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A method for unbalanced transportation problems in fuzzy environment

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Abstract. In this paper, we consider the fully fuzzy unbalanced transportation problem in which the total availability/production is more than the total demand and propose a method to solve it. Such problems are usually solved by adding a dummy destination. Since the dummy destination has no existence in reality, the excess availability is not transported at all and is held back at one or more origins. The method proposed in this paper gives the additional information that to which of the destination(s) the excess availability be transported for future demand at minimum cost. The advantage of the proposed method over the existing method is that the fuzzy optimal solution obtained does not involve the dummy destination. The method has been illustrated with the help of an example.

Keywords. Trapezoidal fuzzy number; fully fuzzy transportation problem; fuzzy optimal solution.

1. Introduction

Among linear programming problems, the transportation problem is very popular. As transportation of goods from one place to another becomes increasingly important in global economics, knowledge of the transportation system is fundamental to the efficient and economical operation of a company. Transportation models play an important role in reducing cost and improving service. It ensures the efficient movement of raw materials and finished goods. It is one of the earliest applications of linear programming problems.

Hitchcock (1941) originally developed the basic transportation problem. Appa (1973) discussed several variations of the transportation problem. In general, transportation problems are solved with the assumptions that the parameters of the transportation problem (i.e., unit cost of transportation from each source to each destination, availability of the product at each source and demand at each destination) are specified in a precise way, i.e., in crisp environment. But in

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practice, the parameters of the transportation problem are not always exactly known and stable. This imprecision may follow from the lack of exact information, uncertainty in judgment etc. Therefore, Zadeh (1965) introduced the concept of fuzzy numbers.

Saad & Abbas (2003) discussed an algorithm for solving the transportation problems in fuzzy environment. Das & Baruah (2007) proposed vogel's approximation method to find the fuzzy initial basic feasible solution of fuzzy transportation problems in which all the parameters are represented by triangular fuzzy numbers. Basirzadeh (2011) used the classical algorithms to find the fuzzy optimal solution of fully fuzzy transportation problems by transforming the fuzzy parameters into crisp parameters.

Pandian & Natrajan (2010) proposed a new algorithm, namely fuzzy zero point method, to find the fuzzy optimal solution of fuzzy transportation problems, where trapezoidal fuzzy numbers represent all the parameters. Later, based on fuzzy zero point method, they proposed a new method to find more-for-less fuzzy optimal solution for such fuzzy transportation problems which have mixed constraints and parameters as trapezoidal fuzzy numbers (Pandian & Natrajan 2010a). De & Yadav (2010) modified the existing method (Kikuchi 2000) by using trapezoidal fuzzy numbers instead of triangular fuzzy numbers. Kaur & Kumar (2011) proposed a new method for the fuzzy transportation problems using ranking function. Using simplex- type algorithm proposed by Arsham & Khan (1989), Gani *et al* (2011) obtained the fuzzy optimal solution of fuzzy transportation problems having parameters as trapezoidal fuzzy numbers.

While solving unbalanced transportation problems we come across two type of cases. Either the total availability is more than the total demand or vice-versa. In case of excess availability, some times it may happen that we do not have enough storage place for the excess commodity at the source(s) and wish to transport it to the destination(s) for the future demand. To solve such problems a dummy destination is added, where the excess availability is transported. Since the dummy destination does not have any existence in reality, so it is not possible to find that the excess available product should be transported to which destination at a minimum cost. In this paper, a method is proposed to get the fuzzy optimal solution in terms of original sources and destinations only and we also get the above said information. To illustrate the proposed method, a fully fuzzy transportation problem is solved using the proposed method and the obtained results are discussed.

2. Prelminaries

In this section, some basic definitions, arithmetic operations and comparison of trapezoidal fuzzy numbers are presented.

2.1 *Basic definitions*

Definition 1 (Kaufmann & Gupta 1985). A fuzzy number $\widetilde{A}$ defined on the universal set of real numbers $\mathbb{R}$, denoted as $\widetilde{A} = (a, b, c, d)$, is said to be a trapezoidal fuzzy number if its membership function $\mu_{\widetilde{A}}(x)$ is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \le x < b \\ 1, & b \le x \le c \\ \frac{(d-x)}{(d-c)}, & c < x \le d \\ 0, & \text{otherwise.} \end{cases}$$

Graphically, a trapezoidal fuzzy number can be represented as (figure 1):

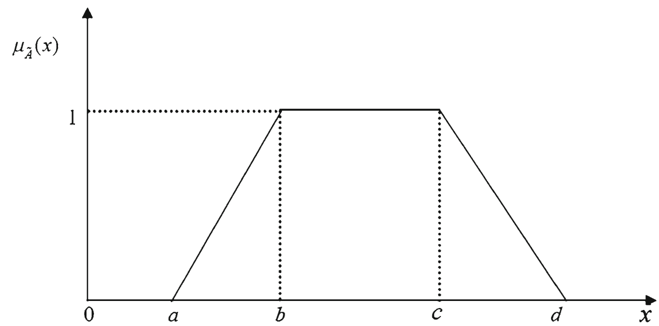

Figure 1. Trapezoidal fuzzy number.

Definition 2 (Liou & Wang 1992). Let $F(\mathbb{R})$ be a set of fuzzy numbers defined on the set of real numbers $\mathbb{R}$ and let $\widetilde{A} = (a, b, c, d) \in F(\mathbb{R})$. Then

$$\Re(\widetilde{A}) = \frac{(a + b + c + d)}{4},$$

is called a ranking function, which maps each fuzzy number into real line.

Remark 1. A trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d)$ is said to be zero trapezoidal fuzzy number if and only if $\Re(\widetilde{A}) = 0$ and is denoted by $\widetilde{0}$.

Remark 2. A trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d)$ is said to be non-negative trapezoidal fuzzy number if and only if $\Re(\widetilde{A}) \geq 0$ and is denoted by $\widetilde{A} \succeq \widetilde{0}$.

2.2 *Arithmetic operations* (Kaufmann & Gupta 1985)

Let $\widetilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\widetilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Then

(i) $\widetilde{A}_1 \oplus \widetilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$,

(ii) $\widetilde{A}_1 \ominus \widetilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$,

(iii) $k\widetilde{A}_1 = \begin{cases} (ka_1, kb_1, kc_1, kd_1), & k \geq 0 \\ (kd_1, kc_1, kb_1, ka_1), & k \leq 0, \end{cases}$

(iv) $\widetilde{A}_1 \otimes \widetilde{A}_2 = (a, b, c, d)$,

where $a = \min(a_1a_2, a_1d_2, d_1a_2, d_1d_2)$, $b = \min(b_1b_2, b_1c_2, c_1b_2, c_1c_2)$, $c = \max(b_1b_2, b_1c_2, c_1b_2, c_1c_2)$, $d = \max(a_1a_2, a_1d_2, d_1a_2, d_1d_2)$.

2.3 *Comparison of trapezoidal fuzzy numbers* (Liou & Wang 1992)

Let $F(\mathbb{R})$ be a set of fuzzy numbers. Let $\widetilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\widetilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Then

$\widetilde{A}_1 \succ \widetilde{A}_2$ if $\Re(\widetilde{A}_1) > \Re(\widetilde{A}_2)$
$\widetilde{A}_1 \prec \widetilde{A}_2$ if $\Re(\widetilde{A}_1) < \Re(\widetilde{A}_2)$
$\widetilde{A}_1 \approx \widetilde{A}_2$ if $\Re(\widetilde{A}_1) = \Re(\widetilde{A}_2)$.

3. Fuzzy linear programming formulation

Consider an unbalanced FFTP having m sources S_i with fuzzy availability $\widetilde{a}_i$ $(1 \le i \le m)$, n destinations D_j with fuzzy demand $\widetilde{b}_j$ $(1 \le j \le n)$. Each of the m sources S_i can transport to any of the n destinations D_j at a transportation cost of $\widetilde{c}_{ij}$ per unit. Let $\widetilde{x}_{ij}$ be the fuzzy quantity of the product that should be transported from source S_i to destination D_j. The linear programming formulation to minimize the total fuzzy transportation cost is as follows:

$$
\begin{aligned}
(LP1) \qquad \text{Minimize} \quad & \sum_{i=1}^{m}\sum_{j=1}^{n}(\widetilde{c}_{ij} \otimes \widetilde{x}_{ij}), \quad \text{subject to} \\
& \sum_{j=1}^{n} \widetilde{x}_{ij} \succeq \widetilde{a}_i \quad (1 \le i \le m), \\
& \sum_{i=1}^{m} \widetilde{x}_{ij} \succeq \widetilde{b}_j \quad (1 \le j \le n), \\
& \widetilde{x}_{ij} \succeq \widetilde{0}.
\end{aligned}
$$

4. Solution by existing methods

Example 1. Consider the transportation problem with 3 sources S_i $(1 \le i \le 3)$ and 3 destinations D_j $(1 \le j \le 3)$ with unit costs of transporting the product from source S_i to destination D_j as

$$
[\widetilde{c}_{ij}] = \begin{bmatrix} (4, 5, 8, 15) & (1, 1, 2, 4) & (1, 2, 3, 6) \\ (1, 1, 2, 4) & (7, 9, 12, 20) & (4, 5, 8, 15) \\ (0, 1, 1, 2) & (7, 12, 15, 26) & (1, 1, 2, 4) \end{bmatrix}.
$$

The fuzzy availability of the product at first, second and third sources is (9,11, 20,40), (3,4,7,14) and (3,4,5,8), respectively and (7,12,15,26), (4,8,10,18) and (2,5,6,11) is the fuzzy demand of the product at first, second and third destinations, respectively. The product can be shipped from any source to any destination. One has to find the fuzzy quantity of the product that should be transported from each source to each destination such that the total fuzzy cost of transportation is minimum.

The chosen problem can be represented in tabular form as shown in table 1.

Table 1. Unbalanced fully fuzzy transportation problem.

Destinations → Sources ↓	D_1	D_2	D_3	Fuzzy availability
S_1	(4,5,8,15)	(1,1,2,4)	(1,2,3,6)	(9,11,20,40)
S_2	(1,1,2,4)	(7,9,12,20)	(4,5,8,15)	(3,4,7,14)
S_3	(0,1,1,2)	(7,12,15,26)	(1,1,2,4)	(3,4,5,8)
Fuzzy demand	(7,12,15,26)	(4,8,10,18)	(2,5,6,11)	

For the above transportation problem the total fuzzy availability is more than the total fuzzy demand. So we first convert it to a balanced problem by adding a dummy destination (D_4) with the corresponding unit transportation costs to be zero trapezoidal fuzzy numbers and convert the constraint inequalities to equations. Now on solving this balanced problem by the existing methods (Basirzadeh 2011; Pandian & Natrajan 2010, 2010a), the following fuzzy optimal solution is obtained:

$\widetilde{x}_{11} = (-15, 0, 7, 20)$, $\widetilde{x}_{12} = (4, 8, 10, 18)$, $\widetilde{x}_{14} = (-40, -12, 7, 49)$, $\widetilde{x}_{21} = (3, 4, 7, 14)$, $\widetilde{x}_{31} = (3, 4, 5, 8)$ and the remaining $\widetilde{x}_{ij}$ to be zero trapezoidal fuzzy numbers. Total fuzzy transportation cost is found to be $(-216, 26, 113, 510)$.

From the above results, we find that the fuzzy quantity $(-40, -12, 7, 49)$ is to be transported to dummy destination D_4 from source S_1 to balance the excess availability. Since, dummy destination has no significance in reality, so the existing methods (Basirzadeh 2011; Pandian & Natrajan 2010, 2010a) do not give any further information.

5. Proposed method

In this section, a method is proposed to solve the unbalanced FFTPs in which total availability is more than total demand. We first prove the following result.

Theorem 1. Let $(LP2)$ be the fuzzy linear formulation of balanced FFTP obtained by adding a dummy source S_{m+1} with unit fuzzy transportation costs $\widetilde{c}_{(m+1)j} = \min_{1 \le i \le m} (\widetilde{c}_{ij}),\ 1 \le j \le n$, a dummy destination D_{n+1} with $\widetilde{c}_{i(n+1)} = \min_{1 \le j \le n} (\widetilde{c}_{ij}),\ 1 \le i \le m$ to $(LP1)$. Also, let $(\widetilde{u}, \widetilde{v})$ and $(\widetilde{u}', \widetilde{v}')$ be the optimal solutions of the duals of $(LP1)$ and $(LP2)$, respectively, where $(\widetilde{u}, \widetilde{v}) = (\widetilde{u}_1, \widetilde{u}_2, \ldots, \widetilde{u}_m, \widetilde{v}_1, \widetilde{v}_2, \ldots, \widetilde{v}_n)$ and $(\widetilde{u}', \widetilde{v}') = (\widetilde{u}'_1, \widetilde{u}'_2, \ldots, \widetilde{u}'_{m+1}, \widetilde{v}'_1, \widetilde{v}'_2, \ldots, \widetilde{v}'_{n+1})$. Then $\widetilde{u}_i = \widetilde{u}'_i,\ 1 \le i \le m$ and $\widetilde{v}_j = \widetilde{v}'_j,\ 1 \le j \le n$ provided $\widetilde{u}'_{m+1} = \widetilde{0}$.

Proof. The problem (*LP2*) is:

$$\text{Minimize} \quad \sum_{i=1}^{m+1}\sum_{j=1}^{n+1}(\widetilde{c}_{ij} \otimes \widetilde{x}_{ij}), \ \text{subject to}$$
$$\sum_{j=1}^{n+1}\widetilde{x}_{ij} \approx \widetilde{a}_i \quad (1 \le i \le m+1),$$
$$\sum_{i=1}^{m+1}\widetilde{x}_{ij} \approx \widetilde{b}_j \quad (1 \le j \le n+1),$$
$$\widetilde{x}_{ij} \succeq \widetilde{0},$$

where $\widetilde{a}_{m+1} = \sum_{i=1}^{m}\widetilde{a}_i$ and $\widetilde{b}_{n+1} = \sum_{i=1}^{m}\widetilde{a}_i \oplus$ excess supply.

Since $(\widetilde{u}, \widetilde{v})$ is the optimal solution of dual of (*LP1*), so $\widetilde{u}_i \oplus \widetilde{v}_j \preceq \widetilde{c}_{ij}, \forall\, 1 \le i \le m, 1 \le j \le n$ and $\widetilde{u}_i \succeq \widetilde{0}, \forall\, 1 \le i \le m$.

To show that the $(\widetilde{u}'_1, \widetilde{u}'_2, ..., \widetilde{u}'_m, \widetilde{v}'_1, \widetilde{v}'_2, ..., \widetilde{v}'_n)$ is also an optimal solution of the dual of (*LP1*), it is sufficient to show that

(i) $\widetilde{u}'_i \oplus \widetilde{v}'_j \preceq \widetilde{c}_{ij}, \forall\, 1 \le i \le m,\ 1 \le j \le n$ and
(ii) $\widetilde{u}'_i \succeq \widetilde{0}, \forall\, 1 \le i \le m$ and $\widetilde{v}'_j \succeq \widetilde{0}, \forall\, 1 \le j \le n$.

(i) Since $(\widetilde{u}', \widetilde{v}')$ is the optimal solution of the dual to problem (*LP2*), so
$\widetilde{u}'_i \oplus \widetilde{v}'_j \preceq \widetilde{c}_{ij}, \forall\, 1 \le i \le m+1, 1 \le j \le n+1$,
so $\widetilde{u}'_i \oplus \widetilde{v}'_j \preceq \widetilde{c}_{ij}, \forall\, 1 \le i \le m, 1 \le j \le n$.

(ii) Since $\widetilde{u}'_i \oplus \widetilde{v}'_j \preceq \widetilde{c}_{ij}$, we have
$\widetilde{u}'_{m+1} \oplus \widetilde{v}'_j \preceq \widetilde{c}_{(m+1)j}\ \forall\, 1 \le j \le n+1$ and $\widetilde{u}'_i \oplus \widetilde{v}'_{n+1} \preceq \widetilde{c}_{i(n+1)},\ \forall\, 1 \le i \le m+1$.

$\widetilde{u}'_{m+1} \oplus \widetilde{v}'_j \preceq \widetilde{c}_{(m+1)j}\ \forall\, 1 \le j \le n+1$ implies
$\widetilde{v}'_j \preceq \widetilde{c}_{(m+1)j}$ (since $\widetilde{u}'_{m+1} = \widetilde{0}$)
$\Rightarrow \widetilde{v}'_j \preceq \widetilde{c}_{ij}$ for all i (since $\widetilde{c}_{(m+1)j} = \min_{1 \le i \le m} (\widetilde{c}_{ij})$)
$\Rightarrow \widetilde{c}_{ij} \ominus \widetilde{v}'_j \succeq \widetilde{0}$
$\Rightarrow \widetilde{u}'_i \succeq \widetilde{0}$ (since corresponding to each $\widetilde{v}'_j$, there exist $\widetilde{u}'_i$ such that $\widetilde{u}'_i \oplus \widetilde{v}'_j = \widetilde{c}_{ij}$).

Similarly from $\widetilde{u}'_i \oplus \widetilde{v}'_{n+1} \preceq \widetilde{c}_{i(n+1)} \forall\, 1 \le i \le m+1$, it can be proved that $\widetilde{v}'_j \succeq \widetilde{0}$ (since $\widetilde{u}'_{m+1} = \widetilde{0}$, so $\widetilde{v}'_{n+1} = \widetilde{0}$ as $\widetilde{b}_{n+1}$ are so chosen that in any feasible solution $\widetilde{x}_{(m+1)(n+1)}$ must be a basic variable). □

This completes the proof.

5.1 *Steps of the proposed method*

Step 1: Balance the given FFTP by adding a dummy source ($(m+1)^{th}$ source) with availability equal to total availability as well as dummy destination ($(n+1)^{th}$ destination) with demand equal to sum of total availability and excess supply. That is

$$\widetilde{a}_{m+1} = \sum_{i=1}^{m} \widetilde{a}_i \quad \text{and} \quad \widetilde{b}_{n+1} = \sum_{i=1}^{m} \widetilde{a}_i\ \oplus \text{ excess supply.}$$

The unit transportation costs are taken as follows:

$$\widetilde{c}_{i(n+1)} = \min_{1 \le j \le n} (\widetilde{c}_{ij}),\ 1 \le i \le m, \quad \widetilde{c}_{(m+1)j} = \min_{1 \le i \le m} (\widetilde{c}_{ij}),\ 1 \le j \le n,$$
$$\widetilde{c}_{ij} = \widetilde{c}_{ij},\ 1 \le i \le m, 1 \le j \le n \quad \text{and} \quad \widetilde{c}_{(m+1)(n+1)} = (0, 0, 0, 0).$$

Step 2: Apply any of the existing methods (Basirzadeh 2011; Pandian & Natrajan 2010, 2010a) to the balanced FFTP obtained in Step 1. Let the fuzzy optimal solution obtained be $\widetilde{x}_{ij}, 1 \le i \le m+1, 1 \le j \le n+1$.

Step 3: Find the values of all the dual variables $\widetilde{u}'_i, 1 \le i \le m$ and $\widetilde{v}'_j, 1 \le j \le n+1$ by assuming $\widetilde{u}'_{m+1} = (0, 0, 0, 0)$ and using the relation $\widetilde{u}'_i \oplus \widetilde{v}'_j = \widetilde{c}'_{ij}$ for basic variables.

Table 2. Balanced fully fuzzy transportation problem.

Destinations → Sources ↓	D_1	D_2	D_3	D_4	Fuzzy availability
S_1	(4,5,8,15)	(1,1,2,4)	(1,2,3,6)	(1,1,2,4)	(9,11,20,40)
S_2	(1,1,2,4)	(7,9,12,20)	(4,5,8,15)	(1,1,2,4)	(3,4,7,14)
S_3	(0,1,1,2)	(7,12,15,26)	(1,1,2,4)	(0,1,1,2)	(3,4,5,8)
S_4	(0,1,1,2)	(1,1,2,4)	(1,1,2,4)	(0,0,0,0)	(15,19,32,62)
Fuzzy demand	(7,12,15,26)	(4,8,10,18)	(2,5,6,11)	(−25, 7, 39, 111)	

Step 4: By Theorem 1, $\widetilde{u}'_i = \widetilde{u}_i,\ 1 \le i \le m$ and $\widetilde{v}'_j = \widetilde{v}_j,\ 1 \le j \le n$. Find those dual variables $\widetilde{u}_i$ and $\widetilde{v}_j$ which have rank zero. Now, the fuzzy optimal solution of the problem in terms of original sources and destinations is obtained as follows:

Let $\widetilde{x}_{(m+1)p}$ for some p and $\widetilde{x}_{q(n+1)}$ for some q be the basic variables in the fuzzy optimal solution obtained in Step 2. Also, let $\widetilde{u}_i$ and $\widetilde{v}_j$ have rank zero for $i \in I$ and $j \in J$.

Then increase the value of the basic variable in the cell with $\min_{i \in I} \widetilde{c}_{ip}$ by $\widetilde{x}_{(m+1)p}$ and the value of $\min_{j \in J} \widetilde{c}_{qj}$ by $\widetilde{x}_{q(n+1)}$ (because availability of only these sources can be increased (Ebrahimnejad & Nasseri 2009)). Break the tie in minimum value(s) arbitrarily.

If the minimum cost cell is non-basic in the optimal solution obtained in Step 2 then it may become basic in the final solution.

6. Numerical example

In this section, the fully fuzzy transportation problem considered in section 4 is solved by the proposed method.

Step 1: Applying Step 1 of the proposed method to the problem considered in section 4, we obtain the balanced FFTP shown in the table 2.

Step 2: Solving this balanced fuzzy transportation problem by using any of the existing methods (Basirzadeh 2011; Pandian & Natrajan 2010, 2010a), following fuzzy optimal solution is obtained:

$\widetilde{x}_{12} = (4, 8, 10, 18)$, $\widetilde{x}_{13} = (2, 5, 6, 11)$, $\widetilde{x}_{14} = (-102, -25, 27, 116)$, $\widetilde{x}_{21} = (3, 4, 7, 14)$, $\widetilde{x}_{31} = (3, 4, 5, 8)$, $\widetilde{x}_{41} = (-15, 0, 7, 20)$, $\widetilde{x}_{44} = (-5, 12, 32, 77)$.

Table 3. Fuzzy optimal solution for Example 1.

Existing methods (Basirzadeh 2011; Pandian & Natrajan 2010, 2010a)	Proposed method
$\widetilde{x}_{11} = (-15, 0, 7, 20)$, $\widetilde{x}_{12} = (4, 8, 10, 18)$ $\widetilde{x}_{13} = (2, 5, 6, 11)$, $\widetilde{x}_{14} = (-40, -12, 7, 49)$ $\widetilde{x}_{21} = (3, 4, 7, 14)$, $\widetilde{x}_{31} = (3, 4, 5, 8)$	$\widetilde{x}_{12} = (-98, -17, 37, 134)$, $\widetilde{x}_{13} = (2, 5, 6, 11)$ $\widetilde{x}_{21} = (3, 4, 7, 14)$, $\widetilde{x}_{31} = (-12, 4, 12, 28)$
Total fuzzy transportation cost = (−216, 26, 113, 510)	Toatl fuzzy transportation cost = (−411, −6, 118, 714)

Step 3: Assuming $\widetilde{u}'_4 = (0, 0, 0, 0)$ and using the relation $\widetilde{u}'_i \oplus \widetilde{v}'_j = \widetilde{c}_{ij}$ for the basic variables, the values of dual variables $\widetilde{u}'_i$, $(1 \leq i \leq 3)$ and $\widetilde{v}'_j$, $(1 \leq j \leq 4)$ and are found to be: $\widetilde{u}'_1 = (1, 1, 2, 4)$, $\widetilde{u}'_2 = (-1, 0, 1, 4)$, $\widetilde{u}'_3 = (-2, 0, 0, 2)$, $\widetilde{v}'_1 = (0, 1, 1, 2)$, $\widetilde{v}'_2 = (-3, -1, 1, 3)$, $\widetilde{v}'_3 = (-3, 0, 2, 5)$, $\widetilde{v}'_4 = (0, 0, 0, 0)$.

Step 4: By Step 4 of the proposed method, we find that the $\Re(\widetilde{u}_3) = 0$ and $\Re(\widetilde{v}_2) = 0$ and also there is only single basic variable corresponding to each of S_3 and D_2 i.e. $\widetilde{x}_{31}$ corresponding to source S_3 and $\widetilde{x}_{12}$ corresponding to destination D_2 . So increase $\widetilde{x}_{31}$ and $\widetilde{x}_{12}$ by the fuzzy quantities $\widetilde{x}_{41}$ and $\widetilde{x}_{14}$, respectively. Hence the fuzzy optimal solution is:

$\widetilde{x}_{12} = (4, 8, 10, 18) \oplus (-102, -25, 27, 116) = (-98, -17, 37, 134)$, $\widetilde{x}_{13} = (2, 5, 6, 11)$, $\widetilde{x}_{21} = (3, 4, 7, 14)$, $\widetilde{x}_{31} = (3, 4, 5, 8) \oplus (-15, 0, 7, 20) = (-12, 4, 12, 28)$.

So, the total fuzzy transportation cost is $(-411, -6, 118, 714)$.

7. Conclusions

The fuzzy optimal solution of Example 1, by the existing methods (Basirzadeh 2011; Pandian & Natrajan 2010, 2010a) as well as the proposed method is shown in table 3.

From this table, it is clear that the fuzzy optimal solution obtained by the proposed method tells us that the excess availability be supplied to the destinations D_1 and D_2. Also, the total transportation cost by the proposed method is less than the existing methods (Basirzadeh 2011; Pandian & Natrajan 2010, 2010a).

Symbols

m: number of sources
n: number of destinations
$\widetilde{a}_i$: the fuzzy availability of the product at source S_i
$\widetilde{b}_j$: the fuzzy demand of the product at destination D_j
$\widetilde{c}_{ij}$: the fuzzy cost for transporting one unit of the product from S_i to D_j
$\widetilde{x}_{ij}$: the fuzzy quantity of the product to be transported from S_i to D_j
$\widetilde{u}_i$: dual variable corresponding to ith source in $(LP1)$
$\widetilde{v}_j$: dual variable corresponding to jth destination in $(LP1)$
$\widetilde{u}'_i$: dual variable corresponding to ith source in $(LP2)$
$\widetilde{v}'_j$: dual variable corresponding to jth destination in $(LP2)$

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