A computationally efficient approach for template matching-based image registration

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Abstract. Image registration using template matching is an important step in image processing. In this paper, a simple, robust and computationally efficient approach is presented. The proposed approach is based on the properties of a normalized covariance matrix. The main advantage of the proposed approach is that the image matching can be achieved without calculating eigenvalues and eigenvectors of a covariance matrix, hence reduces the computational complexity. The experimental results show that the proposed approach performs better in the presence of various noises and rigid geometric transformations.

Keywords. Normalized cross-correlation; similarity measure; eigenvalue; covariance matrix; template matching.

1. Introduction

Image registration is a process of matching two images that contain common visual information and used to find the location of sub-image (template image) within the reference image. These images may be collected from multiple photographs, different sensors or different viewpoints. The aim of the template matching is to find the geometric transformations of the template image (T^I) in the reference image (I^I), where, $T^I(i, j) = \Gamma \{I^I(i', j')\}$ and Γ is a 2-D geometric transformation related with the I^I image coordinates (i', j') (Matungta *et al* 2009). Registration process is extensively useful in computer vision, medical imaging, automatic target recognition in military and analyzing the remote images obtained from the satellites (Zitová & Flusser 2003). In the last 10 years, many image registration methods have been proposed which are broadly classified into feature-based approach and area-based approach. The feature-based approach uses only the features of an image like color gradient, edges, geometric features or main feature points

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(Zokai & Wolberg 2005). Area-based approaches put emphasis on the feature detection rather than the feature matching. In the case of area-based approaches, Normalized Cross-Correlation (NCC) also called as image similarity measures are particularly used for image registration. Similarity measure identifies the degree of closeness between two images (Penney *et al* 1998). The Sum of Squared Difference (SSD) and Mutual Information (MI) are some of the area-based similarity measure techniques commonly used for image registration (Stefano & Matteccia 2003; Rohr *et al* 2004; Gao *et al* 2008; Sarvaiya *et al* 2009). However, these methods give false results for variations in angle, different illumination and noise in the images. To overcome such problems, Tsai & Yang (2005) suggested a smallest eigenvalue-based similarity measure, which is successfully applied to detect the dissimilarity between two Printed Circuit Board (PCBs) images. Further, Sun *et al* (2008) proposed an object recognition method using eigenvalues, covariance matrix and back-propagation neural network. In these methods, the matching of reference image with template image is achieved by calculating the smallest eigenvalue of a covariance matrix. Moreover, Austvoll & Kwolek (2010) proposed a covariance matrix-based object tracking algorithm for similarity measure between face images of same color and shapes. Recently, Gaidhane *et al* (2012a) proposed an efficient similarity measure based on Gerschgorin circle theorem and applied successfully in rigid as well as nonrigid medical image registration.

In this paper, a new computationally efficient image registration method is proposed. It is based on the properties of covariance matrix. In this approach, it is shown that the template matching-based registration can be achieved by checking only simple conditions without calculating the covariance matrix, real eigenvalues and eigenvectors.

2. Template matching methods

In literature, various template matching (registration) methods have been proposed (Zitová & Flusser 2003). The NCC method is one of the successful template matching methods and it is widely used in image matching application. Suppose $I(j, k)$ is the reference image of size $M \times M$ pixel and $T(j - u, k - v)$ is the template image of size $N \times N$ pixel. The Normalized Cross-Correlation (μ) between these images for every shift position is given by (Zitová & Flusser 2003; Sarvaiya *et al* 2009)

$$\mu(u, v) = \frac{\sum_{j=0}^{N-1} \sum_{k=0}^{N-1} I(j, k) T(j - u, k - v)}{\sqrt{\sum_{j=0}^{N-1} \sum_{k=0}^{N-1} I^2(j, k) \sum_{j=0}^{N-1} \sum_{k=0}^{N-1} T^2(j - u, k - v)}}, \quad (1)$$

where $u, v = 0, 1, 2, \dots, M-N$. Among all the computed values of μ , the largest value shows the best match between the reference image and template image. Generally, the NCC value ranges from 0 to 1. The perfect matching between template image $T(j - u, k - v)$ and the reference image $I(j, k)$ is achieved when the maximum value of NCC is unity ($\mu = 1$ or $1 - \mu = 0$). A pixel with maximum value of μ , which is above the predetermined threshold value, is classified as matched pixel. When such type of measure is applied for an image registration, it may not be responsive to the small variations and results into false registration decision. To overcome these problems, Tsai & Yang (2005) proposed an eigenvalue-based approach and it is applied successfully by Sun *et al* (2008) for invariant 2-D objects recognition. In these

methods, a covariance matrix $[A]$ of the reference image $I(j, k)$ and a template image $T(j, k)$ of size $M \times N$ pixel is represented as (Tsai & Yang 2005)

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}. \quad (2)$$

The resultant covariance matrix $[A]$ has only two eigenvalues, i.e., the smallest eigenvalue and the largest eigenvalue. These eigenvalues are denoted as λ_s and λ_L , respectively (Tsai & Yang 2005; Gaidhane *et al* 2012b).

$$\lambda_s = \frac{(a_{11} + a_{22}) - \sqrt{(a_{11} - a_{22})^2 + 4a_{12}^2}}{2}, \quad (3)$$

$$\lambda_L = \frac{(a_{11} + a_{22}) + \sqrt{(a_{11} - a_{22})^2 + 4a_{12}^2}}{2}, \quad (4)$$

$$\lambda_s \leq \lambda_L. \quad (5)$$

The smallest eigenvalue (λ_s) is used as a similarity measure for template image registration. In this method, the template matching for image registration is achieved by following conditions:

- (1) If $\lambda_s < 0$ or $\lambda_s > 0$, then the template image is not matching.
- (2) If $\lambda_s \cong 0$, then the template image is matching.

However, the limitation of the smallest eigenvalue-based method (λ_s) and NCC (μ) method is inadequate response to the small image transformation. Moreover, the computational complexity is higher due to the covariance matrix and eigenvalues (SVD) calculation.

Let $\mathbf{B} \in \mathbb{R}^{m \times m}$, where $\mathbf{B} = [b_1, b_2, \dots, b_m]$ then the covariance matrix $[A]$ of matrix $\mathbf{B}$ is given as

$$[A] = \frac{1}{m-1} \mathbf{B} \mathbf{B}^T. \quad (6)$$

The covariance matrix is a $m \times m$ square matrix. For eigenvalue calculation problem, the covariance matrix is decomposed using Singular Value Decomposition (SVD) as

$$\mathbf{A} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T, \quad (7)$$

where $\mathbf{U} = (u_1, u_2, u_3, \dots, u_m)$ is the orthogonal vector. Similarly, $\mathbf{\Sigma}$ represents the real and positive diagonal singular eigenvalues and $\mathbf{V}^T$ represents the transpose of real complex unitary matrix.

The proposed work is motivated from the fact that the eigenvalues in the above reported methods are calculated using the covariance matrix as well as SVD which contribute most of the computational complexity in the algorithms. The computation of the covariance matrix $[A]$ of a $N \times N$ pixel image requires multiplications of the order $O(N)^3$, and summation of the order $O(N)^3$, respectively. Moreover, for SVD operation, the computational complexity is also of the order $O(N)^3$. Thus, the computational complexity of smallest eigenvalue-based matching approach increases to the order $2 \times O(N)^3$. Therefore, the most of the computational complexity is due to the covariance matrix calculation and SVD decomposition. However, the computational complexity can be reduced by avoiding the covariance matrix calculation in image registration process.

The MI method is also a well known area-based method. It is a statistical approach which measures the similarity between the two images or datasets. Mathematically, MI between two random variables can be calculated as (Viola & Wells 1997)

$$MI(A, B) = H(B) - H(B/A) = H(A) + H(B) - H(A, B), \quad (8)$$

where $H(A) = -E_X(\log(P(A)))$ represents entropy of random variable A and $P(A)$ is the probability distribution of variable A . Further, Li & Yuan (2005) also proposed an algorithm to extract the direct features from image matrices for image matching based on Fisher's Linear Discriminant criterion. The Fisher's discrimination criterion J_c can be obtained using the mean values ($\bar{s}$, $\bar{t}$) and standard deviation (σ_s , σ_t) of reference image and template image, respectively. The Fisher's discrimination criterion J_c can be represented as

$$J_c = \frac{(\bar{s} - \bar{t})^2}{\sigma_s^2 + \sigma_t^2}. \quad (9)$$

Fisher's discriminant criterion is a well known method for feature extraction and discrimination. However, it is not an absolute criterion when used for deriving a set of discriminant vectors (Xu et al 2004).

In image registration applications, the desired patterns can be obtained in the reference image by sliding the windowed template image and calculating the similarity relation (similarity measure) between these two windowed subimages. The zero value of the above discussed similarity measures indicates the occurrence of template image in the reference image. A windowed image with small value of similarity measure below some predefined threshold value is classified as a registered image. Thus, the value of similarity measure should be as small as possible for similar images and larger for dissimilar images. However, the similarity measures value close to predefined threshold value creates the confusion and results into false registration decision. Moreover, the performance is degraded in the presence of noise. To avoid these drawbacks, recently Gerschgorin circle theorem-based graphical method is proposed (Gaidhane et al 2012a). In this approach, the template matching is carried out by considering the Gerschgorin bound of a covariance matrix as a similarity measure of two compared images. The Gerschgorin bound is zero for perfect matching between two compared images and very large for dissimilar images.

In above discussed approaches, it is necessary to compare the calculated similarity measures value with the predefined threshold value. However, the selection of threshold value is not an easy task and depends on the type of similarity measures used in the applications. Therefore, it is required to overcome these difficulties during the image registration. In this paper, a simple and computationally efficient approach for image registration is proposed. It is based on the properties of a covariance matrix and the previously proposed method based on Gerschgorin circle theorem (Gaidhane et al 2012a).

3. Mathematical preliminaries

In general, if the true position of a feature points (p, q) is given by $(\bar{p}, \bar{q})$, then the error can be represented by $\Delta p = p - \bar{p}$ and $\Delta q = q - \bar{q}$. The error terms Δp and Δq are considered as

the random variables and their covariance matrix for zero noise level is given as (Kanazawa & Kanatani 2001)

$$[C] = \begin{bmatrix} E(\Delta p^2) & E(\Delta p \Delta q) \\ E(\Delta q \Delta p) & E(\Delta q^2) \end{bmatrix}, \quad (10)$$

where $E(\cdot)$ represents the expected result. The covariance matrix of two compared images is a symmetric matrix of order 2×2 and it can be represented in the form $\rho^2 \hat{C}$, where ρ^2 is the noise level and $\hat{C}$ is the normalized covariance matrix. The covariance matrix $\hat{C}$ can be calculated from the gray levels of the images. Since the noise characteristics do not depend on the image orientation or pixels positions, the optimal matching can be achieved by the covariance matrix $\hat{C}$ (Irani & Anandan 2000). Moreover, the accuracy of computation can be improved by optimizing only the normalized covariance matrix elements.

Suppose $\mathbf{R}(i, j)$ is the gray-level of pixels $(i, j) \in N_s$, where N_s is the squared neighborhood centered on a point s . Then the pixel (p, q) is to obtained such that $\mathbf{R}(i, j) \cong \mathbf{R}(i + p, j + q), (i, j) \in N_s$. Now using the Taylor expansion, the term $\mathbf{R}(i + p, j + q)$ can be approximated as

$$\mathbf{R}(i + p, j + q) = \frac{1}{2} \sum_{(i, j) \in N_s} (\mathbf{R}_{ij} - \mathbf{R} - \mathbf{R}_{ip} - \mathbf{R}_{jq})^2, \quad (11)$$

where $\mathbf{R}_{ip}$ and $\mathbf{R}_{jq}$ represents the partial derivatives of $\mathbf{R}(i, j)$. Using the optical flow detection theory and gradient constraints (Ohta 1991), the terms in the Eq. (11) can be represented as

$$\mathbf{R}_{ij} - \mathbf{R} = \mathbf{R}_{ip} + \mathbf{R}_{jq}. \quad (12)$$

Thus, the term $\mathbf{R}(i + p, j + q)$ approaches to the minimum value. Using above mathematical preliminaries, the maximum likelihood estimation function L can be approximated using the Taylor expansion as

$$L = \frac{1}{2} \sum_{(i, j) \in N_s} (\mathbf{R}_{ip} + \mathbf{R}_{jq})^2, \quad (13)$$

$$= \frac{1}{2} (pq) \mathbf{H} \begin{pmatrix} p \\ q \end{pmatrix},$$

where $\mathbf{H}$ is a Hessian matrix, which is given as

$$\mathbf{H} = \begin{bmatrix} \sum_{p, q \in N_s} \mathbf{R}_p^2 & \sum_{p, q \in N_s} \mathbf{R}_p \mathbf{R}_q \\ \sum_{p, q \in N_s} \mathbf{R}_p \mathbf{R}_q & \sum_{p, q \in N_s} \mathbf{R}_q^2 \end{bmatrix}. \quad (14)$$

The normalized covariance matrix $[\hat{C}]$ can be defined as the inverse of Hessian matrix $\mathbf{H}$ (Förstner 1987).

$$\hat{C} = \mathbf{H}^{-1} = \begin{bmatrix} \sum_{p, q \in N_s} \mathbf{R}_p^2 & \sum_{p, q \in N_s} \mathbf{R}_p \mathbf{R}_q \\ \sum_{p, q \in N_s} \mathbf{R}_p \mathbf{R}_q & \sum_{p, q \in N_s} \mathbf{R}_q^2 \end{bmatrix},$$

$$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}. \quad (15)$$

In previous work (Gaidhane *et al* 2012a), the elements a_{11} , a_{12} , a_{21} and a_{22} of such covariance matrix are calculated. It is observed that the covariance matrix represented by Eq. (3) is approximately equal to the covariance matrix shown by Eq. (2). Based on this observation, Lemma 1 can be represented as follows;

Lemma 1. Suppose $x(u, v)$ is gray-level reference image and $y(u, v)$ is a gray-level template image which is to be tested using the proposed algorithm. Then the normalized gray values (p^2 , q^2) and mean gray values ($\bar{p}$, $\bar{q}$) of reference image as well as template image at each feature point can be calculated as

$$p^2 = \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x^2(u, v), \quad \bar{p} = \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v), \quad (16)$$

$$q^2 = \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y^2(u, v), \quad \bar{q} = \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v). \quad (17)$$

For these two images, using Eqs. (3), (16) and (17), the normalized covariance matrix can be approximated as

$$[\hat{C}] = \begin{bmatrix} \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x^2(u, v) - [\bar{p}]^2 & \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \cdot y(u, v) - (\bar{p} \cdot \bar{q}) \\ \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \cdot x(u, v) - (\bar{q} \cdot \bar{p}) & \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y^2(u, v) - [\bar{q}]^2 \end{bmatrix}. \quad (18)$$

Using the above Lemma 1, a simple and computationally efficient approach is proposed which avoids the calculation of covariance matrix and hence reduces the computational complexity.

4. Proposed approach for template matching based image registration

It is stated that the template image is matched with the reference image when the magnitude of smallest eigenvalue of a covariance matrix $[A]$ is zero (Tsai & Yang 2005). Moreover, in previous work (Gaidhane *et al* 2012a), it is reported that the Gerschgorin left bound is zero for two completely matched images. These similarity measures are obtained by considering the determinant of covariance matrix $[A]$ equal to zero. Based on this assumption, a new computationally efficient approach is proposed. In this approach, five mathematical conditions are presented. These conditions are obtained using only Eqs. (16), (17) and (18), which are given below in Eq. (19)–(23).

Condition 1:

$$\left\{ \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x^2(u, v) \right] - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \right]^2 \right\} - \left\{ \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \cdot x(u, v) - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \cdot \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \right] \right\} = 0. \quad (19)$$

Condition 2:

$$\left\{ \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \cdot y(u, v) \right] - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \cdot \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \right] \right. \\ \left. - \left\{ \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y^2(u, v) \right] - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \right]^2 \right\} \right\} = 0. \quad (20)$$

Condition 3:

$$\left\{ \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x^2(u, v) - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \right]^2 \right\} \\ - \left\{ \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \cdot y(u, v) - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \cdot \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \right] \right\} = 0. \quad (21)$$

Condition 4:

$$\left\{ \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \cdot x(u, v) - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \cdot \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \right] \right\} \\ - \left\{ \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y^2(u, v) - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \right]^2 \right\} = 0. \quad (22)$$

Condition 5:

$$\left\{ \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x^2(u, v) - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \right]^2 \right\} \times \\ \left\{ \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y^2(u, v) - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \right]^2 \right\} = \\ \left\{ \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \cdot x(u, v) - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \cdot \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \right] \right\} \times \\ \left\{ \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \cdot y(u, v) - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \cdot \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \right] \right\} \\ = \left\{ \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \cdot y(u, v) - \left[\frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} x(u, v) \cdot \frac{1}{m \times n} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} y(u, v) \right] \right\}^2. \quad (23)$$

Based on these five conditions, a simple algorithm for image registration is proposed. It is explained below in following steps:

Step 1: Read the reference image $x(u, v)$ and normalized by its energy to make the image contrast independent.

Step 2: Select the template image and determine its size.

Step 3: Extend the peripheral area of reference image with zeros so that the template image center coincides on the first pixel of the reference image.

Step 4: Move the template image on the entire area of the reference image.

Step 5: Calculate the values of Eq. (16) and Eq. (17).

- (i) If any of the above conditions (**Condition 1 to 5**), i.e., Eq. (19) – (23) is satisfied, then the template image is perfectly matched with the reference image.
- (ii) If all the conditions, i.e., Eq. (19) – (23) are not satisfied, then the template image is not matched with the reference image.

Step 6: Find the position of pixel co-ordinates, where, the template image is perfectly matched with the reference image.

Step 7: Obtain and show the template image within the area of a reference image.

Here, it is important to note that all the proposed conditions are obtained using only Eq. (16) and (17). These conditions are calculated from the mean value and gray-level of the normalized images. The gray-level of each pixel belongs to the squared neighborhood centered on a particular point. Moreover, the noise characteristics do not depend on the image orientation or pixels position. Therefore, the proposed approach is less affected by the noise and small rotation.

In proposed algorithm, the complex calculations such as covariance matrix computation and SVD decomposition are avoided. Moreover, it is observed from Eq. (19)–(23) that the computational complexity is of the order $O(N)^2$ as there is only a square term in the Eq. (19)–(23). Thus, the computational complexity is reduced from order $O(N)^3$ to the order $O(N)^2$.

The proposed approach is demonstrated using figure 1. Figure 1(a) shows the reference image and figure 1(b) represents the rotated image. Suppose figure 1(a) considered a reference image and figure 1(b) as a template image. Using Eq. (1), the NCC between these two images is $(1 - \mu) = 0.231$. Similarly, eigenvalue-based similarity measure can be obtained by using the covariance matrix shown by Eq. (2) as:

$$[A] = \begin{bmatrix} 0.4769 & 0.112 \\ 0.11203 & 0.49297 \end{bmatrix}. \quad (24)$$

Figure 1. MRI images. (a) Reference image, (b) template image rotated by $\theta = 10^\circ$.

From Eq. (24), the smallest eigenvalue is $\lambda_s = 0.3727$. Now, using the proposed approach, the conditions from (1) to (5) can be verified as follows. From Eq. (16) and (17), we get, $P2 = 17.033$, $q^2 = 16.261$, $\bar{p} = 0.335$ and $\bar{q} = 0.3178$.

Condition 1: $16.89 - 14.69 = 2.2 \neq 0$

Condition 2: $14.69 - 12.48 = 2.21 \neq 0$

Condition 3: $16.8901 - 14.6913 = 2.19 \neq 0$

Condition 4: $14.6913 - 12.4848 = 2.21 \neq 0$

Condition 5: $16.8901 \times 12.4848 \neq (14.6913)^2$

Here, none of the conditions get satisfied and the difference in each condition is higher than the value of NCC ($1 - \mu = 0.231$) and the smallest eigenvalue ($\lambda_s = 0.3727$). Thus, the higher values show the stronger evidence of mismatching between these two images.

5. Experimental results and discussion

In this section, the experimental results for evaluating the effectiveness of the proposed approach are presented. A set of 256×256 pixel size reference images ([USC-SIPI Image Database](#)) is considered for experimentations.

In this experiment, figures 2(a1–a3) are considered as the reference images and figures 2(b1–b3) are considered as the template images. The entire area of a template image is used for registration. To study the effect of rigid transformation such as rotation, the rotation parameter (θ) is applied to the template images. The template images are rotated by rotation parameter $\theta = 0^\circ$, 90° and 180° , respectively. Then the NCC, eigenvalue-based similarity measure and proposed approach are tested for different rotation parameter. Here, it is important to note that the value of similarity measures between reference image and template image should be less than the predefined threshold value ($\varepsilon \rightarrow 0$) for similar images and larger for dissimilar images. The selection of a threshold value (ε) plays an important role in final template matching decision. However, the proposed approach is independent of threshold value selection and comparison process. The registration results using proposed approach are shown in figures 2(c1–c3) and the NCC plot are shown in figures 2(d1–d3). In NCC plot, the value of NCC ranges from 0 to 1 is shown on the z-axis and the position of recovered pixel co-ordinates where template perfectly matched with the reference image are shown on x-axis and y-axis, respectively. Before experimentations, it is important to note that the value of similarity measures should be minimum (zero) for similar images and larger for dissimilar images. The smaller value of similarity measure may results into false matching decision.

In first experiment, figure 2(b1) is considered as a template image without rotation, i.e., $\theta = 0^\circ$. This template image is compared with a reference image, i.e., figure 2(a1). It is observed that the value of NCC is near to zero ($1 - \mu = 0.06$) and the smallest eigenvalue is zero ($\lambda_s = 0$). Moreover, using proposed approach, all the stated conditions in Eqs. (19)–(23) are also satisfied. Therefore, it is concluded that the template image, i.e., figure 2(b1) is matched with the reference image, i.e., figure 2(a1). The actual matched image area is shown by gray rectangle within the area of reference image in figure 2(c1). In this experiment NCC, the smallest eigenvalue and proposed method show that the template image is perfectly matched with the reference image.

In second experiment, the template image figure 2(b2) is rotated by an angle $\theta = 90^\circ$ which changes the geometry of template image. Now, the rotated template image is compared with the reference image figure 2(a2). We know that the template image is differing from the reference image as it is rotated by $\theta = 90^\circ$. Therefore, the values of NCC and the smallest eigenvalue

Figure 2. Image registration results. (a1–a3) the reference images, (b1–b3) template images rotated by an angle $\theta = 0^\circ, 90^\circ$ and 180° respectively, (c1–c3) registration results using proposed approach (gray rectangular area indicates the registration result), and (d1–d3) NCC plots.

should be large. However, in this case, the NCC is found to be $(1 - \mu) = 0.202$ and the smallest eigenvalue is $\lambda_s = 0.279$ which is very small. Here, the value of NCC and the smallest eigenvalue is finite and close to threshold value. Suppose the threshold value is $\varepsilon = 0.25$, a template image is said to be matched when the NCC and the smallest eigenvalue is less than the threshold value, i.e., $\varepsilon = 0.25$. In this experiment, as the reference image and template image is dissimilar, the NCC ($1 - \mu$) and the smallest eigenvalue (λ_s) should be greater than the predefined threshold value i.e., $\varepsilon = 0.25$. However, in this experiment, the value of NCC ($1 - \mu$) and λ_s is close to predefined threshold value ($\varepsilon = 0.25$). Therefore, using existing approaches, it is very difficult to take the final image registration decision which may further results into false decision. Now consider the proposed approach. In this, none of the stated conditions given by Eqs. (19)–(23) are satisfied. Hence, using proposed approach, it is easy to conclude that the template image is not matched with the reference image. Thus, the proposed approach avoids the confusion and shows dissimilarity between two compared images.

In third experiment, the template image figure 2(b3), of 130×130 pixel size is rotated with $\theta = 180^\circ$. It is well known that the template image is completely differing from the reference image as it is rotated by $\theta = 180^\circ$. In this, the NCC and small eigenvalue should be large. Unfortunately, it is found to be very small, i.e., $(1 - \mu) = 0.243$ and $\lambda_s = 0.291$, respectively, which are close to threshold value, i.e., $\varepsilon = 0.25$. Therefore, NCC and small eigenvalue-based methods may results into false registration decision. Now, the same experiment is also carried out using proposed algorithm. In this, none of the proposed condition given by Eqs. (19) – (23) is satisfied. Hence, it is concluded that the template image is dissimilar and mismatched with the reference image. The comparative results for template matching using the reported measures like NCC, smallest eigenvalue-based similarity measure and proposed approach are shown in table 1. From table 1, it is observed that the proposed approach is simple, as the final registration decision is taken by calculating only the proposed conditions.

The performance of various reported similarity measure methods is also influenced by the template window size and noise. The small template window size contains small structure of the image content and generates the noise in its original pixel. On other hand, larger template window size increases the computational complexity. Thus, in conventional methods, the selection of template window size is a challenging task. Therefore, to evaluate the effect of window size on the proposed approach, the different sizes of neighborhood window are used in the presence of noise and small rotations. In this experiment, a set of 256×256 pixel size reference images is taken from the image dataset (The whole Brain, Atlas (Johnson & Becker 1999)) and the template images are obtained from these standard brain images by selecting the defective area. Adobe Photoshop 7.0.1 software is used for selection and cropping of the defective area from an image. In this experiment, the noise and rotation parameters are randomly selected. The image registration results for different template window size in the presence of noise and small rotational transformation is shown in figure 3.

The template matching-based registration results for image having the brain tumor disease is shown in row 1 of figure 3. The defective area (tumor) extracted from the standard brain image is shown by the template image of size 40×40 pixel and rotated with the small angle $\theta = 3^\circ$ which is invisible to the human eyes. This template image then matched with the standard image. Similarly, row 2 shows the standard Pick's disease brain image, extracted template image and registration results. Here, the standard image is convolved with a Salt-and-Peppers noise ($d = 0.02$). This noisy image registered with the template image of size 50×50 pixel.

In row 3, a brain image (AIDS dementia disease) is considered as a standard image. The template image of size 45×45 pixel is extracted from the standard brain image and convolved with the Point Spread Function (PSF) of $h = 8 \times 8$ window size. Then the template image is matched with standard brain image. Further, row 4 shows the registration results for the image

Table 1. Comparative result of Normalized cross-correlation ($1 - \mu$), Smallest eigenvalue (λ_s) and Proposed approach.

Similarity measure	Figure 2(a1) and figure 2(b1)	Figure 2(a2) and figure 2(b2)	Figure 2(a3) and figure 2(b3)
NCC ($1 - \mu$)	0.06	0.202	0.243
Smallest eigenvalue-based method (λ_s)	0	0.279	0.291
Proposed method (checking conditions Eq. (19) – (23))	All conditions are satisfied	All conditions are not satisfied	All conditions are not satisfied

Figure 3. Image registration results. (a) Reference image, (b) template image (c) registration results, and (d) NCC plot.

having brain tumor disease. In this, the template image of size 55×55 pixel is extracted from the reference image. Then the reference image is convolved with the Gaussian noise (variance $v = 0.01$) and registered with the extracted template image of size 55×55 pixel.

Table 2. Comparative results of NCC ($1 - \mu$), Smallest eigenvalue (λ_s), Fisher criterion (J_c), Mutual information (MI), Gerschgorin bound (D) and proposed method for figure 3.

Figure 3	Template size	x, y Co-ordinates	$(1 - \mu)$	(λ_s)	(J_c)	(MI)	(D)	Proposed approach (Cond. 1 to 5)
Row 1	40×40	(117, 177)	0.102	0.149	2.675	6.298	0.0049	All satisfied
Row 2	50×50	(130, 100)	0.124	0.326	0.368	5.863	0.0081	All satisfied
Row 3	45×45	(155, 130)	0.453	0.174	0.856	5.098	0.0032	All satisfied
Row 4	55×55	(162, 124)	0.276	0.342	0.458	4.942	0.0062	All satisfied

Figure 4. Sample of fingerprint images of ten individuals from the dataset.

In these experiments, the different template images along with noise effects and small geometric transformation are considered. The reported similarity measures as well as proposed approach are calculated and summarized in table 2. Moreover, the template matching position x - y coordinates are also shown in table 2.

In these experiments, the similarity measures such as NCC ($1 - \mu$), smallest eigenvalue (λ_s), Fisher criterion (J_c), mutual information (MI), and Gerschgorin bound (D) are calculated. Ideally, the values of these similarity measures should be zero for similar images. From table 2, it is observed that the values of similarity measures are greater than zero. It is due to the noise and geometric transformation added to the reference images or template images. Thus, performance of the conventional similarity measures such as $1 - \mu$, λ_s , J_c , and MI are degraded due to the noise and geometric transformation which is a curse for image registration. Thus, using these conventional methods, it is not easy to match a defective area in the reference image. Although, Gerschgorin bound (D) found suitable for template matching applications, it exhibit the higher computational complexity. However, in proposed approach, all the stated conditions are satisfied. Moreover, there is no need of calculation of any numerical threshold value for making final decision on image registration. Thus, the proposed approach performs better in the presence of different noisy conditions as well as small geometric transformation. The proposed approach is also suitable for any size of template window as there is no need of complex computations.

Further, the image registration is performed on large fingerprint image dataset (Maltoni *et al* 2009) using the proposed approach as well as the reported methods. The fingerprint image dataset contains total 80 images of 10 individuals with 8 different views. Thus, dataset forms

Table 3. Registration accuracy of NCC ($1 - \mu$), Smallest eigenvalue (λ_s), Fisher criterion (J_c), Mutual information (MI), Gerschgorin bound (D) and Proposed method for fingerprint dataset.

Method	NCC ($1 - \mu$)	Eigenvalue (λ_s)	Fisher criterion (J_c)	Mutual information (MI)	Gerschgorin bound (D)	Proposed approach (cond. 1 to 5)
Registration accuracy (%)	86.25	94.70	92.20	98.50	99.50	99.85

Table 4. Computational time and computational complexity of proposed approach, MI , Fisher criterion, Smallest eigenvalue and NCC similarity measure.

	Proposed approach cond. (1)-(5)	Gerschgorin bound (D)	Mutual information (MI)	Fisher criterion (J_c)	Smallest eigenvalue (λ_s)	NCC ($1 - \mu$)
Computational Time (seconds)	0.191116	0.2231	0.251151	0.268805	0.240375	0.253710
Computational Complexity	$(55)^2$	$(55)^2 + 4$	$(55)^3 + (55)^2$	$(55)^3 + (55)^3$	$(55)^3 + (4)^3$	$(55)^3 + (55)^3$

the map of $10 \times 8 = 80$ image size. The size of each image is of 640×480 pixel, with 0 to 255 gray-level values. Figure 4 shows the sample of fingerprint image dataset of 10 individuals. All the images are cropped into small size of 96×96 pixel. In this experiment, these 10 fingerprint images are used for training and remaining 70 images are used for testing. The quality of a proposed approach as well as the reported methods is measured in the form of registration accuracy. A sub-window of 48×48 pixel is considered for experimentations. The registration accuracy of various reported similarity measures and proposed approach is shown in table 3. It is observed from table 3 that the registration accuracy of proposed approach is higher than the various reported methods.

All experiments are carried out on an Intel(R) Core(TM) i3 CPU with 2.40 GHZ frequency and 4 GB RAM. The reported similarity measures and proposed method are tested on a Matrix Laboratory (MATLAB) platform of version 2011a. The comparison of computational time as well as the computational complexity of proposed method, D , MI , J_c , λ_s and $1 - \mu$ is shown in table 4. The windowed region of size 55×55 pixel of reference image shown in figure 1(a) and the template image shown in figure 1(b) is considered for these calculations. From table 4, it is observed that the proposed method requires less computational time as compared to other similarity measures. Moreover, the proposed approach is computationally efficient as the computational complexity is of the order $(55)^2$ only.

6. Conclusion

In this paper, a simple, robust and efficient method for template matching is proposed. The proposed approach is computationally efficient since there is no need to calculate a covariance matrix as well as eigenvalues and eigenvectors. Moreover, it is robust and performs better in the presence of noise and small geometric transformations. Experimental results have shown that the proposed method requires less computational time as compared to traditional methods. This approach can be applied in various defect detection and medical imaging applications.

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