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Elastic buckling strength of corroded steel plates

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Abstract. Corrosion makes structures more vulnerable to buckling and yielding failures. It is common practice to assume a uniform thickness reduction for general corrosion. To estimate the remaining strength of corroded structures, typically a much higher level of accuracy is required, since the actual corroded structures have irregular surfaces. Elastic buckling of simply supported rectangular corroded plates are studied with one- and both-sided irregular surfaces. Eigenvalue analysis by using finite element method (FEM) is employed for computing Euler stress. The influence of various geometric and corrosion characteristics are investigated and it is found that the aspect ratio of the plate, the average thickness diminution, the standard deviation of thickness diminution and the amount of corrosion loss have influence on the reduction of buckling strength of the corroded plates. Buckling strength of one- and both-sided corroded plates are the same. In plates with low value of aspect ratio, reduction of buckling strength is negligible. Reduction of buckling strength is more prominent in plates with higher aspect ratio. Reduction of buckling strength is very sensitive to the amount of corrosion loss; the higher the amount of corrosion loss, the more reduction of buckling strength. Reduction of buckling strength is less sensitive to the standard deviation of thickness diminution.

Keywords. Corroded steel plate; Euler critical stress; FEM; irregular surface.

1. Introduction

Deterioration of aged structures due to corrosion is a common problem in steel structures. For structural safety assessment of corroded structures, residual strength should be determined as a function of time to plan repairs and replacements. Two main corrosion mechanisms, namely, general corrosion and pitting corrosion are recognized. Pitting is a localized corrosion in the form of deep holes and general corrosion which occurs in the relatively larger areas, is due to coalescence of the pits.

Many research works are devoted on strength evaluation of corroded structures. Nakai *et al* (2006) have performed a series of nonlinear finite element analysis (FEA) in plates with pit corrosion subjected to in-plane compressive loads and bending moments. Jiang & Guedes-Soares (2011) and Huang *et al* (2010) have studied ultimate strength of pitted plates under biaxial compression by using nonlinear FEA approach. Wang *et al* (2005) have reported strength reduction of corroded deck plates in 20 years old ships under uniform longitudinal compression.

Significant relevant work has been done in the area of residual strength evaluation of corroded structures. However, none of them have involved the sort of simulated corroded surfaces applied in the current work. Actual thickness distribution of the corroded plate would be a time dependent variable and should be expressed as a function of corrosion degree. Strength analysis of such a plate could yield some acceptance criteria to assist surveyors or designers in repairs and replacement planning. Rahbar-Ranji (2001) has proposed a spectrum for random simulation of geometry of a corroded surface based on the mean and standard deviation of thickness diminution. Rahbar-Ranji (2012a) has used this spectrum to analyse plastic collapse load of one- and both-sided corroded plates with irregular surfaces. He has concluded that one-sided corroded plates have maximum reduction of ultimate plastic load. Rahbar–Ranji (2012b, 2013) has also used this spectrum to study ultimate strength and shear buckling strength of corroded plates with irregular surfaces.

The main aim of the present work is to investigate the buckling strength of simply supported plates with one- and both-sided corroded surfaces. Undulated surfaces are generated based on the power spectrum of corroded surface and Euler stress is calculated by using ANSYS code (version *5.6*).

2. Geometry of corroded surface

General corrosion develops in large areas and yields an irregular surface with variable thickness. Rahbar-Ranji (2001) has proposed a spectrum for geometry of a corroded surface in the following form:

$$S(k) = \begin{cases} \dfrac{11.88\alpha\beta\sigma}{k^2}\mathrm{Exp}\left[-\dfrac{2}{3}\left(\dfrac{\beta}{2.97\sigma\,|k|}\right)^3\right] & \Delta t_{\mathrm{avr}} \le 2.97\sigma \\[2ex] \dfrac{\alpha\beta\,(\Delta t_{\mathrm{avr}} + 2.97\sigma)^2}{\Delta t_{\mathrm{avr}} k^2}\mathrm{Exp}\left[-\dfrac{2}{3}\left(\dfrac{\beta}{\Delta t_{\mathrm{avr}}\,|k|}\right)^3\right] & \Delta t_{\mathrm{avr}} \ge 2.97\sigma \end{cases}, \quad (1)$$

where k is the wave number, Δt_{avr} and σ are the average and standard deviation of the thickness diminution, respectively, and α and β are two fitting constants which depend on the corrosion condition and lie in the following ranges:

$$\begin{aligned} \alpha &= 0.01 - 0.15 \\ \beta &= 0.02 - 0.15. \end{aligned}$$

These two constants characterize corrosion conditions and are defined in such a way that the statistical characteristics of a simulated corroded surface be the same as the target surface.

Equation 1 is derived based on the type I asymptotic distribution rule for calculation of the extreme values of thickness diminution. Maximum thickness diminution is assumed as the extreme largest corrosion depth with cumulative probability of 95% and minimum thickness

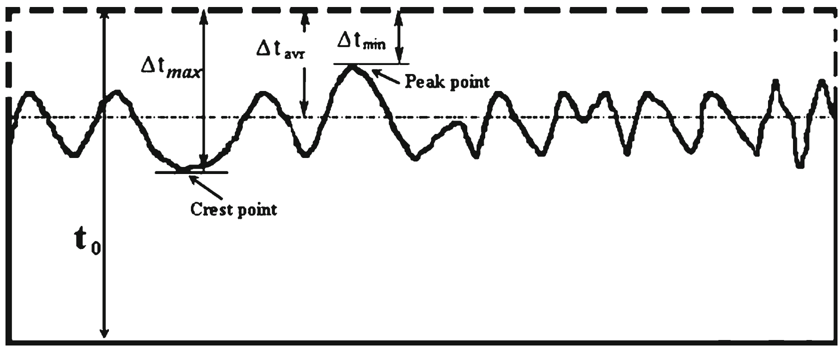

Figure 1. Definition of peak and crest point, Δt_{Max}, Δt_{avr} and Δt_{min} for corroded plate with irregular surface.

diminution is assumed as the smallest corrosion depth with cumulative probability of 5%. According to the type I asymptotic distribution rule, these parameters are calculated as follows

$$\begin{cases} \Delta t_{\text{Max}} = \Delta t_{\text{avr}} + 2.97\sigma \\ \Delta t_{\text{min}} = \Delta t_{\text{avr}} - 2.97\sigma \end{cases}. \tag{2}$$

Minimum corrosion depth is defined as uniform thickness diminution (figure 1).

The average and standard deviation of thickness diminution are two corrosion parameters which are given for any environments. Guo *et al* (2008) have given equations for calculation of the mean and standard deviation of corrosion wastage in deck plates of single hull tankers as a function of ships age based on measured data. Having the mean and standard deviation of thickness diminution, the spectrum of corroded surface would be known. An isotropic spectrum in two directions is expressed by Equation (1), since the stochastic characteristics of the corroded surfaces in all directions are identical. An equivalent wave number is defined as follows

$$k_{eq} = \sqrt{k_1^2 + k_2^2}. \tag{3}$$

Three-dimensional geometry of corroded surface is simulated as follows (Goda 1970)

$$\begin{aligned} \zeta(x_1, x_2) = {} & \sqrt{2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \sqrt{2S\left(k_{1i}, k_{2j}\right) \Delta k_1 \Delta k_2} \\ & \times \left[Cos\left(k_{1i}x_1 + k_{2j}x_2 + \varphi_{1ij}\right) + Cos\left(k_{1i}x_1 - k_{2j}x_2 + \varphi_{2ij}\right)\right], \end{aligned} \tag{4}$$

where N_1 and N_2 are the number of discretization of the spectrum in the x_1 and x_2 directions, respectively, φ_{1ij} and φ_{2ij} are random phase angles uniformly distributed between 0 and 2π, Δk_1 and Δk_2 are wave number increments in the x_1 and x_2 directions, respectively and $k_{1i} = i\Delta k_1$ and $k_{2j} = j\Delta k_2$.

2.1 *One-sided corroded plate*

For one-sided corroded plate all thickness reductions occur at one side of the plate. After generating a corroded surface by using Equation (4), this surface and a flat surface on opposite side are placed in such a way that the average and standard deviation of thickness diminution of the simulated corroded plate and initial values be the same. Irregular thickness of a one-sided corroded plate is determined as follows

$$t\,(x_1, x_2) = t_0 - \Delta t_{\text{avr}} + \zeta\,(x_1, x_2)\,. \tag{5}$$

2.2 *Both-sided corroded plate*

To simulate both-sided corroded plate, two undulated surfaces must be generated and positioned in such a way that the average and standard deviation of the thickness diminution of the simulated corroded plate and initial values be the same. Irregular thickness of a both-sided corroded plate is determined as follows:

$$t\,(x_1, x_2) = t_0 - \Delta t_{\text{avr}} + \zeta^+\,(x_1, x_2) + \zeta^-\,(x_1, x_2)\,, \tag{6}$$

where ζ^+ and ζ^- are ordinates of upper and lower surfaces, respectively. Some statistical studies are needed to determine how thickness reductions are divided between two opposite surfaces, since corrosion information are limited to thickness reduction rather than surface undulation. Another parameter which could have influence on strength of corroded plate is relative position of the crest points on each surface (figure 1).

In this study, both-sided corroded plates with the same thickness reduction parameters at each side and the crest points at the same position are considered.

3. Elastic buckling analysis of plates

Euler stress for buckling of a rectangular plate with four edges simply supported is as follows (Timoshenko & Gere 1961)

$$\sigma_{PE} = \left(m\frac{b}{a} + \frac{1}{m}\frac{a}{b}\right)^2 \frac{\pi^2 D_P}{b^2 t_P}, \tag{7}$$

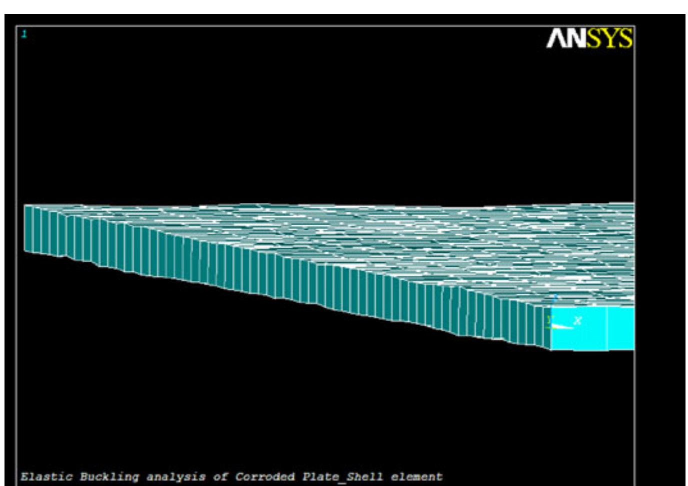

Figure 2. Finite element model of plate with both-sided corroded surfaces (shell elements).

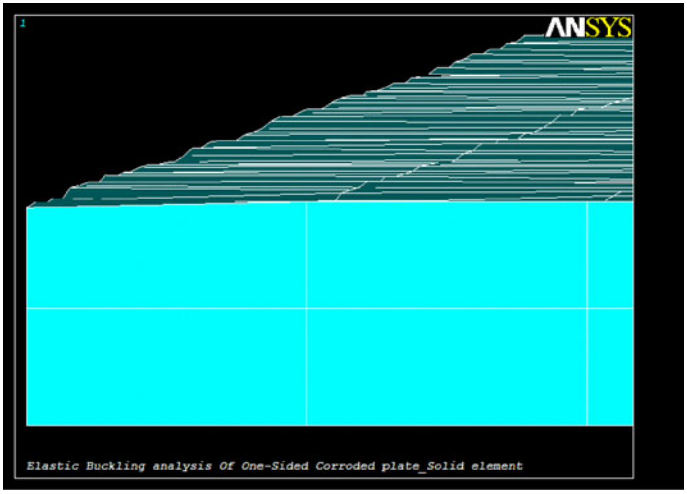

Figure 3. Finite element model of plate with one-sided corroded surfaces (solid elements).

where D_P is the plate bending rigidity and is defined as follows:

$$D_P = \frac{Et_p^3}{12(1-\nu^2)}, \tag{8}$$

where t_p is the thickness of the plate, a is the length of the plate, b is the width of the plate, E is Young's modulus, ν is Poisson's ratio, and m is the number of half waves in longitudinal direction and should be determined in such a way that Euler stress be minimum.

Elastic buckling assessment of a corroded plate with irregular surfaces is evaluated only on the basis of numerical analysis with FEM. A computer code in Fortran 90 is developed to generate irregular surfaces based on the mean and standard deviation of thickness diminution. By using shell elements with variable thickness at each node, both-sided corroded plates with the same irregular surfaces at each side are generated. In this case the mid-plane of corroded plates is not dislocated (figure 2). Solid elements should be used to model one-sided corroded plates (figure 3).

4. Numerical analysis and discussions

To demonstrate the detrimental effect of corrosion with irregular surfaces on buckling strength of corroded plates, a series of FEM eigenvalue analyses are performed. The computer code ANSYS (version *5.6*) has been used for this analysis. One- and both-sided corroded plates are modelled by using SHELL63 and SOLID45 elements, respectively. A uniformly distributed normal stress is applied over one end while holding the other end. Five different corrosion conditions (table 1) are considered and for each case, random irregular surfaces are generated.

Based on studies of Rahbar–Ranji & Zakeri (2010) corrosion changes the mechanical properties of the steel plate; however, Young's modulus and Poisson's ratio remain unchanged. In this study, material is considered as mild steel with $E = 206$ GPa and $\upsilon = 0.3$, width of plate is

Table 1. Corrosion conditions considered in this work.

Mean value of thickness diminution (mm)	Standard deviation of thickness diminution (mm)
0.5	0.1
1.0	0.15
1.5	0.20
2.0	0.25
2.5	0.30

assumed to be 600 mm, thickness is 8 mm and length of plate varies from 600 mm to 3200 mm. Statistical characteristics of the generated surface for different corrosion conditions together with parameters α and β are given in table 2. In this table, CTP is the distance between maximum and minimum points of thickness reduction, or crest-to-peak (CTP) distance which is the maximum height of surface irregularities ($\Delta t_{\mathrm{Max}} - \Delta t_{\min}$). As can be seen, parameters α and β are chosen in such a way that Δt_{avr} and σ of the simulated surface be near to target values.

Table 2. Statistical characteristics of simulated surfaces.

Target surface		Plate	Simulated surface						
Δt_{avr} (mm)	σ (mm)	dimensions (mm)	α	β	$\Delta t_{\min}$ (mm)	Δt_{avr} (mm)	$\Delta t_{\max}$ (mm)	CTP (mm)	σ (mm)
0.5	0.1	600 × 600	0.02	0.02	0.203	0.502	0.867	0.664	0.101
		1200 × 600	0.02	0.02	0.203	0.517	0.893	0.690	0.104
		1800 × 600	0.02	0.02	0.203	0.501	0.892	0.689	0.103
		2400 × 600	0.02	0.02	0.203	0.496	0.881	0.678	0.102
		3000 × 600	0.02	0.02	0.203	0.516	0.913	0.710	0.095
1.0	0.15	600 × 600	0.03	0.02	0.554	1.071	1.681	1.127	0.150
		1200 × 600	0.02	0.03	0.554	1.050	1.687	1.133	0.152
		1800 × 600	0.022	0.024	0.554	1.056	1.605	1.050	0.150
		2400 × 600	0.02	0.027	0.554	1.015	1.621	1.067	0.151
		3000 × 600	0.022	0.027	0.554	1.033	1.591	1.036	0.154
1.5	0.20	600 × 600	0.025	0.035	0.906	1.504	2.333	1.427	0.200
		1200 × 600	0.024	0.030	0.906	1.517	2.340	1.517	0.205
		1800 × 600	0.027	0.028	0.906	1.485	2.246	1.340	0.200
		2400 × 600	0.027	0.027	0.906	1.524	2.337	1.430	0.202
		3000 × 600	0.028	0.028	0.906	1.540	2.281	1.375	0.204
2.0	0.25	600 × 600	0.035	0.030	1.257	2.027	3.145	1.888	0.251
		1200 × 600	0.030	0.032	1.257	2.079	3.119	1.861	0.249
		1800 × 600	0.031	0.030	1.257	2.085	3.024	1.767	0.249
		2400 × 600	0.032	0.030	1.257	2.069	3.011	1.753	0.250
		3000 × 600	0.050	0.020	1.257	2.076	3.124	1.866	0.257
2.5	0.3	600 × 600	0.040	0.030	1.609	2.505	3.601	1.992	0.301
		1200 × 600	0.039	0.030	1.609	2.529	3.844	2.235	0.298
		1800 × 600	0.038	0.030	1.609	2.581	3.784	2.175	0.303
		2400 × 600	0.040	0.030	1.609	2.585	3.867	2.258	0.301
		3000 × 600	0.040	0.030	1.609	2.570	3.692	2.080	0.309

Euler stress for plates with variable thickness at each node are calculated by using FEM and compared with Euler stress of plates with uniform thickness using Equation (7). A reduction factor is defined as follows

$$R_d = \frac{(\sigma_{PE})_{Rough}}{(\sigma_{PE})_{Flush}}, \tag{9}$$

where σ_{PE} is Euler stress for plate buckling. To express the effect of corrosion on the reduction of buckling strength, corrosion loss is introduced as follows

$$\text{Corrosion loss} = \frac{\text{lost weight}}{\text{Initial weight}} \times 100. \tag{10}$$

Table 3 gives Euler stresses, the amount of corrosion loss and reduction factor for plates with both-sided corroded surfaces and different corrosion conditions.

As can be seen, for plates with aspect ratio less than four and amount of corrosion loss less than 30 percent, the reduction factor is very near to one. This means that for these cases the difference between variable thickness assumption and uniform thickness assumption are negligible. For plates with aspect ratio of five, the reduction factor is near to one when the amount of corrosion

Table 3. Euler stress for both-sided corroded plates.

Target surface		Plate dimensions (mm)	Corrosion loss ratio (%)	Euler stress (MPa)		Reduction factor
Δt_{avr} (mm)	σ (mm)			Irregular thickness	Uniform thickness	
0.5	0.1	600 × 600	6.80	115.86	115.15	1.01
		1200 × 600	6.80	114.79	114.91	1.00
		1800 × 600	6.75	115.18	115.13	1.00
		2400 × 600	6.64	117.93	115.41	1.02
		3000 × 600	6.80	115.85	115.00	1.01
1.0	0.15	600 × 600	13.26	99.34	99.61	1.00
		1200 × 600	13.83	98.10	98.29	1.00
		1800 × 600	14.00	97.92	97.90	1.00
		2400 × 600	13.41	98.99	99.28	1.00
		3000 × 600	13.06	99.39	100.07	0.99
1.5	0.20	600 × 600	19.90	84.30	84.95	0.99
		1200 × 600	19.95	85.01	84.84	1.00
		1800 × 600	19.52	85.75	85.76	1.00
		2400 × 600	20.01	84.24	84.71	0.99
		3000 × 600	19.46	84.93	85.88	0.99
2.0	0.25	600 × 600	26.25	71.58	72.01	0.99
		1200 × 600	26.76	71.10	71.02	1.00
		1800 × 600	26.54	71.74	71.44	1.00
		2400 × 600	26.29	71.47	71.93	0.99
		3000 × 600	26.05	68.71	72.41	0.96
2.5	0.3	600 × 600	32.91	59.95	59.60	1.01
		1200 × 600	33.53	58.74	58.52	1.00
		1800 × 600	33.23	58.71	59.03	0.99
		2400 × 600	33.49	57.42	58.58	0.98
		3000 × 600	32.43	58.70	60.44	0.97

Table 4. Euler stress for one-sided and both-sided corroded plate, corrosion loss 6.60–6.80% ($\Delta t_{avr} = 0.5$ mm, $\sigma = 0.1$ mm).

	Euler stress (MPa)	
Plate dimensions (mm)	One-sided corroded plate (solid elements)	Both-sided corroded plate (shell elements)
600 × 600	114.94	115.86
1200 × 600	114.80	114.79
1800 × 600	115.19	115.18
2400 × 600	115.05	117.93
3000 × 600	115.05	115.85

loss is less than 20 percent. For higher amount of the corrosion loss, the reduction factor is lower than one.

Table 4 shows reduction factor of buckling strength for one- and both-sided corroded plates with constant corrosion loss.

As can be seen, the reduction of buckling strength for one- and both-sided corroded plates, are very close. Therefore, buckling strength of the corroded plates is independent of how corrosion has occurred at each side of plates.

Figure 4 shows the effect of standard deviation of thickness diminution (roughness of surface) on the reduction of buckling strength of plates with dimensions 3000 × 600 mm, and average thickness diminution 2.5 mm. As can be seen, the roughness of surface has little effect on the reduction of buckling strength. Generally speaking, by increasing the standard deviation of thickness diminution (roughness of surface), reduction factor is decreased.

Figure 5 shows the relation between reduction factor of buckling strength and the average thickness diminution. In this figure, plate with dimension 3000 × 600 mm and standard deviation of thickness diminution 0.3 mm is considered. As can be seen, the average thickness diminution has influence on reduction factor. However, its influence is not definite and cannot be expressed explicitly. By increasing the average thickness diminution, the reduction factor of buckling strength maybe increased or decreased.

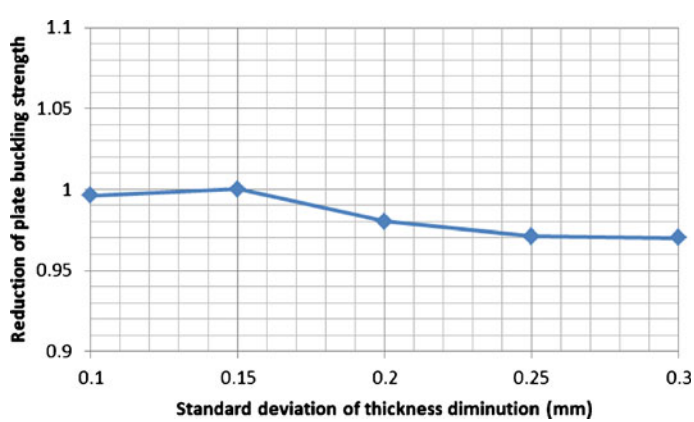

Figure 4. Reduction factor of buckling strength of corroded plate as a function of standard deviation of thickness diminution (plate dimension 3000 × 600 mm, average thickness diminution 2.50 mm).

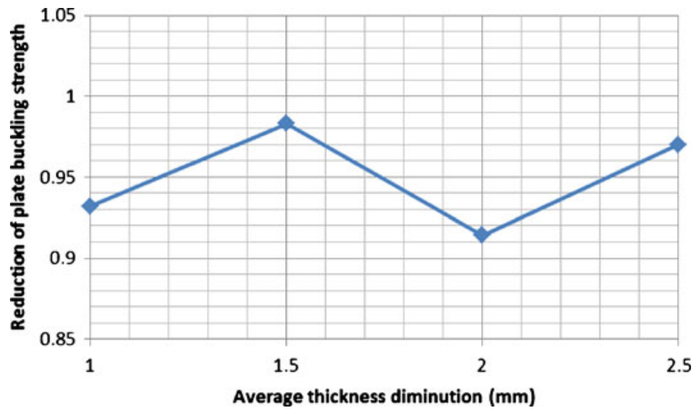

Figure 5. Reduction factor of buckling strength of corroded plate as a function of average thickness diminution (plate dimension 3000 × 600 mm, standard deviation 0.30 mm).

Figure 6 shows the relation between reduction factor of buckling strength and the amount of corrosion loss. Plates with dimensions of 3000 × 600 mm, 2400 × 600 mm, and 1200 × 600 mm, and the average thickness diminution 2.0 mm, and standard deviation of thickness diminution 0.25 mm are considered.

As can be seen, buckling strength of the corroded plates is very sensitive to the amount of corrosion loss and aspect ratio of plates. As concluded before, for plates with lower aspect ratio, say two, the reduction of buckling strength is negligible irrespective of the amount of corrosion loss. For plates with higher aspect ratio, reduction of buckling strength is more prominent. By increasing the amount of corrosion loss, reduction factor decreases. When corrosion loss reaches to 50 percent, the reduction factor is about 0.8 for plate with aspect ratio of five (3000 × 600 mm), about 0.9 in plate with aspect ratio of four (2400 × 600 mm), and about 0.98 in plate with aspect

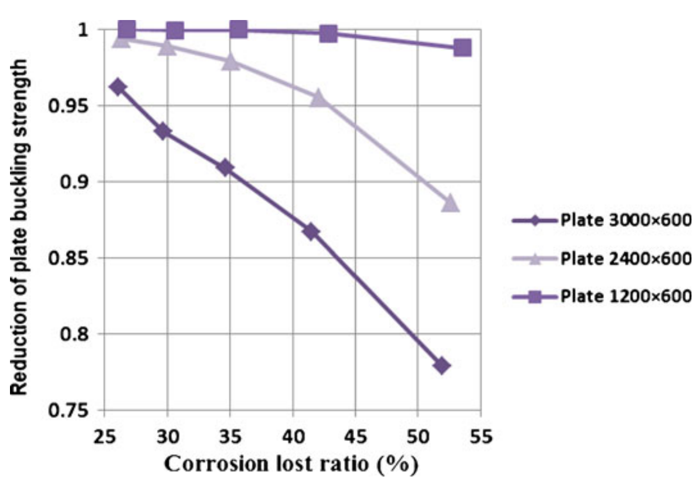

Figure 6. Reduction factor of buckling strength of corroded plate as a function of corrosion loss ratio (average thickness diminution 2.0 mm and standard deviation 0.25 mm).

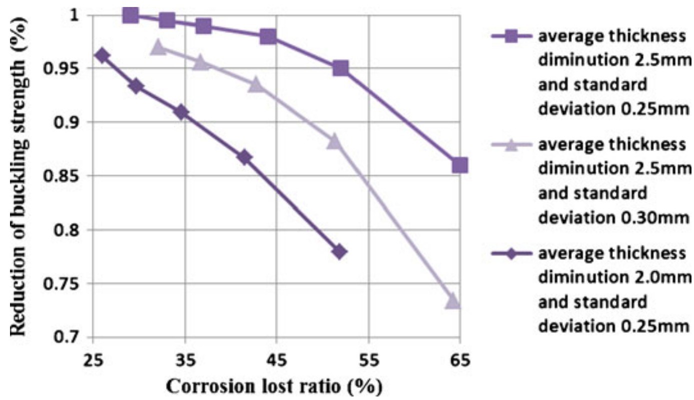

Figure 7. Reduction factor of buckling strength of corroded plate as a function of corrosion loss ratio, plate dimension 3000 × 600 mm.

ratio of two (1200 × 600 mm). Therefore, for thin plate with higher aspect ratio, uniform thickness assumption could lead up to 20 percent overestimation of buckling strength when 50 percent of thickness of plate is reduced due to corrosion.

Figure 7 shows the effect of the amount of corrosion loss on the reduction of buckling strength for different corrosion conditions. In this figure, plate dimension is assumed as 3000 × 600 mm.

Regardless of corrosion conditions, by increasing the amount of corrosion loss, reduction factor is decreased. Comparing two cases of the average thickness diminution 2.5 mm and standard deviation 0.25 mm with the average thickness diminution 2.5 mm and standard deviation 0.3 mm, it implies that by increasing roughness of surface (standard deviation of thickness diminution), reduction factor can be decreased. Comparing two cases of the average thickness diminution 2.5 mm and standard deviation 0.25 mm with the average thickness diminution 2.0 mm and standard deviation 0.25 mm, it implies that by decreasing the average thickness diminution the reduction factor is decreased.

5. Conclusions

There is little study on the strength of corroded plates with irregular surfaces especially as a function of corrosion parameters. Eigenvalue analysis using FEM is employed for Euler stress analysis of corroded plates with one- and both-sided irregular surfaces. It is found that buckling strength of one- and both-sided corroded plates are the same when corrosion loss is constant. Influential parameters are studied and it is found that the amount of corrosion loss, the average thickness diminution, aspect ratio of plate and standard deviation of thickness diminution have influence on reduction of buckling strength. It is found that reduction of buckling strength is very sensitive to the amount of corrosion loss and plate aspect ratio. For plates with aspect ratio less than two, the reduction of buckling strength is negligible irrespective of the amount of corrosion loss. For plates with higher aspect ratio, the reduction of buckling strength is more prominent. Regardless of corrosion conditions, by increasing the amount of corrosion loss, reduction factor is decreased. Reduction of buckling strength is less sensitive to the standard deviation of the thickness diminution. However, by increasing the standard deviation of thickness diminution the reduction of buckling strength is increased. Influence of average thickness diminution on reduction factor of buckling strength is not definite.

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