} \quad (37)$$

case I : the control approach and case II : the SDRE, to verify the investigated outcomes along with the benchmark.

3.1a The basic materials and results: The state space model regarding the SDRE is first given by [22]

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{x})\mathbf{x}(t) + \mathbf{B}(\mathbf{x})\mathbf{u}_{SDRE}(t) \quad (34)$$

where $\mathbf{x}(t)$ and the $\mathbf{u}_{SDRE}(t)$ are taken as

where $\mathbf{0}_{m \times n}$ indicates the $m \times n$ -order zero matrix and also ω_{x0}, ω_{y0} and ω_{z0} denote the initial values of the angular rates of the spacecraft. The $\mathbf{u}_{SDRE}(t)$ is designed to minimize the performance indices via Eq. (39):

$$\begin{cases} J_x(\mathbf{x}(t), \mathbf{u}_{SDRE}(t)) = \int_0^{t_{sm}} \mathbf{x}^T(t) \mathbf{Q}(\mathbf{x}(t)) \mathbf{x}(t) dt \\ J_u(\mathbf{x}(t), \mathbf{u}_{SDRE}(t)) = \int_0^{t_{sm}} \mathbf{u}^T(t) \mathbf{R}(\mathbf{x}(t)) \mathbf{u}_{SDRE}(t) dt \end{cases} \quad (38)$$

Table 5. The verification of the control approach based upon *SDRE*.

	Cases/ performances	Steady state errors for $\varphi^\circ, \theta^\circ, \psi^\circ$	Steady state errors for $\omega_{x,y,z}^\circ/s$	J
1	Case I	$\pm[10.10.190.17] \times 10^{-1}$	$\pm[8.22.53.8] \times 10^{-2}$	4.13
2	Case II	$\pm[14.30.740.85] \times 10^{-1}$	$\pm[10.712.314.5] \times 10^{-2}$	7.54

$$\mathbf{u}_{SDRE}(t) = -\mathbf{R}^{-1}(\mathbf{x}(t))\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}(\mathbf{x}(t))\mathbf{x}(t) \quad (39)$$

where $\mathbf{P}(\mathbf{x}(t))$ is acquired via the Riccati equation

$$\begin{aligned} &\mathbf{P}(\mathbf{x}(t))\mathbf{A}(\mathbf{x}(t)) + \mathbf{A}^T(\mathbf{x}(t))\mathbf{P}(\mathbf{x}(t)) + \mathbf{Q}(\mathbf{x}(t)) \\ &- \mathbf{P}(\mathbf{x}(t))\mathbf{B}(\mathbf{x}(t))\mathbf{R}^{-1}(\mathbf{x}(t))\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}(\mathbf{x}(t)) \\ &= 0. \end{aligned} \quad (40)$$

Now, in order to consider the efficiencies of the control approach, the *cases I* and *II* are synchronously carried out using the following parameters:

$$\begin{cases} \varphi_0 = 0^\circ, \theta_0 = -10^\circ, \psi_0 = 20^\circ \\ \omega_{x0} = \omega_{y0} = \omega_{z0} = 0.2^\circ/s \\ \mathbf{R}(\mathbf{x}(t)) = \frac{1}{2}\mathbf{I}_{de6}. \end{cases} \quad (41)$$

Also, the below parameters are the same as those of the wholes cases:

$$\begin{cases} t_{sm} = 35s \\ I_x = 20, I_y = 40, I_z = 45 \text{ kg m}^2 \\ \varphi_{co} = \theta_{co} = \psi_{co} = 0^\circ \\ \omega_{cx0} = \omega_{cy0} = \omega_{cz0} = 0^\circ/s \\ \mathbf{Q}(\mathbf{x}(t)) = \mathbf{I}_{de6} \end{cases} \quad (42)$$

where $\omega_{cx0}, \omega_{cy0}$ and ω_{cz0} denote the initial parameters regarding the referenced commands. Moreover, t_{sm} and $\mathbf{I}_{de6}$ denote, respectively, the simulation time and the six-order identity matrix. Table 5 shows the present verification process. In case of the steady state errors for $\varphi^\circ, \theta^\circ, \psi^\circ$ and $\omega_{x,y,z}^\circ/s$ as well as the results of $J(\mathbf{x}(t), \mathbf{u}(t)) = J_x(\mathbf{x}(t), \mathbf{u}_{SDRE}(t)) + J_u(\mathbf{x}(t), \mathbf{u}_{SDRE}(t))$, the performance of the control approach is verified.

4. Conclusion

High-resolution three-dimensional manoeuvres are considered in the present research through a realization of control approach in connection with the trajectories optimization for overactuated spacecraft. The proposed manoeuvres control is investigated along with the design of the double closed control loops, in order to cope with the three-axis rotational angles and the corresponding angular rates, synchronously, in line with the related referenced ones, in finite burn. The proposed high-thrust trajectory optimization is significant in orbital transfer by providing the desirable three-axis referenced commands,

at each instant of time. In fact, it transfers the aforementioned overactuated spacecraft from initial orbit to final orbit via the present control approach with a focus on a number of the orbital parameters, including the inclination, the eccentricity, the angular momentum, the semi-major axis and so on. The performance of the three-axis rotational angles tracking in line with the corresponding angular rates should be guaranteed, provided that the desirable accuracies are instantly acquired. The key motivation of the research is to organize an integration of the high-thrust trajectory optimization under the new configuration of the three-axis attitude control approach to efficiently manage the procedure of high-resolution orbital transfer process for overactuated spacecraft. A set of experiments are finally carried out to verify the performance of the investigated results, clearly.

Acknowledgements

The authors would like to express the best and the warmest regards to the respected editors of Sadhana and Springer Publisher as well as the respected anonymous reviewers, for suggesting their impressive, constructive, desirable and technical comments on the present investigation. Moreover, Dr. Mazinan sincerely appreciates the Islamic Azad University (IAU), South Tehran Branch, Tehran, Iran, which played a significant role in the process of paper investigation and organization. Maryam Aghaei Sarchali, Miss Mohadeseh Mazinan and also Mr. Mohammad Mazinan in the same area are greatly appreciated.

References

- [1] Olds A D, Kluever C A and Cupples M L 2007 Interplanetary mission design using differential evolution. *J. Spacecr. Rockets* 44(5): 1060–1070
- [2] Balkanyi L R and Spurlock O F 1993 *DUKSUP—a high-thrust trajectory optimization code*. NASA Lewis Research Center, Cleveland, OH
- [3] Yue X, Ma Y, Zhang X and Geng Z 2013 Trajectory optimization of continuous-thrust spacecraft round trip flights via pseudo spectral method. In: *Proceedings of the 32nd Chinese control conference (CCC)*, 2240–2244
- [4] Yam C H, Di Lorenzo D and Izzo D 2010 Constrained global optimization of low-thrust interplanetary trajectories. In:

- Proceedings of the IEEE Congress on Evolutionary Computation (CEC)*, 1–7
- [5] Jiang X Y, Zhang H B and Tang G J 2013 A novel low-thrust trajectory optimization approach based on virtual gravitational body. In: *Proceedings of the Chinese automation congress (CAC)*, 295–299
 - [6] Jiang X Y, Lian Y J, Zhang H B and Tang G J 2013 A multi-impulse extended method for low-thrust trajectory optimization. In: *Proceedings of the international conference on recent advances in space technologies (RAST)*, 36–368
 - [7] Bando M and Yamakawa H 2010 Low-thrust trajectory optimization using second-order generating functions. In: *Proceedings of the SICE annual conference*, 804–810
 - [8] Dai R, Maximoff J and Mesbahi M 2013 Optimal trajectory generation for establishing connectivity in proximity networks. *IEEE Trans. Aerosp. Electron. Syst.* 49(3): 1968–1981
 - [9] Tian B, Fan W, Su R and Zong Q 2015 Real-time trajectory and attitude coordination control for reusable launch vehicle in reentry phase. *IEEE Trans. Ind. Electron.* 62(3): 1639–1650
 - [10] Curtis H D 2010 Orbital mechanics for engineering students. *Elsevier Aerospace Engineering Series*, Butterworth-Heinemann, Burlington
 - [11] Sidi M J 1997 *Spacecraft dynamics and control: a practical engineering approach*. Aircraft Industries Ltd. and Aviv University, Cambridge Aerospace Series 7. Cambridge University Press, Cambridge
 - [12] He S, Wang W, Lin D and Lei H 2016 Robust output feedback control design for autonomous spacecraft rendezvous with actuator faults. *Proc. Inst. Mech. Eng. I: J. Syst. Control Eng.* 230(6): 578–587
 - [13] Huang X, Yan Y, Zhou Y and Xu H 2015 Output feedback sliding mode control of Lorentz-augmented spacecraft hovering using neural networks. *Proc. Inst. Mech. Eng. I: J. Syst. Control Eng.* 229(10): 939–948
 - [14] Song Z, Li H and Sun K 2014 Finite-time control for non-linear spacecraft attitude based on terminal sliding mode technique. *ISA Trans.* 53(1): 117–124
 - [15] Lu K and Xia Y 2013 Adaptive attitude tracking control for rigid spacecraft with finite-time convergence. *Automatica* 49(12): 3591–3599
 - [16] Yang Y 2012 Spacecraft attitude determination and control: quaternion based PDLQR method. *Annu. Rev. Control* 36(2): 198–219
 - [17] Mazinan A H, Pasand M and Soltani B 2015 Full quaternion based finite-time cascade attitude control approach via pulse modulation synthesis for a spacecraft. *ISA Trans.* 58: 567–585
 - [18] Zou A M and Kumar K D 2011 Adaptive fuzzy fault-tolerant attitude control of spacecraft. *Control Eng. Pract.* 19(1): 10–21
 - [19] Hu Q, Li B and Zhang Y 2013 Robust attitude control design for spacecraft under assigned velocity and control constraints. *ISA Trans.* 52(4): 480–493
 - [20] Cai H and Huang J 2014 The leader-following attitude control of multiple rigid spacecraft systems. *Automatica* 50(4): 1109–1115
 - [21] Kuo Y L and Wu T L 2012 Open-loop and closed-loop attitude dynamics and controls of miniature spacecraft using pseudo wheels. *Comput. Math. Appl.* 64(5): 1282–1290
 - [22] Zhang X, Liu X and Zhu Q 2014 Attitude control of rigid spacecraft with disturbance generated by time varying exo-systems. *Commun. Nonlinear Sci. Numer. Simul.* 19(7): 2423–2434