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Fuzzy linear programming approach for solving transportation problems with interval-valued trapezoidal fuzzy numbers

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Abstract. Transportation problem (TP) is an important network structured linear programming problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in this problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins in a crisp environment. In real life situations, the decision maker may not be sure about the precise values of the coefficients belonging to the transportation problem. The aim of this paper is to introduce a formulation of TP involving interval-valued trapezoidal fuzzy numbers for the transportation costs and values of supplies and demands. We propose a fuzzy linear programming approach for solving interval-valued trapezoidal fuzzy numbers transportation problem based on comparison of interval-valued fuzzy numbers by the help of signed distance ranking. To illustrate the proposed approach an application example is solved. It is demonstrated that study of interval-valued trapezoidal fuzzy numbers transportation problem gives rise to the same expected results as those obtained for TP with trapezoidal fuzzy numbers.

Keywords. Fuzzy linear programming; transportation problem; interval-valued trapezoidal fuzzy numbers; signed distance ranking.

1. Introduction

Transportation problem is an important network structured linear programming problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in this problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. Transportation problem can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling and many others. In general, transportation problems are solved with the assumptions that the transportation costs and values of supplies and demands are specified in a precise way i.e., in crisp environment. However, in many cases the decision maker has no crisp information about the coefficients belonging to the transportation problem. If the nature of the information is vague, that is, if it has some lack of precision, the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy sets, and thus fuzzy transportation problems arise.

Several researchers have carried out investigations on fuzzy transportation problem (FTP). Zimmermann [1] developed Zimmermann's fuzzy linear programming into several fuzzy optimization methods for solving the transportation problems. Oheigeartaigh [2] proposed an algorithm for solving transportation problems where the supplies and demands are fuzzy sets with linear or triangular membership functions. Chanas *et al* [3] investigated the transportation problem with fuzzy supplies and demands and solved them via the parametric programming technique. Their method provided solution which simultaneously satisfies the constraints and the goal to a maximal degree. In addition, Chanas *et al* [4] formulated the classical, interval and fuzzy transportation problem and discussed the methods for solution for the FTP. Chanas & Kuchta [5] discussed the type of transportation problems with fuzzy cost coefficients and converted the problem into a bicriterial transportation problem with crisp objective function. Their method only gives crisp solutions based on efficient solutions of the converted problems. Jimenez & Verdegay [6, 7] investigated the fuzzy solid transportation problem in which supplies, demands and conveyance capacities are represented by trapezoidal fuzzy numbers and applied a parametric approach for finding the fuzzy solution. Liu & Kao [8] developed a procedure, based on extension principle to derive the fuzzy objective value of FTP, in that the cost coefficients and the supply and demand quantities are fuzzy numbers. Gani & Razak [9] presented a two-stage cost minimizing FTP in which supplies and demands are as trapezoidal fuzzy numbers and used a parametric approach for finding a fuzzy solution with the aim of minimizing the sum of the transportation costs in the two stages.

Li *et al* [10] proposed a new method based on goal programming for solving FTP with fuzzy costs. Lin [11] used genetic algorithm for solving transportation problems with fuzzy coefficients. Dinagar & Palanivel [12] investigated FTP, with the help of trapezoidal fuzzy numbers and applied fuzzy modified distribution method to obtain the optimal solution in terms of fuzzy numbers. Pandian & Natarajan [13] introduced a new algorithm namely, fuzzy zero point method for finding fuzzy optimal solution for such FTP in which the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers. Kumar & Kaur [14] proposed a new method based on fuzzy linear programming problem for finding the optimal solution of FTP. Gupta *et al* [15] proposed a new method named as Mehar's method, to find the exact fuzzy optimal solution of fully fuzzy multi-objective transportation problems. Ebrahimnejad [16] applied a fuzzy bounded dual algorithm for solving bounded transportation problems with fuzzy supplies and demands. Shanmugasundari & Ganesan [17] developed the fuzzy version of Vogel's and MODI methods for obtaining the fuzzy initial basic feasible solution and fuzzy optimal feasible solution, respectively, without converting them into classical transportation problem. Also, Sudhagar & Ganesan [18] proposed an algorithm to find an optimal solution of a FTP, where supply, demand and cost coefficients all are fuzzy numbers. Their algorithm provides decision maker with an effective solution in comparison to existing methods. Ebrahimnejad [19] using an example showed that their method will not always lead to a fuzzy optimal solution. Moreover, Kumar & Kaur [20] pointed out the limitations and shortcomings of the existing methods for solving fuzzy solid transportation problem and to overcome these limitations and shortcomings proposed a new method to find the fuzzy optimal solution of unbalanced fuzzy solid transportation problems. In addition, Ebrahimnejad [21] proposed a two-step method for solving FTP where all of the parameters are represented by non-negative triangular fuzzy numbers.

Some researchers applied generalized fuzzy numbers for solving transportation problems. Kumar & Kaur [22] proposed a new method based on ranking function for solving FTP by assuming that transportation cost, supply and demand of the commodity are represented by generalized trapezoidal fuzzy numbers. After that, Kaur & Kumar [23] introduced a similar algorithm for solving a special type of FTP by assuming that a decision maker is uncertain about the precise values of transportation cost only but there is no uncertainty about the supply and demand of the product. Ebrahimnejad [24] demonstrated that once the ranking function is chosen, the FTP introduced by Kaur & Kumar [23] is converted into crisp one, which is easily solved by the standard transportation algorithms.

However, there are only few papers dealing with the problems involving interval-valued fuzzy numbers. Chiang [25] pointed out that it is better to represent the availability and demand as (λ, φ) interval-valued triangular fuzzy numbers instead of normal fuzzy numbers and proposed a method to find the optimal solution of single objective transportation problems by representing the availability and demand as (λ, φ) interval-valued triangular fuzzy numbers. Gupta & Kumar [26] pointed out the shortcomings of the Chiang's method and to overcome them proposed a new method for finding the solution of a linear multi-objective transportation problem by representing the values of cost, supply and demand as (λ, φ) interval-valued triangular fuzzy numbers. In this paper, the shortcomings of some existing methods for solving FTP are pointed out and to overcome these shortcomings a new method based on fuzzy linear programming approach is introduced for finding the optimal solution of TP by representing all the parameters as (w^L, w^U)-interval-valued trapezoidal fuzzy numbers. How to convert an unbalanced interval-valued trapezoidal fuzzy numbers into a balanced one is explored in this study. Moreover, it is demonstrated that study of interval-valued trapezoidal fuzzy numbers transportation problem gives rise to the same expected results as those obtained for TP with trapezoidal fuzzy numbers.

The remainder of this paper is organized as follows. In Section 2, some basic definitions and arithmetic operations are reviewed. In Section 3, we attempt to introduce a formulation of transportation problem with (w^L, w^U)-interval-valued trapezoidal fuzzy numbers. In Section 4, the shortcomings of the existing methods are pointed out. In Section 5, a fuzzy linear programming approach is proposed for solving interval-valued trapezoidal fuzzy numbers transportation problem based on comparison of interval-valued trapezoidal fuzzy numbers by the help of signed distance ranking. Also, the computational complexity of the proposed algorithm has been computed in this section. Section 6 is devoted to illustration of the proposed method using two application examples. Obtained results as well as the main advantages and disadvantages of the proposed method over the existing methods are discussed in Section 7. We present our conclusions in Section 8.

2. Preliminaries

In this section, some basic definitions, the arithmetic operations and the comparison of the level (w^L, w^U)-interval-valued trapezoidal fuzzy numbers are reviewed.

DEFINITION 2.1 ([23])

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers $\mathbb{R}$, is said to be a generalized fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}} : \mathbb{R} \to [0, w]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a_1] \cup [a_4, \infty)$.
3. $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a_1, a_2]$ and strictly decreasing on $[a_3, a_4]$.
4. $\mu_{\tilde{A}}(x) = w$ for all $x \in [a_2, a_3]$, where $0 < w \leq 1$.

DEFINITION 2.2 ([27])

A level w-trapezoidal fuzzy number $\tilde{A}$ or a generalized trapezoidal fuzzy number $\tilde{A}$, denoted by $\tilde{A} = (a_1, a_2, a_3, a_4; w)$, $0 < w \leq 1$, is a fuzzy number with the membership function as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} w\dfrac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2, \\ w, & a_2 \leq x \leq a_3, \\ w\dfrac{a_4-x}{a_4-a_3} & a_3 \leq x \leq a_4, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Let $F_{TN}(w)$ be the family of all level w-trapezoidal fuzzy numbers, that is,

$$F_{TN}(w) = \left\{\tilde{A} = (a_1, a_2, a_3, a_4; w), a_1 \leq a_2 \leq a_3 \leq a_4\right\}, \quad 0 < w \leq 1. \tag{2}$$

DEFINITION 2.3 ([28])

Let $\tilde{A}^L \in F_{TN}(w^L)$ and $\tilde{A}^U \in F_{TN}(w^U)$. A level (w^L, w^U)-interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$, denoted by $\tilde{\tilde{A}} = \left[\tilde{A}^L, \tilde{A}^U\right] = \langle(a_1^L, a_2^L, a_3^L, a_4^L; w^L), \left(a_1^U, a_2^U, a_3^U, a_4^U; w^U\right)\rangle$ is an interval-valued fuzzy set on $\mathbb{R}$ with the lower trapezoidal fuzzy number $\tilde{A}^L$ expressing by

$$\mu_{\tilde{A}^L}(x) = \begin{cases} w^L\dfrac{x-a_1^L}{a_2^L-a_1^L}, & a_1^L \leq x \leq a_2^L, \\ w^L, & a_2^L \leq x \leq a_3^L, \\ w^L\dfrac{a_4^L-x}{a_4^L-a_3^L}, & a_3^L \leq x \leq a_4^L, \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

and the upper trapezoidal fuzzy number $\tilde{A}^U$ expressing by

$$\mu_{\tilde{A}^U}(x) = \begin{cases} w^U\dfrac{x-a_1^U}{a_2^U-a_1^U}, & a_1^U \leq x \leq a_2^U, \\ w^U, & a_2^U \leq x \leq a_3^U, \\ w^U\dfrac{a_4^U-x}{a_4^U-a_3^U}, & a_3^U \leq x \leq a_4^U, \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

where $a_1^L \leq a_2^L \leq a_3^L \leq a_4^L, a_1^U \leq a_2^U \leq a_3^U \leq a_4^U, 0 < w^L \leq w^U \leq 1, a_1^U \leq a_1^L$ and $a_4^L \leq a_4^U$. Moreover, $\mu_{\tilde{A}^L}(x) \leq \mu_{\tilde{A}^U}(x)$.

This means that the grade of membership x belongs to interval $\tilde{\tilde{A}} = \left[\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)\right]$, the latest and greatest grade of membership at x are $\mu_{\tilde{A}^L}(x)$ and $\mu_{\tilde{A}^U}(x)$, respectively.

Let $F_{IVTN}(w^L, w^U)$ be the family of all level (w^L, w^U)-interval-valued trapezoidal fuzzy numbers, that is,

$$F_{IVTN}(w^L, w^U) = \left\{\tilde{\tilde{A}} = \left[\tilde{A}^L, \tilde{A}^U\right] = \left\langle\left(a_1^L, a_2^L, a_3^L, a_4^L; w^L\right), (a_1^U, a_2^U, a_3^U, a_4^U; w^U)\right\rangle : \tilde{A}^L \in F_{TN}(w^L), \tilde{A}^U \in F_{TN}(w^U), a_1^U \leq a_1^L, a_4^L \leq a_4^U\right\}, 0 < w^L \leq w^U \leq 1.$$

DEFINITION 2.4

A (w^L, w^U)-interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}} = \left[\tilde{A}^L, \tilde{A}^U\right] = \langle(a_1^L, a_2^L, a_3^L, a_4^L; w^L), (a_1^U, a_2^U, a_3^U, a_4^U; w^U)\rangle$ is said to be a non-negative (w^L, w^U)-interval-valued trapezoidal fuzzy number if and only if $a_1^U \geq 0$. The set of all non-negative (w^L, w^U)-interval-valued trapezoidal fuzzy number is denoted by $F^+_{IVTN}(w^L, w^U)$.

DEFINITION 2.5

Two (w^L, w^U)-interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}} = \langle(a_1^L, a_2^L, a_3^L, a_4^L; w^L), (a_1^U, a_2^U, a_3^U, a_4^U; w^U)\rangle$ and $\tilde{\tilde{B}} = \langle(b_1^L, b_2^L, b_3^L, b_4^L; w^L), (b_1^U, b_2^U, b_3^U, b_4^U; w^U)\rangle$ is said to be equal, i.e. $\tilde{\tilde{A}} = \tilde{\tilde{B}}$ if and only if $a_i^L = b_i^L (i = 1, 2, 3, 4)$ and $a_i^U = b_i^U (i = 1, 2, 3, 4)$.

DEFINITION 2.6

Let $\tilde{\tilde{A}} = \left[\tilde{A}^L, \tilde{A}^U\right] = \langle(a_1^L, a_2^L, a_3^L, a_4^L; w^L), \left(a_1^U, a_2^U, a_3^U, a_4^U; w^U\right)\rangle$ and $\tilde{\tilde{B}} = \left[\tilde{B}^L, \tilde{B}^U\right] = \langle(b_1^L, b_2^L, b_3^L, b_4^L; w^L), (b_1^U, b_2^U, b_3^U, b_4^U; w^U)\rangle$ belong to $F_{IVTN}(w^L, w^U)$ and k be a non-negative real number. Then the exact formulas for the extended addition and the scalar multiplication and the approximate formula for the extended multiplication are defined as follows (see Wei & Chen [29]):

$$\tilde{\tilde{A}} \oplus \tilde{\tilde{B}} = \left\langle(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; w^L), (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_1^U; w^U)\right\rangle$$

$$k\tilde{\tilde{A}} = \begin{cases} \langle(ka_1^L, ka_2^L, ka_3^L, ka_4^L; w^L), (ka_1^U, ka_2^U, ka_3^U, ka_4^U; w^U)\rangle, & k > 0, \\ \langle(ka_4^L, ka_3^L, ka_2^L, ka_1^L; w^L), (ka_4^U, ka_3^U, ka_2^U, ka_1^U; w^U)\rangle, & k < 0, \\ \langle(0, 0, 0, 0; w^L), (0, 0, 0, 0; w^U)\rangle = \tilde{\tilde{0}}, & k = 0, \end{cases}$$

$$\tilde{\tilde{A}} \otimes \tilde{\tilde{B}} = \left\langle(a_1^L b_1^L, a_2^L b_2^L, a_3^L b_3^L, a_4^L b_4^L; w^L), (a_1^U b_1^U, a_2^U b_2^U, a_3^U b_3^U, a_4^U b_1^U; w^U)\right\rangle, a_1^U, b_1^U \geq 0.$$

DEFINITION 2.7 ([27])

Let $r, 0 \in \mathbb{R}$. The signed distance from r to 0 is defined as $d(r, 0) = r$.

DEFINITION 2.8 ([30])

Let $\tilde{\tilde{A}} \in F_{IVTN}(w^L, w^U)$. The $\alpha-$cut set of $\tilde{\tilde{A}}$ denoted by $\tilde{\tilde{A}}(\alpha)$, is defined as follows (see figure 1):

$$\tilde{\tilde{A}}(\alpha) = \left[\tilde{A}^L(\alpha), \tilde{A}^U(\alpha)\right] = \begin{cases} \left[\tilde{A}_l^U(\alpha), \tilde{A}_l^L(\alpha)\right] \cup \left[\tilde{A}_r^L(\alpha), \tilde{A}_r^U(\alpha)\right] & 0 \le \alpha \le w^L \\ \left[\tilde{A}_l^U(\alpha), \tilde{A}_r^U(\alpha)\right] & w^L \le \alpha \le w^U \end{cases}$$

where

$$\begin{aligned} \tilde{A}_l^L(\alpha) &= a_1^L + (a_2^L - a_1^L)\frac{\alpha}{w^L}, \tilde{A}_r^L(\alpha) = a_4^L \\ &\quad + (a_4^L - a_3^L)\frac{\alpha}{w^L}, \\ \tilde{A}_l^U(\alpha) &= a_1^U + (a_2^U - a_1^U)\frac{\alpha}{w^U}, \tilde{A}_r^U(\alpha) = a_4^U \\ &\quad + (a_4^U - a_3^U)\frac{\alpha}{w^U}, \end{aligned}$$

Theorem 2.1 [30]. *Let $\tilde{\tilde{A}} \in F_{IVTN}(w^L, w^U)$. The signed distance of $\tilde{\tilde{A}}$ from O_1 (y-axis) is given as follows:*

$$d(\tilde{A}, O_1) = \frac{1}{4}\left[a_1 + a_2 + a_3 + a_4\right], \tilde{A}^L = \tilde{A}^U = \tilde{A} \quad (5)$$

$$\begin{aligned} d(\tilde{\tilde{A}}, O_1) &= \frac{1}{8}\left[a_1^L + a_2^L + a_3^L + a_4^L + a_1^U + a_2^U + a_3^U + a_4^U\right], \\ & 0 < w^L = w^U \le 1 \end{aligned} \quad (6)$$

$$\begin{aligned} d(\tilde{\tilde{A}}, O_1) = \frac{1}{8}\Big[& a_1^L + a_2^L + a_3^L + a_4^L + 4a_1^U + 2a_2^U + 2a_3^U \\ & + 4a_4^U + 3(a_2^U + a_3^U - a_1^U - a_4^U)\frac{w^L}{w^U}\Big], \\ & 0 < w^L < w^U \le 1 \end{aligned} \quad (7)$$

Theorem 2.1 describes an efficient approach to order of level (w^L, w^U)-interval-valued trapezoidal fuzzy numbers based on the concept of comparison of fuzzy numbers by the help of signed distance ranking.

DEFINITION 2.9 ([30])

Let $\tilde{\tilde{A}}, \tilde{\tilde{B}} \in F_{IVTN}(w^L, w^U)$. Then the ranking of level (w^L, w^U)-interval-valued trapezoidal fuzzy numbers in $F_{IVTN}(w^L, w^U)$ is defined on the basis of signed distance $d(., O_1)$ as follows:

$$\tilde{\tilde{A}} \prec \tilde{\tilde{B}} \quad \textit{iff} \quad d(\tilde{\tilde{A}}, O_1) < d(\tilde{\tilde{B}}, O_1) \quad (8)$$

$$\tilde{\tilde{A}} \succ \tilde{\tilde{B}} \quad \textit{iff} \quad d(\tilde{\tilde{A}}, O_1) > d(\tilde{\tilde{B}}, O_1) \quad (9)$$

$$\tilde{\tilde{A}} \approx \tilde{\tilde{B}} \quad \textit{iff} \quad d(\tilde{\tilde{A}}, O_1) = d(\tilde{\tilde{B}}, O_1). \quad (10)$$

Notice that the signed distance $d(., O_1)$ provides us a linear ranking function, i.e. for any $\tilde{\tilde{A}}, \tilde{\tilde{B}} \in F_{IVTN}(w^L, w^U)$ and $k \in \mathbb{R}$ we have $d(k\tilde{\tilde{A}} \oplus \tilde{\tilde{B}}, O_1) = kd(\tilde{\tilde{A}}, O_1) + d(\tilde{\tilde{B}}, O_1)$.

In addition, $\left(F_{IVTN}(w^L, w^U), \approx, \prec\right)$ satisfies the law of trichotomy [27], that is, we have $\tilde{\tilde{A}} \prec \tilde{\tilde{B}}$ or $\tilde{\tilde{A}} \approx \tilde{\tilde{B}}$ or $\tilde{\tilde{B}} \prec \tilde{\tilde{A}}$.

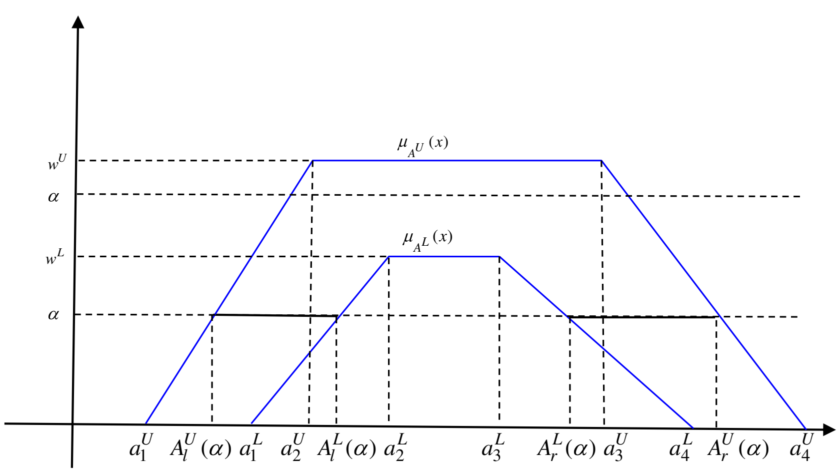

Figure 1. An $\alpha-$ cut of level (w^L, w^U) interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$.

3. Interval-valued trapezoidal fuzzy numbers transportation problem

In this section linear programming formulation of TP in crisp and $F_{IVTN}(w^L, w^U)$ environment are presented.

3.1 *Transportation problem in crisp environment*

The central concept in transportation problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. This problem may be stated mathematically as follows:

$$\begin{aligned}
&\min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}\\
&s.t. \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, ..., m,\\
&\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, ..., n,\\
&x_{ij} \geq 0, \qquad i = 1, 2, ..., m,\ j = 1, 2, ..., n.
\end{aligned} \tag{11}$$

Variable $\tilde{x}_{ij}$ represents the amount of the commodity to be shipped from i^{th} origin to j^{th} destination, c_{ij} is the cost associated with transporting a unit of commodity from i^{th} origin to j^{th} destination, a_i represents the supply of the commodity at i^{th} origin and b_j represents the demand of the commodity at j^{th} destination. Without of loss of generality, it may be assume that $a_i > 0 \forall i, b_j > 0 \forall j, c_{ij} \geq 0 \forall i, j$ and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

3.2 *Transportation problem in $F_{IVTN}(w^L, w^U)$ environment*

A huge number of researchers have shown their interest in solving fuzzy transportation problems where all parameters are considered as triangular or trapezoidal fuzzy numbers. In previous works, it is pointed out that in several situations it is better to use generalized interval-valued fuzzy numbers instead of triangular or trapezoidal fuzzy numbers. Thus some researchers have used interval-valued triangular fuzzy numbers [25, 26] in different types of fuzzy transportation problems. But, as far as we know, there is no method in previous works that focus on the class of fuzzy TP where all parameters as well as the amounts of commodity are represented as the level (w^L, w^U)-interval-valued trapezoidal fuzzy numbers. In this section, a new formulation is given for the same kind of TP.

Assume that all the parameters (cost, supply, demand and amount of commodity) are represented by (w^L, w^U)-interval-valued trapezoidal fuzzy numbers. Then TP (11) in $F_{IVTN}(w^L, w^U)$ environment is

$$\begin{aligned}
&\min \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}\\
&s.t. \sum_{j=1}^{n} \tilde{\tilde{x}}_{ij} = \tilde{\tilde{a}}_i, \quad i = 1, 2, ..., m,\\
&\sum_{i=1}^{m} \tilde{\tilde{x}}_{ij} = \tilde{\tilde{b}}_j, \qquad j = 1, 2, ..., n,\\
&\tilde{\tilde{x}}_{ij} \in F^{+}_{IVTN}(w^L, w^U), i = 1, 2, ..., m, j = 1, 2, ..., n.
\end{aligned} \tag{12}$$

with

$$\sum_{i=1}^{m} \tilde{\tilde{a}}_{ij} = \sum_{j=1}^{n} \tilde{\tilde{b}}_{ij}$$

where

$$\begin{aligned}
\tilde{\tilde{a}}_i &= \left\langle (a^L_{i1}, a^L_{i2}, a^L_{i3}, a^L_{i4}; w^L), (a^U_{i1}, a^U_{i2}, a^U_{i3}, a^U_{i4}; w^U) \right\rangle\\
&\in F^{+}_{IVTN}(w^L, w^U), i = 1, 2, ..., m,
\end{aligned}$$

$$\begin{aligned}
\tilde{\tilde{b}}_j &= \left\langle (b^L_{j1}, b^L_{j2}, b^L_{j3}, b^L_{j4}; w^L), (b^U_{j1}, b^U_{j2}, b^U_{j3}, b^U_{j4}; w^U) \right\rangle\\
&\in F^{+}_{IVTN}(w^L, w^U), j = 1, 2, ..., n,\\
\tilde{\tilde{c}}_{ij} &= \left\langle (c^L_{ij1}, c^L_{ij2}, c^L_{ij3}, c^L_{ij4}; w^L), (c^U_{ij1}, c^U_{ij2}, c^U_{ij3}, c^U_{ij4}; w^U) \right\rangle\\
&\in F^{+}_{IVTN}(w^L, w^U), i = 1, 2, ..., m, j = 1, 2, ..., n,
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\tilde{x}}_{ij} &= \left\langle (x^L_{ij1}, x^L_{ij2}, x^L_{ij3}, x^L_{ij4}; w^L), (x^U_{ij1}, x^U_{ij2}, x^U_{ij3}, x^U_{ij4}; w^U) \right\rangle\\
&\in F^{+}_{IVTN}(w^L, w^U), i = 1, 2, ..., m, j = 1, 2, ..., n.
\end{aligned}$$

4. Shortcomings of the existing methods

There are several methods to find the optimal solution of fuzzy transportation problems. But, there are shortcomings in some existing methods as follows:

(i) The existing method [23] can be applied only to solve FTP, where the transportation costs are represented as generalized trapezoidal fuzzy numbers. This method cannot be used for solving TP, where the transportation costs are represented as generalized interval-valued trapezoidal fuzzy numbers. For example, it is not possible to find the solution of TP, chosen in Example 4.1, by using the existing method as transportation costs are in the form of generalized interval-valued trapezoidal fuzzy numbers.

Example 4.1

$$\min \sum_{i=1}^{2}\sum_{j=1}^{3} \tilde{\tilde{c}}_{ij} x_{ij}$$
$$s.t. \sum_{j=1}^{3} x_{ij} = a_i, \quad i = 1, 2,$$
$$\sum_{i=1}^{2} x_{ij} = b_j, \quad j = 1, 2, 3,$$
$$x_{ij} \geq 0, \quad i = 1, 2, j = 1, 2, 3, \quad (13)$$

where the values of $\tilde{\tilde{c}}_{ij}$ for $i = 1, 2$, $j = 1, 2, 3$, are

$$\begin{aligned}
\tilde{\tilde{c}}_{11} &= \langle(3, 4, 6, 7; 0.8), (2, 3, 8, 9; 1)\rangle, \\
\tilde{\tilde{c}}_{12} &= \langle(2, 4, 8, 9; 0.8), (1, 5, 10, 11; 1)\rangle, \\
\tilde{\tilde{c}}_{13} &= \langle(5, 7, 10, 11; 0.8), (4, 7, 11, 12; 1)\rangle \\
\tilde{\tilde{c}}_{21} &= \langle(7, 8, 12, 13; 0.8), (5, 9, 13, 14; 1)\rangle, \\
\tilde{\tilde{c}}_{12} &= \langle(6, 8, 11, 12; 0.8), (4, 7, 12, 13; 1)\rangle, \\
\tilde{\tilde{c}}_{13} &= \langle(8, 9, 14, 15; 0.8), (7, 10, 15, 16; 1)\rangle.
\end{aligned}$$

The values of supply and demand are $a_1 = 60, a_2 = 80, b_1 = 30, b_2 = 70, b_3 = 40$.

(ii) The existing method [22] can be used for solving FTP with the values of supply and demand as generalized trapezoidal fuzzy numbers, while the transportation costs are represented as real numbers. This method cannot be applied for solving FTP in which the values of supply and demand as generalized interval-valued trapezoidal fuzzy numbers. For example, the existing method [22] cannot be used to find the solution of TP, chosen in Example 4.2, as the values of supply and demand are not represented by generalized trapezoidal fuzzy numbers.

Example 4.2

$$\min \ 2\tilde{\tilde{x}}_{11} + 5\tilde{\tilde{x}}_{12} + 7\tilde{\tilde{x}}_{13} + 4\tilde{\tilde{x}}_{21} + 3\tilde{\tilde{x}}_{22} + 8\tilde{\tilde{x}}_{23}$$
$$s.t. \sum_{j=1}^{3} \tilde{\tilde{x}}_{ij} = \tilde{\tilde{a}}_i, \quad i = 1, 2,$$
$$\sum_{i=1}^{2} \tilde{\tilde{x}}_{ij} = \tilde{\tilde{b}}_j, \quad j = 1, 2, 3,$$
$$\tilde{\tilde{x}}_{ij} F^{+}_{IVTN}(w^L, w^U), i = 1, 2, j = 1, 2, 3, \quad (14)$$

where the values of $\tilde{\tilde{a}}_i$ for $i = 1, 2$ and $\tilde{\tilde{b}}_j$ for $j = 1, 2, 3$ are

$$\begin{aligned}
\tilde{\tilde{a}}_1 &= \langle(30, 40, 60, 70; 0.8), (25, 35, 65, 75; 1)\rangle, \\
\tilde{\tilde{a}}_2 &= \langle(20, 40, 80, 90; 0.8), (15, 35, 85, 95; 1)\rangle \\
\tilde{\tilde{b}}_1 &= \langle(10, 15, 30, 35; 0.8), (9, 15, 30, 40; 1)\rangle, \\
\tilde{\tilde{b}}_2 &= \langle(15, 30, 50, 55; 0.8), (13, 30, 55, 60; 1)\rangle, \\
& \langle(25, 35, 60, 70; 0.8), (18, 25, 65, 70; 1)\rangle.
\end{aligned}$$

(iii) The existing method [27] can be used only to solve FTP in which the values of supply and demand are represented by (λ, φ) interval-valued triangular fuzzy numbers, while the transportation cost and the amounts of commodity to be shipped from origins to destinations are represented as real numbers.

(iv) In the existing method [26] all the parameters (cost, supply and demand) are represented by (λ, φ) interval-valued triangular fuzzy numbers, while the optimal solution of the corresponding FTP as obtained as real numbers. This represents a compromise in terms of fuzzy numbers.

(v) The existing methods [31, 32] can be applied to solve the FTP in which all the parameters (cost, supply and demand) as well as the amounts of commodity to be shipped from origins to destinations are represented by trapezoidal fuzzy numbers. These methods cannot be used to solve FTP (12) in which all the parameters as well as the amounts of commodity are represented by generalized interval-valued trapezoidal fuzzy numbers.

5. Proposed method

In this section, to overcome all the shortcomings of the existing methods, discussed in Section 4, a new method based on fuzzy linear programming formulation is proposed to obtain the interval valued trapezoidal fuzzy optimal solution of interval-valued trapezoidal fuzzy numbers TP (12) in which all the parameters (cost, supply and demand) as well as decision variables are represented by (w^L, w^U)-interval-valued trapezoidal fuzzy numbers. How an unbalanced interval-valued trapezoidal fuzzy numbers TP is converted into a balanced one is explored in the proposed algorithm.

DEFINITION 5.1

The fuzzy optimal solution of interval-valued trapezoidal fuzzy numbers TP (12) is a set of interval-valued trapezoidal fuzzy numbers $\left\{\tilde{\tilde{x}}_{ij}\right\}$ which satisfies the following characteristics:

(i) $\tilde{\tilde{x}}_{ij}$ is a non-negative interval-valued trapezoidal fuzzy number.

(ii) $\sum_{j=1}^{n} \tilde{\tilde{x}}_{ij} = \tilde{\tilde{a}}_i, i = 1, 2, ..., m$ and $\sum_{i=1}^{m} \tilde{\tilde{x}}_{ij} = \tilde{\tilde{b}}_j, j = 1, 2, ..., n$.

(iii) If there exist any other set of non-negative interval-valued trapezoidal fuzzy numbers $\left\{\tilde{\tilde{x}}'_{ij}\right\}$ such that $\sum_{j=1}^{n} \tilde{\tilde{x}}'_{ij} = \tilde{\tilde{a}}_i, i = 1, 2, ..., m$ and $\sum_{i=1}^{m} \tilde{\tilde{x}}'_{ij} = \tilde{\tilde{b}}_j, j = 1, 2, ..., n$, then

$$d\left(\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}, O_1\right) < d\left(\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}'_{ij}, O_1\right).$$

Now, various steps to find the solution of interval-valued trapezoidal fuzzy numbers transportation problem (12) are described as follows

Step 1 Find the total interval-valued fuzzy supply $\sum_{i=1}^{m}\tilde{\tilde{a}}_i$ and the total interval-valued fuzzy demand $\sum_{j=1}^{n}\tilde{\tilde{b}}_j$. Let $\sum_{i=1}^{m}\tilde{\tilde{a}}_i = \langle (m_1^L, m_2^L, m_3^L, m_4^L; w^L), (m_1^U, m_2^U, m_3^U, m_4^U; w^U) \rangle$ and $\sum_{j=1}^{n}\tilde{\tilde{b}}_j = \langle (n_1^L, n_2^L, n_3^L, n_4^L; w^L), (n_1^U, n_2^U, n_3^U, n_4^U; w^U) \rangle$. Examine the problem is balanced or not, i.e., $\sum_{i=1}^{m}\tilde{\tilde{a}}_i = \sum_{j=1}^{n}\tilde{\tilde{b}}_j$ or $\sum_{i=1}^{m}\tilde{\tilde{a}}_i \neq \sum_{j=1}^{n}\tilde{\tilde{b}}_j$.

Case (i) If the problem is balanced, i.e., $\sum_{i=1}^{m}\tilde{\tilde{a}}_i = \sum_{j=1}^{n}\tilde{\tilde{b}}_j$, then go to Step 2.

Case (ii) If $\sum_{i=1}^{m}\tilde{\tilde{a}}_i \neq \sum_{j=1}^{n}\tilde{\tilde{b}}_j$, then convert the unbalanced problem into balanced problem as follows:

Case (a) If $m_1^L \le n_1^L$, $m_2^L \le n_2^L$, $m_3^L \le n_3^L$, $m_4^L \le n_4^L, m_1^U \le n_1^U$, $m_2^U \le n_2^U$, $m_3^U \le n_3^U$ and $m_4^U \le n_4^U$ then introduce a dummy origin with interval-valued fuzzy supply $\langle (n_1^L - m_1^L, n_2^L - m_2^L, n_3^L - m_4^L, n_4^L - m_4^L; w^L), (n_1^U - m_1^U, n_2^U - m_2^U, n_3^U - m_3^U, n_4^U - m_4^U; w^U) \rangle$. Assume the interval-valued fuzzy transportation cost for one unit quantity of the commodity from the introduced dummy origin to all destinations as zero interval-valued trapezoidal fuzzy number, i.e., $\tilde{\tilde{0}}\, \langle (0, 0, 0, 0; 1), (0, 0, 0, 0; 1) \rangle$, then go to Step 2.

Case (b) If $n_1^L \le m_1^L$, $n_2^L \le m_2^L$, $n_3^L \le m_3^L$, $n_4^L \le m_4^L, n_1^U \le m_1^U$, $n_2^U \le m_2^U$, $n_3^U \le m_3^U$ and $n_4^U \le m_4^U$ then introduce a dummy destination with interval-valued fuzzy demand $\langle (m_1^L - n_1^L, m_2^L - n_2^L, m_3^L - n_4^L, m_4^L - n_4^L; w^L), (m_1^U - n_1^U, m_2^U - n_2^U, m_3^U - n_3^U, m_4^U - n_4^U; w^U) \rangle$. Assume the interval-valued fuzzy transportation cost for one unit quantity of the commodity from all origins to the introduced dummy destination as zero interval-valued trapezoidal fuzzy number, i.e., $\tilde{\tilde{0}} = \langle (0, 0, 0, 0; 1), (0, 0, 0, 0; 1) \rangle$, then go to Step 2.

Case (c) If neither Case (a) nor Case (b) is satisfied then introduce a dummy origin with interval-valued fuzzy supply $\langle (A_1^L, A_2^L, A_3^L, A_4^L; w^L), (A_1^U, A_2^U, A_3^U, A_4^U; w^U) \rangle$ and dummy destination with interval-valued fuzzy demand $\langle (B_1^L, B_2^L, B_3^L, B_4^L; w^L), (B_1^U, B_2^U, B_3^U, B_4^U; w^U) \rangle$ where

$$
\begin{aligned}
A_1^L &= \left|n_1^U - m_1^U\right| + \max\left\{0, n_1^L - m_1^L\right\},\\
A_2^L &= A_1^L + \max\left\{0, (n_2^L - n_1^L) - (m_2^L - m_1^L)\right\}\\
A_3^L &= A_2^L + \max\left\{0, (n_3^L - n_2^L) - (m_3^L - m_3^L)\right\},\\
A_4^L &= A_3^L + \max\left\{0, (n_4^L - n_3^L) - (m_4^L - m_3^L)\right\}\\
B_1^L &= \left|n_1^U - m_1^U\right| + \max\left\{0, m_1^L - n_1^L\right\},\\
B_2^L &= B_1^L + \max\left\{0, (m_2^L - m_1^L) - (n_2^L - n_1^L)\right\}\\
B_3^L &= B_2^L + \max\left\{0, (m_3^L - m_2^L) - (n_3^L - n_3^L)\right\},\\
B_4^L &= B_3^L + \max\left\{0, (m_4^L - m_3^L) - (n_4^L - n_3^L)\right\}\\
A_1^U &= \max\left\{0, n_1^U - m_1^U\right\}, A_2^U = \left|n_1^U - m_1^U\right|\\
&\quad + \max\left\{0, n_1^U - m_1^U\right\}\\
&\quad + \max\left\{0, (n_2^U - n_1^U) - (m_2^U - m_1^U)\right\}\\
A_3^U &= A_2^U + \max\left\{0, (n_3^U - n_2^U) - (m_3^U - m_3^U)\right\},\\
A_4^U &= A_3^U + \max\left\{0, (n_4^U - n_3^U) - (m_4^U - m_3^U)\right\}\\
&\quad + \min\left\{0, A_3^U + \max\left\{0, (n_4^U - n_3^U)\right.\right.\\
&\qquad \left.\left. -(m_4^U - m_3^U)\right\} - A_4^L\right\}\\
B_1^U &= \max\left\{0, m_1^U - n_1^U\right\},\\
B_2^U &= B_1^U + \left|n_1^U - m_1^U\right| + \max\left\{0, (m_2^U - m_1^U)\right.\\
&\quad \left. -(n_2^U - n_1^U)\right\}\\
B_3^U &= B_2^U + \max\left\{0, (m_3^U - m_2^U) - (n_3^U - n_3^U)\right\},\\
B_4^U &= B_3^U + \max\left\{0, (m_4^U - m_3^U) - (n_4^U - n_3^U)\right\}\\
&\quad + \min\left\{0, B_3^U + \max\left\{0, (m_4^U - m_3^U)\right.\right.\\
&\qquad \left.\left. -(n_4^U - n_3^U)\right\} - B_4^L\right\}
\end{aligned}
$$

Assume the interval-valued fuzzy transportation cost for one unit quantity of the commodity from the introduced dummy origin to all destinations and form all origins to the introduced dummy destination and as zero interval-valued trapezoidal fuzzy number, i.e., $\tilde{\tilde{0}} = \langle (0, 0, 0, 0; 1), (0, 0, 0, 0; 1) \rangle$, then go to Step 2.

Step 2 Formulate the balanced interval-valued trapezoidal fuzzy numbers transportation problem, obtained in Step 1, into the following interval-valued fuzzy linear programming problem:

$$\min \sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}$$

$$s.t. \sum_{j=1}^{q} \tilde{\tilde{x}}_{ij} = \tilde{\tilde{a}}_i,\ i = 1, 2, ..., p;\ p = morm + 1$$

$$\sum_{i=1}^{p} \tilde{\tilde{x}}_{ij} = \tilde{\tilde{b}}_j,\ j = 1, 2, ..., q;\ q = norn + 1$$
$$\tilde{\tilde{x}}_{ij} \in F^{+}_{IVTN}(w^L, w^U),\ i = 1, 2, ..., p,$$
$$j = 1, 2, ..., q. \quad (15)$$

where p and q are total numbers of origins and destinations, respectively.

Step 3 Now, our objective is to find $\left\{\tilde{\tilde{x}}_{ij}\right\}$ which satisfies the properties of Definition 5.1, i.e.,

$$\min d\left(\sum_{i=1}^{p}\sum_{j=1}^{q} \tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}, O_1\right) = \sum_{i=1}^{p}\sum_{j=1}^{q} d\left(\tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}, O_1\right)$$
$$s.t. \sum_{j=1}^{q} \tilde{\tilde{x}}_{ij} = \tilde{\tilde{a}}_i,\ i = 1, 2, ..., p,$$
$$\sum_{i=1}^{p} \tilde{\tilde{x}}_{ij} = \tilde{\tilde{b}}_j,\ j = 1, 2, ..., q,$$
$$\tilde{\tilde{x}}_{ij} \in F^{+}_{IVTN}(w^L, w^U),\ i = 1, 2, ..., p,$$
$$j = 1, 2, ..., q. \quad (16)$$

Step 4 The interval-valued trapezoidal fuzzy numbers linear programming problem (16) may be rewritten as follows:

$$\min\ d\left(\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}, O_1\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} d\left(\tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}, O_1\right)$$
$$s.t. \left(\sum_{j=1}^{n} x^L_{ij1}, \sum_{j=1}^{n} x^L_{ij2}, \sum_{j=1}^{n} x^L_{ij3}, , \sum_{j=1}^{n} x^L_{ij4}\right) = \left(a^L_{i1}, a^L_{i2}, a^L_{i3}, a^L_{i4}\right),\ i = 1, 2, ..., m,$$
$$\left(\sum_{j=1}^{n} x^U_{ij1}, \sum_{j=1}^{n} x^U_{ij2}, \sum_{j=1}^{n} x^U_{ij3}, , \sum_{j=1}^{n} x^U_{ij4}\right) = \left(a^U_{i1}, a^U_{i2}, a^U_{i3}, a^U_{i4}\right),\ i = 1, 2, ..., m,$$
$$\left(\sum_{i=1}^{m} x^L_{ij1}, \sum_{i=1}^{m} x^L_{ij2}, \sum_{i=1}^{m} x^L_{ij3}, , \sum_{i=1}^{m} x^L_{ij4}\right) = \left(b^L_{j1}, b^L_{j2}, b^L_{j3}, b^L_{j4}\right),\ j = 1, 2, ..., n,$$
$$\left(\sum_{i=1}^{m} x^U_{ij1}, \sum_{i=1}^{m} x^U_{ij2}, \sum_{i=1}^{m} x^U_{ij3}, , \sum_{i=1}^{m} x^U_{ij4}\right) = \left(b^U_{j1}, b^U_{j2}, b^U_{j3}, b^U_{j4}\right),\ j = 1, 2, ..., n,$$
$$\tilde{\tilde{x}}_{ij} \in F^{+}_{IVTN}(w^L, w^U),\ i = 1, 2, ..., p,$$
$$j = 1, 2, ..., q. \quad (17)$$

Step 5 The interval-valued fuzzy problem (17), obtained in Step 4, is converted into the following crisp linear programming problem:

$$\min\ d\left(\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}, O_1\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} d\left(\tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}, O_1\right)$$
$$s.t. \sum_{j=1}^{n} x^L_{ij1} = a^L_{i1}, \sum_{j=1}^{n} x^L_{ij2} = a^L_{i2}, \sum_{j=1}^{n} x^L_{ij3} = a^L_{i3},$$
$$\sum_{j=1}^{n} x^L_{ij4} = a^L_{i4},\ i = 1, 2, ..., m, \quad (18.1)$$
$$\sum_{j=1}^{n} x^U_{ij1} = a^U_{i1}, \sum_{j=1}^{n} x^U_{ij2} = a^U_{i2}, \sum_{j=1}^{n} x^U_{ij3} = a^L_{i3},$$
$$\sum_{j=1}^{n} x^U_{ij4} = a^U_{i4},\ i = 1, 2, ..., m, \quad (18.2)$$
$$\sum_{i=1}^{m} x^L_{ij1} = b^L_{j1}, \sum_{i=1}^{m} x^L_{ij2} = b^L_{j2}, \sum_{i=1}^{m} x^L_{ij3} = b^L_{j3},$$
$$\sum_{i=1}^{m} x^L_{ij4} = b^L_{j4},\ j = 1, 2, ..., n, \quad (18.3)$$
$$\sum_{i=1}^{m} x^U_{ij1} = b^U_{j1}, \sum_{i=1}^{m} x^U_{ij2} = b^U_{j2}, \sum_{i=1}^{m} x^U_{ij3} = b^U_{j3},$$
$$\sum_{i=1}^{m} x^U_{ij4} = b^U_{j4},\ j = 1, 2, ..., n, \quad (18.4)$$
$$x^U_{ij1} \geq 0, x^L_{ij1} - x^U_{ij1} \geq 0, x^L_{ij2} - x^L_{ij1} \geq 0,$$
$$x^L_{ij3} - x^L_{ij2} \geq 0, x^L_{ij4} - x^L_{ij3} \geq 0,\ i = 1, 2, ..., m,$$
$$j = 1, 2, ..., n, (18.5)$$
$$x^U_{ij2} - x^U_{ij1} \geq 0, x^U_{ij3} - x^U_{ij2} \geq 0, x^U_{ij4} - x^U_{ij3} \geq 0,$$
$$x^U_{ij4} - x^L_{ij4} \geq 0,\ i = 1, 2, ..., m,$$
$$j = 1, 2, ..., n. \quad (18.6)$$

(18)

Step 6 Find the optimal solution $x^L_{ij1}, x^L_{ij2}, x^L_{ij3}, x^L_{ij4}, x^U_{ij1}, x^U_{ij2}, x^U_{ij3}, x^U_{ij4}$ by solving the crisp linear programming problem, obtained in Step 5.

Step 7 Find the fuzzy optimal solution $\left\{\tilde{\tilde{x}}_{ij}\right\}$ by putting the values of optimal $x^L_{ij1}, x^L_{ij2}, x^L_{ij3}, x^L_{ij4}, x^U_{ij1}, x^U_{ij2}, x^U_{ij3}, x^U_{ij4}$ in $\tilde{\tilde{x}}_{ij} = \left\langle (x^L_{ij1}, x^L_{ij2}, x^L_{ij3}, x^L_{ij4}; w^L), (x^U_{ij1}, x^U_{ij2}, x^U_{ij3}, x^U_{ij4}; w^U) \right\rangle$.

Step 8 Find the total interval-valued trapezoidal fuzzy number transportation cost by putting the values of optimal $\tilde{\tilde{x}}_{ij}$ in $\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}$.

The following theorem proves that if the level (w^L, w^U)-interval-valued trapezoidal fuzzy numbers reduce to generalized trapezoidal fuzzy numbers, the solution of the proposed algorithm is same the solution obtained from the existing algorithm [32].

Theorem 5.1. *If the level (w^L, w^U)-interval-valued trapezoidal fuzzy numbers reduce to generalized trapezoidal fuzzy numbers, then the optimal solution of generalized interval-valued trapezoidal fuzzy numbers TP obtained based on signed distance ranking is the same with optimal solution of generalized trapezoidal fuzzy numbers TP derived by linear ranking function.*

Proof. Consider the generalized interval-valued trapezoidal fuzzy numbers TP given in (12). The fuzzy optimal solution of this problem according to the proposed algorithm and by the help of signed distance ranking can be obtained by solving the LP problem (18).

Now consider the following generalized trapezoidal fuzzy numbers TP:

$$
\begin{aligned}
&\min \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij} \\
&s.t. \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, \quad i = 1, 2, ..., m, \\
&\quad \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, \quad j = 1, 2, ..., n, \\
&\quad \tilde{x}_{ij} \in F^+_{TN}(w), \quad i = 1, 2, ..., m, \; j = 1, 2, ..., n.
\end{aligned} \tag{19}
$$

where $\tilde{a}_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4}; w) \in F^+_{TN}(w), i = 1, 2, ..., m$, $\tilde{b}_j = (b_{j1}, b_{j2}, b_{j3}, b_{j4}; w) \in F^+_{TN}(w), j = 1, 2, ..., n$,

$$
\begin{aligned}
\tilde{c}_{ij} &= (c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}; w) \in F^+_{TN}(w), \\
&\quad i = 1, 2, ..., m, \; j = 1, 2, ..., n,
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{x}_{ij} &= (x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}; w) \in F^+_{TN}(w), \\
&\quad i = 1, 2, ..., m, \; j = 1, 2, ..., n.
\end{aligned}
$$

The fuzzy optimal solution of generalized trapezoidal fuzzy numbers TP (19) according to the existing algorithm and by the help of linear ranking function can be obtained by solving the following LP problem.

$$
\begin{aligned}
&\min \sum_{i=1}^{m}\sum_{j=1}^{n} \Re(\tilde{c}_{ij} \otimes \tilde{x}_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \Re(c_{ij1}x_{ij1}, c_{ij2}x_{ij2}, \\
&\qquad c_{ij3}x_{ij3}, c_{ij4}x_{ij4}; w) \\
&s.t. \sum_{j=1}^{n} x_{ij1} = a_{i1}, \sum_{j=1}^{n} x_{ij2} = a_{i2}, \sum_{j=1}^{n} x_{ij3} = a_{i3}, \\
&\quad \sum_{j=1}^{n} x_{ij4} = a_{i4}, i = 1, 2, ..., m, \qquad (20.1) \\
&\quad \sum_{i=1}^{m} x_{ij1} = b_{j1}, \sum_{i=1}^{m} x_{ij2} = b_{j2}, \sum_{i=1}^{m} x_{ij3} = b_{j3}, \\
&\quad \sum_{i=1}^{m} x_{ij4} = b_{j4}, j = 1, 2, ..., n, \qquad (20.2) \\
&\quad sx_{ij1} \geq 0, x_{ij2} - x_{ij1} \geq 0, x_{ij3} - x_{ij2} \geq 0, \\
&\quad x_{ij4} - x_{ij3} \geq 0, i = 1, 2, ..., m, \\
&\quad j = 1, 2, ..., n. \qquad (20.3)
\end{aligned} \tag{20}
$$

where $\Re(c_{ij1}x_{ij1}, c_{ij2}x_{ij2}, c_{ij3}x_{ij3}, c_{ij4}x_{ij4}; w) = \frac{w}{4}(c_{ij1}x_{ij1} + c_{ij2}x_{ij2} + c_{ij3}x_{ij3} + c_{ij4}x_{ij4})$.

Now, we are at a position to demonstrate that if the level (w^L, w^U)-interval-valued trapezoidal fuzzy numbers reduce to generalized trapezoidal fuzzy numbers then the optimal solution of problem (18) and problem (19) is the same. For doing this, it is sufficient to show that the objective function and feasible space of these problems are the same in this case.

Suppose that all the parameters represented by (w^L, w^U)-interval-valued trapezoidal fuzzy numbers given in problem (12) are reduced to generalized trapezoidal fuzzy numbers, i.e.

$$
\begin{aligned}
a^L_{i1} &= a^U_{i1} = a_{i1}, a^L_{i2} = a^U_{i2} = a_{i2}, a^L_{i3} = a^U_{i3} = a_{i3}, \\
a^L_{i4} &= a^U_{i4} = a_{i4}, w^L = w^U = w, i = 1, 2, ..., m, \\
b^L_{j1} &= b^U_{j1} = b_{j1}, b^L_{j2} = b^U_{j2} = b_{j2}, b^L_{j3} = b^U_{j3} = b_{j3}, \\
b^L_{j4} &= b^U_{j4} = b_{j4}, j = 1, 2, ..., n, \\
c^L_{ij1} &= c^U_{ij1} = c_{ij1}, c^L_{ij2} = c^U_{ij2} = c_{ij2}, c^L_{ij3} = c^U_{ij3} = c_{ij3}, \\
c^L_{ij4} &= c^U_{ij4} = c_{ij4}, i = 1, 2, ..., m, \\
x^L_{ij1} &= x^U_{ij1} = x_{ij1}, x^L_{ij2} = x^U_{ij2} = x_{ij2}, x^L_{ij3} = x^U_{ij3} = x_{ij3}, \\
x^L_{ij4} &= x^U_{ij4} = x_{ij4}, i = 1, 2, ..., m, j = 1, 2, ..., n.
\end{aligned} \tag{21}
$$

By substituting (21) in constraints of problem (18), constraints (18.1) and (18.2) are reduced to constraints (20.1), constraints (18.3) and (18.4) are reduced to constraints

(20.2) and constraints (18.5) and (18.6) are reduced to constraint (20.3). These confirm that the feasible space of problems (18) and (20) are the same. In a similar way, on substituting (21) into the objective function of problem (18), we get

$$\sum_{i=1}^{m}\sum_{j=1}^{n} d\left(\tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}, O_1\right)$$
$$= \sum_{i=1}^{m}\sum_{j=1}^{n} d\left(\left\langle (c_{ij1}^{L}x_{ij1}^{L}, c_{ij2}^{L}x_{ij2}^{L}, c_{ij3}^{L}x_{ij3}^{L}, c_{ij4}^{L}c_{ij4}^{L}; w^{L}),\right.\right.$$
$$\left.\left.(x_{ij1}^{L}x_{ij1}^{L}, c_{ij2}^{L}x_{ij2}^{L}, c_{ij3}^{L}x_{ij3}^{L}, c_{ij4}^{L}c_{ij4}^{L}; w^{U})\right\rangle, O_1\right).$$

Since the lower and upper trapezoidal fuzzy numbers of the interval-valued trapezoidal fuzzy number $\left(\tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}\right)$ are equal, i.e. $\left(\tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}\right)^{L} = \left(\tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}\right)^{U}$, regarding to relation (5) we conclude that

$$\sum_{i=1}^{m}\sum_{j=1}^{n} d\left(\left\langle (c_{ij1}^{L}x_{ij1}^{L}, c_{ij2}^{L}x_{ij2}^{L}, c_{ij3}^{L}x_{ij3}^{L}, c_{ij4}^{L}c_{ij4}^{L}; w^{L}),\right.\right.$$
$$\left.\left.(c_{ij1}^{U}x_{ij1}^{U}, c_{ij2}^{U}x_{ij2}^{U}, c_{ij3}^{U}x_{ij3}^{U}, c_{ij4}^{U}c_{ij4}^{U}; w^{U})\right\rangle, O_1\right)$$
$$= \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{w}{4}(c_{ij1}x_{ij1} + c_{ij2}x_{ij2} + c_{ij3}x_{ij3} + c_{ij4}x_{ij4})$$

This means that $\sum_{i=1}^{m}\sum_{j=1}^{n} d\left(\tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}, O_1\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} \Re\left(\tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij}\right)$. This ensures that the objective function of problem (18) and (20) is the same in the case of the interval-valued trapezoidal fuzzy numbers are reduced to generalized trapezoidal fuzzy numbers. Thus, the proof is complete. □

In what follows the computational complexity of the proposed algorithm has been discussed. Space and time requirements are generally two criteria used to determine the efficiency of each algorithm. Since today's computers have vast amounts of memory, difference in space requirements of most algorithms is insignificant. With this in mind, here will mainly be concerned with time requirements.

A more reliable approach to the analysis of the efficiency of algorithms is to mathematically measure the speed of the algorithm in terms of "time units." Different types of operations may require different time units. Now, consider an algorithm that finitely solves a linear programming problem. In order to analyze its efficiency or computational complexity it is required to determine an upper bound on the effort required to solve any instance of this problem. This effort may be measured in terms of the number of elementary operations such as additions, multiplications, and comparisons that are required to solve the problem as a function of the size of the problem.

It is worthwhile to note that our proposed algorithm to find the fuzzy optimal solution of the generalized interval-valued trapezoidal fuzzy TP (12) is reduced to solve the crisp LP problem (18). Owing to this, our primary measurement of the computational complexity of the proposed algorithm will be a mathematical analysis of the number of elementary operations such as additions and multiplications and comparisons required to solve the LP problem (18).

Consider the LP problem:

$$\begin{aligned} &\min \quad z = cx \\ &s.t.\ Ax = b, \\ &x \geq 0. \end{aligned}$$

Where A is $m \times n$. The size of an instance of this problem is represented by the entities (m, n, L), where L is the number of binary bits required to record all the data of the problem and is known as the input length of an instance of the problem [33]. It has been shown that this LP problem can be solved by Khachian's ellipsoid algorithm and the Karmarkar's projective algorithm within an effort of $O\left((m+n)^6 L\right)$ and $O(n^4 L)$, respectively [33].

Now consider the generalized interval-valued trapezoidal fuzzy TP (12) with $(m + n)$ fuzzy constraints and mn generalized interval-valued trapezoidal fuzzy decision variables. To find the computational complexity of this problem it is required to compute that of the LP problem (18). Since the LP problem (18) has $(8m + 8n + 8mn)$ constraints and $8mn$ decision variables, thus this problem can be solved by Khachian's ellipsoid algorithm and the Karmarkar's projective algorithm within an effort of $O\left(((8m + 8n + 8mn) + (8mn))^6\, L\right)$ and $O\left((8mn)^4 L\right)$, respectively.

6. Application examples

In this section, our proposed method is illustrated by the help of two application examples.

Example 6.1. *A company has two origins O_1 and O_2, and three destinations D_1, D_2 and D_3; the approximate transportation cost for unit quantity of the commodity from i^{th} source to j^{th} destination, the approximate supply of the commodity at two origins and the approximate demand of the commodity at three destinations are represented by interval-valued trapezoidal fuzzy numbers and shown in table* 1.

The company wants to determine the approximate quantity of the commodity that should be transported from each origin to each destination so that the total approximate transportation cost is minimum.

The interval-valued fuzzy optimal solution of this problem based on the proposed approach is obtained as follows:

Table 1. Data of Example 6.1.

Origin	Destination D1	Destination D2	Destination D3	Supply
O1	$\left\langle(10, 20, 30, 40; \frac{2}{3}), (5, 15, 35, 45; 1)\right\rangle$	$\left\langle(50, 60, 70, 90; \frac{2}{3}), (45, 55, 75, 95, 1)\right\rangle$	$\left\langle(80, 90, 110, 120; \frac{2}{3}), (75, 85, 115, 125; 1)\right\rangle$	$\left\langle(70, 90, 90, 100; \frac{2}{3}), (65, 85, 95, 105; 1)\right\rangle$
O2	$\left\langle(60, 70, 80, 90; \frac{2}{3}), (55, 65, 85, 95; 1)\right\rangle$	$\left\langle(70, 80, 100, 120; \frac{2}{3}), (65, 75, 105, 125; 1)\right\rangle$	$\left\langle(20, 30, 50, 60; \frac{2}{3}), (15, 25, 55, 65; 1)\right\rangle$	$\left\langle(40, 60, 70, 80; \frac{2}{3}), (35, 55, 75, 85; 1)\right\rangle$
Demand	$\left\langle(30, 40, 50, 70; \frac{2}{3}), (25, 35, 55, 75; 1)\right\rangle$	$\left\langle(20, 30, 40, 50; \frac{2}{3}), (15, 25, 45, 55; 1)\right\rangle$	$\left\langle(40, 50, 50, 80; \frac{2}{3}), (35, 45, 55, 85; 1)\right\rangle$	

Step 1: The total interval-valued fuzzy supply and total interval-valued fuzzy demand are $\langle(110, 150, 160, 180; \frac{2}{3}), (100, 140, 170, 190; 1)\rangle$ and $\langle(90, 120, 140, 200; \frac{2}{3}), (75, 105, 155, 215; 1)\rangle$, respectively. Since total interval-valued fuzzy supply and total interval-valued fuzzy demand are not equal, so this is an unbalanced interval-valued fuzzy transportation problem. Now, as described in the proposed method (Case (c) of the Step 1 of the proposed method), unbalanced interval-valued fuzzy transportation problem can be converted into a balanced interval-valued fuzzy transportation problem, by introducing a dummy origin O_3 with interval-valued fuzzy supply $\left\langle(25, 25, 35, 75; \frac{2}{3}), (0, 25, 45, 85; 1)\right\rangle$ and a dummy destination D_4 with interval-valued fuzzy demand $\left\langle(45, 55, 55, 55; \frac{2}{3}), (25, 60, 60, 60; 1)\right\rangle$. Assume the interval-valued fuzzy transportation cost for one unit quantity of the commodity from the introduced dummy origin O_3 to all destinations and from all origins to the introduced dummy destination D_4 are as zero interval-valued trapezoidal fuzzy number, i.e., $\tilde{\tilde{c}}_{14} = \tilde{\tilde{c}}_{24} = \tilde{\tilde{c}}_{31} = \tilde{\tilde{c}}_{32} = \tilde{\tilde{c}}_{33} = \tilde{\tilde{c}}_{34} = \tilde{\tilde{0}}$.

Step 2: The obtained balanced interval-valued fuzzy transportation problem may be formulated into the following interval-valued fuzzy linear programming problem:

$$\min\left\langle\left(10, 20, 30, 40; \frac{2}{3}\right), (5, 15, 35, 45; 1)\right\rangle \otimes \tilde{\tilde{x}}_{11}$$
$$+\left\langle\left(50, 60, 70, 90; \frac{2}{3}\right), (45, 55, 75, 95, 1)\right\rangle \otimes \tilde{\tilde{x}}_{12}$$
$$+\left\langle\left(80, 90, 110, 120; \frac{2}{3}\right), (75, 85, 115, 125; 1)\right\rangle \otimes \tilde{\tilde{x}}_{13}$$
$$+\tilde{\tilde{0}} \otimes \tilde{\tilde{x}}_{14} + \left\langle\left(60, 70, 80, 90; \frac{2}{3}\right) (55, 65, 85, 95; 1)\right\rangle,$$
$$\otimes \tilde{\tilde{x}}_{21} + \left\langle\left(70, 80, 100, 120; \frac{2}{3}\right), (65, 75, 105, 125; 1)\right\rangle$$
$$\otimes \tilde{\tilde{x}}_{22} + \left\langle\left(20, 30, 50, 60; \frac{2}{3}\right), (15, 25, 55, 65; 1)\right\rangle$$
$$\otimes \tilde{\tilde{x}}_{23} + \tilde{\tilde{0}} \otimes \tilde{\tilde{x}}_{24} + \tilde{\tilde{0}} \otimes \tilde{\tilde{x}}_{31} + \tilde{\tilde{0}} \otimes \tilde{\tilde{x}}_{32} + \tilde{\tilde{0}} \otimes \tilde{\tilde{x}}_{33}$$
$$+\tilde{\tilde{0}} \otimes \tilde{\tilde{x}}_{34}$$

$$s.t.\ \tilde{\tilde{x}}_{11} \oplus \tilde{\tilde{x}}_{12} \oplus \tilde{\tilde{x}}_{13} \oplus \tilde{\tilde{x}}_{14}$$
$$= \left\langle\left(70, 90, 90, 100; \frac{2}{3}\right), (65, 85, 95, 105; 1)\right\rangle,$$
$$\tilde{\tilde{x}}_{21} \oplus \tilde{\tilde{x}}_{22} \oplus \tilde{\tilde{x}}_{23} \oplus \tilde{\tilde{x}}_{24}$$
$$= \left\langle\left(40, 60, 70, 80; \frac{2}{3}\right), (35, 55, 75, 85; 1)\right\rangle,$$

$$\tilde{\tilde{x}}_{31} \oplus \tilde{\tilde{x}}_{32} \oplus \tilde{\tilde{x}}_{33} \oplus \tilde{\tilde{x}}_{34} = \left\langle \left(25, 25, 35, 75; \frac{2}{3}\right), (0, 25, 45, 85; 1) \right\rangle,$$
$$\tilde{\tilde{x}}_{11} \oplus \tilde{\tilde{x}}_{21} \oplus \tilde{\tilde{x}}_{31} = \left\langle \left(30, 40, 50, 70; \frac{2}{3}\right), (25, 35, 55, 75; 1) \right\rangle,$$
$$\tilde{\tilde{x}}_{12} \oplus \tilde{\tilde{x}}_{22} \oplus \tilde{\tilde{x}}_{32} = \left\langle \left(20, 30, 40, 50; \frac{2}{3}\right), (15, 25, 45, 55; 1) \right\rangle,$$
$$\tilde{\tilde{x}}_{13} \oplus \tilde{\tilde{x}}_{23} \oplus \tilde{\tilde{x}}_{33} = \left\langle \left(40, 50, 50, 80; \frac{2}{3}\right), (35, 45, 55, 85; 1) \right\rangle,$$
$$\tilde{\tilde{x}}_{14} \oplus \tilde{\tilde{x}}_{24} \oplus \tilde{\tilde{x}}_{34} = \left\langle \left(45, 55, 55, 55; \frac{2}{3}\right), (25, 60, 60, 60; 1) \right\rangle,$$
$$\tilde{\tilde{x}}_{11}, \tilde{\tilde{x}}_{12}, \tilde{\tilde{x}}_{13}, \tilde{\tilde{x}}_{14}, \tilde{\tilde{x}}_{21}, \tilde{\tilde{x}}_{22}, \tilde{\tilde{x}}_{23}, \tilde{\tilde{x}}_{24}, \tilde{\tilde{x}}_{31}, \tilde{\tilde{x}}_{32}, \tilde{\tilde{x}}_{33}, \tilde{\tilde{x}}_{34} \in F^{+}_{IVTN}(w^L, w^U). \tag{22}$$

Step 3: Using Step 3 to Step 5 of the proposed method, the formulated interval-valued fuzzy linear programming problem (22) is converted into the following crisp linear programming problem:

$$\min \frac{1}{8}\Big[10x^L_{11,1} + 20x^L_{11,2} + 30x^L_{11,3} + 40x^L_{11,4} + 10x^U_{11,1} + 60x^U_{11,2} + 140x^U_{11,3} + 90x^U_{11,4}\Big]$$
$$+\frac{1}{8}\Big[50x^L_{12,1} + 60x^L_{12,2} + 70x^L_{12,3} + 90x^L_{12,4} + 90x^U_{12,1} + 220x^U_{12,2} + 300x^U_{12,3} + 190x^U_{12,4}\Big]$$
$$+\frac{1}{8}\Big[80x^L_{13,1} + 90x^L_{13,2} + 110x^L_{13,3} + 120x^L_{13,4} + 150x^U_{13,1} + 340x^U_{13,2} + 460x^U_{13,3} + 250x^U_{13,4}\Big]$$
$$+\frac{1}{8}\Big[60x^L_{21,1} + 70x^L_{21,2} + 80x^L_{21,3} + 90x^L_{21,4} + 110x^U_{21,1} + 260x^U_{21,2} + 340x^U_{21,3} + 190x^U_{21,4}\Big]$$
$$+\frac{1}{8}\Big[70x^L_{22,1} + 80x^L_{22,2} + 100x^L_{22,3} + 120x^L_{22,4} + 130x^U_{22,1} + 300x^U_{22,2} + 420x^U_{22,3} + 500x^U_{22,4}\Big]$$
$$+\frac{1}{8}\Big[20x^L_{23,1} + 30x^L_{23,2} + 50x^L_{23,3} + 60x^L_{23,4} + 30x^U_{23,1} + 100x^U_{23,2} + 220x^U_{23,3} + 340x^U_{23,4}\Big]$$

$$s.t.\ x^L_{11,1} + x^L_{12,1} + x^L_{13,1} + x^L_{14,1} = 70,$$
$$x^L_{11,2} + x^L_{12,2} + x^L_{13,2} + x^L_{14,2} = 90,$$
$$x^L_{11,3} + x^L_{12,3} + x^L_{13,3} + x^L_{14,3} = 90,$$
$$x^L_{11,4} + x^L_{12,4} + x^L_{13,4} + x^L_{14,4} = 100,$$
$$x^U_{11,1} + x^U_{12,1} + x^U_{13,1} + x^U_{14,1} = 65,$$
$$x^U_{11,2} + x^U_{12,2} + x^U_{13,2} + x^U_{14,2} = 85,$$
$$x^U_{11,3} + x^U_{12,3} + x^U_{13,3} + x^U_{14,3} = 95,$$
$$x^U_{11,4} + x^U_{12,4} + x^U_{13,4} + x^U_{14,4} = 105,$$
$$x^L_{21,1} + x^L_{22,1} + x^L_{23,1} + x^L_{24,1} = 40,$$
$$x^L_{21,2} + x^L_{22,2} + x^L_{23,2} + x^L_{24,2} = 60,$$
$$x^L_{21,3} + x^L_{22,3} + x^L_{23,3} + x^L_{24,3} = 70,$$
$$x^L_{21,4} + x^L_{22,4} + x^L_{23,4} + x^L_{24,4} = 80,$$
$$x^U_{21,1} + x^U_{22,1} + x^U_{23,1} + x^U_{24,1} = 35,$$
$$x^U_{21,2} + x^U_{22,2} + x^U_{23,2} + x^U_{24,2} = 55,$$
$$x^U_{21,3} + x^U_{22,3} + x^U_{23,3} + x^U_{24,3} = 75,$$
$$x^U_{21,4} + x^U_{22,4} + x^U_{23,4} + x^U_{24,4} = 85,$$
$$x^L_{31,1} + x^L_{32,1} + x^L_{33,1} + x^L_{34,1} = 25,$$
$$x^L_{31,2} + x^L_{32,2} + x^L_{33,2} + x^L_{34,2} = 25,$$
$$x^L_{31,3} + x^L_{32,3} + x^L_{33,3} + x^L_{34,3} = 35,$$
$$x^L_{31,4} + x^L_{32,4} + x^L_{33,4} + x^L_{34,4} = 75,$$
$$x^U_{31,1} + x^U_{32,1} + x^U_{33,1} + x^U_{34,1} = 0,$$
$$x^U_{31,2} + x^U_{32,2} + x^U_{33,2} + x^U_{34,2} = 25,$$
$$x^U_{31,3} + x^U_{32,3} + x^U_{33,3} + x^U_{34,3} = 45,$$
$$x^U_{31,4} + x^U_{32,4} + x^U_{33,4} + x^U_{34,4} = 85,$$
$$x^L_{11,1} + x^L_{21,1} + x^L_{31,1} = 30,$$
$$x^L_{11,2} + x^L_{21,2} + x^L_{31,2} = 40,$$
$$x^L_{11,3} + x^L_{21,3} + x^L_{31,3} = 50,$$
$$x^L_{11,4} + x^L_{21,4} + x^L_{31,4} = 70,$$
$$x^U_{11,1} + x^U_{21,1} + x^U_{31,1} = 25,$$
$$x^U_{11,2} + x^U_{21,2} + x^U_{31,2} = 35,$$
$$x^U_{11,3} + x^U_{21,3} + x^U_{31,3} = 55,$$
$$x^U_{11,4} + x^U_{21,4} + x^U_{31,4} = 75,$$
$$x^L_{12,1} + x^L_{22,1} + x^L_{32,1} = 20,$$
$$x^L_{12,2} + x^L_{22,2} + x^L_{32,2} = 30,$$
$$x^L_{12,3} + x^L_{22,3} + x^L_{32,3} = 40,$$
$$x^L_{12,4} + x^L_{22,4} + x^L_{32,4} = 50,$$
$$x^U_{12,1} + x^U_{22,1} + x^U_{32,1} = 15,$$
$$x^U_{12,2} + x^U_{22,2} + x^U_{32,2} = 25,$$
$$x^U_{12,3} + x^U_{22,3} + x^U_{32,3} = 45,$$
$$x^U_{12,4} + x^U_{22,4} + x^U_{32,4} = 55,$$
$$x^L_{13,1} + x^L_{23,1} + x^L_{33,1} = 40,$$
$$x^L_{13,2} + x^L_{23,2} + x^L_{33,2} = 50,$$

$$x^L_{13,3}+x^L_{23,3}+x^L_{33,3}=50,$$
$$x^L_{13,4}+x^L_{23,4}+x^L_{33,4}=80,$$
$$x^U_{13,1}+x^U_{23,1}+x^U_{33,1}=35,$$
$$x^U_{13,2}+x^U_{23,2}+x^U_{33,2}=45,$$
$$x^U_{13,3}+x^U_{23,3}+x^U_{33,3}=55,$$
$$x^U_{13,4}+x^U_{23,4}+x^U_{33,4}=85,$$
$$x^L_{14,1}+x^L_{24,1}+x^L_{34,1}=45,$$
$$x^L_{14,2}+x^L_{24,2}+x^L_{34,2}=55,$$
$$x^L_{14,3}+x^L_{24,3}+x^L_{34,3}=55,$$
$$x^L_{14,4}+x^L_{24,4}+x^L_{34,4}=55,$$
$$x^U_{14,1}+x^U_{24,1}+x^U_{34,1}=25,$$
$$x^U_{14,2}+x^U_{24,2}+x^U_{34,2}=60,$$
$$x^U_{14,3}+x^U_{24,3}+x^U_{34,3}=60,$$
$$x^U_{14,4}+x^U_{24,4}+x^U_{34,4}=60,$$
$$x^U_{ij,1}\geq 0,\ x^L_{ij,1}-x^U_{ij,1}\geq 0,\ x^L_{ij,2}-x^L_{ij,1}\geq 0,\ x^L_{ij,3}-x^L_{ij,2}\geq 0,$$
$$x^L_{ij,4}-x^L_{ij,3}\geq 0,\ i=1,2,3,\ j=1,2,3,4,$$
$$x^U_{ij,2}-x^U_{ij,1}\geq 0,\ x^U_{ij,3}-x^U_{ij,2}\geq 0,\ x^U_{ij,4}-x^U_{ij,3}\geq 0,\ x^U_{ij,4}-x^L_{ij,4}\geq 0,\ i=1,2,3,\ j=1,2,3,4. \tag{23}$$

Step 4: The optimal solution of the crisp linear programming problem (23), obtained in Step 3, is as follows:

$$x^L_{11,1}=25,\ x^L_{11,2}=35,\ x^L_{11,3}=35,\ x^L_{11,4}=45,\ x^U_{11,1}=25,\ x^U_{11,2}=35,\ x^U_{11,3}=45,\ x^U_{11,4}=50,$$
$$x^L_{12,1}=15,\ x^L_{12,2}=25,\ x^L_{12,3}=25,\ x^L_{12,4}=25,\ x^U_{12,1}=15,\ x^U_{12,2}=20,\ x^U_{12,3}=20,\ x^U_{12,4}=25,$$
$$x^L_{13,1}=0,\ x^L_{13,2}=0,\ x^L_{13,3}=0,\ x^L_{13,4}=0,\ x^U_{13,1}=0,\ x^U_{13,2}=0,\ x^U_{13,3}=0,\ x^U_{13,4}=0,$$
$$x^L_{21,1}=0,\ x^L_{21,2}=0,\ x^L_{21,3}=10,\ x^L_{21,4}=20,\ x^U_{21,1}=0,\ x^U_{21,2}=0,\ x^U_{21,3}=10,\ x^U_{21,4}=20,$$
$$x^L_{22,1}=0,\ x^L_{22,2}=0,\ x^L_{22,3}=0,\ x^L_{22,4}=0,\ x^U_{22,1}=0,\ x^U_{22,2}=0,\ x^U_{22,3}=0,\ x^U_{22,4}=0,$$
$$x^L_{23,1}=35,\ x^L_{23,2}=45,\ x^L_{23,3}=45,\ x^L_{23,4}=45,\ x^U_{23,1}=35,\ x^U_{23,2}=35,\ x^U_{23,3}=45,\ x^U_{23,4}=45,$$
$$x^L_{24,1}=5,\ x^L_{24,2}=15,\ x^L_{24,3}=15,\ x^L_{24,4}=15,\ x^U_{24,1}=0,\ x^U_{24,2}=20,\ x^U_{24,3}=20,\ x^U_{24,4}=20,$$
$$x^L_{31,1}=5,\ x^L_{31,2}=5,\ x^L_{31,3}=5,\ x^L_{31,4}=5,\ x^U_{31,1}=0,\ x^U_{31,2}=0,\ x^U_{31,3}=0,\ x^U_{31,4}=5,$$
$$x^L_{32,1}=5,\ x^L_{32,2}=5,\ x^L_{32,3}=15,\ x^L_{32,4}=25,\ x^U_{32,1}=0,\ x^U_{32,2}=5,\ x^U_{32,3}=25,\ x^U_{32,4}=30,$$
$$x^L_{33,1}=5,\ x^L_{33,2}=5,\ x^L_{33,3}=5,\ x^L_{33,4}=35,\ x^U_{33,1}=0,\ x^U_{33,2}=10,\ x^U_{33,3}=10,\ x^U_{33,4}=40,$$
$$x^L_{34,1}=10,\ x^L_{34,2}=10,\ x^L_{34,3}=10,\ x^L_{34,4}=10,\ x^U_{34,1}=0,\ x^U_{34,2}=10,\ x^U_{34,3}=10,\ x^U_{34,4}=10.$$

Step 5: Putting the values of $x^L_{ij1}, x^L_{ij2}, x^L_{ij3}, x^L_{ij4}, x^U_{ij1}, x^U_{ij2}, x^U_{ij3}, x^U_{ij4}$ in $\tilde{\tilde{x}}_{ij}=\left\langle (x^L_{ij1}, x^L_{ij2}, x^L_{ij3}, x^L_{ij4}; w^L), (x^U_{ij1}, x^U_{ij2}, x^U_{ij3}, x^U_{ij4}; w^U)\right\rangle$, the interval-valued fuzzy optimal solution is obtained as follows:

$$\tilde{\tilde{x}}_{11}=\left\langle\left(25,35,35,45;\frac{2}{3}\right),(25,35,45,50;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{12}=\left\langle\left(15,25,25,25;\frac{2}{3}\right),(15,20,20,25;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{13}=\left\langle\left(0,0,0,0;\frac{2}{3}\right),(0,0,0,0;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{14}=\left\langle\left(30,30,30,30;\frac{2}{3}\right),(25,30,30,30;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{21}=\left\langle\left(0,0,10,20;\frac{2}{3}\right),(0,0,10,20;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{22}=\left\langle\left(0,0,0,0;\frac{2}{3}\right),(0,0,0,0;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{23}=\left\langle\left(35,45,45,45;\frac{2}{3}\right),(35,35,45,45;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{24}=\left\langle\left(5,15,15,15;\frac{2}{3}\right),(0,20,20,20;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{31}=\left\langle\left(5,5,5,5;\frac{2}{3}\right),(0,0,0,5;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{32}=\left\langle\left(5,5,15,25;\frac{2}{3}\right),(0,5,25,30;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{33}=\left\langle\left(5,5,5,35;\frac{2}{3}\right),(0,10,10,40;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{34}=\left\langle\left(10,10,10,10;\frac{2}{3}\right),(0,10,10,10;1)\right\rangle. \tag{24}$$

Step 8 The total interval-valued trapezoidal fuzzy number transportation cost is achieved by putting the values of optimal $\tilde{\tilde{x}}_{ij}$ given in Eq. (24) in $\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{\tilde{c}}_{ij}\otimes\tilde{\tilde{x}}_{ij}$ as follows:

$$\sum_{i=1}^{3}\sum_{j=1}^{4}\tilde{\tilde{c}}_{ij}\otimes\tilde{\tilde{x}}_{ij}=\left\langle\left(1700,3550,5850,8950;\frac{2}{3}\right),(1325,3350,6400,9450;1)\right\rangle. \tag{25}$$

Example 6.2 (Adopted from Kumar & Kaur [32]*). The data is collected from a trader which supplies a product (TMT), which is made from raw material INGOT and BILLET,* *to different centers after taking the product from different plants. The trader supplies the product from three plants, Fortune Metals (Mandi Gobindgarh), Kamdhenu Saria (Bhiwadi) and Goel Group (Raipur) to four different centers Ludhiana, Delhi, Himachal Pradesh and Leh Ladakh. On the basis of the perception of the trader the approximate transportation cost per ton (rupees in thousands), approximate availability of the product (in tons) at different plants and the approximate demand of the product (in tons) at different centers are represented by trapezoidal fuzzy numbers. In this regards, these approximate parameters can be represented as the level* $(1, 1)$*-interval-valued trapezoidal fuzzy numbers and shown in table 2. For convenience, we represent* $\langle (a_1, a_2, a_3, a_4; w), (a_1, a_2, a_3, a_4; w) \rangle$ *by* $\left\langle \overline{(a_1, a_2, a_3, a_4; w)} \right\rangle$.

The trader wants to determine the approximate quantity of the product that should be transported from each plant to each center so that the total approximate transportation cost is minimized. This problem can be formulated into the following interval-valued trapezoidal fuzzy numbers transportation problem:

$$\begin{aligned}
&\min \left\langle \overline{(19, 20, 21, 22; 1)} \right\rangle \otimes \tilde{\tilde{x}}_{11} + \left\langle \overline{(59, 62, 63, 65; 1)} \right\rangle \otimes \tilde{\tilde{x}}_{12} \\
&\quad + \left\langle \overline{(90, 95, 97, 99; 1)} \right\rangle \otimes \tilde{\tilde{x}}_{13} + \left\langle \overline{(150, 160, 165, 170; 1)} \right\rangle \\
&\quad \otimes \tilde{\tilde{x}}_{14} + \left\langle \overline{(97, 99, 103, 105; 1)} \right\rangle \otimes \tilde{\tilde{x}}_{21} \\
&\quad + \left\langle \overline{(15, 17, 19, 21; 1)} \right\rangle \\
&\quad \otimes \tilde{\tilde{x}}_{22} + \left\langle \overline{(110, 112, 115, 119; 1)} \right\rangle \otimes \tilde{\tilde{x}}_{23} \\
&\quad + \left\langle \overline{(190, 210, 220, 240; 1)} \right\rangle \otimes \tilde{\tilde{x}}_{24} \\
&\quad + \left\langle \overline{(260, 262, 264, 270; 1)} \right\rangle \otimes \tilde{\tilde{x}}_{31} \\
&\quad + \left\langle \overline{(240, 247, 249, 255; 1)} \right\rangle \otimes \tilde{\tilde{x}}_{32} \\
&\quad + \left\langle \overline{(272, 274, 279, 290; 1)} \right\rangle \otimes \tilde{\tilde{x}}_{33} \\
&\quad + \left\langle \overline{(320, 326, 332, 340; 1)} \right\rangle \otimes \tilde{\tilde{x}}_{34} \\
&\quad s.t.\ \tilde{\tilde{x}}_{11} \oplus \tilde{\tilde{x}}_{12} \oplus \tilde{\tilde{x}}_{13} \oplus \tilde{\tilde{x}}_{14} = \left\langle \overline{(3500, 3555, 3580, 4000; 1)} \right\rangle, \\
&\quad \tilde{x}_{21} \oplus \tilde{x}_{22} \oplus \tilde{x}_{23} \oplus \tilde{x}_{24} = \left\langle \overline{(3125, 3175, 3190, 3200; 1)} \right\rangle, \\
&\quad \tilde{x}_{31} \oplus \tilde{x}_{32} \oplus \tilde{x}_{33} \oplus \tilde{x}_{34} = \left\langle \overline{(2475, 2995, 3275, 3400; 1)} \right\rangle, \\
&\quad \tilde{x}_{11} \oplus \tilde{x}_{21} \oplus \tilde{x}_{31} = \left\langle \overline{(2050, 2500, 2700, 3050; 1)} \right\rangle, \\
&\quad \tilde{x}_{12} \oplus \tilde{x}_{22} \oplus \tilde{x}_{32} = \left\langle \overline{\left(40, 50, 50, 80; \frac{2}{3}\right), (3000, 3050, 3100, 3200; 1)} \right\rangle, \\
&\quad \tilde{x}_{13} \oplus \tilde{x}_{23} \oplus \tilde{x}_{33} = \left\langle \overline{(2100, 2150, 2190, 2250; 1)} \right\rangle, \\
&\quad \tilde{x}_{14} \oplus \tilde{x}_{24} \oplus \tilde{x}_{34} = \left\langle \overline{(1950, 2025, 2055, 2100; 1)} \right\rangle, \\
&\quad \tilde{\tilde{x}}_{11}, \tilde{\tilde{x}}_{12}, \tilde{\tilde{x}}_{13}, \tilde{\tilde{x}}_{14}, \tilde{\tilde{x}}_{21}, \tilde{\tilde{x}}_{22}, \tilde{\tilde{x}}_{23}, \tilde{\tilde{x}}_{24}, \tilde{\tilde{x}}_{31}, \tilde{\tilde{x}}_{32}, \tilde{\tilde{x}}_{33}, \tilde{\tilde{x}}_{34} \\
&\quad \in F^{+}_{IVTN}(w^L, w^U).
\end{aligned} \tag{26}$$

Since the total fuzzy supply= $\left\langle \overline{(9110, 9725, 10045, 10600; 1)} \right\rangle$ = total fuzzy demands and so this problem is a balanced interval-valued trapezoidal fuzzy numbers transportation problem.

Table 2. Data of Example 6.2.

Source	Destination				Supply (in tons)
	Ludhiana	Delhi	Kullu	Leh Ladakh	
Gobindgarh	$\left\langle\overline{(19, 20, 21, 22; 1)}\right\rangle$	$\left\langle\overline{(59, 62, 63, 65; 1)}\right\rangle$	$\left\langle\overline{(90, 95, 97, 99; 1)}\right\rangle$	$\left\langle\overline{(150, 160, 165, 170; 1)}\right\rangle$	$\left\langle\overline{(3500, 3555, 3580, 4000; 1)}\right\rangle$
Bhiwadi	$\left\langle\overline{(97, 99, 103, 105; 1)}\right\rangle$	$\left\langle\overline{(15, 17, 19, 21; 1)}\right\rangle$	$\left\langle\overline{(110, 112, 115, 119; 1)}\right\rangle$	$\left\langle\overline{(190, 210, 220, 240; 1)}\right\rangle$	$\left\langle\overline{(3125, 3175, 3190, 3200; 1)}\right\rangle$
Raipur	$\left\langle\overline{(260, 262, 264, 270; 1)}\right\rangle$	$\left\langle\overline{(240, 247, 249, 255; 1)}\right\rangle$	$\left\langle\overline{(272, 274, 279, 290; 1)}\right\rangle$	$\left\langle\overline{(320, 326, 332, 340; 1)}\right\rangle$	$\left\langle\overline{(2475, 2995, 3275, 3400; 1)}\right\rangle$
Demand (in tons)	$\left\langle\overline{(2050, 2500, 2700, 3050; 1)}\right\rangle$	$\left\langle\overline{(3000, 3050, 3100, 3200; 1)}\right\rangle$	$\left\langle\overline{(2100, 2150, 2190, 2250; 1)}\right\rangle$	$\left\langle\overline{(1950, 2025, 2055, 2100; 1)}\right\rangle$	

Using the steps of the proposed method, the interval-valued fuzzy optimal solution of problem (26) is achieved as follows:

$$\begin{aligned}
\tilde{\tilde{x}}_{11} &= \left\langle\overline{(2050, 2105, 2130, 2480; 1)}\right\rangle \\
&= (2050, 2105, 2130, 2480), \\
\tilde{\tilde{x}}_{12} &= \left\langle\overline{(0, 0, 0, 10; 1)}\right\rangle = (0, 0, 0, 10), \\
\tilde{\tilde{x}}_{13} &= \left\langle\overline{(1450, 1450, 1450, 1510; 1)}\right\rangle \\
&= (1450, 1450, 1450, 1510), \\
\tilde{\tilde{x}}_{14} &= \left\langle\overline{(0, 0, 0, 0)}\right\rangle = (0, 0, 0, 0), \\
\tilde{\tilde{x}}_{21} &= \left\langle\overline{(0, 0, 0, 0)}\right\rangle = (0, 0, 0, 0), \\
\tilde{\tilde{x}}_{22} &= \left\langle\overline{(3000, 3050, 3065, 3075; 1)}\right\rangle \\
&= (3000, 3050, 3065, 3075), \\
\tilde{\tilde{x}}_{23} &= \left\langle\overline{(125, 125, 125, 125; 1)}\right\rangle \\
&= (125, 125, 125, 125), \\
\tilde{\tilde{x}}_{24} &= \left\langle\overline{(0, 0, 0, 0)}\right\rangle = (0, 0, 0, 0), \\
\tilde{\tilde{x}}_{31} &= \left\langle\overline{(0, 395, 570, 570; 1)}\right\rangle \\
&= (0, 395, 570, 570), \\
\tilde{\tilde{x}}_{32} &= \left\langle\overline{(0, 0, 35, 115; 1)}\right\rangle = (0, 0, 35, 115), \\
\tilde{\tilde{x}}_{33} &= \left\langle\overline{(525, 575, 615, 615; 1)}\right\rangle = (525, 575, 615, 615), \\
\tilde{\tilde{x}}_{34} &= \left\langle\overline{(1950, 2025, 2055, 2100; 1)}\right\rangle \\
&= (1950, 2025, 2055, 2100).
\end{aligned}$$

Putting the above interval-valued fuzzy optimal solution in the objective function of problem (26) the interval-valued trapezoidal fuzzy number transportation cost is achieved as follows:

$$\begin{aligned}
&\sum_{i=1}^{3}\sum_{j=1}^{4} \tilde{\tilde{c}}_{ij} \otimes \tilde{\tilde{x}}_{ij} \\
&= \left\langle\overline{(999500, 1166890, 1271030, 1359725; 1)}\right\rangle \\
&= (999500, 1166890, 1271030, 1359725).
\end{aligned}$$

The solution is matched with the fuzzy solution obtained based on proposed method by Kumar & Kaur [32].

7. Results and discussions

In this section, the main advantages of the proposed method over the existing methods are explored. Here, we shall point out that the method discussed in this paper is not based on classical transportation methods. This may be regarded as a disadvantage of the proposed method. Therefore, further research on proposing a new method to overcome this shortcoming is an interesting stream of future research. We shall report the significant results of this ongoing project in the near future.

Let us explore the main advantages of the proposed method.

(i) The transportation problem with interval-valued trapezoidal fuzzy transportation cost, chosen in Example 4.1, which may not be solved by the existing method [23], can be solved by using the proposed method.

(ii) The transportation problem with values of supply and demand as interval-valued trapezoidal fuzzy numbers, chosen in Example 4.2, which may not be solved by the existing method [22], can be solved by using the proposed method.

(iii) The proposed approach given in this study not only can be applied for solving the transportation problem with values of supply and demand as (λ, φ) interval-valued triangular fuzzy numbers and rest of parameters as real numbers, but also can be used for solving transportation problem with values of supply and demand as (w^L, w^U)-interval-valued trapezoidal fuzzy numbers. It is worth noting that (λ, φ) interval-valued triangular fuzzy numbers are special cases of (w^L, w^U)-interval-valued trapezoidal fuzzy numbers in which $w^L = \lambda, w^U = \varphi, a_2^L = a_3^L, a_2^U = a_3^U$.

(iv) The final results based on our proposed method are as (w^L, w^U)-interval-valued trapezoidal fuzzy numbers, while the optimal solution based on existing method [26] are as real numbers. In fact, the (λ, φ)-interval-valued triangular fuzzy transportation problem studied by Gupta & Kumar [26] is not in the form of a problem whose model involves fuzzy decision variables. The proposed algorithm in this contribution overcomes this shortcoming.

Remark 7.1. Given a LR flat fuzzy number $\tilde{A}$, parameterized by $\tilde{A} = (A_1, A_2, \alpha, \beta)_{LR}$ with the membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{x-(A_1-\alpha)}{\alpha}\right), & A_1 - \alpha \leq x \leq A_1, \\ 1, & A_1 \leq x \leq A_2, \\ R\left(\frac{(A_1+\beta)-x}{\beta}\right), & A_2 \leq x \leq A_1 + \beta, \end{cases}$$

Kumar & Kaur [31] considered the following ranking function, which first proposed by Yager [34]:

$$\Re(\tilde{A}) = \frac{1}{2}\left[\int_0^1 \left(A_1 - \alpha L^{-1}(\lambda)\right) + d\lambda + \int_0^1 \left(A_2 + \beta R^{-1}(\lambda)\right) + d\lambda\right].$$

Noting that a trapezoidal fuzzy number $\tilde{A} = (A_1, A_2, \alpha, \beta)$ is a special case of a LR flat fuzzy number $\tilde{A} = (A_1, A_2, \alpha, \beta)_{LR}$ with $L(x) = R(x) = \max\{0, 1 - x\}$, the value of Yager's ranking index for any trapezoidal fuzzy number may be obtained as follows:

$$\Re(\tilde{A}) = \frac{1}{2}\left[A_1 + A_2 + \frac{\beta - \alpha}{2}\right].$$

Taking into account the parameterized form of a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; 1)$ given in Definition 2.1 together with Eq. (5) of Theorem 2.1 yields that

$$\begin{aligned} d(\tilde{A}, O_1) &= \frac{1}{4}[a_1 + a_2 + a_3 + a_4] \\ &= \frac{1}{4}[(A_1 - \alpha) + A_1 + A_2 + (A_2 + \beta)] \\ &= \frac{1}{2}\left[A_1 + A_2 + \frac{\beta - \alpha}{2}\right] = \Re(\tilde{A}). \end{aligned}$$

(v) By Remark 7.1, we conclude that the linear ranking function $\Re(.)$, which is employed by Kumar & Kaur [31] to define the orders on trapezoidal fuzzy numbers, is a special version of $d(., O_1)$ if a level (w^L, w^U)-interval valued trapezoidal fuzzy number reduces to a trapezoidal fuzzy number. With this in mind, if the proposed method is applied to Examples 1 and Example 2 in Kumar & Kaur [31], it leads to the exact same results as obtained before. This demonstrates that study of transportation problem with interval-valued trapezoidal fuzzy numbers gives rise to the same expected results as those obtained for linear programming with trapezoidal fuzzy numbers. The results of Example 6.1 and Example 6.2 confirm the validity of this claim.

8. Concluding remarks and future research directions

These days a number of researchers have shown interest in the area of fuzzy transportation problems and various attempts have been made to study the solution of these problems. In this paper, to overcome the shortcomings of the existing methods we introduced a new formulation of transportation problem involving interval-valued trapezoidal fuzzy numbers for the transportation costs and values of supplies and demands. We proposed a fuzzy linear programming approach for solving interval-valued trapezoidal fuzzy transportation problem based on comparison of interval-valued fuzzy numbers by the help of signed distance ranking. To show the advantages of the proposed methods over existing methods, some fuzzy transportation problems, may or may not be solved by the existing methods, are solved by using the proposed methods and it is shown that it is better to use the proposed methods as compared to the existing methods for solving the transportation problems.

Finally, we feel that, there are many other points of research and should be studied later on. Some of these points are discussed below.

(i) The solid transportation problem considers the supply, the demand, and the conveyance to satisfy the transportation requirement in a cost-effective manner. Thus, research on the topic for developing the proposed method to derive the fuzzy objective value of the fuzzy solid transportation problem when the cost coefficients, the supply and demand quantities and conveyance capacities are interval-valued trapezoidal fuzzy numbers, is left to the next research work.

(ii) Further research on introducing a new formulation of interval-valued trapezoidal fuzzy numbers transportation problem that lead to a method for solving this problem based on the classical transportation algorithms is an interesting stream of future research.

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