Investigation of schemes for incorporating generator Q limits in the fast decoupled load flow method

LAKSHMI SUNDARESH* and P S NAGENDRA RAO

Department of Electrical Engineering, Indian Institute of Science, Bangalore 560 012, India
e-mail: lakshmi.sundaresh@gmail.com

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Abstract. Fast Decoupled Load Flow (FDLF) is a very popular and widely used power flow analysis method because of its simplicity and efficiency. Even though the basic FDLF algorithm is well investigated, the same is not true in the case of additional schemes/modifications required to obtain adjusted load flow solutions using the FDLF method. Handling generator Q limits is one such important feature needed in any practical load flow method. This paper presents a comprehensive investigation of two classes of schemes intended to handle this aspect i.e. the bus type switching scheme and the sensitivity scheme. We propose two new sensitivity based schemes and assess their performance in comparison with the existing schemes. In addition, a new scheme to avoid the possibility of anomalous solutions encountered while using the conventional schemes is also proposed and evaluated. Results from extensive simulation studies are provided to highlight the strengths and weaknesses of these existing and proposed schemes, especially from the point of view of reliability.

Keywords. FDLF; adjusted load flow solutions; generator Q limits; bus type switching method; sensitivity method.

1. Introduction

Fast decoupled Load Flow method (FDLF) proposed by Stott & Alsac (1974) is considered to be the work horse of the power industry. It is a part of most of the analysis, planning and operation related studies of power transmission systems. Most of the attractions of this method stem from the fact that this is a constant matrix load flow method and in spite of being an approximate version of the Newton Raphson (NR) method proposed by Tinney & Hart (1967), it is seen to be as reliable as the NR method. Because of its wide spread use, there have been investigations carried out by Wu (1977); Rao *et al* (1984) to understand the reasons contributing to the attractive features of the method. The work by Wu (1977) attempts to analyze the behaviour of this method in the functional analysis framework in an abstract theoretical setting whereas the results given by Rao *et al* (1984) provide methods to find quantitative indicators regarding the

*For correspondence

convergence behaviour. However, these investigations are limited to only the basic unadjusted load flow solutions and do not encompass adjusted solutions.

In any practical situation, the utility of a load flow technique not only depends on the strength of the basic scheme but also on its ability to incorporate a range of adjustments such as generator Q limits and transformer tap limits. This paper is concerned with schemes for handling one of these adjustments; the generator Q limit adjustments. Two approaches have been suggested by Stott & Alsac (1974) for handling this adjustment. The first is the bus type switching scheme, essentially adopted from earlier methods such as NR and Gauss–Seidel (GS) methods as explained by Arrillaga & Arnold (1990). The second approach, the sensitivity-based scheme is proposed exclusively for use with the FDLF method. This latter method enables the use of the constant matrices used in the basic FDLF method during the adjustments iterations without the need for refactoring. This is not possible in the bus type switching scheme.

There have not been many attempts to develop alternate schemes for incorporating this adjustment in FDLF method. One stray attempt is reported by Allan & Arruda (1982). The focus in the work by Allan & Arruda (1982) is about Q violations due to transformer taps rather than generator Q limits. Moreover, it is shown by Rao (1983) that even though the philosophy behind the scheme proposed by Allan & Arruda (1982) is sound, the computational algorithm proposed is based on some implicit assumptions that restrict its use to only a few special cases. A method suggested for handling generator Q limits in FDLF by Rao (1983) is essentially an alternate implementation of the bus type switching scheme and the convergence behaviour of this scheme would be exactly identical to that of the traditional implementation of the bus type switching scheme.

Even though both bus type switching and sensitivity schemes were proposed along with the basic FDLF scheme, there is very little recorded literature that provides an evaluation of the relative performance of these two schemes. The aim of this paper is to first assess the performance of these two schemes for a range of test systems from the point of view of their reliability and convergence rate. Subsequently, it is shown here that the sensitivities computed in the existing sensitivity-based method are in fact only approximate. We alternatively propose the use of two other sets of sensitivities which are more meaningful than the sensitivities used by Stott & Alsac (1974) and assess the effectiveness of these sensitivities for incorporating generator Q limits in the FDLF method. The schemes proposed for obtaining the new sensitivities here are novel and these techniques themselves could be of considerable use in other situations as well.

While investigating the performance of both the conventional Q limit handling schemes several cases are encountered wherein the methods either fail to converge or converge to an improper solution. This latter situation is similar to what has been reported by Zhao *et al* (2008) and Sundaresh & Rao (2014) in the context of the NR method as anomalous solutions. Sundaresh & Rao (2014) have considered a solution to be anomalous if at a violating PV bus the voltage at the solution point is greater than the specified value and its Q is set at the maximum limit or a reciprocal situation (voltage being lower than the specified value and Q set at the minimum limit). Two modifications to the conventional adjustment scheme are also presented here which are intended to pre-empt the possibility of anomalous solutions during the Q limit adjustment process and their effectiveness has been investigated.

The rest of the paper is organised as follows. Fast Decoupled Load Flow is briefly discussed in Section 2. The conventional sensitivity method is discussed in Section 3 and the modified sensitivity methods are discussed in Section 4. Additional schemes to inhibit anomalous solutions are discussed in Section 5. The simulation results are presented and assessed in Section 6. The work is concluded in Section 7. Computational schemes to obtain the new sensitivities and details about the implementation of various modification schemes are given in Appendix A and Appendix B respectively.

2. Fast decoupled load flow

Fast Decoupled Load Flow method proposed by Stott & Alsac (1974), essentially involves iteratively solving the following sets of linear equations obtained from the nonlinear node power balance equations:

$$[\Delta P/V] = [B'] [\Delta \delta]. \quad (1)$$

$$[\Delta Q/V] = [B''] [\Delta V]. \quad (2)$$

The real power and reactive power equations are decoupled and the coefficient matrices in (1) and (2) are constant matrices that depend only on the network parameters. B' is obtained by only considering the reactances and neglecting resistances of the lines. B'' is obtained by considering the imaginary part of the admittance matrix as suggested by Stott & Alsac (1974). The triangular factors of the matrices are computed and stored at the beginning and they are used repeatedly to calculate the changes in voltages and angles. Incorporating Q limit adjustments into this basic FDLF method will involve changes in the B'' matrix in every iteration depending on the number of violated generator buses. The triangular factors of B'' computed in the beginning cannot be readily used during adjustments. The adjustment schemes are intended to handle this problem. The bus type switching scheme proposed by Stott & Alsac (1974) finds modified factors by considering the added columns of B'' whereas the sensitivity-based scheme attempts to avoid refactoring.

3. Existing schemes for handling generator Q limits

3.1 Bus type switching method

The bus type switching method involves identifying the violated generator buses in every iteration and switching the violating PV buses to PQ buses. At such PV buses, the violated Q limit is considered as the specified reactive power at the bus for this particular iteration. Hence, in this method, depending on the violated buses, the structure of the B'' matrix changes in every iteration. This scheme for FDLF is essentially adopted from earlier load flow methods such as the NR and GS method proposed by Tinney & Hart (1967) and Arrillaga & Arnold (1990). Even though this procedure has been known, it must be pointed out that only a broad outline of the scheme is widely known and there are significant differences across implementations of this approach. As shown by Stott (1974), the fact that these differences are critical for the convergence of the method is well-known, but full details of implementations are not available in the public domain.

3.2 Sensitivity method

This approach is suggested by Stott & Alsac (1974) for handling Q limit violations in FDLF method as an alternative to the bus type switching method. The most important feature of this method is that it avoids the need to refactor B'' matrix in each iteration. Yet, this method does not seem to be very popular and there is no known literature that assesses the strengths and weaknesses of this method vis-a-vis the bus type switching method.

In this approach, the required change in the voltage of a violating generator bus is estimated by using the bus sensitivity factor (S_k) which is considered to be $\partial V_k / \partial (Q_k / V_k)$ for the k^{th} bus. If the reactive power error is $\Delta Q_k = (Q_k^{\text{limit}} - Q_k^{\text{cal}})$, where Q_k^{limit} is the violated limit and

Q_k^{cal} is the computed Q at the bus, the required modification for the PV bus voltage is estimated as

$$\Delta V_k = S_k * (\Delta Q_k) / V_k \quad (3)$$

The bus is continued to be treated as a PV bus but at a modified specified voltage $V_k^{sp} + \Delta V_k$.

The most critical aspect of this approach is to calculate the sensitivities efficiently. The procedure suggested by Stott & Alsac (1974) for computing sensitivity factors is discussed below. S_k is obtained for each of the violating PV bus as the diagonal element of the inverse of a modified B''_{new} matrix, the matrix obtained by augmenting B'' matrix by an additional row and column corresponding to each of the violating generator buses at a time. Consider a system with n buses and m PV buses. The B'' matrix corresponds to only PQ type buses. The size of the B'' matrix will be $n' = n - m - 1$. In order to find the sensitivity factor for the k^{th} generator bus, the B'' is modified as B''_{new} as

$$B''_{new} = \begin{bmatrix} B'' & C_1 \\ C_1^T & c_2 \end{bmatrix}. \quad (4)$$

The size of the B''_{new} matrix is $n' + 1$. The new row $[C_1^T \ c_2]$ corresponds to the k^{th} PV bus. The triangular factors are known for the B'' matrix and let these be such that $B'' = [U]^T [D]^{-1} [U]$ where U is a unit upper triangular matrix and D a diagonal matrix with d_i as its i^{th} row diagonal element.

Using these factors, the required sensitivity S_k is obtained as follows:

First, the forward substitution process is carried out on the vector C_1 using the lower triangular matrix $[U^T]^{-1}$ to obtain C'_1 and C'_1 is multiplied by the diagonal matrix $[D]$ to obtain the vector G .

$$[G] = [D][U^T]^{-1}[C_1] = [u_1, u_2 \dots u_{n'}]^T. \quad (5)$$

It is easy to see that this vector G corresponds to the additional column of the upper triangular factor of B''_{new} .

Representing the factors of B''_{new} as

$$U_{new} = \begin{bmatrix} U & G \\ 0 & 1 \end{bmatrix} \quad D_{new} = \begin{bmatrix} D & 0 \\ 0 & d_{n'+1} \end{bmatrix}, \quad (6)$$

where $d_{n'+1}$ is to be determined.

$$B''_{new} = \begin{bmatrix} B'' & C_1 \\ C_1^T & c_2 \end{bmatrix} = [U_{new}]^T [D_{new}]^{-1} [U_{new}] = \begin{bmatrix} U^T & 0 \\ G^T & 1 \end{bmatrix} \begin{bmatrix} D^{-1} & 0 \\ 0 & 1/d_{n'+1} \end{bmatrix} \begin{bmatrix} U & G \\ 0 & 1 \end{bmatrix}. \quad (7)$$

It is easy to see that

$$c_2 = \begin{bmatrix} G^T & 1 \end{bmatrix} \begin{bmatrix} D^{-1} & 0 \\ 0 & 1/d_{n'+1} \end{bmatrix} \begin{bmatrix} G \\ 1 \end{bmatrix} = \frac{u_1^2}{d_1} + \frac{u_2^2}{d_2} + \dots + \frac{u_{n'}^2}{d_{n'}} + \frac{1}{d_{n'+1}} \quad (8)$$

solving for $d_{n'+1}$ we get

$$d_{n'+1} = \left[c_2 - \left(\frac{u_1^2}{d_1} + \frac{u_2^2}{d_2} + \dots + \frac{u_{n'}^2}{d_{n'}} \right) \right]^{-1}. \quad (9)$$

Hence, the sensitivity factor for the k^{th} generator bus is

$$S_k = d_{n'+1}. \quad (10)$$

This procedure is repeated for each of the violating PV buses the first time it violates the limit and the computed sensitivity factor is reused in the subsequent iterations. The FDLF scheme using these sensitivities for adjustment is referred as Sensitivity Scheme I.

4. Modified Sensitivity computation by including all the violating generator buses simultaneously

The sensitivity calculation scheme proposed by Stott & Alsac (1974) discussed in Section 3 can only provide approximate sensitivities. This is because, in any given iteration more than one PV bus can violate their limits. In such a situation, the sensitivities should correspond to the condition where all of them are considered simultaneously and not one by one as in Section 3. We show here that the precise sensitivity factors can be computed with almost the same amount of computational effort. The modified B'' matrix for a system with M number of PV buses violating their limit can be represented as

$$B''_{\text{new}} = \begin{bmatrix} B'' & C \\ C^T & H \end{bmatrix}. \quad (11)$$

This is similar to (4) except that C has M columns and H is a $M \times M$ matrix whereas c_2 was a scalar. The sensitivity used in Stott & Alsac (1974) is the approximate value obtained by augmenting B'' with one column of C at a time. The correct sensitivities are actually the elements of the inverse of B''_{new} corresponding to the diagonal position of H in (11). The first new scheme proposed here is using these as the S_k values instead of what is indicated in Section 3. This scheme is referred by us as Sensitivity Scheme II.

The second new alternative proposed here is to consider all the elements of the inverse (not just the diagonal elements) in the locations corresponding to H in (11). Let this matrix be $[H']^{-1}$. The method of obtaining H' from (11) is given in Appendix A. In this case, the estimates for the voltage changes required at all PV buses are found simultaneously as

$$[\Delta V] = H'^{-1}[\Delta Q/V], \quad (12)$$

where ΔV (estimated voltage change required) and ΔQ (vector Q mismatches at violating buses) are vectors of size M . This approach ensures accounting for the mutual effects of changes in voltages on Q 's at all violating PV buses. This sensitivity-based scheme is referred to as Sensitivity Scheme III. The details of the schemes for computing the new sensitivities are given in Appendix A.

5. Schemes to avoid anomalous solutions

FDLF algorithm when used for systems with normal operating conditions converging to anomalous solution is a new phenomenon being observed here. It has not been reported by other investigators previously. As it appears, such a phenomenon is not a very rare possibility. An attempt has been made to modify the logic of Q limit adjustment to explicitly check and avoid

this possibility during the adjustment process. Anomalous solutions could arise in both the conventional adjustment schemes.

We first consider how the conventional adjustment scheme could push the convergence process towards an anomalous solution and then suggest and study two schemes to avoid it. The two schemes have a lot in common and differ only in some aspects. The adjustment process can be viewed as a sequence of three steps.

- (i) Identify the limit violating buses.
- (ii) Find the reactive power mismatches at the violating buses.
- (iii) Find a correction to the specified voltage at these buses depending on the amount of reactive power mismatches at these buses.

The two conventional adjustment schemes differ only in the third step. In the bus type switching scheme, the Q equations of such PV buses are augmented to the Q equations of PQ buses and the correction for the voltages of all the buses are obtained simultaneously. In the sensitivity scheme, the voltage corrections are directly computed using the sensitivity for each of the violating PV buses. Hence, we focus on the first two steps which are common for both the schemes.

While implementing the first step, Q at a PV bus is calculated using only the specified voltage of the bus. If this value is outside the limits then the PV bus is considered to violate its limit for this iteration.

The second step is to find the reactive power mismatch for limit violating PV buses. First, we check if the bus had violated any of its Q limits in the previous iteration. If the PV bus has not violated its Q limits in the previous iteration, then the Q mismatch is calculated as $(Q^{limit} - Q^{calc})$ i.e. we choose the violated Q limit as the Q specified for that iteration. For the buses that had violated their Q limits in the previous iteration, another value of Q calculated (other than the one calculated in the first step) corresponding to the modified value of the PV bus voltage is obtained. If this value also violates the limits then, the Q specified is chosen as this violated limit. If this value does not violate the Q limit, the Q specified is kept unchanged at the value that had been chosen in the previous iteration. It is observed that it is at this stage, the possibility for an anomalous solution emerges. In the conventional schemes, no explicit check is made at this stage to ensure that the present voltage of the bus and chosen Q-specified are consistent. Hence, we propose two modified implementations to incorporate this additional check so as to avoid the possibility of anomalous solutions.

In the first scheme (modified scheme 1), if the Q calculated corresponding to the specified voltage is not within the limits, the Q specified for this bus is decided based on the value of the modified voltage of the bus. If the modified voltage is lesser than the specified value, the maximum limit (Q^{max}) is chosen as the specified value and if the modified voltage is greater than the specified value, the minimum limit (Q^{min}) is taken as the specified limit. Even though this scheme helps avoid anomalous solutions, this is seen to result in failure for some cases that would converge to a proper solution without this logic. Hence a modified scheme 2 is also proposed and investigated.

In the modified scheme 2, the Q calculated corresponding to the modified voltage at the bus is examined to see whether it is within or outside the limits. If it is outside the limits, the Q specified is chosen as the violated limit. If this value is within the limits, the Q specified for this bus will be decided based on the value of the modified voltage of the bus. If the modified voltage is lesser than the specified value, the maximum limit is chosen as the specified value and if the modified voltage is greater than the specified value, the minimum limit is taken as the specified limit.

The key steps in implementing these two modification schemes are given in Appendix B.

6. Simulation results

All the algorithms introduced earlier (both the existing and proposed schemes) have been extensively tested. These test results are also compared with the results obtained by the FDLF implementation in MATPOWER software which is available in public domain and the details about this software have been discussed in length by Zimmerman *et al* (2011). For the investigations reported here, we have used several test systems from the MATPOWER database. Using the basic data of these systems, we have created a large number of test cases by changing the load, range of Q limits at PV buses, line parameter values and also considering line outages. In all the investigations, we use an initial guess solution of $1 + j0$ for all PQ buses and $V^{sp} + j0$ for all PV buses where V^{sp} is the specified voltage of these buses. The convergence tolerance used is 10^{-5} p.u. throughout. As it is known that the Q limit adjustments must be initiated after achieving reasonable convergence, the limit violations are considered starting from the third iteration. This approach has been used to decide the point of initiation instead of considering the maximum ΔP or ΔQ mismatch, for the sake of uniformity across all systems/examples/algorithms. For ease of reference, we refer to the conventional sensitivity method as Sensitivity I. Sensitivity II is one of the methods proposed in this paper, in which the sensitivities used are only the diagonal values of inverse of the B''_{new} matrix corresponding to the rows of PV buses violating their limits while considering all the violating PV buses simultaneously. Sensitivity III is the method in which the full sensitivity matrix of the violated buses is used to determine the magnitude of voltage corrections at all violating PV buses simultaneously.

The results obtained by considering the base case loading for a number of test systems are presented in table 1. From table 1, it is seen that for the smaller systems, there is not much of a difference in the performances of all the methods, because at the final solution no violations are present. However, for the two larger systems, bus type switching scheme converges to a solution in fewer iterations as compared to the sensitivity methods. All the sensitivity-based methods show a comparable performance. A perusal of results in table 1 shows that the number of violations is very small for these cases. It has also been observed that in these cases there are no back off situations encountered where a PV bus has to be reconsidered as a PV bus after it has been considered as a PQ bus in one or more iterations. Hence, it appears as though finding adjusted solutions in these cases is not a tough proposition.

In order to get a better comparison, a large number of cases have been generated by varying some of the data of the 39, 118 and 300 bus systems. Results of a few of such cases are summarised in tables 2, 3, and 4 respectively. The modifications made to the basic data in order to

Table 1. Performance comparison for various systems.

System	No of violated PV buses	Bus type switching	Sensitivity I	Sensitivity II	Sensitivity III	Matpower
14 bus	0	4.5	4.5	4.5	4.5	4.5
30 bus	0	4.5	4.5	4.5	4.5	4.5
39 bus	0	6	7	7	7	9
57 bus	1	5.5	5.5	5.5	5.5	5.5
118 bus	6	6.5	8.5	8.5	6.5	10.5
300 bus	9	9.5	11.5	10.5	9.5	15

Table 2. 39 Bus system: test case specifications and performance comparison.

Parameters	Case 1	Case 2	Case 3	Case 4
Test conditions				
Load multiplication factor for reactive power	0.6	0.85	1.25	1.3
Load multiplication factor for real power	0.6	0.85	1.25	1.3
Q limit multiplication factor	0.4	0.4	1	1
Line resistance multiplication factor	1.6	1.5	1	1
Line reactance multiplication factor	1	1	1	1
Total number of violated buses	6	3	4	5
Number of iterations				
Bus type switching method	8.5	6.5	10.5	40.5
Sensitivity scheme I	24.5	13.5	28.5	145.5
Sensitivity scheme II	20.5	12	DNC	F
Sensitivity scheme III	14.5	12	20.5	DNC
Matpower	F	20.5	26	DNC

DNC, Does not converge even after 250 iterations, bus mismatches do not increase;

F, Fails to converge and mismatches blow up.

Table 3. 118 Bus system: test case specifications and performance comparison.

Parameters	Case 1	Case 2	Case 3	Case 4	Case 5
Test conditions					
Load multiplication factor for reactive power	1.5	0.8	0.7	0.6	1.81
Load multiplication factor for real power	1.6	0.75	0.75	1.15	1.5
Q limit multiplication factor	0.5	0.4	0.25	0.25	0.5
Line resistance multiplication factor	1.0	0.7	0.7	0.7	1.45
Line reactance multiplication factor	1	0.85	1.4	1.5	0.85
Total number of violated buses	34	25	24	23	38
Number of iterations					
Bus type switching method	13.5	6.5	6.5	8.5	F
Sensitivity scheme I	93.5	54.5	26.5	28.5	F
Sensitivity scheme II	F	F	F	F	F
Sensitivity scheme III	15.5	7.5	8.5*	9.5*	F
Matpower	38.5	16.5*	14.5*	17.5*	DNC

*-Anomalous solution.

DNC, Does not converge even after 250 iterations, bus mismatches do not increase;

F, Fails to converge and mismatches blow up.

generate the test cases are also provided in these Tables in order to facilitate not only verification of these results but also to serve as test cases for other investigations/investigators. Considering the results presented in table 2 for a few cases obtained from the 39 bus system, it is seen that for these cases bus type switching method performs the best. The Sensitivity scheme I succeeds in all the cases but requires considerably more iterations than the bus type switching scheme. It is seen that Sensitivity scheme II fails to converge in two cases. Sensitivity scheme III is slightly better than the other sensitivity schemes except in Case 4.

Results in table 3 for the 118 bus cases show a similar behaviour. Even here, the bus type switching method appears best even though it fails to provide a solution for Case 5. Sensitivity scheme I provides a reasonable solution for all the cases requiring significantly more number of

Table 4. 300 Bus system: test case specifications and performance comparison.

Parameters	Case 1	Case 2	Case 3
	Test conditions		
Load multiplication factor for reactive power	1.02	1.05	1.08
Load multiplication factor for real power	1.05	1.05	1.05
Q limit multiplication factor	1	1	1
Total number of violated buses	23	26	29
	Number of iterations		
Bus type switching method	18.5	31.5	52.5
Sensitivity scheme I	28.5	71.5	117.5
Sensitivity scheme II	26.5	DNC	DNC
Sensitivity scheme III	24.5	46.5	77.5
Matpower	43.5	F	F

DNC, Does not converge even after 250 iterations, bus mismatches do not increase;

F, Fails to converge and mismatches blow up.

iterations than bus type switching except Case 5 whereas Sensitivity scheme II fails throughout. Sensitivity scheme III converges to a solution (number of iterations comparable to bus type switching) for all the cases but it provides anomalous solutions for cases 3 and 4. As highlighted in the earlier work by Sundaresh & Rao (2014), a solution is anomalous if at the converged solution the Q limits at which PV buses are set are inconsistent with the bus voltages from a practical point of view. MATPOWER provides a reasonable solution only in Case 1. It converges to an anomalous solution in three cases and does not converge (DNC) in one case (continues to oscillate but the mismatches do not grow large). The results for 300 bus system in table 4 follow a similar pattern. Performance of Sensitivity scheme II is similar to what is seen in 39 bus system case (table 2) whereas the conventional sensitivity scheme takes more number of iterations than Sensitivity scheme III in all the cases taking the least number of iterations. The bus type switching method is seen to converge to a solution taking least number of iterations as compared to other methods.

From the results in tables 1–4, the performance of MATPOWER implementation of the bus type switching scheme is seen to be different from the conventional scheme. It generally performs poorly as compared with the conventional implementation. Even in cases where it provides a solution, it takes considerably more iterations to converge than the conventional implementation. The reason appears to be that in MATPOWER implementation, Q limits are not checked in every iteration. It is done only when the solution reaches a pre-specified tolerance in every adjustment cycle. The other difference is that the method fails to provide a solution in several cases where the conventional method succeeds. The reason for this behaviour could be the logic used in the MATPOWER code. In the MATPOWER implementation of the bus type switching, no logic has been incorporated to check whether a PV bus once converted to PQ type has to be reconverted to PV type (back off situation). Hence, a PV bus once converted to PQ type continues to be treated as such till convergence and this seems to result in many failures/anomalous solutions. If the logic to include back off situation is incorporated in the bus type switching method, it is quite possible to obtain a solution even in cases where MATPOWER implementation of bus type switching fails to provide a reasonable one.

In order to provide a more thorough comparison of methods, we use 14127 test cases by considering all viable double line contingencies in the 118 bus system that have a feasible load flow solution. The results of this investigation are summarised in table 5. From these results it is seen

Table 5. 118 Bus system: contingency cases.

Algorithm	Bus type switching method	Sensitivity I	Sensitivity II	Sensitivity III
Number of cases converged	13804	14094	12991	13804
Avg Number of iterations	6.571	11.590	9.412	7.1092
Number of anomalous cases	323	30	150	323
Number of failed cases	0	3	986	0

Table 6. 118 Bus system: contingency cases (Modified scheme 1).

Algorithm	Bus type switching method	Sensitivity I	Sensitivity II	Sensitivity III
Number of cases converged	13801	13641	12982	13801
Avg Number of iterations	6.573	11.636	9.468	7.1104
Number of anomalous cases	0	0	0	0
Number of failed cases	326	486	1145	326

Table 7. 118 Bus System: contingency cases (Modified scheme 2).

Algorithm	Bus type switching method	Sensitivity I	Sensitivity II	Sensitivity III
Number of cases converged	13947	14123	12999	13953
Avg Number of iterations	6.608	11.588	9.414	7.1527
Number of anomalous cases	163	0	0	8
Number of failed cases	17	4	1128	166

that all the methods (conventional as well as two new sensitivity schemes) could converge to anomalous solutions in some cases. It can be observed that on an average, bus type switching method takes the least number of iterations to converge to a solution, but the maximum number of anomalous solutions (323) is seen in this method and Sensitivity scheme III. Among the sensitivity methods, Sensitivity scheme III takes on an average lesser number of iterations to converge. Sensitivity scheme II scheme fails to converge in 986 cases. In order to test the effectiveness of the proposed modifications to avoid anomalous solutions, the same set of test cases is used and the results are summarised in tables 6 and 7.

From table 6, it is seen that Modified scheme 1 succeeds to avoid anomalous solutions in all cases. However, comparing results with those in table 5, it is seen that the number of failure cases increases significantly for all the algorithms. This implies that this modification even though helps to avoid anomalous solutions could also result in failure for some cases which would have converged without this modification. Results in table 7 appear to be slightly better especially for the bus type switching scheme and Sensitivity scheme I. But the issue of concern is that anomalous solutions could still result as seen for the bus type switching scheme. Number of failures seen with this modification are significantly less as compared to the results in table 6. The results in table 7 could be considered to be marginally better than those of the results for modification in table 6. However, further investigations seem necessary to clearly conclude which of these options is the better one.

Based on the large number of simulation studies the following observations were made:

1. The conventional bus type switching scheme is quite efficient in cases where it gives a solution.

2. The MATPOWER implementation of the bus type switching scheme is different from the conventional scheme. The violations are not checked/adjusted in every iteration and no provision is made to identify and handle back off situations suitably and hence performs poorly as compared to the conventional scheme.
3. The conventional sensitivity scheme is slightly slower as compared to bus type switching scheme.
4. Both the conventional schemes fail to converge to a solution or converge to an anomalous solution in several cases.
5. The sensitivity scheme II proposed here to improve the reliability of the conventional sensitivity scheme by using more accurate sensitivities and incorporating the mutual effect other limit violating PV buses seems to perform poorly as compared to the conventional sensitivity scheme. This outcome is counter-intuitive and further investigations are needed to understand this phenomenon
6. The sensitivity scheme III provides some improvement over the conventional sensitivity scheme. But the performance is still marginally inferior to that achieved by the bus type switching scheme.
7. The new schemes proposed here to eliminate the possibility of anomalous solution, succeed in avoiding them. But in a good number of cases they diverge. This approach certainly improves the reliability of the conventional schemes but does not guarantee solutions in all cases. Even though Modified scheme 2 appears better than Modified scheme 1, further tests are needed to conclude this with more certainty.

7. Conclusion

A comprehensive study of the performance of two known techniques for handling generator Q limits in the FDLF method is presented. It is shown that the widely used bus type switching method is not always reliable as it may diverge or provide an improper solution in several cases where a proper solution exists. The lesser used sensitivity approach is slightly more reliable in general than the bus type switching scheme but requires more iterations than the former scheme.

Noting that the sensitivities used in existing adjustment schemes are only approximate, new variants of the conventional sensitivity approach have been investigated. The sensitivities used are more precise than the ones in use. The surprising outcome is the performance of the sensitivity based adjustment using one of the new sensitivities (Sensitivity scheme II) does not seem to provide any significant advantages over the existing scheme based on approximate sensitivities. Further investigations are necessary to understand this non-intuitive outcome. The other new scheme (Sensitivity scheme III) proposed here seems to provide some improvement over the conventional sensitivity (Sensitivity scheme I) scheme. But, even with this improvement no advantage is seen over the bus type switching scheme for this method from the point of view of convergence behaviour.

Two new modifications for the conventional adjustment schemes are proposed and it is shown that even though they eliminate the possibility of anomalous solutions to a great extent, they could result in divergence in some of the cases.

The paper also gives details of a large number of test cases for which the conventional Q adjustment schemes are unable to provide a solution. It is believed that these cases will be extremely useful to other researchers to test out new schemes and such data is rarely available in literature.

In a broad sense, this work opens up a number of new questions while trying to answer some old ones. The most significant outcome of this paper is that it establishes that the problem of handling generator Q limits in the context of FDLF is not a solved one and further investigations are necessary to find schemes that are more reliable than the existing schemes and also make them computationally efficient.

Appendix

A. Computation of the new sensitivities

Obtaining the new sensitivities requires finding a few elements of the inverse of the matrix B''_{new} in (11). Equation (11) is reproduced here as follows:

$$B''_{new} = \begin{bmatrix} B'' & C \\ C^T & H \end{bmatrix}, \quad (13)$$

where the augmented columns of C and H correspond to the PV buses violating their Q limits. The computational problem is to obtain the elements of B''_{new}^{-1} corresponding to the diagonal positions of H (the first approach proposed) or all the elements in the positions of H (the second approach proposed). The $U^T D^{-1} U$ factors of B'' are known as they are computed at the beginning of the FDLF iterations. The columns of the factors of B''_{new} corresponding to the columns of C can be obtained (one by one) as explained in Section 3, using the computed factors of B'' . Once computed they can be stored till the final solution is obtained because they are sparse columns and this way the need to compute them repeatedly in every iteration can be avoided.

An alternate approach would be to augment the columns of B'' with columns corresponding to all PV buses during the factorisation of B'' itself (before the beginning of the iterations) and going through the factorisation process even for the augmented columns. It may be noted that before the beginning of the iteration, the PV buses which violate their limits are not known. Hence, we augment B'' with $\hat{C}$ consisting of columns corresponding to all the PV buses. The C matrix in (13) would change from iteration to iteration depending on the PV buses that violate their limits in that iteration but always will be made up of the columns of $\hat{C}$. Denoting the modified elements (due to factorisation) of $\hat{C}$ as $\hat{C}'$ in every iteration we build a matrix C' by choosing the columns of $\hat{C}'$ corresponding to the buses that violate their limits in that iteration. This way, at the end of the factorisation of B'' , all the columns of the factors of B''_{new} (corresponding to positions of C in various iterations) will be available with only a marginal increase in computational effort. In each iteration one has to compute a matrix H'

$$H' = H - C'^T \times D^{-1} \times C', \quad (14)$$

where the columns of C' and H correspond to PV buses that violate their limit. This computation in (14) corresponds to a partial Gaussian elimination process of making elements in the rows of C'^T matrix in (13) zero. The computation in (14) is not demanding as C' is a sparse matrix. As the interest here is in the elements of the inverse of H' , at this point one can find the triangular factors of H' (exploiting sparsity of H' if it is advantageous).

If only the diagonal elements of the inverse of H' are needed, they can be obtained efficiently using the sparse Z bus approach proposed by Takahashi *et al* (1973) using the factors of H' . If one is implementing the second scheme, (using all the elements of H' inverse) the required changes in the voltages at all violating PV buses can be computed by operating on the column of

$\Delta Q/V$ using the factors of H' to obtain all ΔV s as a repeat solution of a system of Eq. in (12) with the computed factors of the coefficient matrix H' .

B. Implementation of the proposed modifications for avoiding anomalous solutions

The proposed modifications can be applied to both the conventional adjustment schemes. It is easy to see that these schemes (bus type switching and sensitivity) are different in only how the changes in voltages of violating PV buses are calculated. But the method of identifying the violating buses and choosing the Q specified for these buses for that iteration is common for both. The proposed modifications are intended to change the conventional algorithms, in how the Q specified is chosen for the violating PV buses. Hence, only these critical steps for the proposed modified algorithms are given below. These steps can be incorporated into bus type switching as well as all the sensitivity schemes.

B.1 Modified scheme 1

Case (i) The PV bus has been treated as a PV bus in the previous iteration.

Calculate the Reactive power (Q^{cal}) at the bus

- If this Q^{cal} at the bus is within the specified limits, the bus is treated as a PV bus.
- Else, this bus is switched to a PQ bus with the Q specified for this bus set at the violated limit.

Case (ii) The PV bus has been treated as a PQ bus in the previous iteration.

In this case, there are two voltages corresponding to this bus at this stage. Its original specified voltage and the computed voltage after treating it as PQ bus. *Calculate the reactive power (Q_1) at the bus with the voltage at this bus assumed to be at the specified value.*

- If this Q_1 is within limits, consider it as a PV bus with the voltage set at the specified value.

Otherwise,

- (***) If the present voltage value is lower than the specified value, the maximum limit of Q is considered as the value of the Q specified value at this bus.
- (***) If the present voltage value is higher than the specified value, the minimum limit of Q is considered as the value of the Q specified value at this bus.

B.2 Modified scheme 2

This scheme is same as the previous scheme up to the last two steps (shown as ***) . The last two steps (shown as ***) are to be replaced by the following steps.

- Calculate Q_2 at this bus corresponding to its present modified voltage
- If Q_2 is outside the specified limits, this bus is considered as a PQ bus with the Q specified at the bus set at this violated limit.

- In case Q_2 is within limits, this bus is considered as a PQ bus and the value of Q specified value at this bus is chosen as follows:
 - If the present voltage value is lower than the specified value, the maximum limit of Q is considered as the value of the Q specified value at this bus.
 - If the present voltage value is higher than the specified value, the minimum limit of Q is considered as the value of the Q specified value at this bus.

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