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Residual strength evaluation of concrete structural components under fatigue loading

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Abstract. This paper presents methodologies for residual strength evaluation of concrete structural components using linear elastic and nonlinear fracture mechanics principles. The effect of cohesive forces due to aggregate bridging has been represented mathematically by employing tension softening models. Various tension softening models such as linear, bilinear, trilinear, exponential and power curve have been described with appropriate expressions. These models have been validated by predicting the remaining life of concrete structural components and comparing with the corresponding experimental values available in the literature. It is observed that the predicted remaining life by using power model and modified bi-linear model is in good agreement with the corresponding experimental values. Residual strength has also been predicted using these tension softening models and observed that the predicted residual strength is in good agreement with the corresponding analytical values in the literature. In general, it is observed that the variation of predicted residual moment with the chosen tension softening model follows the similar trend as in the case of remaining life. Linear model predicts large residual moments followed by trilinear, bilinear and power models.

Keywords. Plain concrete; fracture mechanics; fatigue loading; tension softening; remaining life; residual strength.

1. Introduction

Concrete is a widely used material that is required to withstand a large number of cycles of repeated loading in structures such as highways, airports, bridges, flyovers and other infrastructural engineering structures. The cyclic load may cause structural fatigue failure and there may be significant changes on the characteristics of materials such as stiffness, toughness and durability. Concrete contains numerous flaws, such as holes or air pockets, precracked aggregates,

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lack of complete bond between aggregate and matrix, etc., from which cracks may originate. In concrete members, cracking take place beyond the tensile strength of a material and generally propagate in a direction, which is perpendicular to the maximum tensile stress. The failure of many concrete structures is mainly caused by the fatigue ruptures of concrete. The fracture behaviour of concrete is greatly influenced by the fracture process zone (FPZ). The variation of FPZ along the structure thickness or width is usually neglected. The inelastic fracture response due to the presence of FPZ may then be taken into account by cohesive pressure acting on the crack faces. Figure 1 shows FPZ in ductile-brittle materials and quasi-brittle materials (Bazant 2002).

When the structural components are subjected to repetitive live loads of high-stress amplitude, according to classical theory, applied loads result in in-plane tensile stresses at the bottom of the components. The stress state in such structures is often simulated by conducting three-point bending tests. Plain concrete subjected to flexural loading fails owing to crack propagation. Repeated loading results in a steady decrease in the stiffness of the structure, eventually leading to failure. It is of interest to characterize the material behaviour subjected to such loading and study the crack propagation, remaining life and residual strength resulting from such loading. The current approaches used to evaluate fatigue performance of concrete members are mainly empirical. Fatigue equations based on the well-known S–N curve approach have been developed. A severe limitation of S–N curve approach is the inherent empiricism as it does not use fundamental material parameters that can be determined for use in design or evaluation.

Few experimental investigations on fatigue crack propagation in concrete have been reported (Stuart 1982; Baluch *et al* 1987; Bazant & Xu 1991; Gerstle Walter *et al* 1992; Ramsamooj 1994; Zhang Binsheng & Keru Wu 1997; Toumi Bascos & Turatsinze 1998; Takashi Matsumoto & Victor 1999; Subramaniam *et al* 2000). The rate of fatigue crack growth in concrete exhibits an acceleration stage that follows an initial deceleration stage. In the deceleration stage the rate of crack growth decreases with increasing crack length, whereas in the acceleration stage there is a steady increase in crack growth rate up to failure. They have attempted to apply the fracture mechanics principles to describe the crack growth during the acceleration stage of fatigue crack growth in concrete (Stuart 1982; Baluch *et al* 1987; Bazant & Xu 1991; Gerstle Walter *et al* 1992; Ramsamooj 1994; Zhang Binsheng & Keru Wu 1997; Toumi Bascos & Turatsinze 1998; Takashi Matsumoto & Victor 1999; Subramaniam *et al* 2000). Gerstle Walter *et al* (1992) developed a flexural cracking model to predict crack width, length, strength and cracking stability of plain and reinforced concrete beams using fracture mechanics principles. Design equations and charts were presented for normalized resisting moment. Zhang & Wu (1997) proposed a formula for predicting the residual fatigue strength of ordinary concrete by conducting flexural

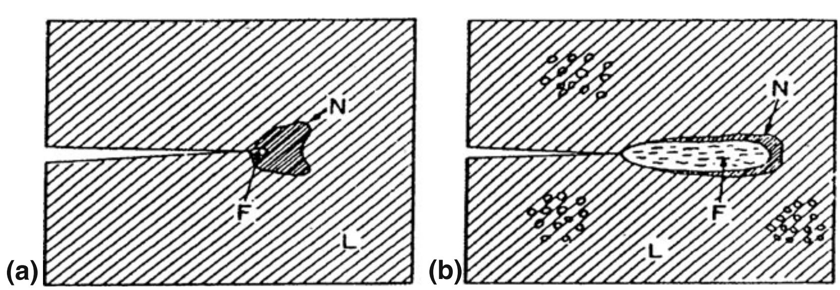


Figure 1. FPZ in ductile and brittle materials. (**a**) Ductile-brittle (metals). (**b**) Quasi-brittle (concrete).

tests on concrete beam specimens. Prasad & Krishnamoorthy (2002) developed a 2D computational model for investigation of crack formation and crack growth in plain and RC plane stress members. Gasser Thomas *et al* (2005) described methodologies for modelling 3D crack propagation in plain concrete. It was mentioned that tensile failure involves progressive micro-cracking, debonding and other complex irreversible processes of internal damage. Wu *et al* (2006) proposed an analytical model to predict the effective fracture toughness of concrete based on the fictitious crack model. The equilibrium equations of forces in the section were derived in combination with the plane section assumption. Slowik *et al* (2006) presented a method for determining tension softening curves of cementitious materials based on an evolutionary algorithm. Sain & Chandra Kishen (2007) conducted numerical studies on three-point bending concrete specimens considering tension softening effect. Ultimate moment capacity was calculated based on equivalent strain concept, by using the fundamental equilibrium equation for the progressive failure of concrete beams. It has been observed from the literature that there are numerous tension softening models to account for softening effect. Using these models the research work carried out towards crack growth analysis, remaining life and residual strength is limited (Stuart 1982; Baluch *et al* 1987; Bazant & Xu 1991; Gerstle Walter *et al* 1992; Ramsamooj 1994; Zhang Binsheng & Keru Wu 1997; Toumi Bascos & Turatsinze 1998; Takashi Matsumoto & Victor 1999; Subramaniam *et al* 2000; Prasad & Krishnamoorthy 2002; Gasser Thomas *et al* 2005; Wu *et al* 2006; Slowik *et al* 2006; Sain & Chandra Kishen 2007). There is a scope to conduct crack growth analysis, remaining life and residual strength prediction of concrete structural components accounting for tension softening effect.

This paper presents methodologies for residual strength evaluation of concrete structural components using linear elastic and nonlinear fracture mechanics principles. The effect of cohesive forces due to aggregate bridging has been represented mathematically by employing tension softening models. Various tension softening models such as linear, bilinear, trilinear, exponential and power curve have been described with appropriate expressions. These models have been validated by predicting the remaining life of concrete structural components and comparing with the corresponding experimental values available in the literature. It is observed that the predicted remaining life by using power model and modified bilinear model is in good agreement with the corresponding experimental values. Residual strength has also been predicted using these tension softening models and observed that the predicted residual strength is in good agreement with the corresponding analytical values in the literature. In general, it is observed that the variation of predicted residual moment with the chosen tension softening model follows the similar trend as in the case of remaining life. Based on the studies, it can be concluded that the predicted residual moment using modified bilinear model may be correct.

2. Residual strength evaluation

It is known that LEFM-based theory does not incorporate the presence of process zone in front of the crack tip. The basic assumptions made for fatigue modelling on residual moment are as follows:

- After a dominant fatigue crack is created, the bridging behaviour within the fracture zone is governing the rate of fatigue crack advancement.
- The stress at the crack tip remains constant and is equal to the material tensile strength.

Material properties outside the fracture zone are unchanged during fatigue loading. Fictitious crack surfaces remain plane after deformation.

Table 1. Different types of bridging force along the cracked surface.

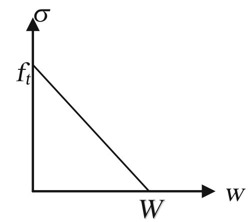
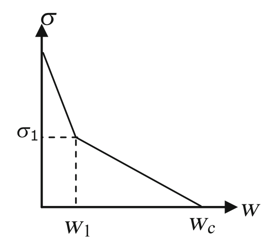
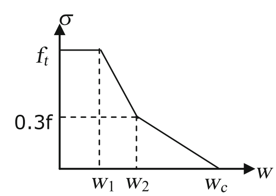
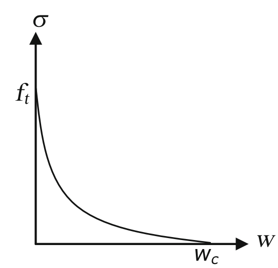
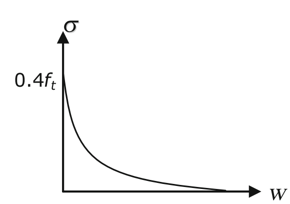
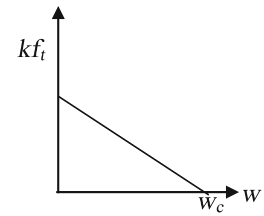
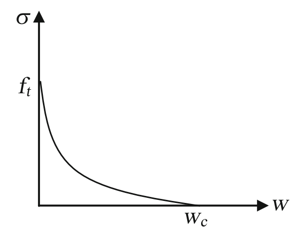
Type	Expression	Shape
Linear curve — Hillerborg *et al* (1976)	$\sigma = f_t(1 - w/w_c)$	
Bilinear curve — Roelfstra & Wittmann (1986)	$\sigma = \begin{cases} f_t - (f_t - \sigma_1)w/w_1 & \textit{for } w \le w_1 \\ \sigma_1 - \sigma_1(w - w_1)/(w_c - w_1) & \textit{for } w_1 > w \end{cases}$	
Trilinear curve — Liaw *et al* (1990)	$\sigma = \begin{cases} f_t & \textit{for } w \le w_1 \\ f_t - 0.7 f_t(w - w_1)(w_2 - w_1) & \textit{for } w_1 < w < w_2 \\ 0.3 f_t(w_c - w)/(w_c - w_2) & \textit{for } w_2 < w \le w_c \end{cases}$	
Exponential curve — Footer *et al* (1986)	$\sigma = \left(1 - \frac{w}{w_c}\right)^n$, where n is a fitting parameter.	
Reinhardt (1985)	$\sigma = f_t\left\{1 - \left(\frac{w}{w_c}\right)^n\right\}$, where $0 < n < 1$ is a fitting parameter	
Gopalaratnam & Shah (1985) similar relationship was also suggested by Cedolin *et al* (1987)	$\sigma = f_t \exp(kw^{ë})$, where k and λ are material parameters $k = -0.06163$ and $\lambda = 1.01$ for concrete with $f'c$ values of 33–47 MPa.	
Power curve — Du *et al* (1990)	$\sigma = 0.4\ f_t(1 - w/w_c)1.5$	

Table 1. (continued.)

Type	Expression	Shape
Bilinear curve with $w_1 = 0$ — Figueiras & Owen (1984)	$\sigma = \mathrm{kf_t}\,(1 - w/w_c)$, where, k = constant.	
Powere curve — Hordijk (1991)	$\sigma = f_c \left\{ \left[1 + \left(a_1 \frac{w}{w_c} \right)^3 \right]_{\exp} \left(-a_2 \frac{w}{w_c} \right) - \frac{w}{w_c} \left(1 + a_1^3 \right) \exp(-a_2) \right\}$, where a_1 and a_2 are fitting parameters	

It is known that once the crack starts propagating under fatigue loading, the aggregates provide bridging force along the cracked surface. Due to aggregate interlocking, initially the bridging force increases along with the crack length. Hence, resistance to crack propagation increases, as an extra force/moment is required to overcome this aggregate bridging force, which hinders the crack propagation. The development of this bridging force along the cracked surface can be described according to tension-softening law of concrete (refer table 1). For example, linear softening law (refer table 1) has been assumed to describe the bridging phenomenon. According to the figure, tensile stress along the crack surface varies with crack opening displacement (w). At a critical value of this opening commonly known as critical crack tip opening displacement or CTODc, the tensile force diminishes to zero. This section describes the evaluation of residual moment using LEFM and nonlinear fracture mechanics principles.

2.1 *Residual moment using LEFM principles*

To assess the residual moment carrying capacity of a pre-notched beam as a function of increasing crack length under fatigue loading, assuming the initial notch a_o to be stressfree, the stress and stain distribution corresponding to ($a_o + \Delta_a$) is shown in figure 2, where h is depth of the beam, B is width of the beam, δ is crack opening displacement, f_c, f_t, are bending stress in compression and tension, respectively. ε_c and ε_t are compressive strain and tensile strain, respectively.

The postpeak response is considered to be neutral axis depth βh can be obtained using the force equilibrium equation assuming linear stress distribution, as follows.

The expressions for total tension, T and total compression C forces can be deduced from figure 2 and are as follows:

$$T = \frac{1}{2}.B.f_t(\beta h - a_o) \tag{1}$$

$$C = \frac{1}{2}B.f_c\,(h - \beta h). \tag{2}$$

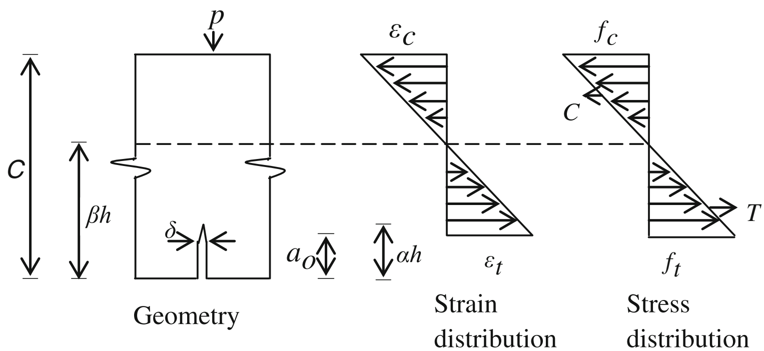


Figure 2. Bending stress and strain distribution.

The expression for C can further be modified as (figure 2)

$$C = \frac{1}{2} B f_c (h - \beta h) = \frac{1}{2} \left(\frac{\beta h^2 f_t}{\beta h - a_o} \right) (1 - \beta)^2. \tag{3}$$

By equating the total tensile and compressive force, the neutral axis depth βh can be obtained as,

$$\beta h = \frac{h + a_o}{2}. \tag{4}$$

The moment of resistance M_R is given by

$$M_R = T x (\text{lever arm}) = T \frac{2}{3} (h - a_o). \tag{5}$$

2.2 *Residual moment using NLFM principles*

Fictitious crack model (FCM) has been employed to study the tensile cracking behaviour of plain concrete member under bending. The FCM has the potential to be very useful in understanding the fracture and failure of concrete structures. It assumes that the FPZ at the tip of a crack is long and infinitesimally narrow. The FPZ is characterized by a normal stress versus crack opening displacement curve, which is considered to be a material property.

Consider a simply supported rectangular beam with width B, depth h, initial crack depth a and span L that is subjected to an external load P. The failure process of the beam can be divided into two stages: (i) a linear elastic stage and (ii) a fictitious crack developing stage. The assumed stress distribution in the cracked section of the beam for the fictitious crack developing stage is shown in figure 3.

Using equilibrium conditions (figure 3), the residual moment can be obtained as follows. In the first stage, according to classical elastic theory, first crack moment, $M_{fc} = \frac{Bh^2}{6} f_t$, where f_t is tensile strength of material.

In the second stage, the crack length αh, crack mouth opening displacement (CMOD), crack opening displacement (δ) and residual moment can be related through the analysis below.

$$M = \int_0^{\alpha h} \sigma_I(x)(h - x) B dx + \int_{\alpha h}^{h} \sigma_{II}(x)(h - x) B dx, \tag{6}$$

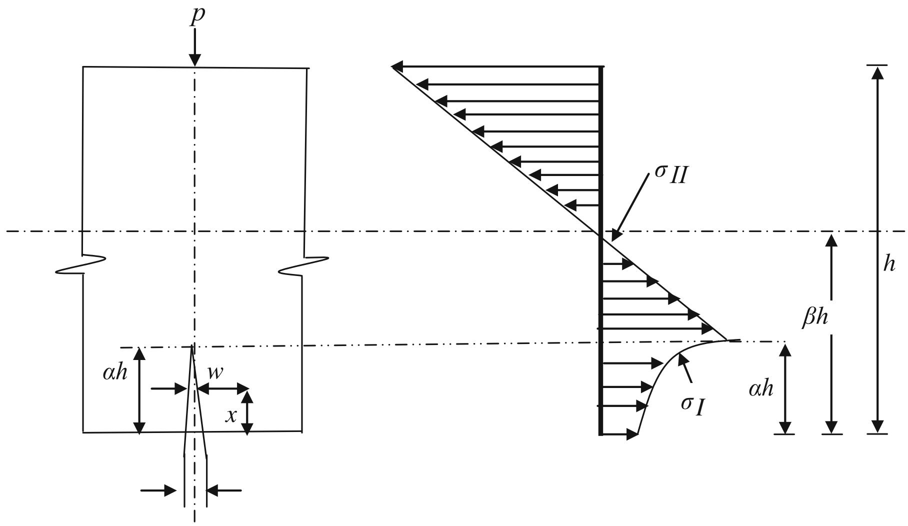


Figure 3. Distribution of normal stress in the cracked section.

where $\sigma_I(x)$, $\sigma_{II}(x)$ are the normal stress function in the cracked and uncracked parts, respectively.

The criterion used for computing the ultimate moment capacity is the crack tip opening displacement, w at the tip of each incremental crack length reaching the critical crack tip opening displacement, CTODc, which is a material parameter.

2.3 *Computation of* $\sigma_I(x)$

Figure 4 shows modelling of cohesive crack and schematic diagram of crack opening displacement. The crack opening displacement w at any point x is assumed to follow linear relationship (figure 4b) and can be expressed as,

$$w = \delta\left(\frac{a_o - x}{\Delta a} + 1\right) \qquad a_o \le x \le a_{\text{eff}}. \tag{7}$$

where δ is the crack opening displacement and a_o is initial crack length.

As an example, let us consider linear softening law (refer table 1).

$$\sigma_I(x) = f_t\,(1 - w/w_c)\,, \tag{8}$$

where f_t = tensile strength of concrete and w_c = critical crack tip opening displacement.

Substituting for w from Eq. (7) in the linear softening law given by Eq. (8), one can obtain,

$$\sigma_I(x) = f_t\left\{1 - \frac{\delta}{w_c}\left(\frac{a_o - x}{\Delta a} + 1\right)\right\}. \tag{9}$$

The crack opening displacement at any point ($\delta(x)$) can be calculated using the following equation

$$\delta(x) = \text{CMOD}\,g_3\left(\frac{a}{b}, \frac{x}{a}\right)$$

$$\text{where } g_3\left(\frac{a}{b}, \frac{x}{a}\right) = \left\{\left(1 - \frac{x}{a}\right)^2 + \left(1.081 - 1.149\frac{a}{b}\right)\left[\frac{x}{a} - \left(\frac{x}{a}\right)^2\right]\right\}^{1/2}, \tag{10}$$

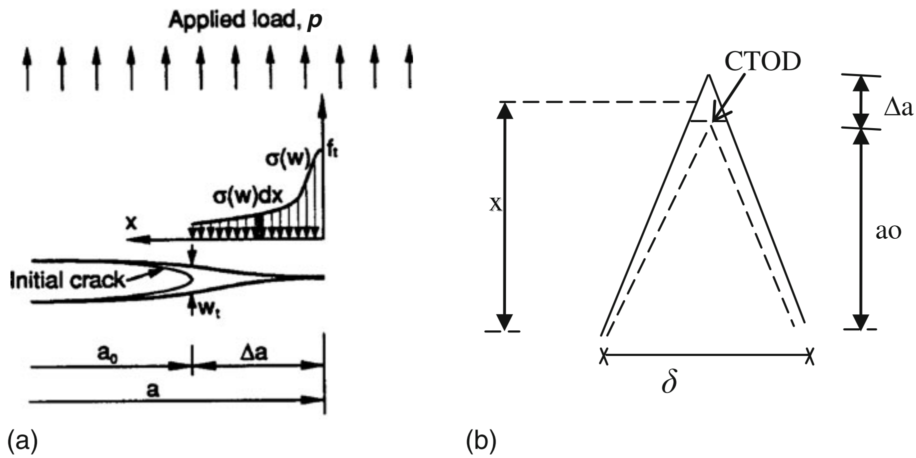


Figure 4. Modelling of cohesive crack. (**a**) Modelling of quasi-brittle crack with crack surfaces in contact. (**b**) Schematic diagram of crack opening displacement.

where CMOD is crack mouth opening displacement and is calculated using the following formula.

$$\mathrm{CMOD} = \frac{4\sigma a}{E} g_2\left(\frac{a}{b}\right), \tag{11}$$

where $g_2(a/b)$ is geometric factor, depends on the ratio of span to depth of the beam and is given below for $S = 2.5b$

$$g_2(a/b) = \frac{1.73 - 8.56a/b + 31.2(a/b)^2 - 46.3(a/b)^3 + 25.1(a/b)^4}{(1 - a/b)^{3/2}}. \tag{12}$$

2.4 *Computation of* $\sigma_{II}(x)$

From the assumed stress distribution at the uncracked part, $\sigma_{II}(x)$ can be related to αh, βh and δ by

$$\sigma_{II}(x) = f_t\left(1 - \frac{x - \alpha h}{\beta h - \alpha h}\right), \tag{13}$$

where βh is the depth of tensile zone, $\beta \in [0,1]$.

3. Numerical studies

Towards validation of methodologies described for residual strength evaluation, three example problems are presented here. The details of the problems are given below.

(i) Problem 1 (Bazant & Xu 1991): Residual strength prediction has been carried out using NLFM principles for concrete three-point bending specimens (figure 5) under constant amplitude loading. This problem was experimentally studied by Bazant & Xu (1991) and numerical predictions were carried out by Sain & Chandra Kishen (2007). The details of the studies are presented below. Modulus of elasticity = 27120 MPa.

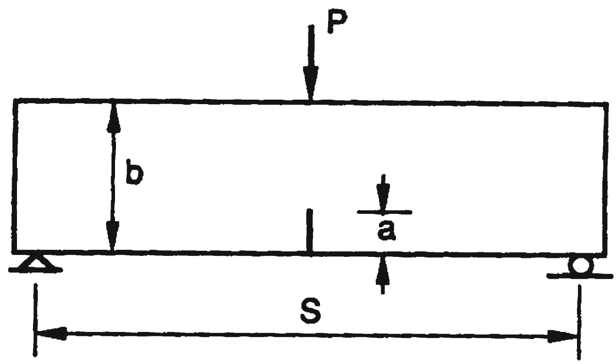


Figure 5. Three point bending specimen.

As a part of validation of various tension softening models, remaining life has been predicted for small, medium and large size beams.

The predicted remaining life using various tension softening models are shown in table 2. From table 2, it can be observed that the predicted remaining life using power curve model and modified bilinear model with $k = 0.6$ and 0.7 is in good agreement with the corresponding experimental values available in the literature.

Input details are given below.

Sl. no.	Specimen dimensions (mm)	Load (N) and f_t (MPa)	Tension softening models		
			Linear	Bilinear	Power
1.	Thickness, $t = 38.1$ Depth, $b=38.1$ Span, $s=95.25$ (Small size specimen) Initial notch depth, $a_o=6.35$	$P=1451.82$ $f_t=3.35$	$w_c=0.0435$ mm	$w_c=0.078$ mm $w_1=2/9$* $w_c=0.01733$ mm $\sigma_1=f_t/3=1.117$ MPa	$w_c=0.11$ mm $n=0.248$
2.	$t=38.1$, $b=76.2$ $s = 190.5$, $a_o=12.7$ (Medium size specimen)	$P=2387.68$ $f_t=3.86$	$w_c=0.0436$ mm	$w_c=0.078$ mm $w_1=0.01733$ mm $\sigma_1=1.287$ MPa	
3.	$t=38.1$, $b=127$ $s=317.5$, $a_o=21.17$ (Large size specimen)	$P=4145.54$ $f_t=4.68$	$w_c=0.0436$ mm	$w_c=0.0785$ mm $w_1=0.01744$ mm $\sigma_1=1.56$ MPa	

Residual strength has also been predicted for all the beams. In each case, residual strength has been predicted by using linear, bilinear and power models and compared with the corresponding analytical values available in the literature (Sain & Chandra Kishen 2007). Figures 6–8 show the normalized residual strength vs. crack depth for small, medium and large size beams, respectively and the comparison with the corresponding analytical values (Sain & Chandra Kishen 2007). It can be observed that the predicted residual strength is in good agreement with

Table 2. Predicted remaining life using various tension softening models.

Max. stress MPa	Remaining life using Linear	Bilinear	Trilinear	Expo. model by footer	Expo. model by Reinhardt	Power curve	Exptl. (Bazant & Xu 1991)
0.291	34862	33496	34129	34982	34672	31982	33409
0.07422	7789	7498	7662	7801	7754	7162	7450
0.01915	42812	41146	41970	42798	42486	39102	40867
Max. stress	Remaining life using modified Bilinear model, $\sigma = kf_t\left(1.0 - \frac{w}{w_c}\right)$ with $w_1 = 0$						Exptl. (Bazant & Xu 1991)
	$k = 0.9$	$k = 0.8$	$k = 0.7$	$k = 0.6$	$k = 0.5$		
0.291	34117	33546	32983	32484	32101		33409
0.07422	7692	7512	7368	7213	7146		7450
0.01915	41848	41098	40315	39684	39128		40867

the corresponding literature values. Further, it can be observed that the normalized moment carrying capacity is strongly dependent on the choice of approximation of the softening law. The predicted residual moment using linear softening law is larger, where as bilinear softening law predicts lesser moment followed by power law. In general, it can be observed that the variation of predicted residual moment with the chosen tension softening model follows the similar trend as in the case of remaining life.

(ii) Problem 2 (Toumi & Turatsinze 1998): This problem was studied by Toumi & Turatsinze (1998) for three-point bending concrete specimen. Details of the problem are shown below.

Length $(S) = 320$ mm, Depth $(b) = 80$ mm, Thickness $(t) = 50$ mm
Initial crack length $(a_o) = 4$ mm, Compressive strength $= 57$ MPa
Tensile strength $= 4.2$ MPa, Fracture toughness $= 0.63$ MPa$\sqrt{}$m, Minimum load $= 198.72$ N.

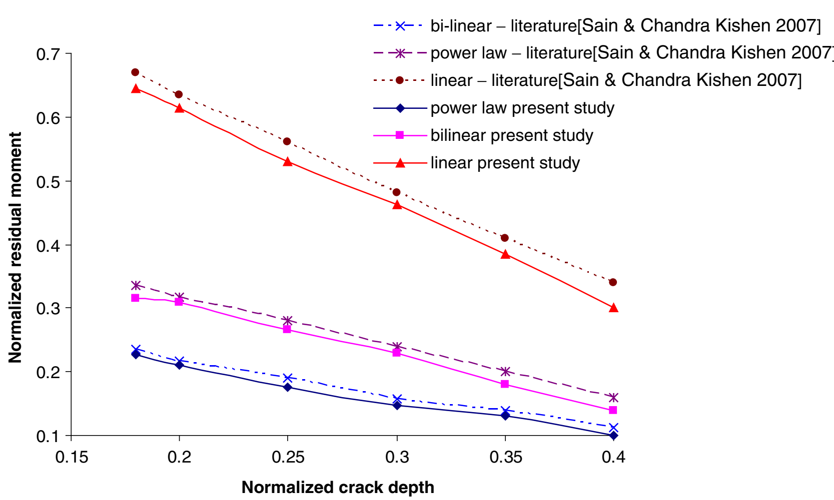


Figure 6. Normalized crack depth vs. Normalized residual moment (small size specimen).

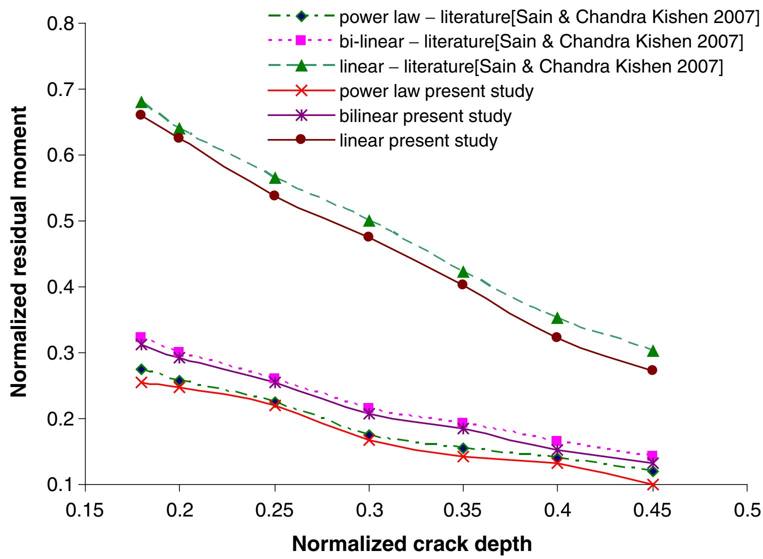


Figure 7. Normalized crack depth vs. Normalized residual moment (medium size specimen).

As a part of validation of various tension softening models, remaining life has been predicted for various loading cases. The predicted remaining life using various tension softening models are shown in table 3. From table 3, it can be observed that the predicted remaining life using

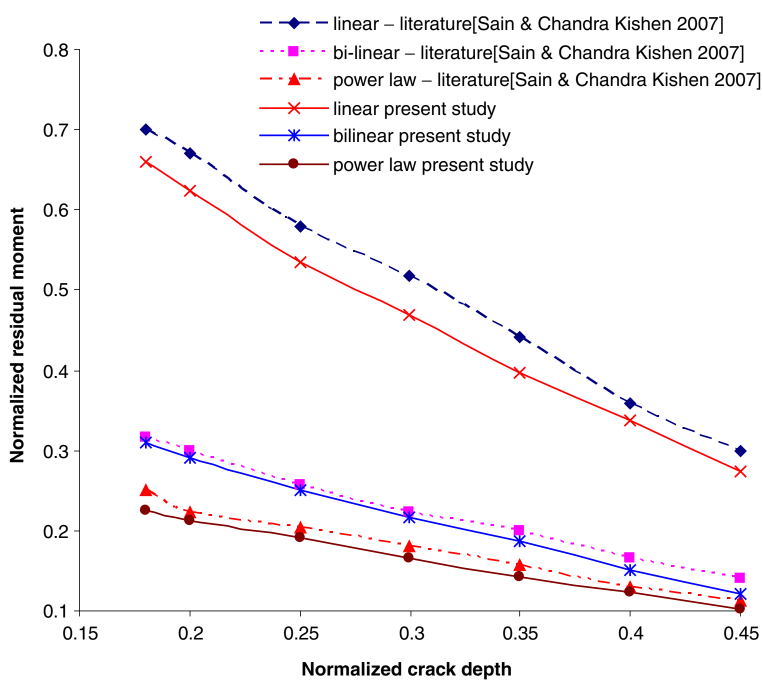


Figure 8. Normalized crack depth vs. Normalized residual moment (large size specimen).

Table 3. Predicted remaining life using various tension softening models.

Max. stress MPa	Remaining life using Linear	Bilinear	Trilinear	Expo. model by footer	Expo. model by Reinhardt	Power curve	Exptl. (Toumi & Turatsinze 1998)
1.125	33304	32251	32942	33308	33102	30887	32222
1.05	66747	63892	65032	66781	65348	61011	63611
0.975	74775	69692	70998	74791	71346	66592	69444
0.9	19102	18479	18801	19116	18892	17612	18333
Max. stress	Remaining life using modified bilinear model, $\sigma = kf_t\left(1.0 - \frac{w}{w_c}\right)$ with $w_1 = 0$						Exptl. (Bazant & Xu 1991)
	$k = 0.9$	$k = 0.8$	$k = 0.7$	$k = 0.6$	$k = 0.5$		
1.125	32809	32310	31826	31352	30982		32222
1.05	64842	63862	62740	61893	61063		63611
0.975	71102	69672	68412	67568	66510		69444
0.9	18678	18441	18096	17806	17417		18333

power curve model and modified bilinear model with $k = 0.6$ and 0.7 is in good agreement with the corresponding experimental values available in the literature (Toumi & Turatsinze 1998).

Residual moment has been predicted by using LEFM and NLFM principles. Various tension softening models such as linear, bilinear, trilinear, power and modified bilinear with $K = 0.6$ and 0.7 have been considered for residual strength prediction. Figure 9 shows the plot of predicted residual moment by using LEFM, linear, bilinear, trilinear, power and modified bilinear with $K = 0.6$ and 0.7. From figure 9, it can be observed that the variation of predicted residual moment with the assumed tension softening model follows the similar trend as in the case of remaining life prediction (refer table 3).

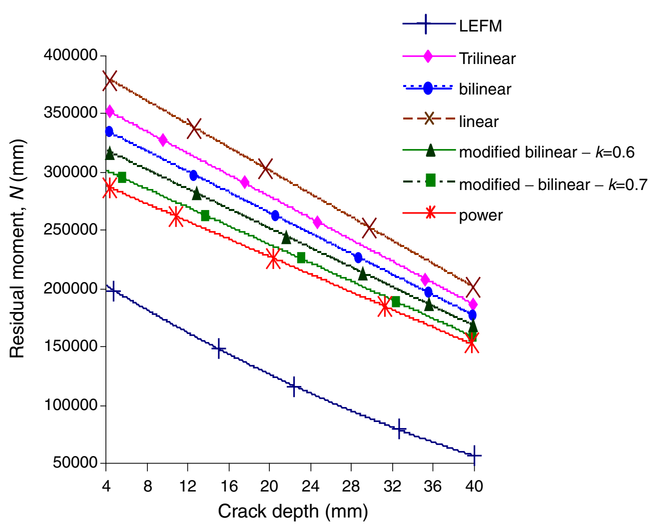


Figure 9. Crack depth vs residual moment (NLFM).

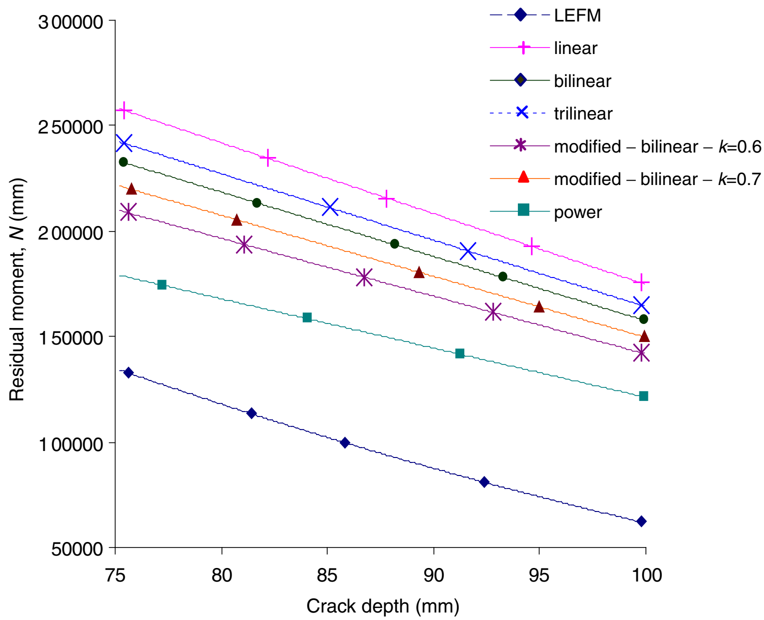


Figure 10. Crack depth vs residual moment (LEFM & NLFM).

(iii) Problem 3 (Baluch *et al* 1987): Another example problem has been chosen for residual strength prediction. This problem was studied by Baluch *et al* (1987). The details of the problem are given below.

Length of supported span (s) = 1360 mm
Thickness (t) = 51 mm
Depth (b) = 152 mm
Fracture toughness = 1.16 × 106 $N/m^{3/2}$
Max. stress = 0.5194 Mpa
Stress ratio = 0.1
Initial crack depth = 75 mm.

Residual moment has been predicted by using LEFM and NLFM principles. Figure 10 shows the plot of predicted residual moment by using LEFM, linear, bilinear, trilinear, power and modified bi-linear with $K = 0.6$ and 0.7. From figure 10, it can be observed that the variation of predicted residual moment follows the similar trend as observed in the previous problems.

4. Summary and conclusions

Methodologies for residual strength evaluation of concrete structural components using linear elastic fracture mechanics and nonlinear fracture mechanics principles have been presented. The effect of cohesive forces due to aggregate bridging is approximated mathematically through tension softening models. Various tension softening models such as linear, bilinear, trilinear, exponential and power curve have been presented with appropriate expressions. These models have been validated by predicting the remaining life of concrete structural components and comparing with the corresponding experimental values available in the literature. It is observed that the predicted remaining life by using power model and modified bi-linear model is in good agreement with the corresponding experimental values. Ultimate moment capacity has been computed by assuming the crack tip opening displacement reaches the critical crack tip opening displacement at each incremental crack depth. Residual strength has also been predicted using these tension softening models and observed that the predicted residual strength is in good agreement with the corresponding analytical values in the literature. Further, it is observed that the normalized moment carrying capacity is strongly dependent on the choice of approximation of the softening law. In general, it is observed that the variation of predicted residual moment with the chosen tension softening model follows the similar trend as in the case of remaining life. Based on the studies, it can be concluded that the predicted residual moment using modified bi-linear model may be correct.

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