Give-and-take based peer-to-peer content distribution networks

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MS received 23 April 2013; revised 12 February 2014; accepted 25 March 2014

Abstract. Content Distribution Networks (CDNs) are widely used to distribute data to large number of users. Traditionally, content is being replicated among a number of surrogate servers, leading to high operational costs. In this context, Peer-to-Peer (P2P) CDNs have emerged as a viable alternative. An issue of concern in P2P networks is that of *free riders*, i.e., selfish peers who download files and leave without uploading anything in return. Free riding must be discouraged. In this paper, we propose a criterion, the *Give-and-Take (G&T) criterion*, that disallows free riders. Incorporating the G&T criterion in our model, we study a problem that arises naturally when a new peer enters the system: viz., the problem of downloading a ‘universe’ of segments, scattered among other peers, at low cost. We analyse this $\mathcal{NP}$ -hard problem, and characterize the optimal download cost under the G&T criterion. We propose an optimal algorithm, and provide a sub-optimal algorithm that is *nearly optimal*, but runs much more quickly; this provides an attractive balance between running time and performance. Finally, we compare the performance of our algorithms with that of a few existing P2P downloading strategies in use. We also study the computation time for prescribing the strategy for initial segment and peer selection for the newly arrived peer for various existing and proposed algorithms, and quantify cost–computation time trade-offs.

Keywords. Peer-to-peer networks; algorithms.

1. Introduction

A Content Distribution Network (CDN) contains copies of data placed at various surrogate servers in the network so that users across the world can access the content (figure 1). The problem

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of *scaling* CDNs with the number of users is an important issue (Vakali & Pallis 2003). Proposed solutions were observed to be economically infeasible (Choffnes & Bustamante 2008; Plissonneau et al 2005).

The peer-to-peer network emerged as an alternative for single source data networks that can work over low bandwidth links. Compared to traditional content distribution schemes, peer-to-peer networks permit significant reduction in the distributor's hardware, bandwidth resource costs and reduce the dependence on the original distributor (Menasché et al 2010), as these costs are shared among users who desire to download files.

In a peer-to-peer content distribution network (P2PCDN), each user downloads segments of files from other users (mostly) and is expected to upload to other users as well (figure 2). The peers are also encouraged to exchange among themselves as the bandwidth cost at the surrogate server is usually high (Plissonneau et al 2005).

However, most practical P2PCDN systems face difficulties because of *free riders*. These are selfish peers who download files, uploading negligible data or nothing in return (Feldman & Chuang 2005; Feldman et al 2006; Typpi 2009; Karakaya et al 2009). P2P networking is based on the idea of willing participation of peers, and free riders violate this principle. For effective operation of P2PCDNs, free riding must be discouraged.

Various policies and methods have been proposed (Feldman et al 2006; Feldman & Chuang 2005; Locher et al 2006; Rahman et al 2010; Nishida & Nguyen 2010; Karakaya et al 2009) to discourage free riding. All these policies have weaknesses that can be exploited. For example, in the whitewashing attack (Feldman et al 2006), a peer assumes different identities, and pretending to be a new peer downloads some segments without uploading anything in return. Over time, a malicious user can accumulate substantial amount of data by exploiting the system. Peers can also falsify their reputation values and spoof identities for their benefit (Karakaya et al 2009).

In this paper, we propose the *Give-and-Take criterion (G&T)*, which prohibits free riding. Any existing policy can be augmented to incorporate the G&T criterion. Essentially, the idea is that a peer *A* can download segments from another peer *B* if and only if *A* offers *at least*

Figure 1. Content distribution network.

Figure 2. Peer-to-peer content distribution network (enabling exchange between peers decreases the load on surrogate server which allows more users to connect to surrogate server).

one segment for B to download. In this work, we consider the special case in which a peer is willing to trade *all* the segments it has in exchange for even *one* segment that he does not have. In other words, peers ‘give liberally,’ even if they take just one new segment. Because of this, after exchange, each peer possesses a segment set that is the *union* of the segment sets before exchange.

The G&T criterion does not depend on identity of the peers involved. To meet the criterion, peer A must offer at least one segment to peer B , and vice versa. The criterion remains unaffected even if A is the mutated identity of some other peer. This is why a whitewasher derives no benefit: even with a changed identity, a whitewasher had provided some segments in return to the other peer. Hence, a whitewasher cannot keep downloading small amounts and accumulate segments.

Incorporating the G&T criterion in our model, we study a problem that arises naturally when a new peer (namely, peer 0) enters the system — the problem of downloading a ‘universe’ of segments, scattered among other peers, at least cost. The cost of exchange between peers 0 and j is fixed. The cost can depend on a number of factors such as the delay incurred in completion of exchanges (with higher delays implying higher costs). Reputation of the peers can also influence cost; for example, if peer j has a history of selfishness, a high cost can be assigned — this discourages exchanges with j and serves as a punitive measure to peer j for selfishness.

Peer 0 has no segments with him, the G&T criterion implies that exchanges with peers cannot even begin. To get started, peer 0 downloads some segments from the *seed or surrogate server*, which is a storehouse of all segments. We note that the seed is *not* a peer, and the G and T criterion does not apply to downloading of segments from the seed. Peer 0’s overall objective is to acquire the universe of segments at low aggregate cost. However, the cost of downloading from the seed is very high. Thus, peer 0’s strategy would be to download a few segments from the seed such that G&T compliant exchanges with peers can begin. For the purpose of analysis

in this paper, we assume that system is changing at a slow rate, effectively making it static. If the system is more dynamic, given the current state of the system, the analysis in this paper can be used to find the best possible action (for example, the decision to join the network or not) at that instant.

We propose an optimal algorithm to solve this problem. We compare the performance of a few commonly used P2P downloading strategies with the optimal algorithm.

This paper makes the following contributions. Firstly, we propose a criterion called Give and Take (G&T) that prohibits free riding. Secondly, we formulate and solve a problem where a new peer wishes to download a universe of objects at low cost, while satisfying the G&T criterion at each exchange.

Section 2 discusses the system model in detail, followed by an analysis of the problem, optimal and sub-optimal algorithms in section 3. Section 4 compares the results with other algorithms used in P2P networks.

2. System model

We consider a scenario with m peers, a newly arrived peer, denoted ‘peer 0’ and *one Seed* (or *Surrogate server*). The objective of peer 0 is to collect the universe at minimum cost. The cost of fetching the segment set with peer j is C_j .

2.1 Notation

- **Complete segment set:** $N = \{1, 2, \dots, n\}$
- **Peer set:** $M = \{1, 2, \dots, m\}$
- **Segment container sets:** Segments with peer j

$$O_j = \{i \in N : \text{peer } j \text{ has segment } i\} \forall j \in M$$

- $[P, C_{opt}] = \text{setcover}(N_{sub}, M_{sub}, C(M_{sub}))$
 setcover is a function with the following parameters:
 N_{sub} : Set of segments which needs to be fetched; $N_{sub} \subseteq N$
 M_{sub} : Subset of peers; $M_{sub} \subseteq M$
 $C(M_{sub})$: Cost vector for peers in M_{sub}
 P : Set of peers providing a minimum cost set cover, $P \subseteq M_{sub}$ and $\cup_{j \in P} O_j = N_{sub}$
 C_{opt} : Cost of a minimum cost set cover

The objective of the set cover is to select a collection of minimum cost subsets such that each element of the universe is present in at least one of the subsets. There may be several collections (P) which cover the universe at the least cost. Computation of minimum cost set cover is a $\mathcal{NP}$ -hard problem (Caprara et al 2000), but various approximation algorithms are available (Caprara et al 2000; Chavatal 1979; Feige 1998; Alon et al 2006).

2.2 Assumptions

- A1: The cost of fetching any segment from the seed is same, and is much higher than the cost of fetching the segment from a peer. This assumption is made as usually segments are of the same size, and the bandwidth cost at the surrogate server is high (Aggarwal et al 2007).

Figure 3. Cost structure for peer 0.

The surrogate server has to serve a large number of users, which leads to increased cost of resources at the surrogate server tremendously (figure 3):

$$C_s(e) = C_s \gg C_j \quad \forall e \in N \text{ and } j \in M$$

A2: Peer 0 does not have any segment when it enters the system.

A3: **Give-and-Take (G&T) criterion** Peer 0 will be willing to exchange with peer j if peer j has at least one segment which is unavailable with peer 0. Similarly, peer j will be willing to exchange with peer 0 iff peer 0 has a segment which is unavailable with peer j . Let peer 0 have O_0 as the set of segments at the time of exchange with peer j . Then following must be true¹ for exchange to take place:

$$O_0 \cap O_j^c \neq \emptyset \quad \text{and} \quad O_0^c \cap O_j \neq \emptyset.$$

If this criterion is met, then peer 0 gets all the segments that peer j has. Similarly, peer j gets all segments that peer 0 has.

A4: All nodes $j \in M$ have only a strict subset of universe ($O_j \subsetneq N$). Only seed has the universe N . Any number of segments can be fetched from the seed without the need for any exchange criterion as in A3.

2.3 Problem formulation

The problem is analysed from the perspective of peer 0. To begin with, peer 0 has no segments and his objective is to download the universe N at low cost.

¹ X^c denotes the complement of set X .

Peer 0 can download segments either from the seed or from peers in M . For downloading from the seed, peer 0 needs to pay a cost of C_s for each segment, and he need not satisfy the G and T criterion. For downloading from peer $j \in M$, a cost C_j needs to be paid and the G and T criterion should be satisfied between the segments sets available with peer 0 and peer j at the time of the exchange.

Let us assume $O_{0,s} \subseteq N$ is fetched from the seed, followed by exchange with peers² in P_{exc} . Hence, the total cost of downloading the universe is given by:

$$C_T = C_s |O_{0,s}| + \sum_{j \in P_{exc}} C_j.$$

The objective of this paper is to determine an optimal $O_{0,s}$ and a set of peers P_{exc} such that peer 0 has the universe N at the end of all G&T compliant exchanges with peers in P_{exc} , and the total cost C_T , is minimized.

3. Analysis

Let $O_{0,s} \subseteq N$ be the set of segments that is fetched from the seed. Once $O_{0,s}$ is fetched, peer 0 tries to acquire $N \setminus O_{0,s}$ from other peers ($P_{exc} \subseteq M$) at low cost, while conforming to the G&T criterion. Total cost for peer 0 is given by $C_T = C_s |O_{0,s}| + \sum_{j \in P_{exc}} C_j$. This problem is $\mathcal{NP}$ -hard as proved in Lemma 1.

To progress towards a solution, we make the following observations; proofs are straight forward.

Observation 1: Each peer belonging to a minimum cost set cover has at least one unique segment which is not available with any other peer in the set cover.

Observation 2: From Obs.1, it follows that if peer 0 can exchange with any *one* peer in a minimum cost set cover, then he can exchange with all others in the set cover as well.

Observation 3: Minimum number of segments to be fetched from the seed to enable peer 0 to fetch all segments is $\max(|N \setminus \bigcup_{j \in M} O_j|, 1)$. (If all peers do not cover the universe, then one needs to fetch segments which are not there with any of the peers in M . Otherwise, choosing any $O_{0,s} = \{e\}$ such that $e \in (\bigcap_{j \in P_{scp}} O_j)^c$ will enable peer 0 to fetch the universe, here P_{scp} is the standard set cover solution.)

We define instance of our problem as:

Minimum Cost G&T criterion (MCGT) Problem: Instance defined by $(N, M, O_j (j \in M), C_j (j \in M), C_s)$. We need to find $O_{0,s} \subseteq N$ and $P_{exc} \subseteq M$ such that,

Condition 1: $O_{0,s} \cup \left(\bigcup_{j \in P_{exc}} O_j \right) = N$.

Condition 2: Peer 0 can exchange with peers in P_{exc} while complying with the G&T criterion.

²Peer 0's augmented set will always be considered for exchange. Hence, the order of peers in which exchanges are carried out by peer 0 also matters.

Condition 3: $(C_s |O_{0,s}| + \sum_{j \in P_{exc}} C_j)$ is minimized.

We define an instance of the Set cover problem as:

Instance defined by $(\bigcup_{j=1}^m O_j, M, O_j(j \in M), C_j(j \in M))$. We need to find $P_{exc} \subseteq M$ such that,

Condition 1: $\bigcup_{j \in P_{exc}} O_j = \bigcup_{j \in M} O_j$

Condition 2: $\sum_{j \in P_{exc}} C_j$ is minimized.

Lemma 1. *The problem of finding the minimum cost solution (i.e., minimizing the cost C_T) for the MCGT problem is NP-hard.*

Proof. We use the technique of restriction. Consider instances of the MCGT problem satisfying the following:

- (a) There exists an element $e_0 \in N$, such that e_0 is missing from each peer's object set, i.e., $e_0 \notin O_j, \forall j \in M$.
- (b) $\bigcup_{j \in M} O_j = N \setminus \{e_0\}$.

For instance of this type, peer 0 *must* download e_0 from the seed, as e_0 is not available with any peer $j, 1 \leq j \leq m$. With e_0 in hand, peer 0 is in position to exchange with any peer in M as none of the peers has e_0 .

Then, we set $O_{0,s} = \{e_0\}$, and note that condition 2 is satisfied. Further, we observe that problem of obtaining P_{exc} such that conditions 1 and 3 are satisfied is the same as a minimum cost set cover problem, with

Set of elements: $N \setminus \{e_0\} = \bigcup_{j \in M} O_j$

Subsets: $O_j, \forall j \in M$

Costs: $C_j, \forall j \in M$.

Hence, by restriction, the MCGT problem is $\mathcal{NP}$ -hard.

Alternatively, in the language of deterministic polynomial-time reduction, we reduce the set cover problem instance $(\bigcup_{j=1}^m O_j, M, O_j(j \in M), C_j(j \in M))$ to the MCGT problem instance $(\bigcup_{j=1}^m O_j \cup e_0, M, O_j(j \in M), C_j(j \in M), C_s)$, where e_0 is any element not in $\bigcup_{j=1}^m O_j$ and $C_s = \sum_{j=1}^m C_j + 1$.

This reduction takes only linear time in the input set cover instance $(\bigcup_{j=1}^m O_j, M, O_j(j \in M), C_j(j \in M))$.

Now observe that C_{opt} is the minimum cost solution for the set cover problem instance $(\bigcup_{j=1}^m O_j, M, O_j(j \in M), C_j(j \in M))$ if and only if $C_T = C_s + C_{opt}$ is the minimum cost solution for the above-mentioned MCGT problem instance $(\bigcup_{j=1}^m O_j \cup e_0, M, O_j(j \in M), C_j(j \in M), C_s)$. To see the proof of this, suppose C_{opt} is the minimum cost solution for the set cover instance and C_T is the minimum cost solution for the MCGT instance. Peer 0 must download e_0 from the seed by paying a cost of C_s . At this point it has two options — download more objects from the seed or acquire the remaining objects from the other peers at a minimum cost of C_{opt} . But note that if peer 0 downloads any object in addition to e_0 then it pays an extra $C_s = (\sum_{j=1}^m C_j + 1)$ which is greater than C_{opt} . Hence the optimum strategy for peer 0 is

to download e_0 from the seed, followed by acquiring the remaining objects from other peers, making the minimum cost solution for this MCGT instance $C_s + C_{opt}$.

Hence, the set cover problem reduces in deterministic polynomial time to the MCGT problem, implying the latter's NP-hardness. $\square$

We move on to obtain the optimal cost for peer 0.

Based on the segments available with the peers, two cases can occur.

Case 1: All segments not available with the peer group: $\cup_{j=1}^m O_j \neq N$

In this case, peer 0 has to fetch $N \setminus \cup_{j=1}^m O_j$ from the seed anyway. Since fetching $N \setminus \cup_{j=1}^m O_j$ will enable peer 0 to exchange with all other peers, the optimal value of $O_{0,s}$ is given by $O_{0,s} = N \setminus \cup_{j=1}^m O_j$.

With this $O_{0,s}$, the G&T criterion is satisfied automatically for all peers in M , reducing the problem to a standard set cover problem, $[P_{exc}, C_{peer}] = \text{setcover}(\cup_{j=1}^m O_j, M, C)$.

Hence, the optimum cost for peer 0 is $C_T = \sum_{e \in N \setminus \cup_{j=1}^m O_j} C_s(e) + C_{peer}$.

Case 2: All segments available with the peer group: $\cup_{j=1}^m O_j = N$

Suppose that peer 0 fetches segment $e \in N$ from the seed. Let $\Gamma^{*,e}$ represent the optimal cost of acquiring the universe through G&T-compliant exchanges after having obtained e from the seed³. Lemma 2 characterizes the optimal cost for various e .

Lemma 2. Let $O_{0,s} = \{e\}$ and $\mathcal{P}_{e,\min} = \{P_{e,\min} : [P_{e,\min}, C_{e,\min}] = \text{setcover}(N \setminus \{e\}, M, C)\}$ be all the solution sets for acquiring $N \setminus \{e\}$ from peers in M at the low cost. We define $\tilde{N}_e$ as

$$\tilde{N}_e = \left(\bigcap_{P_{e,\min} \in \mathcal{P}_{e,\min}} \bigcap_{j \in P_{e,\min}} O_j \right)^c$$

If $e \in \tilde{N}_e$, then $\Gamma^{*,e} \leq C_s + C_{scp}$, otherwise, $\Gamma^{*,e} > C_s + C_{scp}$.

Proof. If segment e is fetched from the seed at cost C_s , then peer 0 will try to acquire $N \setminus \{e\}$ from the other peers. Thus, $(C_s + C_{e,\min})$ is the low cost of obtaining the universe N when the G&T criterion is not considered. When the G&T criterion is taken into account, the low cost $\Gamma^{*,e}$ cannot be any less. Therefore,

$$\Gamma^{*,e} \geq C_s + C_{e,\min} \quad \forall \quad e \in N. \quad (1)$$

In (1), equality will be achieved whenever exchange is possible with at least one peer in $P_{e,\min}$ (by Observation 2) where $P_{e,\min} \in \mathcal{P}_{e,\min}$. This happens when for some $P_{e,\min}$ at least one peer in $P_{e,\min}$ does not possess e . Figures 4a and 4b correspond to this scenario.

³We note that all minimum cost set cover solutions are considered in arriving at $\Gamma^{*,e}$.

Figure 4. Various possibilities for elements of $\mathcal{P}_{e,\min}$. Each of the circle denotes a peer in M . Circles with e shows presence of segment e with that particular peer.

Also, we note that since every cover of N is also a cover of $N \setminus \{e\}$,

$$\therefore C_{e,\min} \leq C_{scp} \quad \forall e \in N \quad (2)$$

If $e \in \tilde{N}_e$, $\exists P_{e,\min} \in \mathcal{P}_{e,\min}$ such that there is at least one peer in $P_{e,\min}$ who does not have segment e . Hence, peer 0 can exchange with that particular peer and follow it up by exchanges with other peers in $P_{e,\min}$. Thus, peer 0 can acquire the universe N by means of G&T compliant exchanges. Figures 4a and 4b help in visualizing this. Therefore, equality will exist in (1):

$$\Gamma^{*,e} = C_s + C_{e,\min} \quad \forall e \in \tilde{N}_e \quad (3)$$

From (2) and (3),

$$\Gamma^{*,e} \leq C_s + C_{scp} \quad \forall e \in \tilde{N}_e \quad (4)$$

Hence, the first part of the Lemma is proved.

Next, $e \in \tilde{N}_e^c \implies e \in \bigcap_{P_{e,\min} \in \mathcal{P}_{e,\min}} \bigcap_{j \in P_{e,\min}} O_j$. This means that all peers in *any* $P_{e,\min} \in \mathcal{P}_{e,\min}$ have segment e (figure 4c). Exchange is not possible in this case, therefore from (1) we have

$$\Gamma^{*,e} > C_s + C_{e,\min} \quad \forall e \in \tilde{N}_e^c \quad (5)$$

If $e \in \tilde{N}_e^c$, then figure 4c shows that solution sets $P_{e,\min} \in \mathcal{P}_{e,\min}$ cover N as well. Therefore,

$$\begin{aligned} C_{e,\min} &\geq C_{scp} \quad \forall e \in \tilde{N}_e^c \\ C_{e,\min} &= C_{scp} \quad \forall e \in \tilde{N}_e^c \end{aligned} \quad (\text{Using (2)}) \quad (6)$$

Using (5) and (6), we have

$$\Gamma^{*,e} > C_s + C_{scp} \quad \forall e \in \tilde{N}_e^c \quad (7)$$

□

Remark 1. For scenarios corresponding to figure 4a: $\bigcup_{j \in P_{e,\min}} O_j = N \setminus \{e\} \forall P_{e,\min} \in \mathcal{P}_{e,\min}$. For scenarios corresponding to Figure 4b, $\exists P_{e,\min} \in \mathcal{P}_{e,\min}$ such that $\bigcup_{j \in P_{e,\min}} O_j = N$, therefore $C_{e,\min} \geq C_{scp}$. Using (2), we get $C_{e,\min} = C_{scp}$.

Algorithm 1. Optimal segment and peer selection strategy.

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1:  $\hat{N} = \{e : \bigcup_{j \in M_e^c} O_j = N \setminus \{e\}\}$ 
2: if  $\hat{N} \neq \emptyset$  then
3:   for all  $e \in \hat{N}$  do
4:      $[\hat{P}_{e,\min}, \hat{C}_{e,\min}] = \text{setcover}(N \setminus \{e\}, M_e^c, C(M_e^c))$     %  $C(M_e^c)$  are the costs of
       exchanging with peers in  $M_e^c$ 
5:   end for
6:    $e_s = \arg \min_{e \in \hat{N}} (\hat{C}_{e,\min})$  and  $P_{exc} = \hat{P}_{e_s,\min}$ 
7: else
8:    $e_s = \emptyset$  and  $\hat{C}_{e_s,\min} = \infty$ 
9: end if
10:  $[P_{scp}, C_{scp}] = \text{setcover}(N, M, C)$     %  $P_{scp}$  can be any set cover
       solution
11: if  $\hat{C}_{e_s,\min} > C_{scp}$  then
12:    $e_s = \text{Any segment from } \bigcup_{j \in P_{scp}} O_j^c$  and  $P_{exc} = P_{scp}$ 
13: end if
14: Fetch  $e_s$  from seed and exchange among peers in  $P_{exc}$ 

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Lemma 2 helps in identifying ‘good’ candidates for the selection of $O_{0,s}$, and forms the basis of Algorithm 1.

Lemma 3. *Algorithm 1 is optimal.*

Proof. By Lemma 2, $O_{0,s} = \{e\}$ can be a optimal choice iff $e \in \tilde{N}_e$. Algorithm 1 considers all feasible values (defined by Lemma 2) of $O_{0,s}$ and chooses the segment that minimizes the cost of exchange.

We define $\hat{N}$ as feasible subset of segments fulfilling the condition (for scenarios corresponding to figure 4a) in Line 1 of algorithm.

Let $M_e \subseteq M$ be the subset of peers having segment e . We note that if $C_{e,\min}$ is strictly less than C_{scp} , then $P_{e,\min} \subseteq M_e^c$, $\forall P_{e,\min} \in \mathcal{P}_{e,\min}$.

If $\hat{N} \neq \emptyset$, we evaluate the cost of acquiring the remaining segments from peers in M_e^c for each $e \in \hat{N}$ (Lines (3)–(5) of Algorithm 1) and choose e_s that minimizes the cost of acquiring the remaining segments. Otherwise, we define $\hat{C}_{e_s,\min}$ as infinity.

If $\hat{C}_{e_s,\min} > C_{scp}$, then e_s can be any segment from $\bigcup_{j \in P_{scp}} O_j^c$ and exchange with peers P_{exc} . The cost of exchange among peers is C_{scp} .

To summarize, Algorithm 1 first searches for scenarios corresponding to figure 4a in lines (1)–(9). In case search results in cost of exchange $\hat{C}_{e_s,\min} > C_{scp}$, then optimal cost of exchange among peers is C_{scp} as being achieved in scenarios corresponding to figure 4b. Hence, Algorithm 1 is optimal. $\square$

After fetching e_s from seed, peer 0 will initiate exchange with peers in P_{exc} not having segment e_s . Now, using the unique elements of these peers (from Obs. 1), peer 0 can exchange with remaining peers in P_{exc} (from Obs. 2) and get the universe.

Algorithm 1 is built on a procedure to solve the minimum cost set cover problem. It is known that the latter is $\mathcal{NP}$ -hard. Hence, repeated evaluation of the set cover problems in *lines 3 to 5* of Algorithm 1 is computationally most expensive.

Algorithm 2 sacrifices optimality to reduce computation. In Algorithm 2, we solve the problem *setcover* (N, M, C) only once, and identify the unique elements with each peer in P_{scp} (line 2 of Algorithm 2). If each peer has at least 2 unique elements, then it suffices to choose e_s randomly from $\bigcup_{j \in P_{scp}} O_j^c$ (line 4). On the other hand, if one or more peer in P_{scp} has a *single* unique element, then we avoid exchanging with the most ‘expensive’ of such peers (line 7 of Algorithm 2).

Algorithm 2. Sub-optimal cost.

- 1: $[P_{scp}, C_{scp}] \leftarrow \text{setcover}(N, M, C)$ % P_{scp} can be any minimum cost set cover.
 - 2: $O_{p_i}^U \leftarrow O_{p_i} \setminus \left(\bigcup_{p_j \in P_{scp} \setminus p_i} O_{p_j} \right) \forall p_i \in P_{scp}$ % Unique elements available with peer p_i in P_{scp}
 - 3: **if** $|O_{p_i}^U| \geq 2 \quad \forall p_i \in P_{scp}$ **then**
 - 4: Choose e_s randomly in $\bigcup_{j \in P_{scp}} O_j^c$
 - 5: Initiate exchange among peers in P_{scp}
 - 6: **else**
 - 7: $e_s \leftarrow O_{p_{\max}}^U$ where $p_{\max} = \arg \max_{p_i \in P_{scp}, |O_{p_i}^U|=1} C_{p_i}$ % Fetching e_s will avoid exchange with the peer $p_{\max}$, with whom exchange is most expensive
 - 8: Initiate exchange among peers in $P_{scp} \setminus p_{\max}$
 - 9: **end if**
-

4. Results

The objective of this section is to compare the performance of Algorithms 1 and 2⁴ with those based on strategies used in the *Bittorrent* protocol, used widely in peer-to-peer networks. The *Bittorrent* algorithms are modified slightly so that exchanges satisfy the G&T criterion. The question we address is: supposing *all* algorithms allowed *only G&T-compliant exchanges*, what can we say about their relative performance?

For selecting the initial set of segments, *Bittorrent* uses either a random or a *Rarest-First* approach, followed by the *choking* of peers, depending on the history of peer activities (Pouwelse *et al* 2005). The costs C_j were assumed to be a function of all parameters of interest to peer 0. Algorithms 3 and 4 are based on segment and peer selection strategies used in *Bittorrent*.

Algorithm 3 selects the initial segment and peer uniformly at random. Peering happens if the G&T criterion is satisfied.

⁴For solving set cover problem, we write the same as binary Integer program. We use *bintprog* function of the Optimization toolbox, MATLAB 2013b for evaluating the solution. *Bintprog* uses the branch and bound method for solving binary integer program.

Algorithm 3. Bittorrent based uniform segment and peer selection.

```

1:  $O_0 \leftarrow e_s$ : Selecting  $e_s$  uniformly among  $N$       % Initial segment is selected at random
2: while  $O_0 \neq N$  do
3:   Select peer  $j$  uniformly among  $M$ 
4:   if Give-and-Take criterion is satisfied then
5:      $O_0 \leftarrow O_0 \cup O_j$ ,
6:   end if
7: end while

```

Algorithm 4. Bittorrent based rarest first and probabilistic choking.**Require:** Probability vector for peer selection in M

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1:  $O_0 \leftarrow e_s$ :  $e_s$  selection based on Rarest First % Segment available with minimum number
   of peers is chosen
2: while  $O_0 \neq N$  do
3:   Randomly select peer  $j$  among  $M$  % As per the specified probability distribution
4:   if Give-and-Take criterion is satisfied then
5:      $O_0 \leftarrow O_0 \cup O_j$ ,
6:   end if
7: end while

```

Algorithm 4 uses the *Rarest First* strategy for selecting the initial segment, followed by peer selection based on probabilities of peer un-choking (Locher *et al* 2006). First, the segment available with the least number of peers is chosen. Then, one of the peers is unchoked, based on their unchoking probabilities.

For simulation purposes, the probability vectors in Algorithm 4 were defined/generated as follows:

4a: Uniformly at random

4b: At random, but with low probability values assigned to peers with high cost

4c: Prob (peer j is selected) = $\frac{1 - \frac{c_j}{\sum_{j \in M} c_j}}{m-1}$. This form ensures that peers with high cost are chosen with low probability.

The results are shown in figure 5. $|M| = m$ denotes the number of peers and $|N| = n$ the size of the universe. For $j \in M$, the segment set O_j and the cost of exchanging segments with the peer were generated uniformly at random. For each set of bars, the results shown are averages over 1000 sample runs. Algorithm 2 sacrifices optimality for shorter running times, but achieves costs that are very close to optimal. It can be seen that Algorithm 1, which is optimal, achieves much lower cost—on an average 51%, 51%, 42% and 51% over all sets in figure 5, respectively, than algorithms 3, 4a, 4b and 4c.

Figure 6 plots the cost of exchange against the time taken for running each algorithm on 50 randomly generated data points, with 300 segments and 200 peers. From the figure, one can

Figure 5. For all sets, $\bigcup_{j \in M} O_j = N$; Each set is generated independently shows the cost of acquiring the universe for a sample path. For algorithms 3, 4a, 4b and 4c), the same has been averaged over 1000 runs.

Figure 6. Trade-off between computation time and cost for 50 randomly generated datasets with 300 segments and 200 peers. Cost against the time taken for running each algorithm on each of data point is plotted.

Table 1. Mean running time for algorithm 2 (on MATLAB) for different sets of values of m and n averaged over 500 randomly generated object sets (Machine Specifications: Processor: Intel®Core™ 2 Duo E7500 @2.93GHz (2CPUs); Memory: 2004 MB RAM).

Number of peers(m)	Number of Segments(n)	Mean running time for algorithm 2
30	50	0.0298 s
100	100	0.0967 s
150	200	0.2809 s
200	300	0.727 s
300	400	1.1324 s
500	400	2.0078 s
500	600	4.8460 s
600	600	5.7293 s
700	700	16.9082 s
800	800	20.2471 s

observe that costs achieved by both Algorithms 1 and 2 are quite similar, but the running time of Algorithm 2 is orders of magnitude⁵ lower than that of Algorithm 1.

Also, each of the Algorithms 3, 4a, 4b and 4c runs very quickly, but the cost of exchange is higher than that achieved by Algorithms 1 and 2. To summarize, the proposed Algorithm 2 guarantees the best cost of exchange possible, at the expense of roughly one and a half orders of magnitude increase in computation time.

Is Algorithm 2 suitable for practical applications?

Algorithm 2 is built over a procedure to solve an $\mathcal{NP}$ —hard problem. Hence, running time of Algorithm 2 increases drastically as the number of peers or segments is increased. Still, Algorithm 2 can be a practical choice in many situations.

For instance, table 1 shows the mean running time of Algorithm 2 for 500 randomly generated object sets. We compare the observed running times of Algorithm 2 with the reported observations in Sharma *et al* (2013). Visual observations from figure 1 in Sharma *et al* (2013) show that for 10 peers, the system takes around 10–30 s for getting hold of the first chunk. Hence, Algorithm 2 can be a good alternative for coming up with a list of peers, as its running time is below 30 sec for systems having many more than 10 peers. Moreover, it achieves near-optimal cost.

Also, Liao *et al* (2007) reports download completion time of 10000–40000 seconds for different scenarios. Hence, we believe it is quite acceptable for the peer to spend 20 seconds—which is negligible compared to the download completion time—in an initial computation that yields nearly the best possible download cost.

5. Conclusions and future work

In this paper, our objective was to devise a mechanism that would discourage *free riding* in P2P CDNs. To this end, we introduced the G&T criterion. Subsequently, we have analysed a

⁵The complexity for Algorithm 1 and Algorithm 2 is $O(n2^m)$ and $O(2^m)$ respectively.

problem of interest to a newly-arrived peer—that of acquiring the universe, scattered among the segment sets of peers—at minimal cost, and in conformance with the G&T criterion. Our analyses suggested the optimal algorithm. Performance comparison with some commonly used algorithms quantified the extent of improvement possible. We also studied the computation time for prescribing the set of segment and peer selection strategies to the newly arrived peer. For moderate-sized problems, algorithm 2 can lead to near-optimal cost while the running time remains within a few seconds; thus, algorithm 2 emerges as a practical option when the problem size is not too big. For larger problem sizes, our approach indicates the extent of improvement in cost that is possible. This indicates the need to devise algorithms that *reduce the large gap* between optimal cost and the cost achieved by Bittorrent-like algorithms; we would like to take up this as future work.

Further, in the present paper, we considered *one* new peer acquiring segments, while other peers' segment sets were 'frozen'. We are in the process of studying dynamic scenarios in which the system evolves with time. In future, we also would like to study the situation that arises when all peers are involved in exchanges simultaneously.

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