Sādhanā Vol. 38, Part 2, April 2013, pp. 265–280. © Indian Academy of Sciences

Dynamic analysis of rail vehicle axle

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MS received 7 February 2011; revised 3 August 2012; accepted 11 January 2013

Abstract. In this paper, in order to obtain the dynamic forces on the passenger coach axle, a full rail vehicle model with 19-dof (degrees of freedom) has been considered. For a specific example, the variations of these dynamic forces with velocity of the passenger coach, suspension characteristics and way conditions have been examined. Dynamic forces found in the resonance regions at the range of 2–5 m/s (7.2–18 km/h) has been discussed. Theoretical results obtained for the dynamic forces have been successfully compared with the experimental results of German Railways (Deutsche Bahn-DB).

Keywords. Rail vehicle axle; dynamic analysis; fatigue fracture.

1. Introduction

To perform structural dynamics analysis and to predict the ride comfort, supported by a modular design concept, vehicle components for a metro train have been modelled and stored as substructures in a specific vehicle component database (Stribersky *et al* 2002). In order to analyse the behaviour of a suburban train equipped with a pneumatic secondary suspension, Docoquier *et al* (2007) obtained a complete model of the pneumatic circuit and coupled with a multibody model of the train.

Bayraktar *et al* (2009) have investigated the vertical motions of a light rail vehicle serving in Istanbul Transportation Co. by considering the effect of axle stiffness. Finally, time history and frequency responses for displacements and accelerations of the rail vehicle are presented at the end of the study. Similarly, in the study of Guclu & Metin (2009), a rail vehicle system in use in Istanbul traffic is studied by forming a 22 dof model of the vehicle. In an effort to minimize displacement and acceleration of the vibrations obtained in the end of simulations based on time and frequency domains, a fuzzy logic controller is used to actively control vibrations in the simulation environment. In the study of Yagiz & Gursel (2005), the vertical and angular vibrations of the vehicle body with flexible body is controlled by using sliding mode control which calculates the amount of control forces which must be generated by actuators mounted on the front and rear bogies.

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Increasing problems with track damage is another problem which has been observed by researchers and railway companies. Since the origin of this damage is suspected in the mid frequency dynamics, research activities in the frequency range from 50 to 500 Hz have been initiated (Popp *et al* 1999). Also, it is possible to see different train-track models in (Kruse & Popp 2001; Zboifiski 1998; Zhai & Wang 2006).

2. Rail vehicle model

The vehicle body which rests on two bogies each containing two wheel sets is investigated, and the model with 19 dof is shown in figure 1. The sipring and damping elements connecting the wheel set bogie frame are called the primary suspension. The secondary suspension connects the bogie frame and the vehicle body. The aim of this section is to present the model of the full passenger coach analysed in this paper as shown in figure 1. It is assumed that the friction coefficient between the wheel and the rail is constant and equal for right and left wheels. Also, it is considered that passenger coach, its elements and rail are rigid bodies. The mounting of body to the bogies and bogies to wheelsets are flexible. There are 19 degrees of freedom in the model. They include the vertical, horizontal motions and yawing, roll, pitch angles of the car body and the front and rear bogie frames, vertical and horizontal motion and yawing angle of the four wheelset. By analysing a set of ordinary equations, we can achieve dynamical forces and even, behaviours of the vehicle.

The system with 19 differential equations (see appendix A) is given in the following form:

$$M\ddot{\alpha} + C\dot{\alpha} + K\alpha = F.f, \tag{1}$$

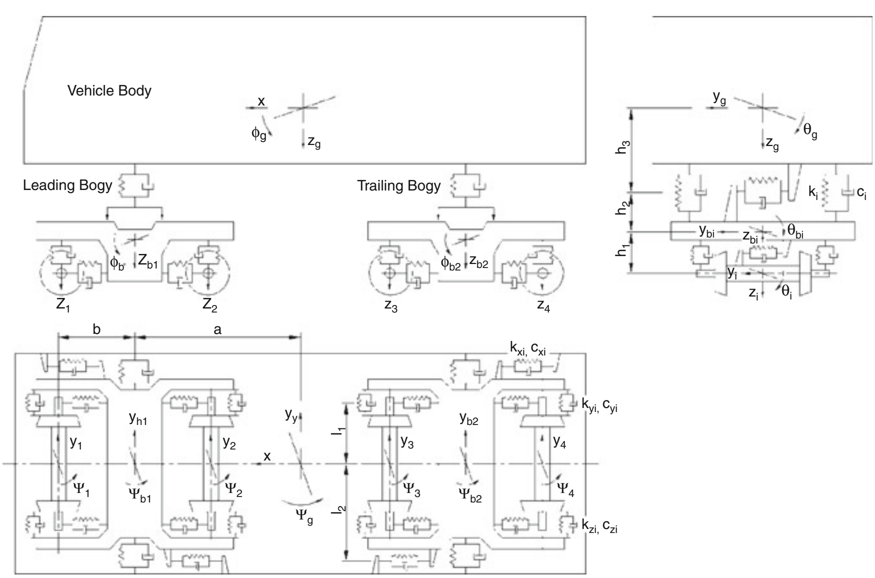


Figure 1. Dynamic model of the rail vehicle (Dikmen 1989).

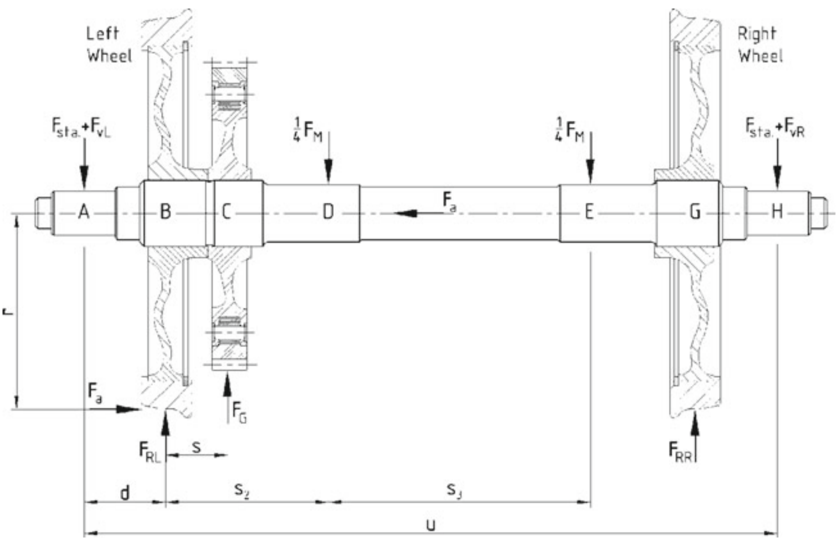


Figure 2. The forces acting the axle (F_M: motor weight, F_G: gear force, F_R: reaction force, F_{sta}: static weight).

M, C and K are mass, damping and stiffness matrices given in the appendix A. F is forcing matrix and f is forcing vector.

$$\alpha = \alpha_{is}.\sin\omega t + \alpha_{ic}.\cos\omega t. \tag{2}$$

In order to calculate the dynamics forces acting the axle, the maximum value of derivations are required.

$$\alpha_i = \sqrt{\alpha_{is}^2 + \alpha_{ic}^2}\ \ (i = 1,19)\,, \tag{3}$$

where α_{is}, α_{ic} are sine and cose components of α_i vector, respectively.

Rail horizontal, vertical and angular irregularities are explained by the Equations (4–6), respectively.

$$y = y_0.\cos(\Omega_y x) \tag{4}$$

$$z = z_0.\cos(\Omega_z x) \tag{5}$$

$$\theta = \theta_0.\cos(\Omega_\theta x)\,, \tag{6}$$

where y_i, z_i, θ_i are rail road excitation inputs (i = 1, 4).

The maximum forces occur when these irregularities are maximum at the same time as shown in figure 2 and as explained by Equation (7). Repetitions of maximum cases occur for each 50 [m], where ly, lz and l_θ are wave lengths as seen in figure 3. Also, other parameters of the irregularity are presented in appendix A.

$$y + z + \theta = \sqrt{\theta_0^2 + A + 2\theta_0\sqrt{A}\cos\left(\Delta\Omega_{yz\theta}.x\right)}.\cos\left(\Omega_\theta + 0.5\Delta\Omega_{yz\theta}\right).x, \tag{7}$$

where $y + z = \sqrt{y_0^2 + z_0^2 + 2y_0 z_0\cos\left(\Delta\Omega_{yz}.x\right)}.\cos\left(\Omega_z + 0.5\Delta\Omega_{yz}\right).x$

and $A = \sqrt{y_0^2 + z_0^2 + 2y_0 z_0\cos\left(\Delta\Omega_{yz}\right)}.$

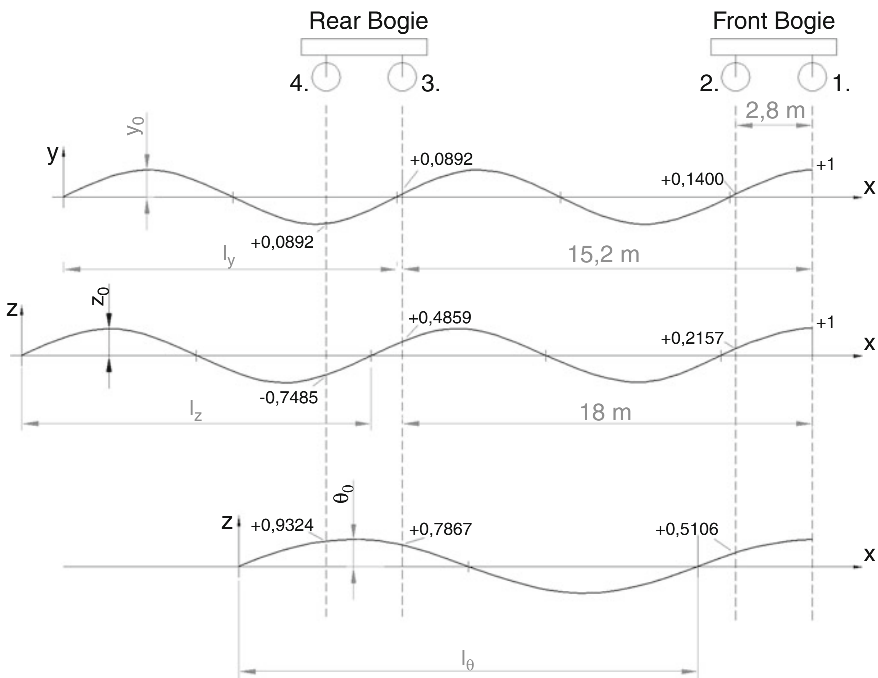


Figure 3. Irregularities related to y, z and θ axes.

3. Forces acting the axle and dynamic equations

The dynamic loads of the railway vehicle are important for the fatigue life prediction of vehicle components. Also, it is possible to reduce the weight of the vehicle structure by predicting dynamic forces (Stichel & Knothe 1998).

Generally, the forces acting on the axle for each axle are passenger coach and motor load, gear forces and the forces resulted from irregularities. These considered forces acting the axle are shown in figure 2. The determination of passenger coach load, motor load and gear forces are related to constructive parameters of the passenger coach. On the other hand, determination of dynamic forces expressed by Equations (8–10) are related to α_i (i = 1…9) which is obtained by the solution of equations of motion.

The equations of dynamic forces in general form

Vertical forces:

$$F_{vL} = \frac{1}{2}K_{z1}\left(\alpha_7 + \alpha_{11}.l_1 + \alpha_{13}.b\right) + \frac{1}{2}C_{z1}\left(\dot{\alpha}_7 + \dot{\alpha}_{11}.l_1 + \dot{\alpha}_{13}.b\right) \tag{8}$$

$$F_{vR} = \frac{1}{2}K_{z1}\left(\alpha_7 - \alpha_{11}.l_1 + \alpha_{13}.b\right) + \frac{1}{2}C_{z1}\left(\dot{\alpha}_7 - \dot{\alpha}_{11}.l_1 + \dot{\alpha}_{13}.b\right), \tag{9}$$

where FvL is left vertical force and F_{vR} is right vertical force.

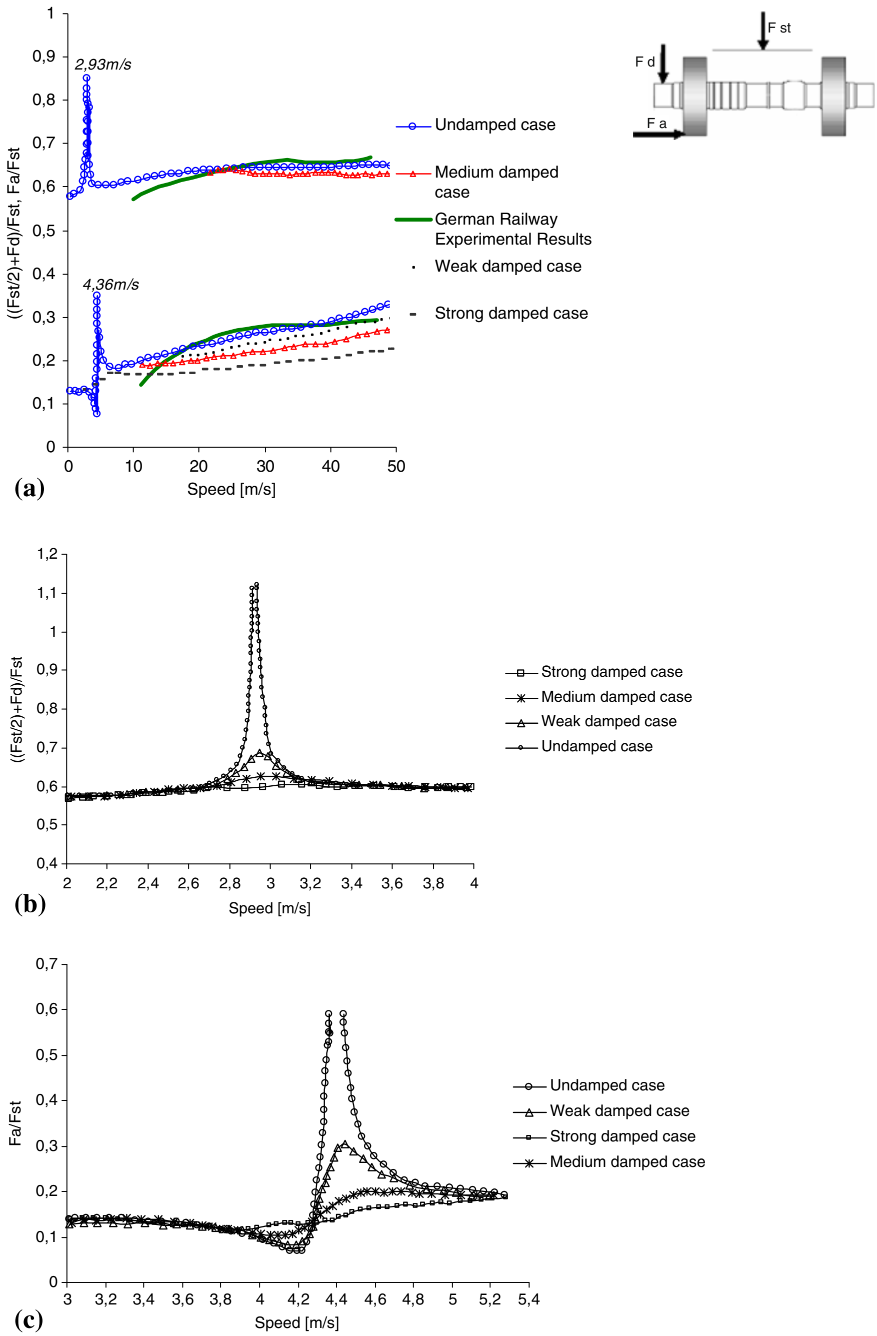


Figure 4. Variation of dynamic forces on axle versus speed (**a**) comparison with German Railway (DB) experimental results, (**b**) vertical force, (**c**) axial force.

Axial force:

$$F_a = K_{y1}\left(\frac{1}{2}\alpha_5 + \alpha_9\right) + C_{y1}\left(\frac{1}{2}\dot{\alpha}_5 + \dot{\alpha}_9\right). \qquad (10)$$

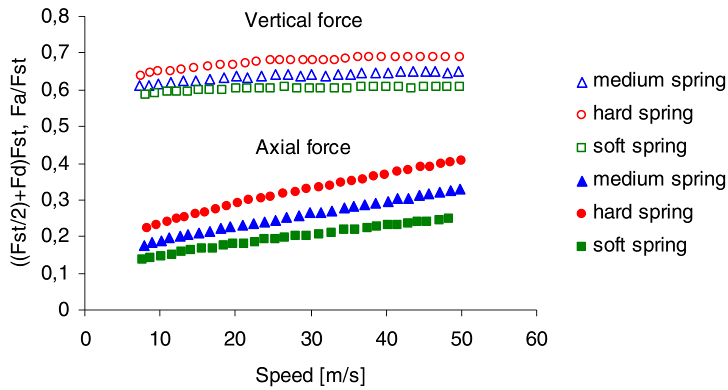


Figure 5. Variation of dynamic forces versus speed for different spring stiffness.

The relations of dynamic forces with passenger coach speed, spring stiffness, way conditions and passenger coach load will be presented. The variation of dynamic forces acting the axle versus speed is presented in figure 4. In order to compare with the experimental results, the proportion of dynamic forces to static load is considered. Static load per axle affects both of the axle ends equally. However, dynamic forces act the right and left sides differently. In the study, the left side is taken into account due to its greatness. Consequently, vertical forces and axial forces are considered as $\left(F_{dleft}+0{,}5.F_{st}\right)/F_{st}$ and $\left(F_a/F_{st}\right)$, respectively.

As seen from figure 4, resonance occurs for 2.93 m/s in vertical force and for 4.36 m/s in axial force. In the case of damping, resonance disappears proportional to damping value. There is no viscous damping in the sample passenger coach considered here. Only dry friction damping exists. The dry friction damping is effective among certain values of irregularity amplitudes (Platin 1983). It means that the extreme peaks in the resonance region turn into small peaks. Although a force increase exists in the critical speed region, this increase can not be as much as causing of sudden brake in the axle. In addition, if these speed regions are passed suddenly, the danger dies out certainly.

In figure 4, the experimental results which are plotted according to experiments made by Deutch Railway (DB) are presented. These diagrams with the code of Fw.28.02.8 in DB are used for the calculations of passenger coach axles with the travelling speed less than 44,4 m/s

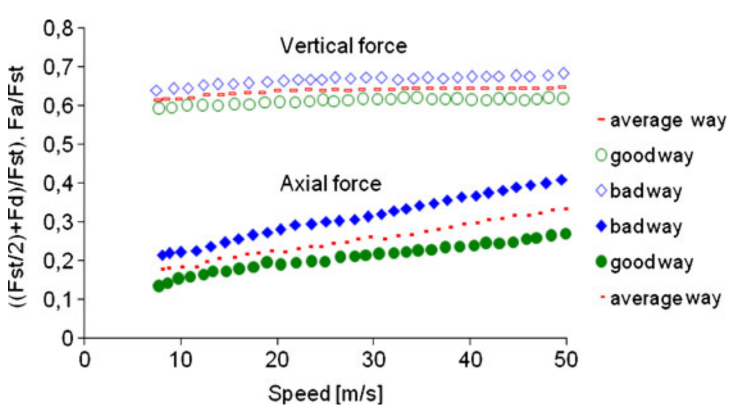


Figure 6. Variation of dynamic forces versus speed related to way conditions.

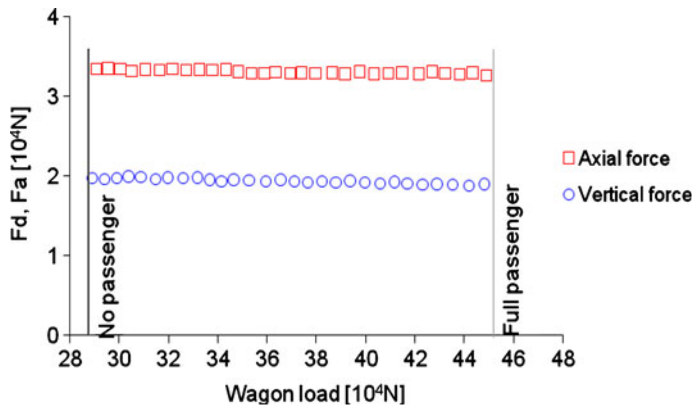


Figure 7. Variation of dynamic forces versus passenger coachload.

(Hanchen & Decker 1967). In fact, these diagrams should be as a curve group due to passenger coach and way conditions. Because of its practicability, average results are used as shown in the figure. Consequently, obtained theorical results reconcile with these diagrams fairly.

The plotted experimental results start with the speed of 10 m/s. That is why the resonance regions seen in theoretical results are not seen in experimental results. In some papers (Mitschke & Helms 1975; Joly 1986), it is possible to see sudden accelerations in low speed regions. The reasons of the differences at speeds of 10–20 m/s between the theoretical and experimental results may be explained in the following. The differences of way conditions and properties of passenger coach, restrictive acceptances during theoretical calculations. And the irregularities are not exact sinusoid.

It is possible to explain the effect of spring stiffness on dynamic forces by means of figure 5. In the case of using soft spring than usual, the forces acting axle are less. This is foreseeable but, it causes reduction of comfort in the system.

Dynamic forces vary due to whether the way conditions are good or bad (figure 6). In the form of good way, the forces acting the axle decrease. It is a prospective result too. However, it should be noted that providing and conserving good quality way conditions will cost too much.

Figure 7 enables to see the variation of dynamic forces acting the axle versus passenger coach load. Dynamic forces decrease for a limited scale with the rise of passenger coach load. Although it seems good, it should be considered that the static load per axle will increase.

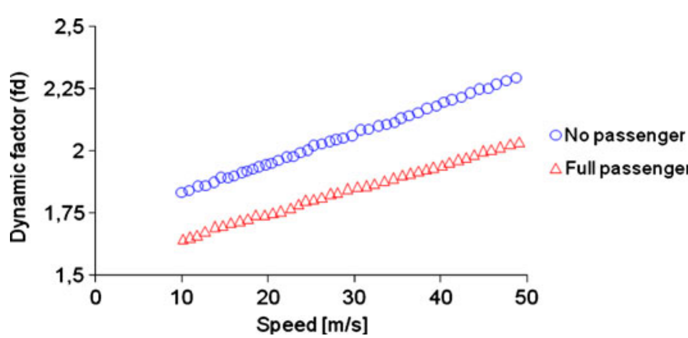


Figure 8. Variation of dynamic factor versus speed.

Dynamic factor is denoted by the ratio of total force or moment to static load or moment in the system. Figure 8 shows that variation of dynamic factor in the critical section of the axle due to passenger coach load whether the passenger coach having full passenger or no passenger. It is a fact that dynamic factor shows nearly a linear increase with travelling speed. Besides, dynamic factor gets great values while the passenger coach has no passenger. The reason of this can be explained by figure 7. In other words, the dynamic forces which act as the axle in the form of passenger coach having full passenger or no passenger nearly remains unchanged. So, it is clear that, the ratio of 'total force/static force' will not be more in the form of passenger coach having no passenger. The dynamic factor is usefull for practical calculations. Tha calculations performed by the product of static load and a proper dynamic factor, provides to make short work of it. But, this proper value is determined by using experiences. For example, considering figure 8, dynamic factor should be taken at least about 1.6–1.7.

4. Conclusion

In order to obtain the forces acting on the axle, a dynamic model with 19 dof is formed. Then, a discussion on dynamic forces acting on the axle is presented in detail. The variation of dynamic forces versus speed is examined and the results are compared with experimental results. Also, damping cases are considered with resonance regions. Additionaly, the variation of forces are analysed due to spring stiffness and railway quality and passenger coach load. The railway conditions should be as good as possible. The forces have minimum effect on the axle in good way as seen in figure 6. When the passenger coach load is too much, it will be usefull to travel with low speed. Also, the soft springs should be chosen properly for the comfort as explained in figure 5. If it is possible, damping should be added to the system. Because, it is clear from figure 4 that in the damping case, sudden force rise can be avoided and also, dynamic forces for other speeds can be decreased. Finally, dynamic factor is explained and the relation of it related to passenger coach speed is presented.

Appendix A

Parameters of vehicle and axle:

Weight of the body (empty): 288000 N
Weight of the body (full): 448000 N
Engine bogy weight: 68000 N
Engine weight: 26000 N
Static weight per axle (empty): 106000 N
Static weight per axle (empty): 146000 N
Gear force: 18000 N
Inertia moment of the pulling bogie (Jbx): 2000 Ns^2 m
Inertia moment of the trailing bogie (Jbx): 2200 Ns^2 m
Inertia moment of the pulling bogie (Jby): 4900 Ns^2 m
Inertia moment of the trailing bogie (Jby): 5500 Ns^2 m
Inertia moment of the pulling bogie (Jbz): 4500 Ns^2 m
Inertia moment of the trailing bogie (Jbz): 5000 Ns^2 m
Inertia moment of the car body (Jcx): 82000 Ns^2
Inertia moment of the car body (Jcy): 1890000 Ns^2 m
Inertia moment of the car body (Jcz): 1870000 Ns^2 m
Stiffness (kx, primary suspension): 4000000 N/m
Stiffness (ky, primary suspension): 4000000 N/m
Stiffness (kz, primary suspension): 1980000 N/m
Stiffness (kx, secondary suspension): 2400000 N/m
Stiffness (ky, secondary suspension): 2400000 N/m
Stiffness (kz, secondary suspension): 2280000 N/m
$l_1 = 1$ m
$l_2 = l_3 = 1{,}2$ m
$h_1 = h_2 = 0{,}1$ m
$h_3 = 1$ m
$a = 7{,}6$ m
$b = 1{,}4$ m

Parameters of rail irregularities:

Amplitude [m;rad]	Wave length [m]	Angular speed [1/m]
$Y_0 = 0{,}006 + 0{,}012.v/50$	$Ly = 12{,}3$	$\Omega y = 0{,}5108280$
$Z_0 = 0{,}006 + 0{,}004.v/50$	$Lz = 13$	$\Omega z = 0{,}4833219$
$\theta_0 = 0{,}010 + 0{,}020.v/50$	$L\theta = 17$	$\Omega\theta = 0{,}3695991$

Equations of forces and moments:

Lateral suspension forces in wheel and bogie:

$$\begin{aligned} P_i &= c_{yi}\,.\,(\dot{y}_i - \dot{y}_{bi}) + k_{yi}\,.\,(y_i - y_{bi}) \\ P_{bi} &= c_{byi}\,.\,(\dot{y}_{bi} - \dot{y}_c) + k_{byi}\,.\,(y_{bi} - y_c) \end{aligned}$$

Vertical suspension forces in wheel and bogie:

$$\begin{aligned} V_i &= c_{zi}\,.\,(\dot{z}_i - \dot{z}_{bi}) + k_{zi}\,.\,(z_i - z_{bi}) \\ V_{bi} &= c_{bzi}\,.\,(\dot{z}_{bi} - \dot{z}_c) + k_{bzi}\,.\,(z_{bi} - z_c) \end{aligned}$$

Pitching suspension moments in wheel and bogie:

$$\begin{aligned} T_i &= \left(c_{xi}.\dot{\phi}_{bi} + k_{xi}.\phi_{bi}\right).h_1^2 \\ T_{bi} &= \left(c_{bxi}.\dot{\phi}_{bi} + k_{bxi}.\phi_{bi}\right).h_2^2 - \left(c_{bxi}.\dot{\phi}_c + k_{bxi}.\phi_c\right).h_2.h_3 \end{aligned}$$

Yawing suspension moments in bogie:

$$\begin{aligned} Q_i &= \left[c_{xi}\,.\left(\dot{\psi}_i - \dot{\psi}_{bi}\right) + k_{xi}\,.\,(\psi_i - \psi_{bi})\right].l_1^2 \\ Q_{bi} &= \left[c_{bxi}\,.\left(\dot{\psi}_{bi} - \dot{\psi}_c\right) + k_{bxi}\,.\,(\psi_{bi} - \psi_c)\right].l_2^2 \end{aligned}$$

Rolling suspension moments in bogie:

$$\begin{aligned} U_i &= \left[c_{zi}\,.\left(\dot{\theta}_i - \dot{\theta}_{bi}\right) + k_{zi}\,.\,(\theta_i - \theta_{bi})\right].l_1^2 \\ U_{bi} &= \left[c_{bzi}\,.\left(\dot{\theta}_{bi} - \dot{\theta}_c\right) + k_{bzi}\,.\,(\theta_{bi} - \theta_c)\right].l_3^2 \end{aligned}$$

Equations of motion for 19 dof rail vehicle system:

Wheelsets:

$$\begin{aligned} i = 1,2 \quad &: \quad J_{zdi}.\ddot{\psi}_i - .l_1^2.c_{xi}\left(\dot{\psi}_i - \dot{\psi}_{bi}\right) - l_1^2.k_{xi}\left(\dot{\psi}_i - \dot{\psi}_{bi}\right) = -J_{zdi}\,(1/r_1 - 1/r_2)\,v^2/2b \\ i = 3,4 \quad &: \quad J_{zdi}.\ddot{\psi}_i - .l_1^2.c_{xi}\left(\dot{\psi}_i - \dot{\psi}_{bi}\right) - l_1^2.k_{xi}\left(\dot{\psi}_i - \dot{\psi}_{bi}\right) = -J_{zdi}\,(1/r_3 - 1/r_4)\,v^2/2b \end{aligned}$$

Bogies:

$$\begin{aligned} i = 1 \quad : \quad & m_{bi}.\ddot{y}_{bi} - c_{byi}\left(\dot{y}_{bi} - \dot{y}_g\right) - \left(c_{y1} + c_{y2}\right)\dot{y}_{bi} - k_{byi}\left(y_{bi} - y_g\right) - \left(k_{y1} + k_{y2}\right)y_{bi} \\ & = -c_{y1}\dot{y}_1 - c_{y2}\dot{y}_2 - k_{y1}y_1 - k_{y2}y_2 - m_{bi}\,(1/r_1 + 1/r_2)\,v^2/2 \\ i = 2 \quad : \quad & m_{bi}.\ddot{y}_{bi} - c_{byi}\left(\dot{y}_{bi} - \dot{y}_g\right) - \left(c_{y3} + c_{y4}\right)\dot{y}_{bi} - k_{byi}\left(y_{bi} - y_g\right) - \left(k_{y3} + k_{y4}\right)y_{bi} \\ & = -c_{y3}\dot{y}_3 - c_{y4}\dot{y}_4 - k_{y3}y_3 - k_{y4}y_4 - m_{bi}\,(1/r_3 + 1/r_4)\,v^2/2 \end{aligned}$$

$$i = 1 \quad : \quad \begin{aligned} & m_{bi}.\ddot{z}_{bi} - c_{bzi}\left(\dot{z}_{bi} - \dot{z}_g\right) - (c_{z1} + c_{z2})\,\dot{z}_{bi} - k_{bzi}\left(z_{bi} - z_g\right) - (k_{z1} + k_{z2})\,z_{bi} \\ & \quad = -c_{z1}\dot{z}_1 - c_{z2}\dot{z}_2 - k_{z1}z_1 - k_{z2}z_2 \end{aligned}$$

$$i = 2 \quad : \quad \begin{aligned} & m_{bi}.\ddot{z}_{bi} - c_{bzi}\left(\dot{z}_{bi} - \dot{z}_g\right) - (c_{z3} + c_{z4})\,\dot{z}_{bi} - k_{bzi}\left(z_{bi} - z_g\right) - (k_{z3} + k_{z4})\,z_{bi} \\ & \quad = -c_{z3}\dot{z}_3 - c_{z4}\dot{z}_4 - k_{z3}z_3 - k_{z4}z_4 - \end{aligned}$$

$$i = 1 \quad : \quad \begin{aligned} & J_{zbi}.\ddot{\psi}_{bi} - .l_2^2.c_{bxi}\left(\dot{\psi}_{bi} - \dot{\psi}_g\right) + l_1^2.c_{x1}\left(\dot{\psi}_1 - \dot{\psi}_{bi}\right) + l_1^2.c_{x2}\left(\dot{\psi}_2 - \dot{\psi}_{bi}\right) \\ & \quad - b\left(c_{y1} + c_{y2}\right)y_{bi} - -l_2^2.k_{bxi}\left(\psi_{bi} - \psi_g\right) + l_1^2.k_{x1}\left(\psi_1 - \psi_{bi}\right) \\ & \quad + l_1^2.k_{x2}\left(\psi_2 - \psi_{bi}\right) - b\left(k_{y1} + k_{y2}\right)y_{bi} \\ & \quad = -b\left(c_{y1}\dot{y}_1 + c_{y2}\dot{y}_2 + k_{y1}y_1 + k_{y2}y_2\right) - J_{zbi}\left(1/r_1 - 1/r_2\right)v^2/2b \end{aligned}$$

$$i = 2 \quad : \quad \begin{aligned} & J_{zbi}.\ddot{\psi}_{bi} - .l_2^2.c_{bxi}\left(\dot{\psi}_{bi} - \dot{\psi}_g\right) + l_1^2.c_{x3}\left(\dot{\psi}_3 - \dot{\psi}_{bi}\right) + l_1^2.c_{x4}\left(\dot{\psi}_4 - \dot{\psi}_{bi}\right) \\ & \quad - b\left(c_{y3} + c_{y4}\right)y_{bi} - l_2^2.k_{bxi}\left(\psi_{bi} - \psi_g\right) + l_1^2.k_{x3}\left(\psi_3 - \psi_{bi}\right) \\ & \quad + l_1^2.k_{x4}\left(\psi_4 - \psi_{bi}\right) - b\left(k_{y3} + k_{y4}\right)y_{bi} \\ & \quad = -b\left(c_{y3}\dot{y}_3 + c_{y4}\dot{y}_4 + k_{y3}y_3 + k_{y4}y_4\right) - J_{zbi}\left(1/r_3 - 1/r_4\right)v^2/2b \end{aligned}$$

$$i = 1 \quad : \quad \begin{aligned} & J_{xbi}.\ddot{\theta}_{bi} - .l_3^2.c_{bzi}\left(\dot{\theta}_{bi} - \dot{\theta}_g\right) + l_1^2.\left(c_{z1} + c_{z2}\right)\dot{\theta}_{bi} - h_2.c_{byi}\left(\dot{y}_{bi} - \dot{y}_g\right) \\ & \quad + h_1\left(c_{y1} + c_{y2}\right)\dot{y}_{bi} - l_3^2.k_{bz1}\left(\theta_{bi} - \theta_g\right) - l_1^2\left(k_{z1} + k_{z2}\right)\theta_{bi} \\ & \quad - h_2 k_{byi}\left(y_{bi} - y_g\right) + h_1\left(k_{y1} + k_{y2}\right)y_{bi} = -l_1^2\left(c_{z1}\theta_1 + c_{z2}\theta_2\right) \\ & \quad + h_1\left(c_{y1}\dot{y}_1 + c_{y2}\dot{y}_2\right) - l_1^2\left(k_{z1}\theta_1 + k_{z2}\theta_2\right) + h_1\left(k_{y1}y_1 + k_{y2}y_2\right) \end{aligned}$$

$$i = 2 \quad : \quad \begin{aligned} & J_{xbi}.\theta_{bi} - .l_3^2.c_{bzi}\left(\dot{\theta}_{bi} - \dot{\theta}_g\right) + l_1^2.\left(c_{z3} + c_{z4}\right)\dot{\theta}_{bi} - h_2.c_{byi}\left(\dot{y}_{bi} - \dot{y}_g\right) \\ & \quad + h_1\left(c_{y3} + c_{y4}\right)\dot{y}_{bi} - l_3^2.k_{bz1}\left(\theta_{bi} - \theta_g\right) - l_1^2\left(k_{z3} + k_{z4}\right)\theta_{bi} \\ & \quad - h_2 k_{byi}\left(y_{bi} - y_g\right) + h_1\left(k_{y3} + k_{y4}\right)y_{bi} = -l_1^2\left(c_{z3}\theta_3 + c_{z4}\theta_4\right) \\ & \quad + h_1\left(c_{y3}\dot{y}_3 + c_{y4}\dot{y}_4\right) - l_1^2\left(k_{z3}\theta_3 + k_{z4}\theta_4\right) + h_1\left(k_{y3}y_3 + k_{y4}y_4\right) \end{aligned}$$

$$i = 1 \quad : \quad \begin{aligned} & J_{ybi}.\ddot{\phi}_{bi} - .b.c_{bzi}\left(\dot{z}_{bi} - \dot{z}_g\right) - b.\left(c_{z1} + c_{z2}\right)\dot{z}_{bi} - h_2^2.c_{bxi}\dot{\phi}_{bi} + h_2h_3c_{bxi}\dot{\phi}_g \\ & \quad - h_1^2\left(c_{x1} + c_{x2}\right)\dot{\phi}_{bi} - b.k_{bzi}\left(z_{bi} - z_g\right) - b\left(k_{z1} + k_{z2}\right)z_{bi} - h_2^2.k_{bxi}.\phi_{bi} \\ & \quad + h_2h_3k_{bxi}.\theta_g - h_1^2\left(k_{x1} + k_{x2}\right)\theta_{bi} = -b\left(c_{z1}.\dot{z}_1 + c_{z2}\dot{z}_2 + k_{z1}z_1 + k_{z2}z_2\right) \end{aligned}$$

$$i = 2 \quad : \quad \begin{aligned} & J_{ybi}.\ddot{\phi}_{bi} - .b.c_{bzi}\left(\dot{z}_{bi} - \dot{z}_g\right) - b.\left(c_{z3} + c_{z4}\right)\dot{z}_{bi} - h_2^2.c_{bxi}\dot{\phi}_{bi} + h_2h_3c_{bxi}\dot{\phi}_g \\ & \quad - h_1^2\left(c_{x3} + c_{x4}\right)\dot{\phi}_{bi} - b.k_{bzi}\left(z_{bi} - z_g\right) - b\left(k_{z3} + k_{z4}\right)z_{bi} - h_2^2.k_{bxi}.\phi_{bi} \\ & \quad + h_2h_3k_{bxi}.\theta_g - h_1^2\left(k_{x3} + k_{x4}\right)\theta_{bi} = -b\left(c_{z3}.\dot{z}_3 + c_{z4}\dot{z}_4 + k_{z3}z_3 + k_{z4}z_4\right) \end{aligned}$$

Car body:

$$m_g.\ddot{y}_g - \left(c_{by1} + c_{by2}\right)\dot{y}_g + c_{by1}.\dot{y}_{b1} + c_{by2}.\dot{y}_{b2} - \left(k_{by1} + k_{by2}\right)y_g + k_{by1}.y_{b1} + k_{by2}.y_{b2} = 0$$

$$m_g.\ddot{z}_g - \left(c_{bz1} + c_{bz2}\right)\dot{z}_g + c_{bz1}.\dot{z}_{b1} + c_{bz1}.\dot{z}_{b2} - \left(k_{bz1} + k_{bz2}\right)z_g + k_{bz1}.z_{b1} + k_{bz2}.z_{b2} = 0$$

$$
\begin{aligned}
J_{zg}.\ddot{\psi}_g &+ l_2^2.c_{bxi}\left(\dot{\psi}_{b1}+\dot{\psi}_g\right) + l_2^2.c_{bx2}\left(\dot{\psi}_{b2}-\dot{\psi}_g\right) + a.c_{by1}\left(\dot{y}_{b1}-\dot{y}_g\right) + a.c_{by2}\left(\dot{y}_{b2}-\dot{y}_g\right) \\
&+ l_2^2.k_{bx1}\left(\psi_{b1}-\psi_g\right) + l_2^2.k_{bx2}\left(\psi_{b2}-\psi_g\right) + a.k_{by1}\left(y_{b1}-y_g\right) \\
&+ a.k_{by2}\left(y_{b2}-y_g\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
J_{xg}.\ddot{\theta}_g &+ l_3^2.c_{bz1}\left(\dot{\theta}_{b1}+\dot{\theta}_g\right) + l_3^2.c_{bz2}\left(\dot{\theta}_{b2}-\dot{\theta}_g\right) - h_3.c_{by1}.\left(\dot{y}_{b1}-\dot{y}_g\right) - h_3.c_{by2}\left(\dot{y}_{b2}-\dot{y}_g\right) \\
&+ l_3^2.k_{bz1}\left(\theta_{b1}-\theta_g\right) + l_3^2.k_{bz2}\left(\theta_{b2}-\theta_g\right) - h_3.k_{by1}\left(y_{b1}-y_g\right) \\
&- h_3.k_{by2}\left(y_{b2}-y_g\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
J_{yg}.\ddot{\phi}_g &+ ac_{bz1}\left(\dot{z}_{b1}+\dot{z}_g\right) + a.c_{bz2}\left(\dot{z}_{b2}-\dot{z}_g\right) + h_3^2.\left(c_{bx1}+c_{bx2}\right)\dot{\phi}_g - h_2h_3\left(c_{bx1}\dot{\phi}_{b1}+c_{bx2}\dot{\phi}_{b2}\right) \\
&+ ak_{bz1}\left(z_{b1}-z_{b2g}\right) + ak_{bz2}\left(z_{b2}-z_g\right) + h_3^2\left(k_{bx1}+k_{bx2}\right)\phi_g \\
&- h_2h_3\left(k_{bx1}\phi_{b1}+k_{bx2}\phi_{b2}\right) = 0
\end{aligned}
$$

Mass, damping and spring matrices:

$$
M=\begin{bmatrix}
J_{zd1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & J_{zd2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & J_{zd3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & J_{zd4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & m_{b1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & m_{b2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & m_{b1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{b1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{zb1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{zb2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{xb1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{xb2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{yb1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{yb2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_g & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_g & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{zg} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{xg} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{yg}
\end{bmatrix}
$$

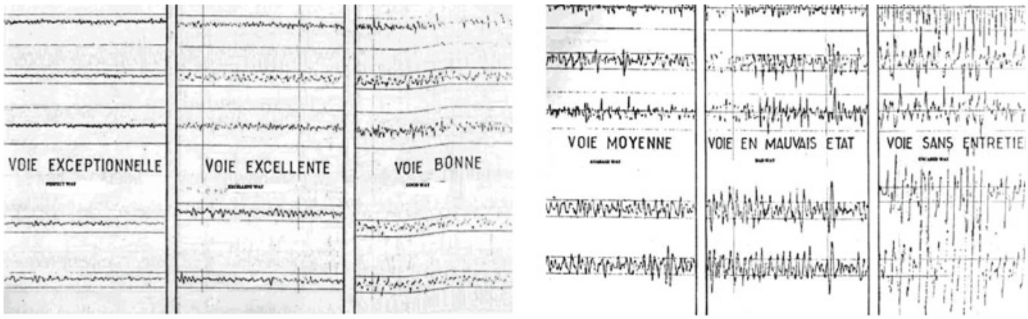
$$C=\begin{bmatrix}
-l_1^2.c_{x1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & l_1^2.c_{x1} & 0\\
0 & -l_1^2.c_{x2} & 0 & 0 & 0 & 0 & 0 & 0 & l_1^2.c_{x2} & 0\\
0 & 0 & -l_1^2.c_{x3} & 0 & 0 & 0 & 0 & 0 & 0 & l_1^2.c_{x3}\\
0 & 0 & 0 & -l_1^2.c_{x4} & 0 & 0 & 0 & 0 & 0 & l_1^2.c_{x4}\\
0 & 0 & 0 & 0 & -c_{by1}\cdot\left(c_{y1}+c_{y2}\right) & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -c_{by2}\cdot\left(c_{y3}+c_{y4}\right) & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -c_{bz1}\cdot(c_{z1}+c_{z2}) & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_{bz2}\cdot(c_{z3}+c_{z4}) & 0 & 0\\
l_1^2.c_{x1} & l_1^2.c_{x2} & 0 & 0 & -b.\left(c_{y1}+c_{y2}\right) & 0 & 0 & 0 & -l_2^2.c_{bx1}-l_1^2.(c_{x1}+c_{x2}) & 0\\
0 & 0 & l_1^2.c_{x3} & l_1^2.c_{x4} & 0 & -b.\left(c_{y3}+c_{y4}\right) & 0 & 0 & 0 & -l_2^2.c_{bx2}-l_1^2.(c_{x3}+c_{x4})\\
0 & 0 & 0 & 0 & -h_2.c_{by1}+h_1.\left(c_{y1}+c_{y2}\right) & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -h_2.c_{by2}+h_1.\left(c_{y3}+c_{y4}\right) & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -b.c_{bz1}-b.(c_{z1}+c_{z2}) & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -b.c_{bz2}-b.(c_{z3}+c_{z4}) & 0 & 0\\
0 & 0 & 0 & 0 & c_{by1} & c_{by2} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & c_{bz1} & c_{bz2} & 0 & 0\\
0 & 0 & 0 & 0 & a.c_{by1} & a.c_{by2} & 0 & 0 & l_2^2.c_{bx1} & l_2^2.c_{bx2}\\
0 & 0 & 0 & 0 & -h_3.c_{by1} & -h_3.c_{by2} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & a.c_{bz1} & a.c_{bz2} & 0 & 0
\end{bmatrix}$$

$$
\left.\begin{array}{ccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & c_{by1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & c_{by2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c_{bz1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c_{bz2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & l_2^2.c_{bx1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & l_2^2.c_{bx2} & 0 & 0 \\
-l_3^2.c_{bz1}-l_1^2.\left(c_{z1}+c_{z2}\right) & 0 & 0 & 0 & h_2.c_{by1} & 0 & 0 & l_3^2.c_{bz1} & 0 \\
0 & -l_3^2.c_{bz2}-l_1^2.\left(c_{z3}+c_{z4}\right) & 0 & 0 & h_2.c_{by2} & 0 & 0 & l_3^2.c_{bz2} & 0 \\
0 & 0 & -h_2^2.c_{bx1}-h_1^2.\left(c_{x1}+c_{x2}\right) & 0 & 0 & b.c_{bz1} & 0 & 0 & h_2.h_3.c_{bx1} \\
0 & 0 & 0 & -h_2^2.c_{bx2}-h_1^2.\left(c_{x3}+c_{x4}\right) & 0 & b.c_{bz2} & 0 & 0 & h_2.h_3.c_{bx2} \\
0 & 0 & 0 & 0 & -\left(c_{by1}+c_{by2}\right) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\left(c_{bz1}+c_{bz2}\right) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -a\left(c_{by1}+c_{by2}\right) & 0 & -l_2^2\left(c_{bx1}+c_{bx2}\right) & 0 & 0 \\
l_3^2.c_{bz1} & l_3^2.c_{bz2} & 0 & 0 & h_3\left(c_{by1}+c_{by2}\right) & 0 & 0 & -l_3^2\left(c_{bz1}+c_{bz2}\right) & 0 \\
0 & 0 & -h_2.h_3.c_{bx1} & -h_2.h_3.c_{bx2} & 0 & -a\left(c_{bz1}+c_{bz2}\right) & 0 & 0 & h_3^2\left(c_{bx1}+c_{bx2}\right)
\end{array}\right]
$$

$$
K = \begin{bmatrix}
-l_1^2.k_{x1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & l_1^2.k_{x1} & 0 \\
0 & -l_1^2.k_{x2} & 0 & 0 & 0 & 0 & 0 & 0 & l_1^2.k_{x2} & 0 \\
0 & 0 & -l_1^2.k_{x3} & 0 & 0 & 0 & 0 & 0 & 0 & l_1^2.k_{x3} \\
0 & 0 & 0 & -l_1^2.k_{x4} & 0 & 0 & 0 & 0 & 0 & l_1^2.k_{x4} \\
0 & 0 & 0 & 0 & -k_{by1}\cdot\left(k_{y1}+k_{y2}\right) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -k_{by2}\cdot\left(k_{y3}+k_{y4}\right) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -k_{bz1}\cdot(k_{z1}+k_{z2}) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_{bz2}\cdot(k_{z3}+k_{z4}) & 0 & 0 \\
l_1^2.k_{x1} & l_1^2.k_{x2} & 0 & 0 & -b.\left(k_{y1}+k_{y2}\right) & 0 & 0 & 0 & -l_2^2.k_{bx1}-l_2^2.(k_{x1}+k_{x2}) & 0 \\
0 & 0 & l_1^2.k_{x3} & l_1^2.k_{x4} & 0 & -b.\left(k_{y3}+k_{y4}\right) & 0 & 0 & 0 & -l_2^2.k_{bx2}-l_1^2.(k_{x3}+k_{x4}) \\
0 & 0 & 0 & 0 & -h_2.k_{by1}+h_1.\left(k_{y1}+k_{y2}\right) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -h_2.k_{by2}+h_1.\left(k_{y3}+k_{y4}\right) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -b.k_{bz1}-b.(k_{z1}+k_{z2}) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -b.k_{bz2}-b.(k_{z3}+k_{z4}) & 0 & 0 \\
0 & 0 & 0 & 0 & k_{by1} & k_{by2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & k_{bz1} & k_{bz2} & 0 & 0 \\
0 & 0 & 0 & 0 & a.k_{by1} & a.k_{by2} & 0 & 0 & l_2^2.k_{bx1} & l_2^2.k_{bx2} \\
0 & 0 & 0 & 0 & -h_3.k_{by1} & -h_3.k_{by2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & a.k_{bz1} & a.k_{bz2} & 0 & 0
\end{bmatrix}
$$

$$
\left.\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_{by1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_{by2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & k_{bz1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & k_{bz2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & l_2^2.k_{bx1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & l_2^2.k_{bx2} & 0 & 0 \\
-l_3^2.k_{bz1}-l_1^2.(k_{z1}+k_{z2}) & 0 & 0 & 0 & h_2.k_{by1} & 0 & 0 & l_3^2.k_{bz1} & 0 \\
0 & -l_3^2.k_{bz2}-l_1^2.(k_{z3}+k_{z4}) & 0 & 0 & h_2.k_{by2} & 0 & 0 & l_3^2.k_{bz2} & 0 \\
0 & 0 & -h_2^2.k_{bx1}-h_1^2.(k_{x1}+k_{x2}) & 0 & 0 & b.k_{bz1} & 0 & 0 & h_2.h_3.k_{bx1} \\
0 & 0 & 0 & -h_2^2.k_{bx2}-h_1^2.(k_{x3}+k_{x4}) & 0 & b.k_{bz2} & 0 & 0 & h_2.h_3.k_{bx2} \\
0 & 0 & 0 & 0 & -\left(k_{by1}+k_{by2}\right) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\left(k_{bz1}+k_{bz2}\right) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -a\left(k_{by1}+k_{by2}\right) & 0 & -l_2^2\left(k_{bx1}+k_{bx2}\right) & 0 & 0 \\
l_3^2.k_{bz1} & l_3^2.k_{bz2} & 0 & 0 & h_3\left(k_{by1}+k_{by2}\right) & 0 & 0 & -l_3^2\left(k_{bz1}+k_{bz2}\right) & 0 \\
0 & 0 & -h_2.h_3.k_{bx1} & -h_2.h_3.k_{bx2} & 0 & -a\left(k_{bz1}+k_{bz2}\right) & 0 & 0 & h_3^2\left(k_{bx1}+k_{bx2}\right)
\end{matrix}\right]
$$

Quality of railway:



(a) perfect (b) very good (c) good (d) average (e) bad (f) uncared

References

Bayraktar M, Guclu R, Metin M and Yazici H 2009 The vibration analysis of light rail vehicle with axle modelling, *14. National Machine Theory Symposium (UMTS 2009)*, 02–04 June 2009, Guzelyurt, Cyprus (In Turkish)

Dikmen F 1989 *Axle fractures and precaution in rail vehicles: computer based dynamic analysis and constructive approach*, PhD Thesis, Yildiz Technical University, Institute of Natural Science, Istanbul (In Turkish)

Docoquier N, Fisette P and Jeanmrt H 2007 Multiphysics modelling of railway vehicles equipped with pneumatic suspensions. *Vehicle System Dynamics* 45(6): 505–524

Guclu R and Metin M 2009 Fuzzy logic control of vibrations of a light rail transport vehicle in use in Istanbul traffic. *J. Vibr. Control* 15(9): 1423–1440

Hanchen R and Decker K H 1967 *Neue festigkeitberechnung für der maschinenbau*, ZEV, C. Hanser Verlag

Joly R 1986 *Schwingungsverhalten eines eisenbahnfahrzeugs*, Schienen der Welt, 23–26, Marz

Kruse H and Popp K 2001 A modular algorithm for linear, periodic, train-track models. *Archive of Applied Mechanics* 71: 473–486

Mitschke M and Helms H 1975 Schwingungen von schienen-und straaefahrzeungen. *ETR: Eisenbahntechnische Rundschau* 24(10): 371–377

Platin B E 1983 The common effect of dry and viscous friction in railway load wagon systems, *3rd National Mechanic Conference* (In Turkish)

Popp K, Kruse H and Kaiser I 1999 Vehicle-track dynamics in the mid-frequency range. *Vehicle System Dynamics* 31: 423–464

Stichel S and Knothe K 1998 Fatigue life prediction for a S-train. *Vehicle System Dynamics* 28: 390–403

Stribersky A, Moser F and Rulka W 2002 Structural dynamics and ride comfort of a rail vehicle system. *Advances in Engineering Software* 33: 541–552

Yagiz N and Gursel A 2005 Active suspension control of a railway vehicle with a flexible body. *Int. J. Vehicle Autonomous Syst.* 3(1): 80–95

Zboifiski K 1998 Dynamical investigation of railway vehicles on a curved track. *Eur. J. Mech. A/Solids* 17: 1001–1020

Zhai W M and Wang K Y 2006 Lateral interactions of trains and tracks on small-radius curves: simulation and experiment. *Vehicle System Dynamics* 44: 520–530