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Variant of a volume-of-fluid method for surface tension-dominant two-phase flows

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Abstract. The capabilities of the volume-of-fluid method for the calculation of surface tension-dominant two-phase flows are explained. The accurate calculation of the interface remains a problem for the volume-of-fluid method if the density ratios of the fluids in different phases are high. The simulations of bubble growth is performed in water at near critical pressure for different degrees of superheat using combined level-set and volume-of fluid (CLSVOF) method. The effect of superheat on the frequency of bubble formation was analyzed. A deviation from the periodic bubble release is observed in the case of superheat of 20 K in water. The vapor-jet-like columnar structure is observed. Effect of heat flux on the slender vapor column has also been explained.

Keywords. Volume of fluid method; level set method; bubble growth; film boiling.

The volume-of-fluid (Welch & Wilson 2000) and the level-set (Osher & Sethian 1998) approaches, are the major routes followed in the paradigm of fixed-grid methods. In the level-set method (Osher & Sethian 1998) the interface is defined by a level-set function ϕ. This function is initialized as a signed distance function from the interface, positive on one side and negative on the other side of the interface. The interface itself is represented by the zero level of ϕ. Having computed the velocity field, the front evolves as a solution of a transport equation for ϕ. This leads to interface smearing and difficulties in preserving the mass conservation. The level-set method has the inherent strength that ϕ varies smoothly across the interface, which leads to accurate calculation of the curvature and the interface normal vector. Owing to its simplicity, the level-set method has been used in solving a wide variety of problems (Sethian 1999; Sussman 2003).

In the volume-of-fluid (**VOF**) method (Welch & Wilson 2000), the volume of each fluid is calculated in all cells containing portions of the interface. A volume fraction α is defined for one particular fluid inside a cell as its material volume divided by the total cell volume. Hence α is zero or unity in pure fluid (vapor or liquid) cells and has a value of $0 < \alpha < 1$ in two-phase cells. Volume-of-fluid algorithms consist of three major parts, the first part is, the

interface reconstruction method, which finds an explicit description of the interface in each two-phase cell based on volume fractions at this time step, the second part is *advection algorithm*, which calculates the distribution of α at the next time step by solving an advection equation using the reconstructed interface and the velocity field, and the final part consists of *surface tension model*, which takes account of surface tension effects at the interface. In this context, it is worth mentioning that Popinet & Zaleski (1999) developed a novel front tracking method to compute surface tension dominated flows.

A significant development of the interface representation was achieved by Youngs (1982) by introducing the concept of piecewise-linear method (piecewise-linear interface calculation, **PLIC**). An improved version of phase interface representation (**LVIRA**) was devised by Puckett *et al* (1997) and implemented successfully by Welch & Rachidi (2002) and Agarwal *et al* (2004) to predict bubble formation in film boiling. Tomar *et al* (2005) have extended the method as a variant of CLSVOF (Coupled level set and volume of fluid) method to simulate bubble growth and ebullition cycle in water and R-134a at new critical and far-from-critical pressures. Juric & Tryggvason (1998) have provided novel procedures of computing boiling flows.

The present article is an attempt to consolidate the capabilities of recently published volume-of-fluid methods by our group (Agarwal *et al* (2004), Tomar *et al* (2005), Gerlach *et al* (2006), Tomar *et al* (2008), Chakraborty *et al* (2009) and Ray *et al* (2010, 2012)).

The mass, momentum and energy conservation equations for the incompressible Newtonian fluids for the liquid and vapor phases are given by

$$\frac{\partial U_i}{\partial x_i} = 0 \tag{1}$$

$$\rho\left(\frac{\partial U_j}{\partial t} + \frac{\partial U_i U_j}{\partial x_i}\right) = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_i}\left(\mu\frac{\partial U_j}{\partial x_i}\right) \tag{2}$$

$$\left(\frac{\partial\left(\rho c_p\theta\right)}{\partial t} + \frac{\partial\left(\rho c_p\theta U_j\right)}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left(k\frac{\partial\theta}{\partial x_j}\right) \tag{3}$$

Here U_j, p, c_p, ρ, θ, μ and k are the fluid velocity, pressure, specific heat, density, temperature, viscosity and thermal conductivity, respectively.

A rectangular domian $5\lambda_B \times \lambda_B$ has been chosen for the present investigation. Where $\lambda_B = 2\pi\sqrt{3\sigma/(\rho_l - \rho_g)g}$ is the Taylor's most dangerous wave length. The simulations are two-dimensional. The boundary conditions are symmetry conditions at the left and right boundaries

$$at\ x = 0\ and\ x = 5\lambda_B : u = 0;\ \frac{\partial v}{\partial x} = 0;\ \frac{\partial\theta}{\partial x} = 0$$

Constant wall temperature condition is used on the solid-fluid interface

$$at\ y = 0 \quad \theta = \theta_{sup}$$

Outflow boundary conditions are used on the top surface of the domain

$$at\ y = \lambda_B \quad \frac{\partial u}{\partial y} = \frac{\partial v}{\partial y} = \frac{\partial\theta}{\partial y} = 0;\ P = P_\circ$$

The outlet pressure is the saturation pressure less the hydrostatic pressure difference from the initial film level to the outlet. The boundary condition at the vapor liquid interface is of special

concern in this study. In order to address this issue, a suitable interface tracking method has to be implemented.

The presence of two different phases of the fluid requires handling of the phase interface. We advect the interface using enhancement of VOF method of Hirt & Nichols (1981) due to Youngs (1982). The method of Youngs is implemented at the end of a time cycle to calculate the new density field through complience of conservation of mass for each cell

$$\frac{\partial}{\partial t}\int_{V_c}\rho dV + \int_{S_c}\rho \mathrm{v}.\mathbf{n}\, dS = 0, \tag{4}$$

where V_c is the cell volume and S_c is the cell surface. The symbol **v** is used for the fluid velocity. Once, the new cell densities are found, the cell void fractions are calculated using

$$\alpha = \frac{\rho - \rho_g}{\rho_l - \rho_g}. \tag{5}$$

Here ρ_l and ρ_g are the densities of the saturated liquid and saturated vapor, respectively. The implementation of the method of Youngs has been well documented in Welch & Wilson (2000), Rudman (1997) and Rider & Kothe (1998). The curvature of the iterface within each two phase cell is determined using the procedure described by Welch & Rachidi (2002) based on **LVIRA** method of Puckett *et al* (1997). The method requires minimization of the function

$$G_{ij}(\mathsf{n}) = \sum_{k,l=-1}^{1}\{\alpha_{i+k,j+l} - \hat{\alpha}(\mathbf{n}, l)_{i+k,j+l}\}^2. \tag{6}$$

Here $\alpha_{i,j}$ is the actual void fraction of the cell (i,j) and $\hat{\alpha}(n,l)_{i,j}$ is the trial function that maps the line with surface normal **n** and offset length l into a volume fraction of the cell (i,j). This minimization of the function $G_{ij}(\mathbf{n})$ is a nonlinear problem requiring an initial estimate for **n**. This initial **n** is the value determined by the modified VOF method of Youngs (1982) as

$$\mathbf{n} = \frac{\nabla\alpha}{|\nabla\alpha|} \tag{7}$$

The momentum equations are augmented using the continuum surface tension model of Brackbill *et al* (1992) in the following way

$$\rho(\widetilde{\alpha})\left(\frac{\partial v}{\partial t} + \mathbf{v}.\nabla\mathbf{v}\right) = -\nabla P + \rho(\widetilde{\alpha})\,\mathbf{g} + \nabla\cdot\left[\mu(\widetilde{\alpha})\left(\nabla\mathbf{v} + (\nabla\mathbf{v})^T\right)\right] + \sigma\kappa\nabla\widetilde{\alpha}, \tag{8}$$

where $\widetilde{\alpha}$ is a smoothened void field and κ is the curvature of the surface defined by smoothened void field. The surface tension force is applied to the transition region at the interface. The density and viscosity vary with the void fields as

$$\rho(\widetilde{\alpha}) = \widetilde{\alpha}\rho_l + (1-\widetilde{\alpha})\,\rho_g \tag{9}$$

$$\mu(\widetilde{\alpha}) = \widetilde{\alpha}\mu_l + (1-\widetilde{\alpha})\,\mu_g. \tag{10}$$

For the interface cells, we use the augmented momentum Equation (8), the modified conservation of mass and the energy jump condition. The discontinuity of the velocity field, the velocity

gradients, and the viscosity are treated by smoothening. A detailed formulation is available in Agarwal *et al* (2004).

Using the level-set formulation due to (Chang *et al* 1996) the momentum transport equation for incompressible two-phase flow becomes

$$\rho(\phi)\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}.\nabla\mathbf{v}\right) = -\nabla P + \rho(\phi)g + \nabla\cdot[\mu(\phi)(\nabla\mathbf{v} + (\nabla\mathbf{v})^T)] + \sigma\kappa\nabla H\phi \tag{11}$$

and the LS function advection equation is

$$\frac{\partial \phi}{\partial t} + \mathbf{v}.\nabla\phi = 0 \tag{12}$$

The density and viscosity are derived from the level-set function as

$$\rho(\phi) = \rho_g\,[1 - H(\phi)] + \rho_l H(\phi)$$
$$\mu(\phi) = \mu\,[1 - H(\phi)] + \mu_l H(\phi),$$

where H (ϕ) is the Heaviside function,

$$H(\phi) = \begin{cases} 0 & if\,\phi \le -\in \\ \frac{1}{2}\left[1 + \frac{\phi}{\in} + \frac{1}{\pi}\sin\left(\frac{\pi\phi}{\in}\right)\right] & if\,\phi \le \in \\ 1 & if\,\phi > \in \end{cases}$$

The local mean curvature is given by $\kappa = -\nabla.\mathbf{n}$

When discretizing the LS advection equation (12), the volume-of-fluid function α is also simultaneously solved from the following equation:

$$\frac{\partial \alpha}{\partial t} + \nabla\cdot(\mathrm{v}\alpha) = 0.$$

Figure 1 (a) shows the periodic bubble release patterns with growing interface and varying vapor volume. The periodic growth and release of vapor bubbles is usually called ebullition cycle. Starting with a thin vapor layer, gradually the bubbles grow in size. Finally, the bubbles reach a limit at which they detach. The detached bubbles move upwards and leave the domain of interest. The released bubbles leave a very thin vapor film near the nodal position on the wall. Due to the vapor production at the interface the thin vapor layer again grows in size. The next bubbles tend to grow at the antinodes according to the Taylor's wave length λ_B.

The mechanism for the repeating bubble detachment pattern (not shown here) can be explained in the following way. After the bubbles are released from the surface, the vapor packets left behind experience a downward force due to surface tension. Subsequently the vapor packets are pushed down towards the film. The vapor packets impinge on the horizontal surface. The surface tension induced flow promotes the movement of pressure gradient driven impinging vapor packet towards the antinodes. The vapor turns upward to initiate an identical bubble release cycle. Similar patterns of bubble release were observed by Son & Dhir (1997, 1998) in their axisymmetric simulations of saturated film boiling.

CLSVOF based Simulations were performed with a superheat of 20°C and a notable change in bubble dynamics took place. For a superheat of 10°C, the bubble detachment took place alternatively from node and antinode. But for 20°C superheat, a very tall and slender vapor column formation was observed as shown in figure 1 (b). Similar profiles were observed in

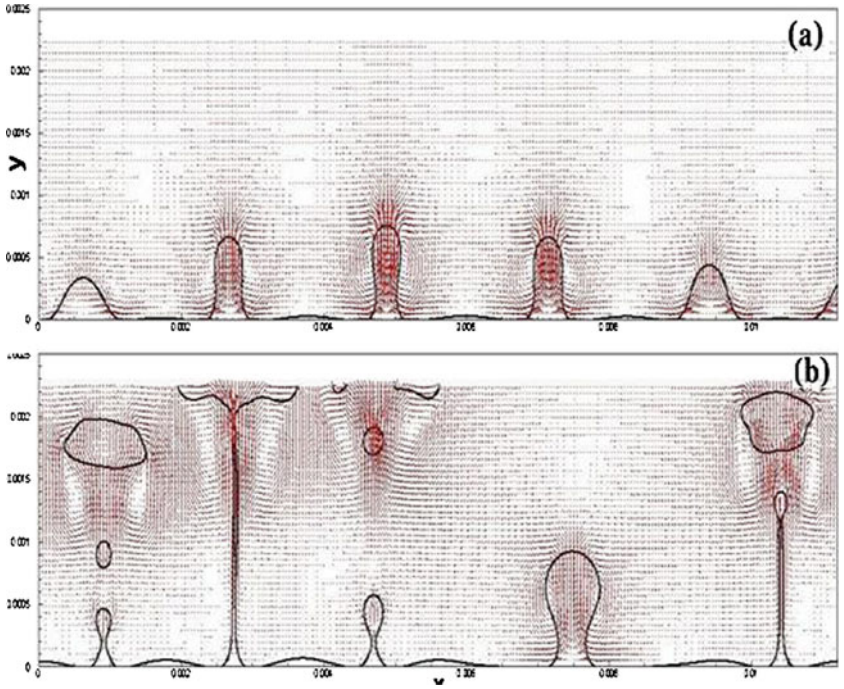

Figure 1. Interface profile with velocity vectors at a surface superheat of (**a**) $\Delta T = 10$ K, (**b**) $\Delta T = 20$ K.

experiments of Reimann & Grigull (1975) when higher values of heat flux beyond a critical value were used. It was observed that after a critical value of heat flux, this change in bubble dynamics took place.

Nomenclature

c_p, c_v	specific heat at constant pressure/volume
Gr	Grashof number
g	gravitational acceleration
H	height of computational domain
h_{lg}	latent heat of vaporization
k	thermal conductivity
l	normal distance from cell center to phase interface
n	interface normal vector, pointing into liquid phase
P_o	saturation pressure in excess of hydrostatic pressure
p	total pressure
q	heat flux vector
S_c	computational cell boundary surface
S_I	phase interface surface
t	time
u	x component of velocity

U_i, U_j velocity vectors
V volume
V_c cell volume
v y component of velocity
v fluid velocity
x spatial coordinate in the horizontal direction
x_i, x_j space vectors
y spatial coordinate in the vertical direction

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