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Numerical optimization of a transcritical CO_2/propylene cascaded refrigeration-heat pump system with economizer in HT cycle

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Abstract. Use of organic refrigerants such as Hydrochlorofluorocarbons and Chlorofluorocarbons have been criticized for their adverse impact on the Earth's protective ozone layer and for their significant global warming potential (GWP). CO_2 has been receiving great concern as an alternative refrigerant. Cascade refrigeration systems employing CO_2 are used for low temperature applications. Being a low critical temperature fluid CO_2 transcritical cascade systems offer low COP for a given application. Parallel compression economization is one of the promising cycle modifications to improve the COP of transcritical CO_2 cascaded systems. In this paper, transcritical CO_2/propylene cascade system with parallel compression economization in the HT cycle has been analysed for cooling/heating applications. An enhancement in COP of 9% has been predicted. Thermodynamic analysis on R744-R1270 cascade refrigeration system has been performed to determine the optimal value of the various design parameters of the system. The design parameters included are: gas cooler outlet temperature and intermediate temperature in the high temperature circuit and evaporator temperature and temperature difference in the cascade condenser in the low temperature circuit.

Keywords. Propylene/CO_2; transcritical; cascade refrigeration system; parallel compression economization.

1. Introduction

Low temperature multi-stage vapor compression refrigeration systems are not suitable for numerous engineering applications such as cryogenic separation in petrochemical industries,

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liquefaction of petroleum vapors and natural gas, manufacturing of dry ice, precipitation hardening of special alloy steel and storage of food, blood, etc. This is due to exceedingly large specific volumes, low solidification temperature, low operating pressure of the refrigerants and problems faced during operation of compressors. These difficulties can be surmounted by using a cascade refrigeration system. The cascade refrigeration cycle has two V-C cycles connected in series. The lower cycle cools the refrigerated space and the upper cycle cools the lower cycle. This is accomplished with a cascade heat exchanger. This heat exchanger acts like a condenser for the lower cycle and like an evaporator for the upper cycle. Many engineering applications require cooling and heating at the same time which cannot be achieved simultaneously and efficiently by a single stage or multistage vapor compression system as a single refrigerant may not provide the required temperature lift. A cascade system is the most excellent option for these applications. A layout of the trancritical cascade system is shown in figure 1. Figures 2 and 3, show the thermodynamic processes for CO_2-propylene transcritical cascade system on P-h and T-s diagrams. It can be seen from the T-s diagram in figure 3 that the compressor work decreases and the amount of heat absorbed from the refrigerated space increases as a result of cascading. For this reason, the coefficients of performance (COP) values of cascade systems are higher than those of the ideal vapor compression system, as in the literature. It should be noted however

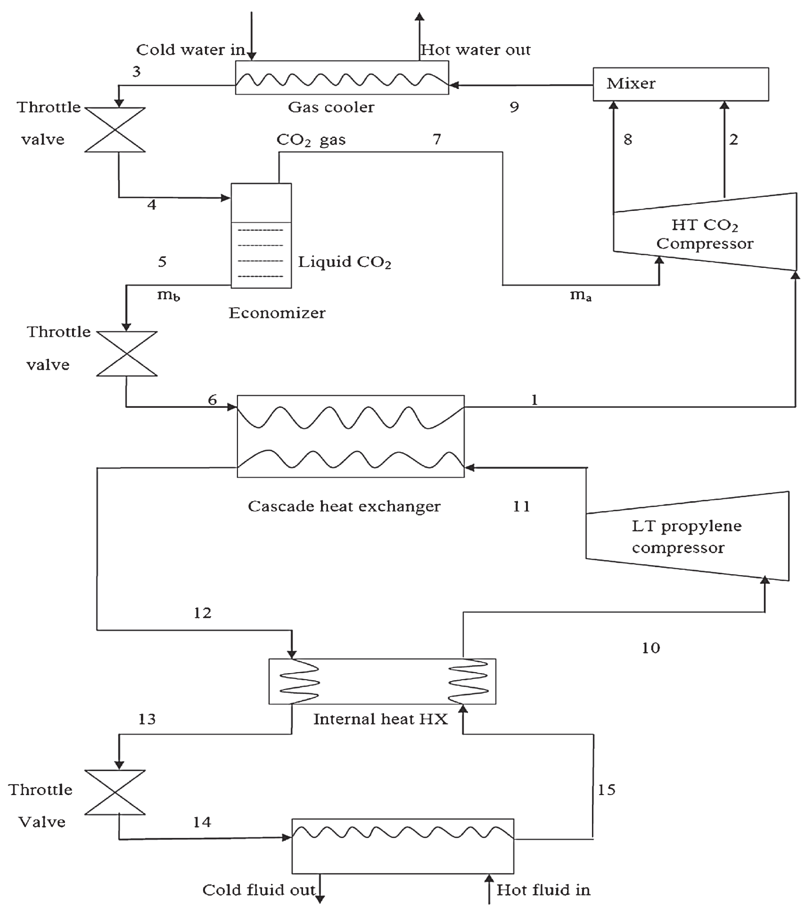

Figure 1. Equipment layout of a transcritical cascade system with economizer.

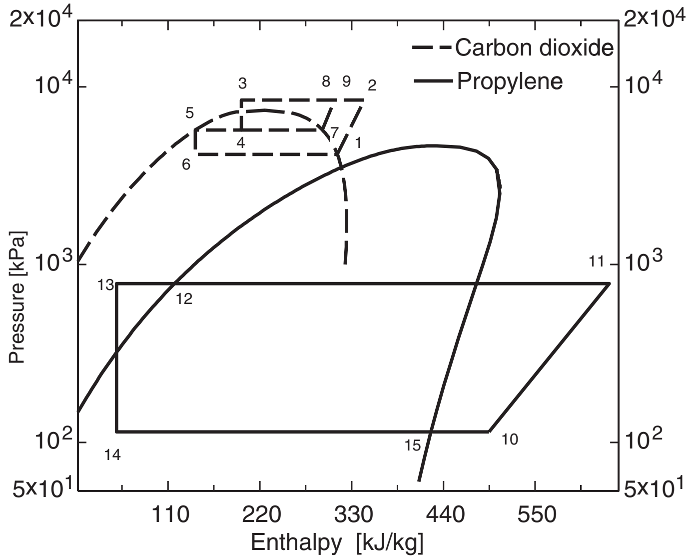

Figure 2. Thermodynamic cycle on P-h plane.

that the initial cost of the cascade system is about double, when compared to a simple vapor compression system.

Use of hydrocarbons such as propylene, propane, butane, isobutane, ethane as refrigerant in subcritical cascade high temperature, HT, circuit may be a serious concern due to its high flammability at a temperature of about 60°C. Because as per the ASHRAE Standard 34 safety

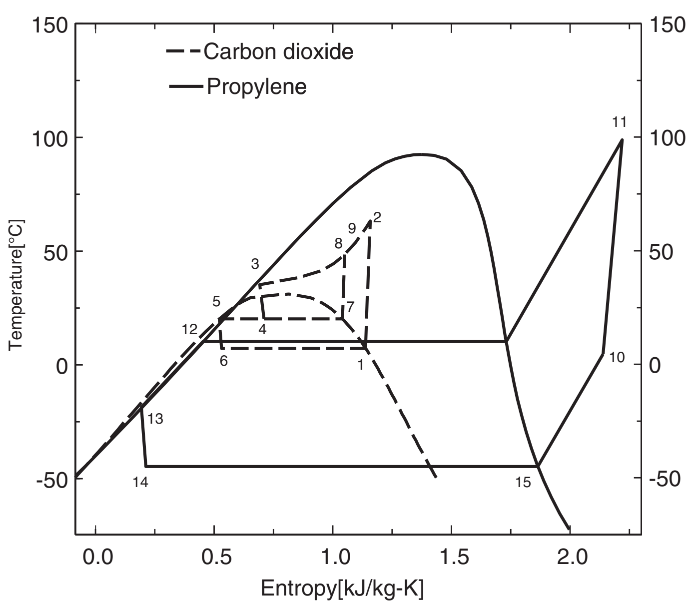

Figure 3. Thermodynamic cycle on T-s plane.

rating of all the above hydrocarbons is A3, which shows higher flammability if LFL or $ETFL_{60} \leq 100$ g/m^3 or heat of combustion (HOC) ≥ 19 MJ/kg at 60°C. The use of propylene as a refrigerant in the low temperature, LT circuit of the transcritical cascade system will prevent the ignition due to lower operating temperature. Use of CO_2 in the LT circuit of subcritical cycle limits the evaporator temperature to the triple point of CO_2 (i.e. −56.6°C). The applications requiring cooling temperature below −56.6°C are not possible with the subcritical cascade system. This issue can also be resolved by using transcritical system with CO_2 in HT circuit and propylene (triple point temperature of −185.2°C) in LT circuit. Propylene has excellent thermodynamic properties, quite similar to those of ammonia. The molar mass of 42 is ideal for turbo compressors and is only about one third of its halocarbon competitors. Propylene is cheaply and universally available. Physical properties of some of the refrigerants are presented in table 1.

The following lines are quoted from Rivera *et al* (2011) to justify the use of CO_2 instead of ammonia or hydrocarbons in the HT cycle for heating applications. As can be deduced from the literature review, CO_2 in the low cycle and NH_3 in the high cycle have been successfully used in cascade systems for cooling where temperatures as low as −323 K are required. However, as shown by Dopazo *et al* (2009) for specific conditions, the COP decreases to about 50% with the increment of the condenser temperature (from 298 to 323 K). Following that tendency, it is clear that the COP can be reduced even more at higher condenser temperatures. Another disadvantage of using ammonia in the high cycle for heating (high condenser temperatures) is the high operating pressure (e.g., P = 26.1 bar at 333 K). On the other hand, flammability is an important aspect to consider if hydrocarbons are used in cascade systems. Safety assessments and appropriate operation range are always recommended. This particular disadvantage can be controlled if the hydrocarbons are applied in the LT cycle of a cascade cycle'.

The conventional fluid pairs used in subcritical cascade systems, employing CO_2 in the LT side and ammonia, propylene, propane, ethanol and butane in the HT side are not suitable for high temperature heating applications. In the transcritical cascade cycle, gas cooler (HT heat

Table 1. Physical properties of refrigerants (http://en.wikipedia.org/wiki/Refrigerant).

	Carbon dioxide (R-744)	Ammonia (R-717)	Propane (R-290)	Propylene (R-1270)
Environmental classification	–	–	HC	HO
Molecular weight	44	17	44	42
Normal boiling point (°C)	−78.4	−33.3	−42.1	−47.7
Critical pressure (bars)	73.72	112.97	42.56	46
Critical temperature (°C)	31.1	133	96.8	94.4
Triple point (°C)	−56.6	−77.66	−187.67	−185.2
Liquid density (kg/m^3)	770	681.9	–	613.9
Critical density (kg/m^3)	467.6	225	218.5	223.39
Vapor density (kg/m^3)	1.977	0.73	2.01	1.81
Heat of vaporization (kJ/mol)	15.326	23.35	15.7	10.427
Specific heat liquid J/(mol K)	–	80.8	98.36	–
Specific heat vapour J/(mol K)	36.33	35.06	73.6	62
Ozone depletion potential (ODP)	0	0	0	0
Global warming potential (GWP)	1	0	3	>0 (smog)
ASHRAE Std 34 safety rating	A1	B2L	A3	A3

(Refrigerant Reference Guide, Fourth Edition, National Refrigerants, Inc.).

exchanger) pressure and temperature are independent of each other unlike the subcritical two-phase region. The high CO_2 vapor pressure in the gas cooler leads not only to a lower pressure ratio in compressor but also improves compressor efficiency, along with high heat-transfer coefficients and lower relative pressure losses. This also results in higher gas cooler temperatures with reasonable compressor power consumption. Therefore, the application of CO_2 in transcritical heat pumps for water heating up to 90°C can be an excellent option. Despite lower COP of the CO_2 transcritical system, it may still compete with other vapor compression systems (single stage, multistage and sub critical cascade) using other refrigerants. Sarkar *et al* (2004) have proposed NH_3–CO_2 cascade system with CO_2 in the HT side for simultaneous cooling and heating applications. In 2000, a new refrigeration technology to build cascade systems with carbon dioxide and propane as refrigerants was implemented in a small supermarket in Denmark. In the present work, carbon dioxide-propylene cascade system with CO_2 in the HT side and C_3H_6 in the LT side has been studied. This system can offer a much larger temperature lift.

A suitable selection of refrigerants in HT and LT cycles of the transcritical cascade system can provide a large temperature lift with better system efficiency. The low temperature circuit of a transcritical cascade refrigerant system can normally be charged with N_2O, propane (R290), butane (R600), R134a, ammonia, carbon dioxide and propylene (R1270) for cooling whereas, carbon dioxide (R744) is used in the high temperature circuit for heating applications. A comparison of cascade system COP using subcritical or transcritical cycle with different combinations of refrigerants under different operating conditions as reported by the researchers in the literature with the transcritical cascade system using CO_2-propylene is presented in table 2. It is observed that COP_{max} in transcritical cycle with CO_2-propylene operating under the same conditions is higher than other cascade cycles due to the superior fluid properties. Even if the heating effect is not considered, the cooling COP is higher compared to the refrigeration systems mentioned in the literature.

Table 2. Comparison of COPsys using CO_2-propylene with COP of other cascade systems.

						References		Present study	
Cooling References	Cascade system	Refrigerants	T_C (°C)	T_E (°C)	ΔT (°C)	T_{opt} (°C)	COP_{max}	T_{opt} (°C)	COP_{max}
Dopazo *et al* (2009)	Subcritical	CO_2/NH_3	29.72	−50	3.48	−20.96	0.92	9.551	1.473
Lee *et al* (2006)	Subcritical	CO_2/NH_3	30	−50	3	−20	1.21	9.082	1.473
Getu & Bansal (2008)	Subcritical	CO_2/NH_3	40	−50	5	−18	1.4	1.327	1.096
Bingming *et al* (2009)	Subcritical	CO_2/NH_3	40	−50	5	−23	1.41	1.327	1.096
Kruse & Russmann (2006)	Transcritical	CO_2/ N_2O	35	−50	−25	5	0.8	7.673	2.009
Kruse & Russmann (2006)	Transcritical	N_2O / N_2O	35	−50	−25	5	0.95	7.673	2.009
Cogeneration									
Bhattacharya *et al* (2005)	Transcritical	CO_2/ Propane	45	−40	5	5	3.31	2.143	3.245
Bhattacharya *et al* (2009)	Transcritical	CO_2/ N_2O	45	−70	5	−12	2	−7.755	2.442
Yari & Mahmoudi (2011)	Transcritical	CO_2/ CO_2	45	−25	5	0.0248	1.484	5.714	1.402*
Rivera *et al* (2011)	Transcritical	CO_2/R717, R290, R600**, R134a	35	−32	5	5	3.94	22.27	4.364

*cooling COP_{max}

**Cascade system using butane (R600) gives maximum COP

The main drawback of carbon dioxide as a refrigerant is its inherent high working pressure, which is much higher than that of other natural and synthetic refrigerants. Since carbon dioxide offers a much higher volumetric capacity, the problem of higher working pressure can be overcome by using optimal design involving compact and stronger components.

Dopazo *et al* (2009) performed a theoretical analysis of a CO_2/NH_3 cascade refrigeration system for cooling applications at low temperatures. Lee *et al* (2006) performed a thermodynamic analysis of optimal condensing temperature of cascade-condenser in CO_2/NH_3 subcritical cascade refrigeration systems. Getu & Bansal (2008) have analysed carbon dioxide-ammonia subcritical cascade system thermodynamically to find out the optimum condensing temperature of CO_2 using different refrigerants such as NH_3, propane, propylene and ethanol in the high temperature circuits. Bingming *et al* (2009) designed and developed an experimental CO_2/NH_3 subcritical cascade refrigeration system using screw compressor. Ma *et al* (2005) measured the performance of a subcritical cascade CO_2–NH_3 cycle using an expander as a replacement for the expansion valve. Dopazo & Seara (2011) experimentally investigated a subcritical cascade refrigeration system with CO_2/NH_3. Yu *et al* (2014) presented a CO_2/NH_3 cascade refrigeration system, in which a falling film evaporator–condenser was used as the cascade heat exchanger. The thermodynamic analysis results of the proposed system show an improvement in the coefficient of performance (COP) due to the smaller temperature difference provided by this type of cascade heat exchanger.

Kruse & Russmann (2006) performed a thermodynamic analysis of cascade refrigerating systems using N_2O as refrigerant for LT stage and compared it with an R23–R134a LT–HT cycle, and concluded that the transcritical carbon dioxide HT cycle with nitrous oxide LT cycle cascade system had a lower COP. A carbon dioxide-propane transcritical cascade system was analysed by Bhattacharyya *et al* (2005), to determine an optimum cascade evaporating temperature of CO_2 in the high temperature circuit for simultaneous heating and cooling applications. Another transcritical cascade system was investigated by Bhattacharyya *et al* (2009) using N_2O for the low temperature cascaded stage and carbon dioxide for the high temperature stage, to determine an optimum cascade evaporating temperature of CO_2 in the HT circuit for cooling applications. Yari & Mahmoudi (2011) presented two new and high performance configurations for ejector-expansion TRCC cascade cycles and their performances were discussed theoretically based on first and second laws of thermodynamics.

Subcritical cascade cycle is useful for cooling applications only because HT condenser temperature ranges between 30°C and 40°C, which is unsuitable for heating purposes. Thus, subcritical cycle has lower COP and temperature lift, hence lower energy efficiency.

Wang *et al* (2013b) conducted an experimental research on a prototype air-source transcritical CO_2 heat pump water heater, in a range of ambient temperatures (15 to 35°C) and water outlet temperatures (55 to 80°C) and illustrated the effect of water outlet temperatures, ambient temperatures and evaporating temperatures on the optimal discharge pressure and system COP. Cecchinato & Corradi (2011) developed an R744 commercial single door bottle cooler that was cost affective and matched with the performance of typical cost optimized R404A and R134a systems. Zhang *et al* (2010) conducted experimental and simulation researches to investigate the relationships between optimum heat rejection pressure and other related operating parameters for a transcritical CO_2 heat pump system with two throttle valves. Wang *et al* (2013a) investigated the effect of geometrical parameters of gas coolers on system performance and the optimal discharge pressure with a developed and experimentally validated numerical model of an air-source transcritical CO_2 heat pump water heater system. Gu & Chen (2005) performed a detailed analysis on the relationship between the optimum high pressure and other systematic parameters for a CO_2 transcritical refrigeration system with internal heat exchangers. Sarkar (2009) designed and developed a working prototype of a transcritical CO_2 heat pump system for simultaneous cooling and heating of water based on numerical simulation studies. Carolina (2012) designed and built a transcritical CO_2 heat pump system that combined a water-to-water CO_2 heat pump with both hot and cold thermal storages known as Thermal Battery (TB).

Different researchers adopted various techniques to enhance the performance of simple transcritical CO_2 cycle. Kim & Kim (2002) carried out simulation and experimental studies on the performance of an autocascade refrigeration system using carbon dioxide as a refrigerant. Cho *et al* (2005) studied the effect of refrigerant charge amount on the performance of a transcritical CO_2 heat pump and compared it with that of other refrigerant systems. Li & Groll (2005) predicted that the COP of a transcritical CO_2 refrigeration cycle could be enhanced by 16% by means of an ejector expansion device. Baek *et al* (2005) investigated the performance improvement of a transcritical CO_2 cycle by using piston–cylinder work producing expansion device.

Groll *et al* (2002) studied the effect of pressure ratios across compressors on the performance of the transcritical CO_2 cycle with two-stage compression and intercooling. Robinson & Groll (1998) evaluated and compared the efficiencies of transcritical CO_2 cycles with and without an expansion turbine. Kim *et al* (2004) presented various methods for performance enhancement in a transcritical CO_2 cycle and its applications.

A fluid such as CO_2 with a lower critical temperature of 31.1°C will tend to have a higher volumetric capacity and a lower COP in the high temperature cycle of the transcritical cascade system due to high levels of irreversibility involved. Parallel compression offers the possibility to increase cooling capacities and efficiencies of medium temperature systems during peak load operation and high ambient temperatures. Parallel compression economization is one of the promising cycle modifications where refrigerant vapor is compressed to a super critical discharge pressure in two separate non-mixing streams, one coming from the economizer and the other coming from the cascade heat exchanger to improve the performance of the transcritical CO_2 refrigeration cycle. Use of parallel compression reduces gas cooler outlet temperature and discharge pressure along with the amount of flash gas inside the receiver. This leads to a lower medium pressure in the system resulting in higher enthalpy differences in evaporator side and increased refrigeration capacities. Increased cooling demand can be catered with the use of this technique through capacity regulation. Consequently, parallel compression is redundant with lower ambient conditions.

Very few attempts have been made to improve the performance of the transcritical cascade system. In this present paper, an attempt has been made to carry out the thermodynamic analysis of propylene–carbon dioxide transcritical cascade system employing parallel compression economization in the HT circuit in order to improve the system performance.

2. System description

A schematic diagram of the transcritical cascade system is shown in figure 1. The cascade refrigeration system is constituted by two single stage systems connected by a cascade heat exchanger, where propylene is used in LT cycle for cooling and CO_2 is used in HT cycle to condense propylene. The waste heat of HT cycle can be used for heating purposes in order to improve the system performance. Propylene evaporator absorbs the heat $\dot{Q}_L$ from the cooling space at temperature T_E. Propylene is then compressed in propylene compressor and condensed in the cascade heat exchanger at temperature T_{12} before it enters into the evaporator after passing through the expansion valve in order to complete the cycle.

In the HT circuit heat $\dot{Q}_H$ is removed from CO_2 gas cooler at gliding temperature varying from T_9 to T_c by an external cooling medium. The liquid and vapour are separated in the economizer after the expansion of transcritical fluid from states 3 to 4 in primary expansion valve. The liquid from the separator is further expanded in another expansion valve to provide cooling effect in the cascade heat exchanger. The saturated vapor from the evaporator and economizer is compressed in the compressor simultaneously to state 2 and 8 respectively. The mixed stream of state 9 enters the gas cooler for heat rejection. Figures 2 and 3 show the thermodynamic processes for CO_2-propylene transcritical cascade system on P-h and T-s diagrams along with their saturation lines.

3. Thermodynamic modelling

The cycle is modelled detailing each individual process of the cycle. Steady flow energy equation and mass balance equation has been employed. A parametric study at various gas cooler exit temperatures (32°C, 35°C, 38°C, 41°C and 44°C), evaporating temperatures (−35°C, −40°C, −45°C, −50°C and −55°C) and for various temperature differences (3°C, 4°C, 5°C, 6°C, and 7°C) across the cascade heat exchanger is conducted to determine the optimum condensing temperature of a cascade heat exchanger in CO_2-propylene cascade refrigeration system. An optimum temperature of 20°C was selected at the economizer inlet as it resulted in maximum COP.

The thermodynamic analysis of the two-stage cascade refrigeration system was performed based on the following general assumptions.

(i) Adiabatic and irreversible compression with an isentropic efficiency of 0.8 for both high and low temperature compressors.
(ii) Refrigerants at the cascade heat exchanger and economizer outlet for HT cycle and evaporator for LT cycle outlet are saturated.
(iii) All the heat released by the low-temperature circuit condenser is rejected to the high-temperature circuit evaporator.
(iv) Negligible pressure and heat losses in the pipe networks or system components.
(v) Heat transfer processes in cascade heat exchanger, evaporator, economizer and gas cooler are isobaric.
(vi) Heat transfer in cascade heat exchanger, evaporator, economizer and gas cooler with the ambient is negligible.

Compressor power consumption for HT cycle can be formulated as

$$\dot{W}_{HT} = \dot{m}_b (h_2 - h_1) + \dot{m}_a (h_8 - h_7) . \tag{1}$$

Mass balance in the mixer is:

$$\dot{m}_c = \dot{m}_b + \dot{m}_a. \tag{2}$$

Energy balance in the mixer is:

$$\dot{m}_c h_9 = \dot{m}_a h_8 + \dot{m}_b h_2. \tag{3}$$

The rate of heat transfer from the HT gas cooler is expressed as:

$$\dot{Q}_H = \dot{m}_c (h_9 - h_3) . \tag{4}$$

Mass balance in the economizer is:

$$\dot{m}_c = \dot{m}_b + \dot{m}_a. \tag{5}$$

Energy balance in the economizer is given as:

$$\dot{m}_c h_4 = \dot{m}_a h_7 + \dot{m}_b h_5. \tag{6}$$

The mass flow ratio can be derived from energy balance at the cascade condenser:

$$\dot{m}_b (h_1 - h_6) = \dot{m}_p (h_{11} - h_{12}). \tag{7}$$

The isentropic efficiency of the HT CO_2 compressor can be represented as:

$$\eta_c = \frac{(h_{2s} - h_1)}{(h_2 - h_1)}. \tag{8}$$

The isentropic efficiency of the LT CO_2 compressor can be represented as:

$$\eta_c = \frac{(h_{8s} - h_7)}{(h_8 - h_7)}. \tag{9}$$

Compressor power consumption for LT cycle can be formulated as:

$$\dot{W}_{LT} = \dot{m}_p (h_1 - h_{10}). \tag{10}$$

The rate of heat absorbed by the LT evaporator is defined by:

$$\dot{Q}_L = \dot{m}_c (h_{15} - h_{14}). \tag{11}$$

Energy balance in the LT internal heat exchanger can be formulated as:

$$(h_{10} - h_{15}) = (h_{12} - h_{13}). \tag{12}$$

The isentropic efficiency of the LT compressor can be formulated as:

$$\eta_c = \frac{(h_{11s} - h_{10})}{(h_{11} - h_{10})}. \tag{13}$$

The expression for effectiveness of the LT internal heat exchanger is represented by:

$$\varepsilon = \frac{(T_{10} - T_{15})}{(T_{12} - T_{15})}. \tag{14}$$

The COP of heating and that of cooling can be defined as:

$$COP_{LTcooling} = \frac{\dot{Q}_L}{\dot{W}_{LT}}; COP_{HTheating} = \frac{\dot{Q}_h}{\dot{W}_{HT}}. \tag{15}$$

Equation for the overall COP of the system is expressed as

$$COP_{sys} = \frac{\dot{Q}_L + \dot{Q}_H}{\dot{W}_{LT} + \dot{W}_{HT}}. \tag{16}$$

4. Results

Engineering equation solver version 6.883 has been used for performing calculations, plotting graphs and regression analysis. COP_{max} is the maximum coefficient of performance of the system at intermediate temperature for given T_C, T_E and ΔT and that intermediate temperature is called T_{opt} (for maximum COP). The parameters that have been assumed constant for the computation of results are mentioned below.

a) Low temperature cycle condensing temperature, $T_5 = 7°C$.
b) Low temperature cycle evaporating temperature, $T_E = -45°C$.
c) Effectiveness of cascade heat exchanger, evaporator and condenser = 1.
d) The economizer inlet temperature, $T_4 = 20°C$.
e) High temperature cycle gas cooler outlet temperature, $T_C = 45°C$.
f) Temperature difference in cascade condenser, $\Delta T = 3°C$.

The parameters which have been varied for the the calculation of results are given below.

a) The cascade condenser temperature difference $\Delta T = 0$ to 20°C.
b) The low temperature cycle condenser temperature is varied from $T_5 = -3$ to 15°C.
c) The economizer inlet temperature is varied from $T_4 = 0°$ to 30°C.
d) The low temperature cycle evaporator temperature is varied from $T_E = -55°C$ to $-25°C$.
e) The high temperature cycle gas cooler outlet temperature is varied from $T_C = 30°C$ to 45°C.
f) The effectiveness of the LT internal heat exchanger is varied from $\varepsilon = 0.5$ to 1.
g) Mass flow rate of refrigerant for HT cycle is assumed as 1 kg/sec.

Figure 4 shows the effect of the condensing temperature T_c on COP_{sys}, COP_{HT} and COP_{LT}. The figure shows that increasing T_c reduces COP_{sys} and COP_{HT} while COP_{LT} remains constant. Rise in condenser temperature increases compressor work of HT circuit due to increase in

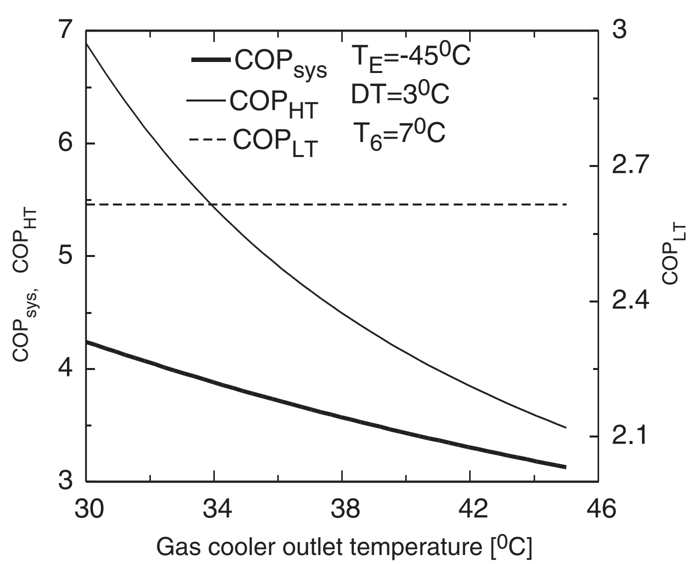

Figure 4. Influence of T_C on COP of system.

pressure ratio across HT circuit and hence total work done by system increases while COP decreases. Theoretically there is no effect of T_c on refrigerating effect of system. Figure 5 represents the effect of the evaporating temperature T_E on COP_{sys}, COP_{HT} and COP_{LT}. The figure shows that increasing T_E increases COP_{sys} and COP_{LT} while COP_{HT} remains constant. There is a decrease in pressure ratio across low temperature circuit with and increase in evaporator temperature. This results in reduced total compressor work by system and increased system refrigerating effect.

Figure 6 shows the effect of gas cooler outlet temperature T_c on COP_{max} and T_{opt}. The optimal intermediate condensing temperature T_{opt} ensures the maximum system COP under the given conditions. In the case of the maximum system COP, the overall system configuration is dependent on the relation between the mass flow rate in HTC and mass flow rate in LTC. The figure shows that increasing T_c decreases both COP_{max} and $T_{opt.}$ figure 7 shows the effect of the intermediate temperature T_6 on COP_{sys}, COP_{HT} and COP_{LT}. As T_6 rises, temperature lift for the LT propylene cycle increases whereas that for the HT carbon dioxide cycle lowers leading to reduction in COP_{LT} and improvement in COP_{HT}. These two opposing phenomena cause a marginal initial increase and then a reduction in COP_{sys} with an optimum value of COP. Figure 8 represents the effect LT internal heat exchanger effectiveness on COP_{sys}. Increase in IHX effectiveness (ε) causes more efficient heat exchange between the cold and hot fluid leading to reduction in both refrigeration capacity and LT compressor power input resulting in a marginal increase in COP_{sys} and COP_{LT} while COP_{HT} remains constant.

Figure 9 represents the effect of temperature differences in the cascade condenser ΔT on COP_{sys}, COP_{HT} and COP_{LT}. Increasing ΔT reduces COP_{sys} and COP_{LT} while COP_{HT} remains constant. This is intuitive as increase in temperature difference causes heat transfer to occur through finite temperature difference resulting in external irreversibility in the system thereby decreasing the system performance. Theoretically, there is no effect of ΔT on refrigerating effect of system but work of compressor increases with increase in ΔT. The figure 10 shows that with

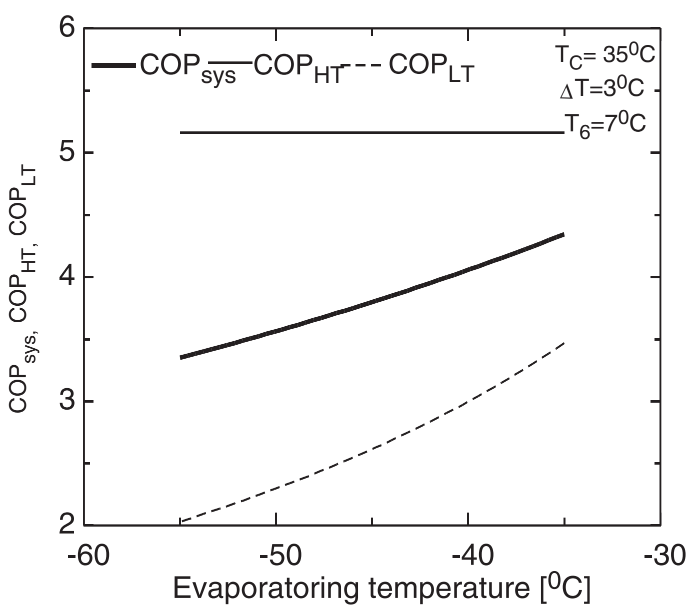

Figure 5. Influence of T_E on COP of system.

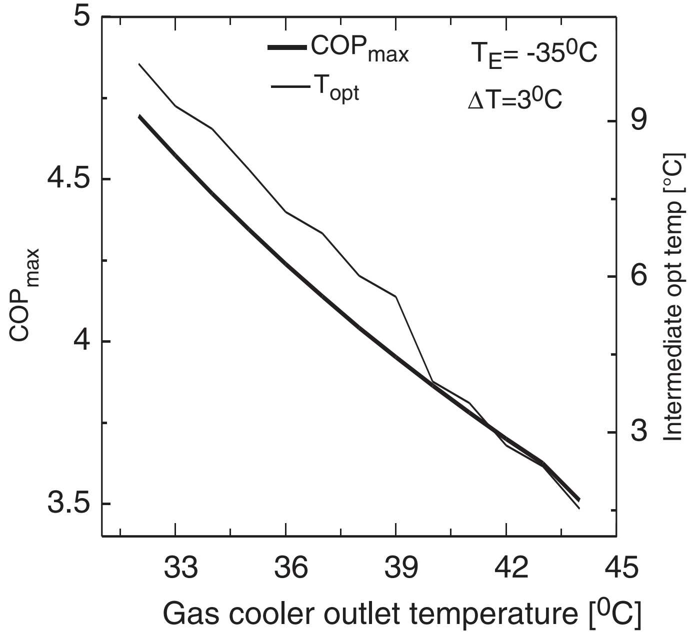

Figure 6. Influence of T_C on COP_{max} and T_{opt}.

increasing T_4, COP_{sys} first increase attains an optimum value (at 20°C) and then decreases. The changes in the mass flow ratio of R1270 to that of R744 (see figure 11) were investigated at identical temperatures. The mass flow ratio of R1270 to that of R744 mass ratio decreased

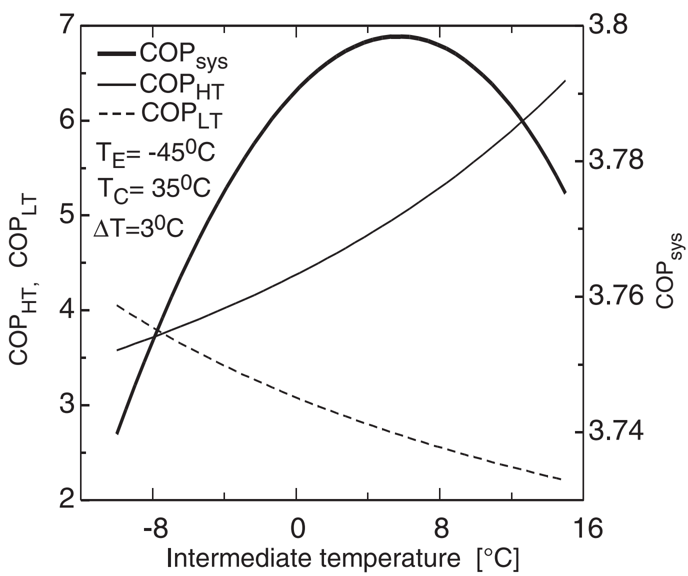

Figure 7. Influence of T_5 on COP of system.

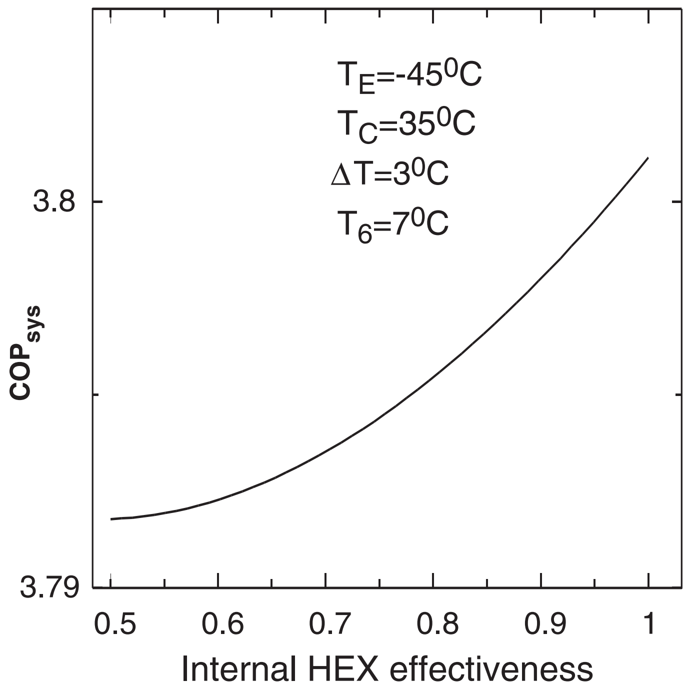

Figure 8. Variation of system COP with ε.

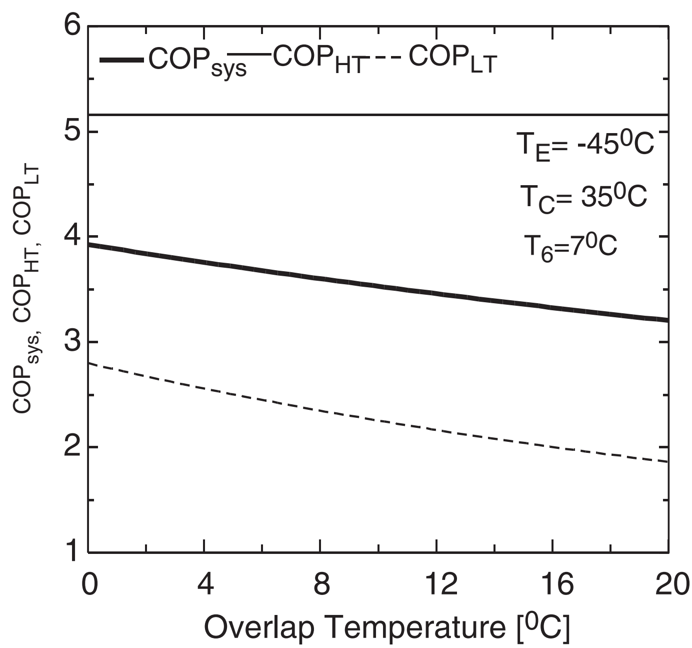

Figure 9. Variation of system COP with ΔT.

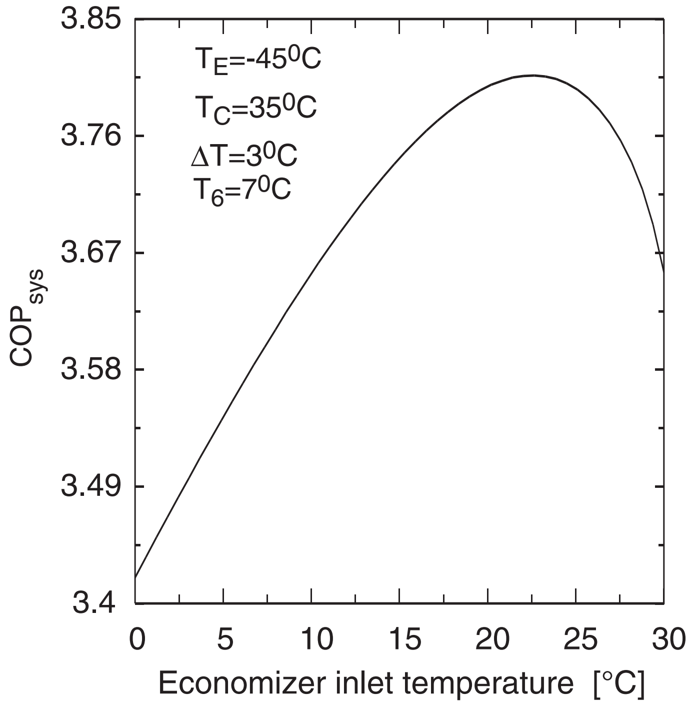

Figure 10. Influence of T_4 on COP of system.

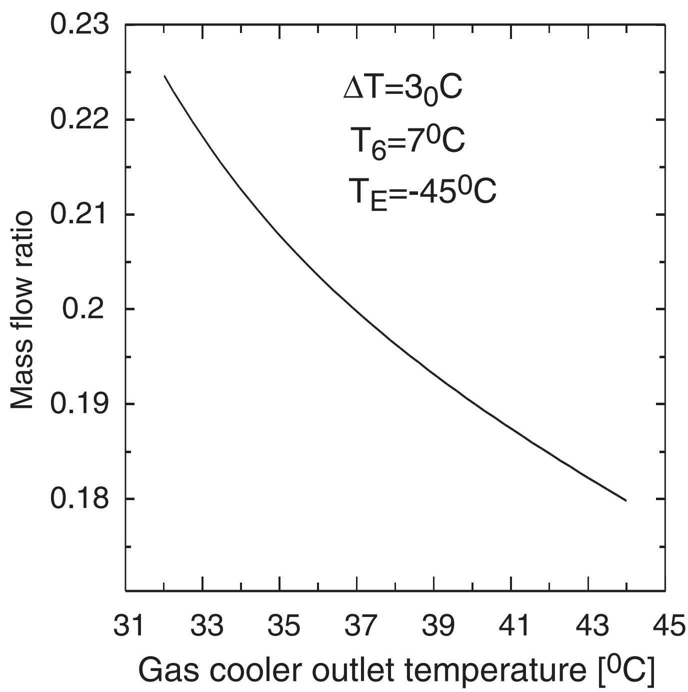

Figure 11. Influence of T_C on Mass flow ratio.

with a rise in condensing temperature, cascade evaporating temperature (T_5) and temperature difference in cascade condenser (ΔT) of the system. However, it increased with an increase in evaporating temperature of R1270 (T_E).

If propylene is replaced by propane in the above transcritical cascade system the COP_{sys} is 3.262. This value of COP is in agreement with the value calculated by Bhattacharyya *et al* (2009). A slightly higher value (3.31) of COP_{sys} was a result of the use of a better optimization technique. The values of other parameters and the trends obtained in the graphs are almost the same. This fact validates the present model.

5. Optimization

Analysis has been carried out to study the effects of T_C, T_E, and ΔT on COP and performance parameters of carbon dioxide–propylene cascade system. In the analysis, five evaporator temperatures ($T_E = -35°C, -40°C, -45°C, -50°C$ and $-55°C$), five gas cooler exit temperatures (32° to 44°C at an interval of 3 each) and five cascade heat exchanger temperatures ($\Delta T = 3°C$, 4°C, 5°C, 6°C, 7°C) were considered and total 125 data points were obtained. The results of analysis for ΔT of 3°C are presented in table 3.These input data are used to develop mathematical equations for optimum performance parameters with multi linear regression method inbuilt in engineering equation solver (EES commercial version 6.883).

Performance parameters of cascade transcritical system, optimum evaporating temperature of the HT CO_2 circuit (T_{opt}), the optimum mass flow ratio of R1270 to that of R744 ($(\dot{m}_L/\dot{m}_H)_{opt}$) and the maximum coefficient of performance (COP_{max}) were regressed as a function of input operating parameters such as evaporating temperature (T_E), gas cooler outlet temperature (T_c) and temperature differences in cascade heat exchanger (ΔT). Regression equations for optimum cascade evaporating temperatures, maximum COP and the optimum mass flow ratio $\dot{m}_L/\dot{m}_H$ opt for R744-R1270 combination are presented as follows.

$$T_{opt} = a_o + a_1 \cdot T_c + a_2 T_E + a_3 \Delta T \tag{17}$$

$$COP_{max} = a_o + a_1 \cdot T_c + a_2 T_E + a_3 \Delta T \tag{18}$$

$$\left(\frac{\dot{m}_L}{\dot{m}_H}\right)_{opt} = a_o + a_1 T_c + a_2 T_E + a_3 \Delta T. \tag{19}$$

Table 3. COP under various design parameters ($\Delta T = 3°$).

Data Point No.	1	2	3	4	5	1	2	3	4	5	1	2	3
T_E(°C)	−55	−55	−55	−55	−55	−50	−50	−50	−50	−50	−45	−45	−45
T_C(°C)	32	35	38	41	44	32	35	38	41	44	32	35	38
T_{opt}(°C)	3.878	1.02	−1.837	−3.878	−5.102	5.102	2.245	1.429	−1.837	−3.469	6.327	3.878	1.837
COP	3.546	3.354	3.182	3.028	2.889	3.787	3.565	3.369	3.193	3.036	4.055	3.798	3.572
$\dot{m}_L/\dot{m}_H$	0.1972	0.1923	0.1863	0.2258	0.2129	0.2041	0.1964	0.1899	0.2286	0.2166	0.2085	0.2013	0.1956

Data Point No.	4	5	1	2	3	4	5	1	2	3	4	5
T_E(°C)	−45	−45	−40	−40	−40	−40	−40	−35	−35	−35	−35	−35
T_c(°C)	41	44	32	35	38	41	44	32	35	38	41	44
T_{opt}(°C)	0.6122	-0.612	9.184	6.327	3.878	2.245	0.3061	10.1	8.061	6.02	3.571	1.531
COP	3.373	3.195	4.357	4.057	3.796	3.568	3.366	4.696	4.345	4.043	3.781	3.551
$\dot{m}_L/\dot{m}_H$	0.2349	0.2213	0.2125	0.2066	0.2009	0.1972	0.1923	0.1863	0.2258	0.2129	0.2041	0.1964

Table 4. Statistical information for Eqs. (17)–(19).

	Linear regression coefficients for T_{opt}		Linear regression coefficients for $(\dot{m}_L/\dot{m}_H)$opt		Linear regression coefficients for COP_{max}	
	Value	Standard error	Value	Standard error	Value	Standard error
a_0	43.60742	0.9004335	0.3584827	0.002377847	8.293449	0.06815297
a_1	−0.6778426	0.01843947	−0.002787539	0.00004869458	−0.06983775	0.001395666
a_2	0.3321077	0.01106368	0.001018498	0.00002921675	0.04255282	0.0008373998
a_3	−0.1336481	0.05531841	−0.0004375918	0.0001460838	−0.03546358	0.004186999
Number of points (n)=125			Number of points (n)=125		Number of points (n)=125	
rms = 0.86734			rms = 0.0022905		rms = 0.065648	
R^2 = 94.95%			R^2 = 97.4%		R^2 = 97.73%	

The linear regression coefficients a_0, a_1, a_2 and a_3 along with other statistical indicators such as standard error, root mean square error and correlation coefficient (R^2) are given in table 4.

It is important to mention that the compressor discharge pressure P_2 (in kPa) has been optimized to yield maximum COP as given by Sarkar & Agrawal (2010):

$$P_{2opt} = 3687.7 + 38.23 \times T_c - 0.004 \times T_6 + 2.7667 \times T_c^2. \tag{20}$$

6. Conclusions

Thermodynamic analysis of a CO_2–propylene based transcritical cascade system has been carried out in the present study using Engineering equation solver commercial version 6.883. In this study, variation of three important design parameters i.e., gas cooler outlet temperature T_C, evaporating temperature T_E and overlap temperature in cascade heat exchanger are considered in order to determine system COP, optimum temperature in cascade heat exchanger and optimum mass flow ratio of LT and HT cycles. The following inferences are drawn from the study.

(i) An improvement in COP of 9% has been estimated by the use of economizer.
(ii) When the intermediate temperature is higher, it is difficult to get the desired evaporator temperature T_E. The results of the analysis suggest keeping the intermediate temperature lower for getting optimum performance of the system. This is done by decreasing gas cooler outlet temperature.
(iii) The overall increase in system COP due to a more effective internal heat exchanger is marginal, hence the use of internal heat exchanger is not recommended.
(iv) The optimum discharge pressure is lowered significantly by the use of economizer in the HT cycle. There is a considerable effect of the gas cooler exit temperature on optimum discharge pressure.
(v) COP of the system improves considerably by the use of economizer at lower evaporator temperature.
(vi) The maximum COP increases with increase in T_E, but decreases as T_c or ΔT increases.
(vii) Regression equations have been developed for T_{opt}, COP_{max} and optimum mass flow ratio to help thermal engineers to design an optimized transcritical cascade system.

Nomenclature

h	specific enthalpy (kJ kg^{-1})	$\dot{Q}$	heat transfer rate (kW)
$\dot{m}_H$	mass flow rate of CO_2(kg s^{-1})	COP	coefficient of performance
$\dot{W}_{HT}$	HT compressor power (kW)	$\dot{m}_L$	mass flow rate of propylene (kg s^{-1})
T	temperature (K)	$\dot{W}_{LT}$	LT compressor power (kW)
ε	effectiveness of internal heat exchanger	η_c	compressor isentropic efficiency
IT	Intermediate temperature	s	entropy
1-6	points of refrigerant (HT side)	7-15	points of refrigerant (LT side)
C	Carbon dioxide	ΔT	overlap temperature
P	pressure	H	Heating
L	cooling	LT	low temperature cycle
HT	high temperature cycle	max	Maximum
opt	optimum	prop	Propylene

References

Baek J S, Groll E A and Lawless P B 2005 Piston–cylinder work producing expansion device in a carbon dioxide cycle. Part II: Theoretical investigation. *Int. J. Refrig.* 28: 152–64

Bhattacharyya S, Garai A and Sarkar J 2009 Thermodynamic analysis and optimization of a novel N_2O–CO_2 cascade system for refrigeration and heating. *Int. J. Refrig.* 32: 1077–84

Bhattacharyya S, Mukhopadhyay S, Kumar A, Khurana R K and Sarkar J 2005 Optimization of a CO_2–C_3H_8 cascade system for refrigeration and heating. *Int. J. Refrig.* 28: 1284–92

Bingming W, Huagen W, Jianfeng L and Ziwen X 2009 Experimental investigation on the performance of NH_3/CO_2 cascade refrigeration system with twin-screw compressor. *Int. J. Refrig.* 32(135865)

Carolina C 2012 Energy Optimization for Transcritical CO_2 Heat Pumps for Combined Heating and Cooling and Thermal Storage Applications, International Refrigeration and Air Conditioning Conference, Paper 1168, Purdue, USA

Cecchinato L and Corradi M 2011 Transcritical carbon dioxide small commercial cooling applications analysis. *Int. J. Refrigeration* 34(1): 50–62

Cho H, Ryu C, Kim Y and Kim H Y 2005 Effects of refrigerant charge amount on the performance of a transcritical CO_2 heat pump. *Int. J. Refrig.* 28: 1266–73

Dopazo J A and Seara J F 2011 Experimental evaluation of a cascade refrigeration system prototype with CO_2 and NH_3 for freezing process applications. *Int. J. Refrig.* 34: 257–267

Dopazo J A, Seara J F, Sieres J and Uhía F J 2009 Theoretical analysis of a CO_2–NH_3 cascade refrigeration system for cooling applications at low temperatures. *Appl. Therm. Eng.* 29: 1577–83

Getu H M and Bansal P K 2008 Thermodynamic analysis of an R744–R717 cascade refrigeration system. *Int. J. Refrig.* 31: 45–54

Groll E A, Baek J S and Lawless P B 2002 Effect of pressure ratios across compressors on the performance of the transcritical CO_2 cycle with two-stage compression and intercooling. Proceedings of compressor engineering conference, Purdue, USA, 11–7

Gu J and Chen Y 2005 The optimum high pressure for CO_2 transcritical refrigeration systems with internal heat exchangers. *Int. J. Refrigeration* 28: 1238–1249

Kim M, Pettersen J and Bullard C W 2004 Fundamental process and system design issues in CO_2 vapor compression systems. *Prog. Energy Combust. Sci.* 30: 119–74

Kim S G and Kim M S 2002 Experiment and simulation on the performance of an autocascade refrigeration system using carbon dioxide as a refrigerant. *Int. J. Refrig.* 25: 1093–101

Kruse H and Russmann H 2006 The natural fluid nitrous oxide –an option as substitute for low temperature synthetic refrigerants. *Int. J. Refrig.* 29(5): 799–806

Lee T S, Liu C H and Chen T W 2006 Thermodynamic analysis of optimal condensing temperature of cascade-condenser in CO_2/NH_3 cascade refrigeration systems. *Int. J. Refrig.* 29: 1100–1108

Li D and Groll E A 2005 Transcritical CO_2 refrigeration cycle with ejector-expansion device. *Int. J. Refrig.* 28: 766–73

Ma Y T, Ning J H, Chen Q and Ma L R 2005 Study on replacement and energy-saving of natural refrigerant CO_2/NH_3 cascade refrigeration cycle. *Proceedings of the International Conference on Compressor and Refrigeration*, Dalian, China

Refrigerant Reference Guide, Fourth Edition, National Refrigerants, Inc.

Rivera W, Colorado D and Hernández J A 2011 Comparative study of a cascade cycle for simultaneous refrigeration and heating operating with ammonia, R134a, butane, propane, and CO_2 as working fluids. *Int. J. Sustainable Energy* 31(6): 365–381

Robinson D M and Groll E A 1998 Efficiencies of transcritical CO_2 cycles with and without an expansion turbine. *Int. J. Refrig.* 21(7): 577–89

Sarkar J, Bhattacharyya S and Ramgopal M 2004 Carbon dioxide based cascade systems for simultaneous refrigeration and heating applications. *Proceedings of 6th IIR-Gustav Lorentzen natural working fluid conference*, Galsgow 2004

Sarkar J and Agrawal N 2010 Performance optimization of transcritical CO_2 cycle with parallel compression economization. *Int. J. Thermal Sci.* 49: 838–843

Sarkar J 2009 A transcritical CO_2 heat pump for simultaneous water cooling and heating: Test results and model validation. *Int. J. Energy Research* 33(1): 100–109

Wang S, He Y, Tuo H, Cao F and Xing Z 2013a Effect of heat transfer area and refrigerant mass flux in a gas cooler on heating performance of air-source transcritical CO_2 heat pump water heater system. *Energy and Buildings* 67: 1–10

Wang S, Tuo H, Cao F and Xing Z 2013b Experimental investigation on air-source transcritical CO_2 heat pump water heater system at a fixed water inlet temperature. *Int. J. Refrigeration* 36(3): 701–716

Yari M and Mahmoudi S M S 2011 Thermodynamic analysis and optimization of novel ejector-expansion TRCC (transcritical CO_2) cascade refrigeration cycles (Novel transcritical CO_2 cycle). *Energy* 36(12): 6839–6850

Yu J, Ma M, Yu J and Wang X 2014 Performance evaluation and optimal configuration analysis of a CO_2/NH_3 cascade refrigeration system with falling film evaporator–condenser. *Energy Conversion and Management* 79: 224–231

Zhang X P, Fan X W, Wang F K and Shen H G 2010 Theoretical and experimental studies on optimum heat rejection pressure for a CO_2 heat pump system. *Appl. Thermal Eng.* 30(16): 2537–2544