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Decentralized linear quadratic power system stabilizers for multi-machine power systems

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Abstract. Linear quadratic stabilizers are well-known for their superior control capabilities when compared to the conventional lead–lag power system stabilizers. However, they have not seen much of practical importance as the state variables are generally not measurable; especially the generator rotor angle measurement is not available in most of the power plants. Full state feedback controllers require feedback of other machine states in a multi-machine power system and necessitate block diagonal structure constraints for decentralized implementation. This paper investigates the design of Linear Quadratic Power System Stabilizers using a recently proposed modified Heffron–Phillip's model. This model is derived by taking the secondary bus voltage of the step-up transformer as reference instead of the infinite bus. The state variables of this model can be obtained by local measurements. This model allows a coordinated linear quadratic control design in multi machine systems. The performance of the proposed controller has been evaluated on two widely used multi-machine power systems, 4 generator 10 bus and 10 generator 39 bus systems. It has been observed that the performance of the proposed controller is superior to that of the conventional Power System Stabilizers (PSS) over a wide range of operating and system conditions.

Keywords. Power system stabilizer; linear quadratic regulator; small-signal stability; transient stability.

1. Introduction

Modern excitation systems considerably enhance the overall transient stability of power systems by fast terminal voltage regulation. However, fast voltage regulation has detrimental effect on the small signal stability of the system (Demello & Concordia 1969). An auxiliary controller called power system stabilizer (PSS) is often needed for damping the low frequency oscillations (Demello & Concordia 1969; Larsen & Swann 1981). The concept of PSS and their tuning procedures are well-explored (Demello & Concordia 1969; Larsen & Swann 1981; Farmer & Agrawal 1983; Kundur *et al* 1989). Though the conventional lead–lag stabilizers have simple robust structures, tuning them is an involved process which requires considerable expertise and also a knowledge of system parameters external to the generating station.

State feedback control techniques have been investigated in the literature as an alternative to conventional stabilizers (Yu 1983). Optimal control theory is applied by Yu *et al* (1970), Anderson (1971), Habibullah & Yu (1974), Stromotich & Fleming (1972), Moussa & Yu (1972a,b), Simoes Costa *et al* (1997) for the design of PSS. Pole placement and output feedback control methods were proposed in Choi & Lim (1982), Mohan *et al* (1978), Davison & Rau (1971), Padiyar *et al* (1980), Arnautovic & Medanic (1987), Chow & Sanchez-Gasca (1989). Though the state feedback controllers have shown better performance over the conventional stabilizers, they have not seen much of practical realization because most of the power system state variables are not measurable. Full state feedback controllers design in multi-machine power systems require feedback of other machine states, model reduction techniques and necessitate the block diagonal structure constraints to be imposed while solving the control problem (Simoes Costa *et al* 1997; Chow & Sanchez-Gasca 1989; Arnautovic & Medanic 1987).

Gurunath & Sen (2008, 2010) have proposed a new approach for the design of conventional power system stabilizers, using a modified Heffron–Phillip's model. This model has been derived by taking the secondary voltage of the step-up transformer as reference instead of the infinite bus voltage. The parameters of this model are independent of the equivalent external reactance and infinite bus voltage. This paper investigates the possibility of designing state feedback controllers using this model. Here linear quadratic regulator (LQR) is used as a state feedback controller. A coordinated LQR design can be obtained with this model and it can be implemented by using the information available within the power plant. The proposed PSS has been evaluated on two widely used multi-machine systems, 4 generator 10 bus and IEEE 10 generator 39 bus system. The proposed controller has shown consistently better performance when compared to the conventional PSS over a wide range of operating and system conditions.

2. Modelling of power system

Small signal stability analysis requires dynamic modelling of major power system components such as the synchronous generator, excitation system, AC network, etc. IEEE Model 1.0, IEEE Task (IEEE Task Force 1986), is used to represent the synchronous generator with a high gain, low time constant static exciter, (IEEE Std. 421.5 2005). In systems equipped with static excitation systems, the complexity of higher order models is largely due to the presence of amortisseur windings which always contribute to positive damping. The adequacy of third order model (IEEE 1.0) for synchronous machines has been experimentally verified recently by Arjona *et al* (2009) and a large number of nonlinear excitation controllers are designed based on this model (Lu *et al* 2001). The dynamic equations governing the SMIB system are as follows.

Generator mechanical equations

$$\dot{\delta} = \omega_B S_m, \tag{1}$$

$$\dot{S}_m = \frac{1}{2H}\{T_{mech} - T_{elec} - DS_m\}. \tag{2}$$

q-Axis flux linkage equation

$$\dot{E}'_q = \frac{1}{T'_{do}}\left\{- E'_q + (X_d - X'_d)i_d + E_{fd}\right\}. \tag{3}$$

Generated electrical torque equation

$$T_{elec} = E'_q i_q + (X'_d - X_q)i_d i_q. \tag{4}$$

Static excitation system equation

$$\dot{E}_{fd} = \frac{1}{T_A}\left\{- E_{fd} + K_A(V_{ref} + V_{pss} - V_t)\right\}. \tag{5}$$

The variables have standard meaning and they are defined in the nomenclature. The stator algebraic equations are given by

$$\begin{aligned} E'_q + X'_d i_d - R_a i_q &= V_q \\ -X_q i_q - R_a i_d &= V_d. \end{aligned} \tag{6}$$

Now consider a single generator connected to the external system through a power transformer as shown in figure 1 (Gurunath & Sen 2010). The rotor angle with respect to the voltage $V_s\angle\theta_s$ of the high voltage bus is defined as $\delta_s = \delta - \theta_s$. The expressions for δ_s, E'_q, i_d and i_q are as follows (Gurunath & Sen 2010; Gurunath 2010).

$$\delta_s = tan^{-1}\frac{P_s\left(X_t + X_q\right) - Q_s\left(R_a + R_t\right)}{P_s\left(R_a + R_t\right) + Q_s\left(X_t + X_q\right) + V_s^2}, \tag{7}$$

where $P_s = V_s I_a \cos\theta_p$ and $Q_s = V_s I_a \sin\theta_p$. From stator algebraic equations (6) one can get the following equation for E'_q

$$\begin{aligned} E'_q = \frac{(X_t+X'_d)}{X_t}\sqrt{V_t^2 - \left(\frac{X_q}{(X_t+X_q)}V_s\sin\delta_s\right)^2}. \\ -\frac{X'_d}{X_t}V_s\cos\delta_s. \end{aligned} \tag{8}$$

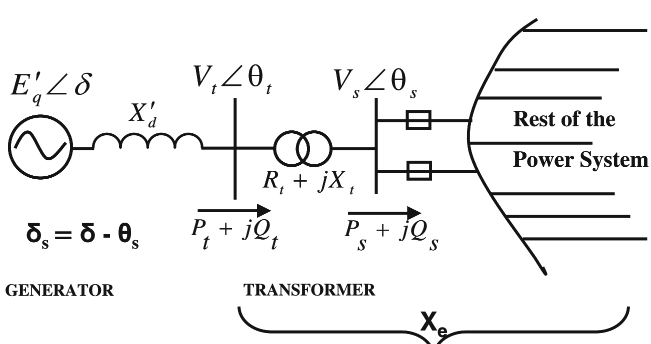

Figure 1. A single machine connected to external network.

For a tap changing transformer the usual π-equivalent representation results in the following transformation between the currents and voltages

$$\begin{bmatrix} I_t \\ I_s \end{bmatrix} = \begin{bmatrix} \frac{y}{a^2} & -\frac{y}{a} \\ -\frac{y}{a} & y \end{bmatrix} \begin{bmatrix} V_t \\ V_s \end{bmatrix}. \tag{9}$$

This transformation matrix is singular, so one can not use this representation. In such cases one should take $X_t = a^2 X_t$ and $V_s = aV_s$. All the equations in this paper are representative for $a = 1$.

3. Modified Heffron–Phillip's model (Gurunath 2010)

The standard linear model of SMIB known as Heffron–Phillip's model (also called K-constant model), Heffron & Phillips (1952) can be obtained by linearizing the system equations around an operating condition. The synchronous machine can be interfaced with the external network by converting machine equations in Park's reference frame to synchronously rotating Kron's reference frame. The equations are given below for a SMIB system.

$$\begin{aligned} V_Q + jV_D &= (V_q + jV_d)e^{j\delta} \\ &= (i_q + ji_d)(R_e + jX_e)e^{j\delta} + E_b\angle 0. \end{aligned} \tag{10}$$

This is simply Kirchhoff's Voltage law between the generator terminal and the infinite bus. The subscripts q and d refer to the q and d-axis, respectively in Park's reference frame. Q and D refer to the Q and D-axis, respectively in Kron's reference frame. Similar equation can be written between transformer bus and the generator terminal voltage and is given below.

$$(V_q + jV_d) = (i_q + ji_d)(R_t + jX_t) + V_s\angle\theta_s\, e^{-j\delta}. \tag{11}$$

This is the modification suggested in Gurunath & Sen (2010) to make the PSS design independent of the external system parameters. Replacing δ by $\delta_s + \theta_s$ in the above equation gives

$$(V_q + jV_d) = (i_q + ji_d)(R_t + jX_t) + V_s\angle - \delta_s. \tag{12}$$

Equating the real and imaginary parts of the above equation gives the modified stator algebraic equations referred to the transformer bus. These equations are true even in multi machine environment for any machine.

$$\begin{aligned} V_q &= R_t i_q - X_t i_d + V_s \cos\delta_s \\ V_d &= R_t i_d + X_t i_q - V_s \sin\delta_s. \end{aligned} \tag{13}$$

Equating (6), (13) and rearranging one can get

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \frac{-1}{\Lambda} \begin{bmatrix} -\left(X_q + X_t\right) & R_t \\ R_t & \left(X_d' + X_t\right) \end{bmatrix} \begin{bmatrix} V_s\cos\delta_s - E_q' \\ -V_s\sin\delta_s \end{bmatrix}, \tag{14}$$

where

$$\Lambda = \left(X_q + X_t\right)\left(X_d' + X_t\right) + R_t^2.$$

Linearizing the equations (1) to (5), (13) and (14) one can get

$$\Delta T_e = G_1 \Delta\delta_s + G_2 \Delta E'_q + G_{V1} \Delta V_s \tag{15}$$

$$\Delta\delta_s = \omega_B S_m - \Delta\theta_s \tag{16}$$

$$\Delta S_m = \frac{1}{2H}[\Delta T_m - \Delta T_e - D\Delta S_m] \tag{17}$$

$$\Delta E'_q = \frac{G_3}{1 + sG_3 T'_{do}}[\Delta E_{fd} - G_4 \Delta\delta_s - G_{V2} \Delta V_s] \tag{18}$$

$$\Delta V_t = G_5 \Delta\delta_s + G_6 \Delta E'_q + G_{V3} \Delta V_s \tag{19}$$

$$\Delta E_{fd} = \frac{K_A}{1 + sT_A}\left[\Delta V_{ref} - \left(G_5 \Delta\delta_s + G_6 \Delta E'_q + G_{V3} \Delta V_s\right)\right]. \tag{20}$$

The constants G_1 to G_6 and G_{V1} to G_{V3} are given in Gurunath & Sen (2010).

Figure 2 shows the modified Heffron–Phillip's model. The detailed derivation of the model is given in Gurunath (2010). The constants G_1 to G_6 are no longer referenced to δ, E_b and the equivalent reactance X_e. They are functions of V_s, δ_s, V_t and machine currents. If the deviations in the transformer voltage are neglected, then this model exactly represents a strong system with the external reactance X_e equal to the transformer reactance X_t (Gurunath 2010). If the nominal system also represents full loading condition, then the GEP from this model will give maximum phase lag. The G-constants can be obtained in real time by load flow information at the transformer and at the generator terminals. So for any PSS design based on this model, the parameters can be easily modified to accommodate major structural changes in the system from time to time by local measurements.

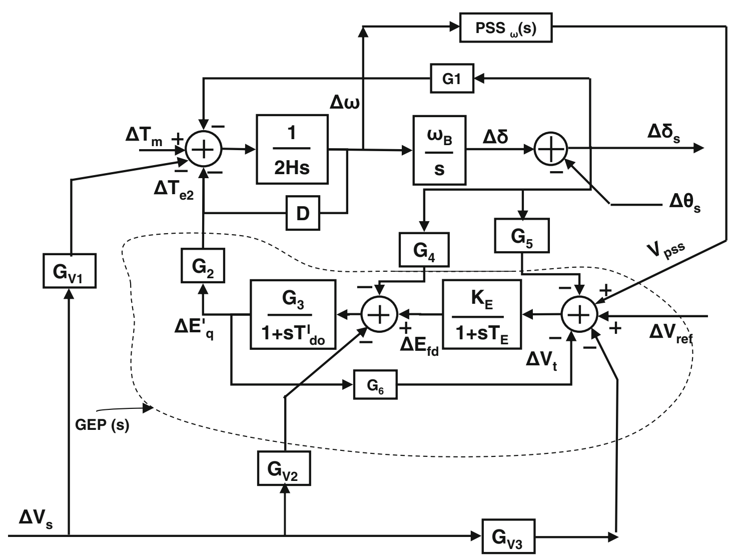

Figure 2. Linearized model of a single machine in a connected network.

4. Linear quadratic power system stabilizer

The linearized state equations of the modified Heffron–Phillip's model are as follows (Gurunath & Sen 2010)

$$\dot{X} = AX + BV_{PSS} + B_1 \begin{bmatrix} \Delta\theta_s \\ \Delta V_s \end{bmatrix}, \tag{21}$$

where

$$A = \begin{bmatrix} 0 & \omega_B & 0 & 0 \\ -\frac{G_1}{2H} & -\frac{D}{2H} & -\frac{G_2}{2H} & 0 \\ -\frac{G_4}{T'_{do}} & 0 & -\frac{1}{G_3 T'_{do}} & \frac{1}{T'_{do}} \\ -\frac{G_A G_5}{T_A} & 0 & -\frac{G_A G_6}{T_A} & -\frac{1}{T_A} \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_A}{T_A} \end{bmatrix};$$

$$B_1 = \begin{bmatrix} -1 & 0 \\ 0 & -\frac{G_{V1}}{2H} \\ 0 & -\frac{G_{V2}}{T'_{do}} \\ 0 & -\frac{K_A G_{V3}}{T_A} \end{bmatrix}. \tag{22}$$

The state variables are $X = [\Delta\delta_s; \Delta S_m; \Delta E'_q; \Delta E_{fd}] = [\delta_s - \delta_{s0}, S_m - S_{m0}, E'_q - E'_{q0}, E_{fd} - E_{fd0}]$. The control input to damp system oscillations is denoted as V_{PSS}. Considering B_1 as disturbance term a linear quadratic regulator is designed to minimize the objective function

$$J = \lim_{\tau\to\infty} \frac{1}{\tau} E \left[\int_0^\tau \left(X^T Q X + V_{PSS}^T R V_{PSS} \right) dt \right]. \tag{23}$$

A detailed analysis of the frequency response of generator electrical torques is carried out in Lam & Yee (1998). Two components were observed in the frequency response between AVR input and the resultant electrical torque at the rotor shaft. One component was shown to be dependent on the associated generator as well as on the network admittance matrix, augmented with the generator admittances. And the other component was shown to be dependent only on the associated generator. The diagonal dominance property of the admittance matrix makes the first component less affected by the external system. It was shown that the required dynamic information for the PSS design is contained mostly within the generating plant. Based on this presumption the modified Heffron–Phillip's model is used in Gurunath & Sen (2010) for the lead–lag PSS design. The B_1 matrix represents the generator and the external system interaction terms. From figure 2 one can observe that these interaction terms do not contribute to the GEP(s) transfer function. Hence neglecting this term for LQR design does not affect the system performance. With this assumption the state feedback controller is obtained as

$$V_{PSS} = -K_{lqr} X, \tag{24}$$

where

$$K_{lqr} = R^{-1} B^T P \tag{25}$$

and the matrix P is the solution of the Algebraic Riccati Equation (ARE)

$$PA + A^T P - PBR^{-1}B^T P + Q = 0. \tag{26}$$

In a multi-machine environment each generator contributes to certain oscillatory modes. Using this modified Heffron–Phillip's model, LQR stabilizers can be designed at each generator such that the corresponding oscillatory modes are well-damped. In this way, a coordinated LQR design can be achieved. This controller tries to control the rotor angle measured with respect to the local bus rather than the angle δ measured with respect to the remote bus. Equations (7) and (8) can be used to compute the state variables δ_s and E'_q from the real and reactive power measurements at the secondary bus of the transformer. The measurements of speed S_m and field voltage E_{fd} are usually available in the generating stations. Therefore, state estimators are not required in the proposed approach. This attributes to the decentralized nature of the proposed controllers.

5. Simulation results

Extensive simulations are carried out on 4 generator 10 bus system and 10 generator 39 bus system to asses the performance of the proposed LQR stabilizer over a wide range of operating conditions. Only a few representative results have been included here. The results are compared with that of a conventional lead–lag PSS (CPSS). The system data for the multi machine test systems is taken from Padiyar (1996).

5.1 *4 Generator 10 bus system*

This is a widely used two-area system for validating various control concepts (Kundur *et al* 1989). Conventional lead–lag speed input stabilizer (CPSS) are tuned for the test system following using the residue method (Pagola *et al* 1989). The system data can be obtained from Padiyar (1996). A power system stabilizer should ideally be designed to not only provide adequate local mode damping but also sufficiently contribute to damping of rotor modes immediately after the first swing following a fault. In the latter case the nonlinear performance of the stabilizer becomes important (Larsen & Swann 1981). With this in view, the linear and nonlinear performance of the stabilizer has been evaluated at two operating conditions, with 400 MW and 600 MW power flows from bus 7 to 8, for the following disturbances

(i) 10% step change in V_{ref}.
(ii) A 3ϕ fault at the transformer terminals cleared in less than 100 ms.

Table 1. CPSS Parameters 4 GEN 10 bus system.

	PSS data		
	T_1	T_2	K_{PSS}
GEN-1	0.26074	0.19424	55
GEN-2	0.38329	0.04924	8.7
GEN-3	0.36287	0.06184	10.5
GEN-4	0.28442	0.1787	79
Output limits $= \pm 0.1$; $T_w = 10s$;			

Table 2. LQRPSS Parameters 4 GEN 10 bus system.

	LQR - PSS data		
	Q (diagonal)	R	K_{lqr}
GEN-1	[0.26; 1.0; 0.0001; 0.00001]	0.2	[−0.17838; −67.09; 0.8858; 0.048562]
GEN-2	[0.26; 1.0; 0.0001; 0.00001]	0.2	[−0.059214; −8.124; 0.98091; 0.0049761]
GEN-3	[0.26; 1.0; 0.0001; 0.00001]	0.2	[−0.10922; −62.341; 0.94517; 0.0049312]
GEN-4	[0.26; 1.0; 0.0001; 0.00001]	0.2	[−0.051369; −57.173; 0.99251; 0.00499061]

For each generator Q = diag([0.26,1,0.0001,0.00001]) and R = 0.2 are selected. Selection of Q matrix requires some trial and error. However, we observed that giving more weight to E'_q and E_{fd} i.e., the third and fourth diagonal elements of Q matrix does not have much effect on the system damping but they introduce more steady state error in terminal voltage. So here these two constants are set at very low values. The second diagonal element of Q (corresponds to slip speed S_m) is kept constant at 1. The first diagonal element of Q (corresponds to δ_s) and R are varied so as to get damping ratio $\zeta >= 0.35$ for the closed loop eigenvalues $eig(A - BK)$. For GEN-1, $A_1 = [0,376.99, 0, 0; -0.10882, 0, -0.11859, 0; -0.31075, 0, -0.58889, 0.125; -521.56, 0, -2104.8, -50]$, $B_1 = [0; 0; 0; 10000]$. The complex open loop eigenvalues are $-0.63812 \pm 5.9428i$. Using Q and R as mentioned above one can get the feedback gain matrix $K_1 = [-0.17838, -67.09, 0.8858, 0.0048562]$ with closed loop complex eigenvalues as $[-3.8776 \pm 9.0447i]$. With this PSS on GEN-1 the overall system eigenvalues are computed. It has been found that the modes contributed by GEN-1 are well-damped. If the corresponding

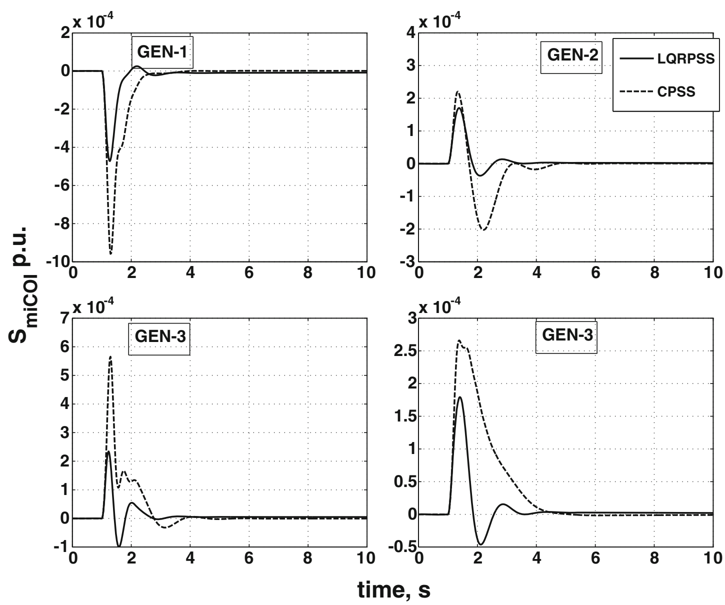

Figure 3. S_{miCOI} Responses of GEN-1 to GEN-4, 10% step change in V_{ref} at GEN-1, 400 MW tie line flow.

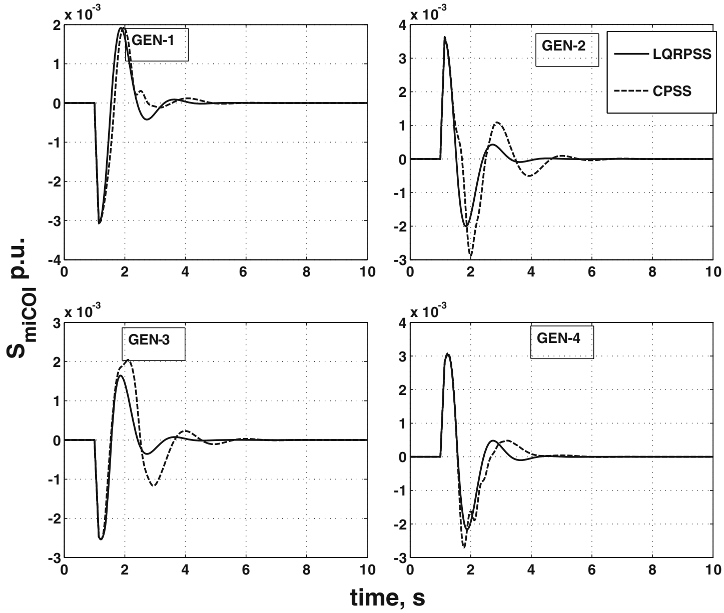

Figure 4. S_{miCOI} Responses of GEN-1 to GEN-4, 3ϕ fault at bus-7 for 142 ms, one line trip, 400 MW tie line flow.

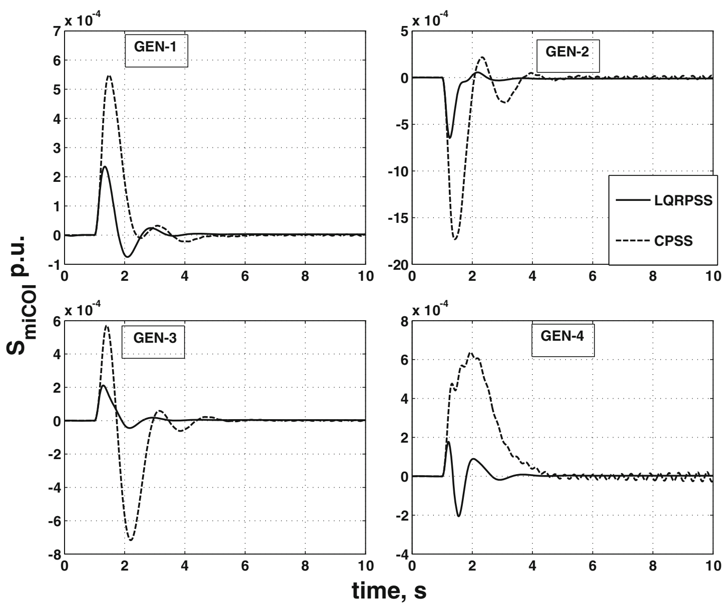

Figure 5. S_{miCOI} Responses of GEN-1 to GEN-4, 10% step change in V_{ref} at GEN-1, 600 MW tie line flow.

Table 3. CPSS Parameters 10 GEN 39 bus system.

Residue design			
GEN	T_1	T_2	K_{pss}
1	0.3469	0.0557	35
3	0.3442	0.0731	4
4	0.3104	0.0667	22
5	0.2934	0.0456	20.5
6	0.3833	0.0422	5
7	0.2866	0.0506	25
8	0.3083	0.0484	27
9	0.3277	0.0841	5.5
10	0.3124	0.0871	19.5
$T_w = 10s$, number of stages = 2, output limits = 0.15, −0.1;			

eigenvalues are not damped properly one can tune Q and R values. Similar procedure is followed for all other generators. The feedback gains for other generators are given in tables 1 and 2.

Figure 3 shows the slip speed responses S_{miCOI} with respect to Center of Inertia (COI) of generator 1 (GEN-1) to GEN-4 for a 10% step change in V_{ref} at GEN-1 under 400 MW condition. It can be observed from the figure that the response of the system with the proposed PSS (LQRPSS) is much faster, reaches steady state quickly with better damping when compared to the performance of the system with the conventional PSS.

Figure 4 shows the S_{miCOI} responses of GEN-1 to GEN-4 for a 3ϕ fault at bus-7 cleared after 142 ms by tripping one of the parallel lines between 7 and 8. It is clear from the figure that the performance of the proposed LQRPSS under nonlinear operating range of power systems is much better than that of the conventional PSS.

Figure 5 shows the S_{miCOI} responses for 600 MW power flow in line 7–8. The responses are provided for a 10% step change at GEN-2. Observe that the responses of GEN-2 and GEN-4 have unstable oscillations of low magnitude. The proposed PSS performance is much better and no oscillations exist. This shows the superiority of the proposed LQRPSS over the conventional PSS.

Table 4. LQRPSS Parameters 10 GEN 39 bus system.

LQR - PSS data			
	Q (diagonal)	R	K_{lqr}
GEN-1	[0:5; 1:0; 0:1; 0:0001]	0.6	[−0.17838; −67.09; 0.8858; 0.048562]
GEN-3	[0:9; 1:0; 0:2; 0:0001]	0.6	[−0.059214; −8.124; 0.98091; 0.0049761]
GEN-4	[1:0; 1:0; 0:2; 0:0001]	0.6	[0:7703; 43:973; 1:9338; 0:0098284]
GEN-5	[1:0; 1:0; 0:2; 0:0001]	0.6	[0:8049; 35:447; 2:1775; 0:01032]
GEN-6	[0:9; 1:0; 0:09; 0:0001]	0.6	[0:91637; 31:172; 1:8104; 0:0075675]
GEN-7	[0:8; 1:0; 0:09; 0:0001]	0.6	[0:77618; 31:42; 1:614; 0:0083423]
GEN-8	[0:8; 1:0; 0:09; 0:0001]	0.6	[0:83902; 28:317; 1:9332; 0:0084123]
GEN-9	[0:8; 1:0; 0:09; 0:0001]	0.6	[0:71024; 41:152; 1:7917; 0:010147]
GEN-10	[0:1; 1:0; 0:09; 0:0001]	0.6	[0:21455; 19:301; 0:72116; 0:0049411]

Table 5. Participation factors.

	Frequency	GEN-1	GEN-2	GEN-3	GEN-4	GEN-5	GEN-6	GEN-7	GEN-8	GEN-9	GEN-10
Mode 1	1.38 Hz	0.0002	0	0.0004	**0.1189**	**0.3644**	0.0009	0.0136	0.0061	0	0
Mode 2	1.32Hz	0	0	0	0.0029	0.0017	**0.1692**	**0.3161**	0.0164	0.0001	0.0004
Mode 3	1.30 Hz	0.002	0.0005	0.0025	0.0058	0.0018	0.0159	0.0033	**0.4383**	0.0128	0.0218
Mode 4	1.14 Hz	**0.2713**	0	**0.2311**	0	0	0.0003	0.0001	0	0	0
Mode 5	1.11 Hz	0.0013	0	0.0006	**0.2204**	0.0382	**0.1701**	0.0761	0	0.0005	0.0002
Mode 6	1.0 Hz	**0.1716**	0.0022	**0.1914**	0.0583	0.022	0.0347	0.0197	0.0007	0.0049	0.0173
Mode 7	0.96 Hz	0.0175	0.0006	0.0228	0.0164	0.0079	0.0139	0.0084	0.0036	**0.3942**	0.0208
Mode 8	0.97 Hz	0.0048	0.0032	0.0059	0.0209	0.0088	0.0176	0.0104	0.0161	0.0161	**0.4842**
Mode 9	0.59 Hz	0.0178	**0.2006**	0.0265	0.0431	0.0379	0.0517	0.0381	0.0134	0.0489	0.0297

5.2 *10 Generator 39 bus system*

This system is also a most widely used to test system for validating several control designs. For this system, PSSs are designed using the proposed method and also using the residue method (Pagola *et al* 1989) for all generators except GEN-2 which is an equivalent of external network. The system data can be obtained from Padiyar (1996). The CPSS data and the LQRPSS data are given in tables 3 and 4.

Table 5 shows the participation factors of the generators. Observe that the generators (GEN) 1,3,4, and 6 are having considerable participation in more than two modes (highlighted with grey colour). The remaining generators (GEN-5,7,8,9, and 10) are having highest participation in only one mode. Table 6 shows the eigenvalues of the system with the addition of PSS on individual generators. This table also contains the eigenvalues without PSS. The modes affected most due to PSS addition are highlighted. The Q and R values are tuned such that the addition of PSS on each generator impacts only one mode in which they participate. The eigenvalues of the system without PSS and with PSS on all generators except on GEN-2 for both the methods are given in table 7. This table also contains additional modes introduced due to the PSS action. It can be observed that the additional modes introduced by residue method are mostly exciter modes, where as with the proposed LQRPSS, no additional modes are introduced. All the inter area modes are well-damped in both the cases. This shows that using the linearized model up to the transformer bus at each generator gives a coordinated LQR design for the whole system without any additional modes. This is a desirable feature for any PSS design method.

The performance of the proposed stabilizers have been tested at varied operating conditions. Three operating scenarios have been presented here. Loading conditions considered here are nominal loading, heavy loading (all loads are increased by 15% and generation at GEN-5 to 10

Table 6. Eigenvalues with PSS on individual generators.

Mode no.	No PSS	PSS on GEN-1	PSS on GEN-3	PSS on GEN-4	PSS on GEN-5
Mode-1	−0.38 ± 8.66i	−0.38 ± 8.66i	−0.38 ± 8.66i	**−0.99 ± 8.34i**	**−2.04 ± 8.89i**
Mode-2	−0.34 ± 8.31i	−0.34 ± 8.31i	−0.34 ± 8.31i	−0.35 ± 8.31i	−0.34 ± 8.31i
Mode-3	−0.25 ± 8.21i	−0.26 ± 8.19i	−0.25 ± 8.20i	−0.26 ± 8.21i	−0.25 ± 8.21i
Mode-4	−0.08 ± 7.59i	**−1.02 ± 7.87i**	**−0.17 ± 7.14i**	−0.08 ± 7.59i	−0.08 ± 7.59i
Mode-5	−0.21 ± 6.96i	−0.21 ± 6.96i	−0.22 ± 6.95i	**−1.29 ± 7.40i**	−0.29 ± 7.00i
Mode-6	−0.07 ± 6.40i	**−0.24 ± 6.47i**	**−1.80 ± 7.23i**	−0.11 ± 6.48i	−0.10 ± 6.42i
Mode-7	0.22 ± 6.03i	0.21 ± 6.04i	0.20 ± 6.05i	0.20 ± 6.06i	0.20 ± 6.05i
Mode-8	−0.53 ± 6.07i	−0.53 ± 6.08i	−0.52 ± 6.07i	−0.57 ± 6.13i	−0.56 ± 6.09i
Mode-9	0.05 ± 3.72i	−0.00 ± 3.71i	−0.03 ± 3.73i	−0.08 ± 3.73i	−0.05 ± 3.72i

Mode no.	PSS on GEN-6	PSS on GEN-7	PSS on GEN-8	PSS on GEN-9	PSS on GEN-10
Mode-1	−0.38 ± 8.66i	−0.38 ± 8.64i	−0.38 ± 8.65i	−0.38 ± 8.66i	−0.38 ± 8.66i
Mode-2	**−1.07 ± 8.23i**	**−1.59 ± 8.53i**	−0.34 ± 8.31i	−0.34 ± 8.31i	−0.34 ± 8.31i
Mode-3	−0.25 ± 8.21i	−0.25 ± 8.21i	**−1.76 ± 8.61i**	−0.30 ± 8.177i	−0.26 ± 8.21i
Mode-4	−0.08 ± 7.59i	−0.08 ± 7.59i	−0.08 ± 7.60i	−0.08 ± 7.59i	−0.08 ± 7.59i
Mode-5	**−0.59 ± 7.18i**	−0.32 ± 7.07i	−0.21 ± 6.96i	−0.21 ± 6.96i	−0.21 ± 6.96i
Mode-6	−0.11 ± 6.42i	−0.09 ± 6.41i	−0.07 ± 6.40i	−0.06 ± 6.40i	−0.07 ± 6.39i
Mode-7	0.20 ± 6.05i	0.20 ± 6.04i	0.21 ± 6.04i	**−1.87 ± 6.14i**	0.21 ± 6.03i
Mode-8	−0.57 ± 6.11i	−0.56 ± 6.09i	−0.54 ± 6.06i	−0.56 ± 6.02i	**−0.79 ± 6.20i**
Mode-9	−0.07 ± 3.72i	−0.04 ± 3.72i	0.02 ± 3.72i	−0.15 ± 3.75i	0.02 ± 3.73i

Table 7. Eigenvalues with PSS and without PSS on all generators (* exciter modes).

No PSS	Proposed LQRPSS	Residue PSS
−0.38 ± 8.66i	−2.57 ± 9.24i	−10.36 ± 29.16i*
−0.25 ± 8.19i	−2.00 ± 8.80i	−19.45 ± 27.45i*
−0.34 ± 8.31i	−1.86 ± 8.63i	−18.42 ± 27.64i*
−0.21 ± 7.20i	−1.66 ± 7.88i	−16.14 ± 26.70i*
−0.21 ± 6.96i	−1.81 ± 7.18i	−17.16 ± 24.24i*
−0.11 ± 6.31i	−1.73 ± 6.95i	−12.77 ± 18.39i*
0.21 ± 6.03i	−1.08 ± 6.30i	−22.62 ± 13.62i
−0.53 ± 6.08i	−2.06 ± 6.24i	−10.03 ± 13.24i
0.04 ± 3.72i	−0.86 ± 3.29i	−12.74 ± 6.31i
		−10.71 ± 7.45i
		−1.16 ± 6.36i
		−1.43 ± 6.06i
		−1.13 ± 5.45i
		−3.11 ± 4.84i
		−2.50 ± 4.49i
		−2.82 ± 4.44i
		−1.91 ± 3.88i
		−2.16 ± 3.44i
		−0.87 ± 2.25i

is increased by 15%) and light loading (all loads are decreased by 15% and generation at GEN-3 to 10 are decreased by 15%). From extensive simulation studies the lines 21–22, 26–29 and 28–29 are found to be critical for system stability.

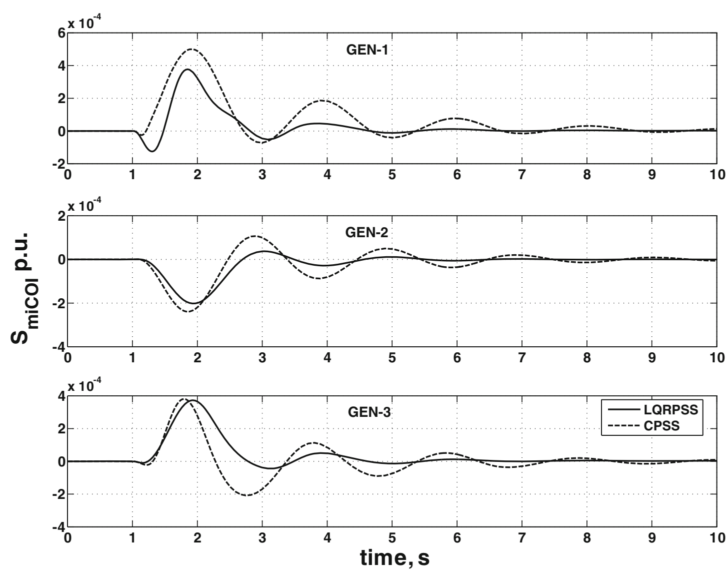

Figure 6. S_{miCOI} Responses of GEN-1 to GEN-3 for a $0.1 p.u.$ step change in V_{ref} at GEN-2, 10 GEN 39 bus system.

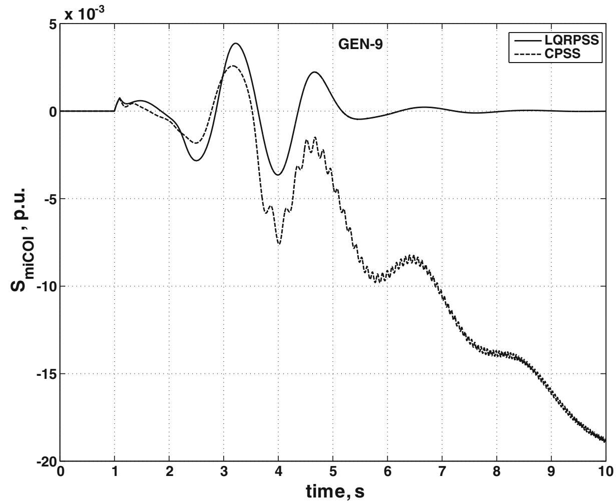

Figure 7. S_{miCOI} Responses of GEN-9 for a 3ϕ fault, 96 ms duration at bus 29 cleared by tripping 29–28, 10 GEN 39 bus system.

Figure 6 shows the slip speed responses S_{miCOI} of GEN-1 to 3 with respect to center of inertia for a small disturbance of $0.1 p.u.$ step change in V_{ref} at GEN-2. One can observe that the response with the proposed LQRPSS is damped within 4 seconds and is much better than that of the conventional PSS.

Figure 7 shows S_{miCOI} responses of GEN-9 for a 3ϕ fault at bus 29 cleared in 96 ms by tripping same line as before. Here the response with CPSS is unstable where as the response with the proposed LQRPSS is stable and well-damped.

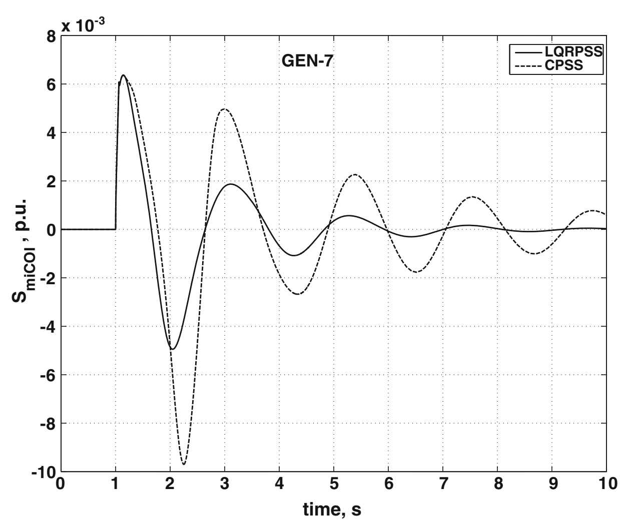

Figure 8. S_{miCOI} Responses of GEN-7 for a 3ϕ fault, 60 ms duration at bus 22 cleared by tripping 21–22, heavy loading, 10 GEN 39 bus system.

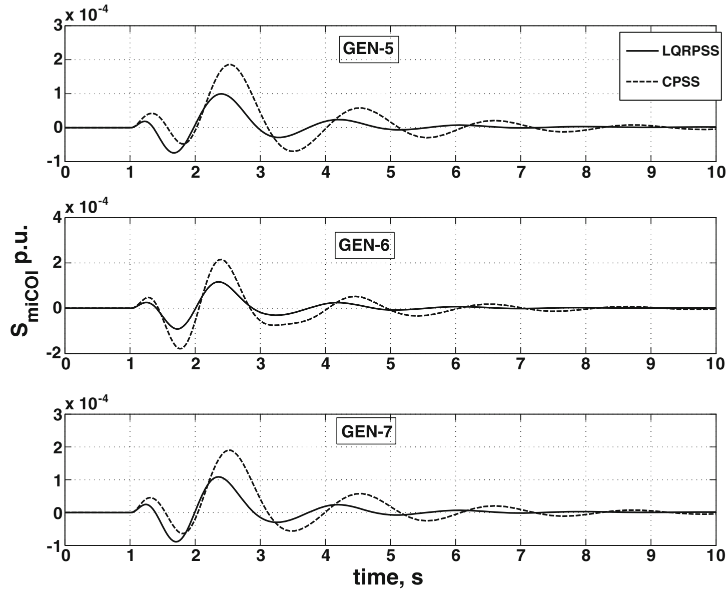

Figure 9. S_{miCOI} Responses of GEN-5 to 7 for $0.1p.u.$ step change in V_{ref} at GEN-9, light loading, 10 GEN 39 bus system.

Figure 8 shows slip speed variation S_{miCOI} of GEN-7 for a 3ϕ fault at bus 22 cleared in 60 ms by tripping 21–22 under heavy loading. Here also the response with the proposed PSS is much better than the conventional PSS. These simulations reflect the ability of the proposed PSS in enhancing transient stability as well as the small signal stability of the system.

Figure 9 shows the slip speed responses S_{miCOI} of GEN-5 to 7 for a $0.1p.u.$ step change in V_{ref} at GEN-9 under light loading conditions. This figure shows the superiority of the proposed PSS under light loading conditions. From these simulation results it is evident that the proposed method has shown better performance over a wide range of operating conditions when compared to that of the conventional method. The proposed LQR design can be implemented with local measurements, does not require any external information for the design and has less computational requirements. It deals only with a 4×4 matrix, at the most a 6×6 matrix if an IEEE Type 1 excitation system considered and needs only steady state information of the plant for the design.

6. Conclusions

A new approach for the design of full state feedback linear quadratic regulator has been proposed for the power system stabilizers in this paper. Recently proposed modified Heffron–Phillip's model, which has been developed by taking the secondary voltage of the step-up transformer as reference instead of the infinite bus, has been used in this paper. The proposed controller can be realized by the local measurements and does not require any external system information for the design. The performance of the proposed LQRPSS has been evaluated on two widely used test systems, 4 generator 10 bus and 10 generator 39 bus test system, over a wide range of system and operating conditions. The performance of the proposed PSS is much better than the conventional PSS in all the tested operating conditions.

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