Analysis of a Lorentz force based vibration exciter using permanent magnets mounted on a piezoelectric stack

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Abstract. This work presents performance analysis of a Lorentz force based non-contact vibration exciter by mounting a couple of permanent magnets on a piezoelectric stack. A conductor is attached to the structure to be excited and is placed midway between unlike poles of a couple of permanent magnets. The permanent magnets are placed on a piezoelectric stack. This stack, because of its nano-positioning capabilities, can impart an accurate and adjustable harmonic vibratory motion to the couple of permanent magnets. The piezoelectric stack, because of its high stiffness remains uncoupled with the dynamics of the structure. Due to the relative motion between the magnets and the conductor, Lorentz force is generated within the conductor. This Lorentz force is responsible for vibration of the structure in a plane parallel to the pole faces of the magnets. This keeps the magnetic field almost independent of the vibration of the structure and the chance of the structure hitting the magnet during large vibration is totally eliminated. If the amplitude of displacement of the stack is kept constant, the non-contact excitation force in this exciter remains proportional to the excitation frequency. Though use of this exciter eliminates mass (apart from that of the conductor attached to the structure) and stiffness coupling, a known damping term gets added to that of the excited structure.

Keywords. Lorentz force; eddy current; finite element; permanent magnet; vibration; non-contact exciter.

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1. Introduction

Based on their calculation and representation in engineering practice, Schweitzer *et al* (2003) have divided non-contact magnetic forces on structure and rotors into two groups. The first group is the reluctance force where force is derived from energy stored in the magnetic field. Reluctance forces act perpendicular to the interfaces. Such reluctance forces have extensively been used in active magnetic bearings. Schweitzer *et al* (2003), Genta (2004), Chiba *et al* (2005) systematically described the extensive research output in this field in their well-known texts. Kreuzinger–Janik and Irretier (2000a, b) have utilized reluctance force for vibration excitation in modal analysis of rotors. They have developed a one-sided magnetic exciter for this purpose and used it for determination of modal parameters of rotors for higher modes. Recently Nandi *et al* (2009) have proposed and analysed a one-sided magnetic actuator for vibration control of rotor. The second group of force is known as the Lorentz force. This force arises due to motion of charges in a magnetic field. This force has long been used in eddy current dampers. In an eddy current damper, magnetic flux through a moving electrically conductive disk generates eddy current in the disk. Lorentz force generated due to eddy currents exerts a strong resistance to the motion, which is essentially viscous in nature (proportional to the disk velocity). Nagaya *et al* (1984) determined the damping co-efficient for a plate conductor with a cylindrical permanent magnet on one side using a semi-analytical technique. Kligerman & Gottlieb (1998) derived a closed form expression for such a damping force where a circular disk moves within a uniform annular magnetic field.

Sodano (2006) first reported a Lorentz force based non-contact vibration exciter for light-weight and flexible structures. A thin conducting plate was attached to a beam and the beam vibrates perpendicular to the pole face of the magnet. In this exciter, the permanent magnet is placed on an electro-dynamic exciter.

The exciter configuration proposed by Sodano (2006), at least theoretically, is a non-linear one. In the configuration proposed by him the magnetic field changes with vibration of the structure. This makes the excitation force a function of the amplitude of vibration. Further, in this configuration, during large amplitude vibration, the structure may hit the magnet. It may also happen that the natural frequency of the electro-dynamic exciter is close to that of the structure. In such condition it becomes difficult to control the excitation force.

The authors propose a linear Lorentz force based exciter configuration. In the proposed configuration, since the structure vibrates parallel to the pole faces of the magnet, the magnetic field does not change with the vibration of the structure. The non-contact force remains proportional to the relative velocity of the conductor attached to the structure. Also, there is no possibility of the structure hitting the magnet during large vibration of the structure. In the present work, the authors place a couple of permanent magnets on a piezoelectric stack. The piezoelectric stacks have natural frequency in the order of several kHz. Since structures generally have first few natural frequencies much lower than this, one need not bother about the dynamics of the stack. In this frequency range of interest, the motion of the stack remains almost proportional to the applied input voltage. Piezoelectric stacks (Preumont 2005) are known for their nano-positioning capabilities. Hence, the motion of the magnets mounted on the stack can be much more precisely controlled.

To evaluate the excitation force for their proposed exciter, one first needs to determine the magnetic field at the location of the conductor. This is performed in this work first by using an integral expression and then solving an axisymmetric quasi-harmonic partial differential equation. Once the field is known, the determination of eddy currents requires solution of another Poisson's equation. Finite element codes are developed in-house for solving the

partial differential equations mentioned above. In order to verify the correctness of the computed magnetic field, the commercial finite element software ANSYS has been used.

Finally, a simplified model of the proposed exciter is developed. A third finite element simulation of a cantilever beam along with the model of the proposed exciter shows the frequency response of the structure.

2. Analysis

The configuration of the proposed exciter is shown in figure 1. Two permanent magnets are placed on a piezoelectric stack. The unlike poles of the magnets face each other. A circular

Figure 1. Schematic diagram of the (a) proposed eddy current exciter and (b) piezoelectric stack with permanent magnets, (c) the beam to be excited with the conductor attached to it.

conductor is attached to a beam. The beam is placed in such a way that the conductor remains midway between the pole faces of the magnets. The magnets are made to vibrate by applying a harmonic voltage to the piezoelectric stack.

The objective of this work is to evaluate the excitation force and its relationship to the displacement of the magnets imparted by the piezoelectric stack. In order to achieve this, first the magnetic field through the conductor is computed using standard relations adapted to the present requirement. This is further verified with a finite element simulation. Then a Poisson's equation is solved using finite elements for determination of the eddy current density in the conductor. Once that is known, the Lorentz force on the conductor can be evaluated using standard relations.

In order to investigate the effect of the proposed exciter on the structure being excited, a model of the exciter is developed. The use of this model along with the finite element or modal model of the structure is then demonstrated.

2.1 Determination of magnetic field

2.1a. Method 1—Using integral expressions: A magnetic material with magnetization $\vec{M}(\vec{r}_o)$ is considered (figure 2). For an infinitesimal volume dV_o of the material, the following relation expresses the magnetic vector potential at a point (Griffiths 2003):

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{V_o} \frac{\vec{M}(\vec{r}_o) \times \hat{r}'}{r'^2} dV_o, \quad (1)$$

where, the vector $\vec{r}'$ denotes the position vector of the point with respect to the location of the infinitesimal volume. The unit vector along $\vec{r}'$ is $\hat{r}'$.

Using $\nabla_o \left(\frac{1}{r'} \right) = \frac{\hat{r}'}{r'^2}$ and integration by parts, relation (1) reduces to a volume and a surface integral as shown below:

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{V_o} \frac{\vec{J}_b(\vec{r}_o)}{r'} dV_o + \frac{\mu_o}{4\pi} \int_{A_o} \frac{\vec{K}_b(\vec{r}_o)}{r'} dA_o, \quad (2)$$

Figure 2. An infinitesimal volume in a permanent magnet and a point where field is sought.

where, $\bar{J}_b = \nabla \times \bar{M}$ and $\bar{K}_b = \bar{M} \times \hat{n}$. The outward unit vector normal to the surface is $\hat{n}$. A differential surface element is denoted by dA_o . For a permanent magnet with uniform magnetization, the first term in equation (2) becomes zero.

$$\bar{B}(\bar{r}) = \nabla \times \bar{A} = \nabla \times \left(\frac{\mu_o}{4\pi} \int_{A_o} \frac{\bar{K}_b(\bar{r}_o)}{r'} dA_o \right) = \frac{\mu_o}{4\pi} \int_{A_o} \frac{\bar{K}_b(\bar{r}_o) \times \hat{r}'}{r'^2} dA_o. \quad (3)$$

The symbol $\bar{B}$ represents the magnetic flux density vector.

For a cylindrical magnet shown in figure 3 with magnetization $\bar{M} = M\hat{k}$

$$\bar{K}_b = M\hat{k} \times \hat{e}_{\rho o} = M\hat{e}_{\theta o}. \quad (4)$$

Therefore,

$$\bar{B}(\bar{r}) = \frac{\mu_o MR}{4\pi} \int_{z_o} \int_0^{2\pi} \frac{\hat{e}_{\theta o} \times \hat{r}'}{r'^2} d\theta_o dz_o. \quad (5)$$

From symmetry conditions it is evident that magnetic field perpendicular to a radial plane is zero. Moreover, the magnetic field distribution is identical for any radial plane. Therefore, without any

Figure 3. Field due to an infinitesimal strip of permanent magnet.

loss of generality, the x - z plane is considered in this work. The field at any point in this plane can be expressed as:

$$\bar{B} = \frac{\mu_o MR}{4\pi} \int_{z_o} \int_0^{2\pi} \frac{(-\sin \theta_o \hat{i} + \cos \theta_o \hat{j}) \times \left\{ (x - R \cos \theta_o) \hat{i} - R \sin \theta_o \hat{j} + (z - z_o) \hat{k} \right\}}{\left\{ (x - R \cos \theta_o)^2 + (R \sin \theta_o)^2 + (z - z_o)^2 \right\}^{3/2}} d\theta_o dz_o. \quad (6)$$

Or,

$$\bar{B} = \frac{\mu_o MR}{4\pi} \int_{z_o} \int_0^{2\pi} \frac{(-\sin \theta_o \hat{i} + \cos \theta_o \hat{j}) \times \left\{ (x - R \cos \theta_o) \hat{i} - R \sin \theta_o \hat{j} + (z - z_o) \hat{k} \right\}}{\left\{ x^2 + R^2 + (z - z_o)^2 + 2xR \cos \theta_o \right\}^{3/2}} d\theta_o dz_o \quad (7)$$

$$\bar{B} = \frac{\mu_o MR}{4\pi} \int_{z_o} \int_0^{2\pi} \frac{(z - z_o) \cos \theta_o \hat{i} + (z - z_o) \sin \theta_o \hat{j} + (R - x \cos \theta_o) \hat{k}}{\left\{ x^2 + R^2 + (z - z_o)^2 + 2xR \cos \theta_o \right\}^{3/2}} d\theta_o dz_o. \quad (8)$$

The second term becomes zero after integration. Therefore, the above relation reduces to

$$B_x = \frac{\mu_o MR}{4\pi} \int_{z_o} \int_0^{2\pi} \frac{(z - z_o) \cos \theta_o}{\left\{ x^2 + R^2 + (z - z_o)^2 + 2xR \cos \theta_o \right\}^{3/2}} d\theta_o dz_o. \quad (9)$$

$$B_z = \frac{\mu_o MR}{4\pi} \int_{z_o} \int_0^{2\pi} \frac{(R - x \cos \theta_o)}{\left\{ x^2 + R^2 + (z - z_o)^2 + 2xR \cos \theta_o \right\}^{3/2}} d\theta_o dz_o. \quad (10)$$

In the present work, the conducting disk is placed midway between unlike poles of two permanent magnets. The field at the disk can be computed by superposition of fields generated by two magnets. From symmetry, the radial component of field at the disk is zero.

It is difficult to evaluate the integral given by equation (10) analytically and a numerical integration scheme is adopted here for this purpose. In any numerical integration scheme, the integrand is sampled at a number of points and the value of the integrand (abscissa) at a sampling point is multiplied by an appropriate weight. All such products of the integrand and the corresponding weight values are summed up to compute the value of the integral. For a given point in the x - z plane,

$$B_z(x, z) = \frac{\mu_o MR}{4\pi} \int_{z_o} \int_0^{2\pi} \frac{(R - x \cos \theta_o)}{\left\{ x^2 + R^2 + (z - z_o)^2 + 2xR \cos \theta_o \right\}^{3/2}} d\theta_o dz_o \quad (11a)$$

$$= \sum_{i=1}^M \sum_{j=1}^N w_i w_j f(z_{oj}, \theta_{oj}) = \sum_{i=1}^M \sum_{j=1}^N W_{ij} f(z_{oj}, \theta_{oj}),$$

where

$$f(z_{oj}, \theta_{oj}) = \frac{\mu_o MR}{4\pi} \frac{(R - x \cos \theta_{oj})}{\left\{ x^2 + R^2 + (z - z_{oj})^2 + 2xR \cos \theta_{oj} \right\}^{3/2}}. \quad (11b)$$

And $W_{ij} = w_i w_j$.

A two-dimensional trapezoidal rule is adopted in the present work.

2.1b. Method 2—Using finite element simulation: A domain is considered here, which is composed of two permanent magnets and surrounding air. Since the frequencies of interest are low, Maxwell's equations for magnetostatics are considered. The equations are as follows:

In absence of free current

$$\nabla \times \vec{H} = \vec{0}, \quad (12a)$$

where

$$\vec{B} = \mu (\vec{H} + \vec{H}_c). \quad (12b)$$

From Equation (12b) one obtains

$$\vec{H} = \frac{1}{\mu} \vec{B} - \vec{H}_c \quad (12c)$$

$$\nabla \bullet \vec{B} = 0, \quad (13)$$

where the symbols $\vec{B}$, $\vec{H}$ and $\vec{H}_c$ stand for flux density, auxiliary field and coercive force respectively. The permeability of the medium is denoted by $\mu = \mu_0 \mu_r$. Permeability of free space and relative permeability of the medium are represented by the symbols μ_0 and μ_r respectively.

The remnant magnetic flux density $\vec{B}_r = \mu \vec{H}_c$.

Using (12c) and (12a), one obtains

$$\nabla \times \vec{H} = \nabla \times \frac{1}{\mu} \vec{B} - \nabla \times \vec{H}_c = \vec{0}. \quad (14)$$

Equation (13) is valid when

$$\vec{B} = \nabla \times \vec{A}, \quad (15)$$

where the vector $\vec{A}$ is known as the magnetic vector potential.

With respect to figure 3, the coordinate x coincides with the radial direction r of the axisymmetric analysis. The z axis remains the same.

Here the flux density vector $\vec{B}$, the auxiliary field vector $\vec{H}$ and the coercive force $\vec{H}_c$ are independent of the θ direction. The above vectors do not have any component in the θ direction. To satisfy the above restriction on the flux density vector $\vec{B}$, the magnetic vector potential has only θ component.

$$\vec{A} = A_\theta \hat{e}_\theta, \quad (16)$$

where the unit vectors in the radial, θ and z directions are denoted by symbols $\hat{e}_r$, $\hat{e}_\theta$ and $\hat{k}$ respectively.

Now,

$$\vec{B} = \nabla \times \vec{A} = -\frac{\partial A_\theta}{\partial z} \hat{e}_r + \frac{1}{r} \frac{\partial}{\partial r} (r A_\theta) \hat{k}. \quad (17)$$

Using relation (14) one obtains,

$$-\left[\frac{\partial}{\partial z} \left(\frac{1}{\mu} \frac{\partial A_\theta}{\partial z} \right) + \frac{\partial}{\partial r} \left\{ \frac{1}{\mu r} \frac{\partial}{\partial r} (r A_\theta) \right\} \right] - \left(\frac{\partial H_{cr}}{\partial z} - \frac{\partial H_{cz}}{\partial r} \right) = 0. \quad (18)$$

Or,

$$-\left(\frac{\partial}{\partial r} \left\{ \frac{1}{r\mu} \frac{\partial}{\partial r} (r A_\theta) \right\} + \frac{\partial}{\partial z} \left\{ \frac{1}{r\mu} \frac{\partial}{\partial z} (r A_\theta) \right\} \right) - \left(\frac{\partial H_{cr}}{\partial z} - \frac{\partial H_{cz}}{\partial r} \right) = 0. \quad (19)$$

The Neumann boundary condition can be written as

$$-\left(\frac{1}{r\mu}\frac{\partial(rA_\theta)}{\partial r}n_r + \frac{1}{r\mu}\frac{\partial(rA_\theta)}{\partial z}n_z\right) = \text{specified.} \quad (20)$$

With shape functions as the weighting functions, the Galerkin finite element starts with the following equation.

$$\int_{A^e} N_i \left[-\left(\frac{\partial}{\partial r} \left\{ \frac{1}{r\mu} \frac{\partial}{\partial r} (rA_\theta^e) \right\} + \frac{\partial}{\partial z} \left\{ \frac{1}{r\mu} \frac{\partial}{\partial z} (rA_\theta^e) \right\} \right) - \left(\frac{\partial H_{cr}}{\partial z} - \frac{\partial H_{cz}}{\partial r} \right) \right] dA^e = 0, \quad (21)$$

where the variable $rA_\theta^e = \Theta = N_j\Theta_j$ is the approximation for the variable rA_θ within an element. The shape functions are denoted by N_j and the nodal unknowns by Θ_j . The symbol A^e denotes the element area. The weak form is then obtained from equation (21).

It can be shown that with no free current inside, the finite element equations become,

$$\int_{A^e} \frac{1}{r\mu} \left(\frac{\partial N_i}{\partial r} \frac{\partial N_j}{\partial r} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right) \Theta_j dA^e = \int_{A^e} \left(\frac{\partial N_i}{\partial r} H_{cz} - \frac{\partial N_i}{\partial z} H_{cr} \right) dA^e. \quad (22)$$

Element equation (22) has to be assembled over the whole system. For the sake of simplicity three-node triangular elements have been considered in the present work. Based on the formulation described above, a code has been developed in-house.

For a three-node triangular element,

$$N_j = a_j + b_j r + c_j z \quad j = 1, 2, 3. \quad (23)$$

The expressions for the coefficients (a_j , b_j and c_j) are available in any standard text (Hutton 2004) on finite element analysis. Over an element the values of the radius r and the components of the coercive force have been assumed constant and equal to the values of the corresponding quantities at the centroid.

In matrix notation, equation (22) now becomes

$$\frac{1}{r_{\text{centroid}}\mu} \begin{bmatrix} b_1^2 & b_1b_2 & b_1b_3 \\ b_2b_1 & b_2^2 & b_2b_3 \\ b_3b_1 & b_3b_2 & b_3^2 \end{bmatrix} A^e \begin{Bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} A^e (H_{cz})_{\text{centroid}} - \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} A^e (H_{cr})_{\text{centroid}}. \quad (24)$$

This matrix equation has to be assembled over all elements to obtain the final system matrix. The far-field boundary including the axis of symmetry can be considered as a constant $\Theta = rA_\theta$ line.

2.1c. ANSYS formulation: A finite element simulation using popular finite element analysis ANSYS is performed in order to validate the results obtained from the finite element code developed in-house. In the axisymmetric finite element magnetostatic simulation the two-dimensional coupled field element PLANE13 is used. The permanent magnet is surrounded by air. The air is also modelled using PLANE13 element but with a zero value of magnetization. The boundary of air is taken so far from the magnet that the field almost becomes zero there. The parallel and

perpendicular boundary conditions at the interface between magnet and air are appropriately taken care of by the software.

2.2 Determination of Lorentz force on a moving conductor

In this proposed exciter the magnets vibrate due to the motion of the piezoelectric stacks. On the other hand, the conductor being attached to the structure, vibrates with it. Therefore, there is always a relative velocity between the conductor and the magnet. In the present work, it is assumed that this relative velocity of the conducting disk within the poles of the magnets is small. When the velocity of the disk is small, the problem can be considered to be a quasi-static one and the time dependent terms in the Maxwell's equations can be neglected. The force generated for a unit relative velocity of the disk parallel to the pole faces of the magnets is considered. The actual force on the conductor would be the force due to unit relative velocity times the actual relative velocity $\bar{v}$ of the conductor with respect to the magnets.

2.2a. Governing equations: Considering the conservations of electric charge in the conductor,

$$\nabla \cdot \bar{j} = 0, \quad (25)$$

where

$$\bar{j} = \text{Current density vector} = \sigma (\bar{E} - \nabla \Phi). \quad (26)$$

σ = Electrical conductivity of disk material.

$$\bar{E} = \bar{v} \times \bar{B}_d. \quad (27)$$

$\bar{v}$ = Relative velocity of the conducting plate with respect to the magnet

$\bar{B}_d$ = Magnetic flux density at disk location

Φ = Scalar potential function of the induced electric field, the electrical field is expressed as $\bar{E}_{\text{ind}} = -\nabla \Phi$

∇ = The vector operator 'del' = $\left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} \right)$ in case of a two-dimensional problem.

Now from equations (26) and (27), equation (25) can be written as:

$$\begin{aligned} \sigma \nabla \cdot (\bar{E} - \nabla \Phi) &= 0 \text{ or} \\ \nabla \cdot \bar{E} &= \nabla^2 \Phi, \text{ or} \\ \nabla^2 \Phi &= \nabla \cdot (\bar{v} \times \bar{B}_d). \end{aligned} \quad (28)$$

Now considering the directions shown in figure 4a

$$\begin{aligned} \bar{v} &= v_y \hat{j} \\ \bar{B}_d &= B_{dz} \hat{k} \\ \bar{v} \times \bar{B}_d &= (v_y \hat{j}) \times (B_{dz} \hat{k}) = (v_y B_{dz} \hat{i}) \end{aligned}$$

Or,

$$\nabla \cdot (\bar{v} \times \bar{B}_d) = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} \right) \cdot (v_y B_{dz} \hat{i}) = \frac{\partial (v_y B_{dz})}{\partial x} = v_y \frac{\partial B_{dz}}{\partial x}. \quad (29)$$

Figure 4. (a) The coordinate system for computation of eddy current and (b) dimensions of magnet and conducting plate.

Therefore, equation 15 reduces to the following Poisson's equation

$$\nabla^2 \Phi = v_y \frac{\partial B_{dz}}{\partial x}. \quad (30)$$

The boundary conditions can be established as follows:

As there cannot be any component of the current density vector $\vec{j}$ perpendicular to the boundary

$$\vec{j} \cdot \hat{n} = 0. \quad (31)$$

Or,

$$\sigma (\vec{E} - \nabla \Phi) \cdot \hat{n} = 0. \quad (32)$$

The vector $\vec{E} = \vec{v} \times \vec{B}_d$ is a null vector at the boundary of the conductor as $\vec{B}_d = \vec{0}$ there. The magnetic flux density vector is confined only in a zone, which is smaller than the diameter of the conductor (figure 1, figure 4b). Therefore, at the boundary the following Neumann boundary condition has to be satisfied.

$$\nabla \phi \cdot \hat{n} = 0 \text{ Or, } \frac{\partial \phi}{\partial n} = 0. \quad (33)$$

A simple finite element code is developed for solving the Poisson's equation (30) along with the boundary condition expressed by equation (33). The value of the flux density component B_{dz} is computed using equation (11) and evaluated at each nodal point of the finite element mesh of the conducting plate. The value of B_{dz} and its derivative $\frac{\partial B_{dz}}{\partial x}$ within the element is computed using the interpolation functions and their derivatives respectively. Therefore, the right hand side of the Poisson's equation (30) now becomes fully known. Though, the right hand side of equation (30) is fully known, it is a quite complicated function of space variables and hence an analytical solution of equation (30) and (31) is difficult. As mentioned earlier, the authors developed an in-house finite element code using triangular elements for this purpose. The details of the finite element formulation are presented below.

2.2b. Galerkin finite element formulation: In Bubnov–Galerkin finite element method the shape functions are considered as the weighting functions. The weighted error within each element is then made zero.

$$\int_{A^e} N_i \left(\nabla^2 \Phi^e dA^e - v_y \frac{\partial B_{dz}^e}{\partial x} \right) = 0. \quad (34)$$

The approximate distribution of the variable ϕ within an element is given by

$$\phi^e = N_j \phi_j. \quad (35)$$

For the sake of simplicity, a three-node triangular finite element is considered in the present finite element formulation. As described in section 2.1b, the shape functions within an element are expressed as

$$N_j = a_j + b_j x + c_j y \quad j = 1, 2, 3. \quad (36)$$

The nodal values of the variable ϕ are denoted by ϕ_j , $j = 1, 2, 3$.

Similarly, the flux density within an element of the disk is expressed by the relation $B_{dz}^e = N_j B_j$ where the nodal values of the z-component of flux density vector are denoted by B_j . The values of the flux density at the nodes can be computed using equation (11). Therefore,

$$\frac{\partial B_{dz}^e}{\partial x} = \frac{\partial N_j}{\partial x} B_j = b_j B_j. \quad (37)$$

Breaking up the first term into a domain and a boundary term using Gauss divergence theorem one obtains

$$\int_{\Gamma_2^e} N_i \left(\frac{\partial \phi^e}{\partial n} \right) d\Gamma_2^e - \int_{A^e} \left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) \phi_j dA^e - v_y b_j B_j \int_{A^e} N_i dA^e = 0, \quad (38)$$

where Γ_2^e is the edge of the element on the Neumann boundary.

Using Neumann boundary condition (33) and after some standard manipulations, equation (28) reduces to:

$$\begin{bmatrix} b_1^2 & b_1 b_2 & b_1 b_3 \\ b_2 b_1 & b_2^2 & b_2 b_3 \\ b_3 b_1 & b_3 b_2 & b_3^2 \end{bmatrix} A^e \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{Bmatrix} = - \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} v_y \frac{(b_1 B_1 + b_2 B_2 + b_3 B_3) A^e}{3}. \quad (39)$$

The element equations given by equation (39) are then assembled to generate the final system equations. The nodal values of the variable ϕ are then obtained from the solution of the system equations.

2.2c. Lorentz force calculation: The Lorentz force over the conductor can be expressed as,

$$\bar{F}_L = \int_{\text{Area}} \bar{j} \times \bar{B}_d t dA, \quad (40)$$

where the symbol t stands for the thickness of the conductor.

In finite element analysis this force is evaluated by summing the contributions over element areas.

$$\bar{F}_L = \sum_{\text{elements}} \int_{A^e} \bar{j} \times \bar{B}_d t dA^e. \quad (41)$$

It is evident that for a relative velocity of the conductor in the positive y direction the Lorentz force on the conductor will be in the negative y-direction. The nature of equation (30) reveals that for a given magnetic flux density the Lorentz force on the conductor is proportional to the relative velocity v_y of the conductor.

A unit value of relative velocity of the conductor is considered. The Lorentz force generated due to this relative velocity can be taken as coefficient connecting the force and relative velocity (between the conducting disk and the stack). This coefficient is denoted using the symbol α .

Therefore, one can write

$$F_{Ly} = -\alpha v_y = \alpha (\dot{u}_p - \dot{u}_d), \quad (42)$$

where the symbols $\dot{u}_p$ and $\dot{u}_d$ stand for velocities (in the y-direction) of the piezoelectric stack and the conducting disk respectively.

2.3 Model of structure with the proposed exciter

In the present work, finite element model of a cantilever beam with the proposed exciter is considered. The equation of the system can be written as follows:

$$[M] \{\ddot{U}\} + [C] \{\dot{U}\} + [K] \{U\} = \{T_c\} \alpha (\dot{u}_p - \dot{u}_m). \quad (43)$$

Now, the conducting disk is attached to the structure at such a location that the velocity of the disk in the y-direction equals the velocity of the m^{th} degree of freedom of the structure. Therefore, one can write

$$\begin{aligned} \dot{u}_d &= \dot{u}_m \\ \text{Or, } [M] \{\ddot{U}\} + [C] \{\dot{U}\} + [K] \{U\} &= \{T_c\} \alpha \dot{u}_p - \{T_c\} \alpha \{T_c\}^T \{\dot{U}\} \\ \text{Or, } [M] \{\ddot{U}\} + \{[C] + \{T_c\} \alpha \{T_c\}^T\} \{\dot{U}\} + [K] \{U\} &= \{T_c\} \alpha \dot{u}_p. \end{aligned} \quad (44)$$

The displacement vector of the finite element model of the beam is denoted by $\{U\}$. The matrices $[M]$, $[K]$ and $[C]$ are the standard mass, stiffness and damping matrices. The proposed exciter is so placed that it applies a force at the m^{th} degree of freedom. The vector $\{T_c\}$ has an entry 1 at the m^{th} row. It has zero entries in all other rows. This vector actually places the force (or velocity) at a specified degree of freedom appropriately in the force vector (or velocity vector).

The displacement of the stack can be expressed as

$$u_p = U_p \sin \omega t. \quad (45)$$

Equation (24) can therefore be expressed as:

$$[M] \{\ddot{U}\} + \{[C] + \{T_c\} \alpha \{T_c\}^T\} \{\dot{U}\} + [K] \{U\} = \{T_c\} \alpha \omega U_p \cos \omega t. \quad (46)$$

From equation (36) it is evident that a harmonic force and a viscous damper (at the location of the exciter) represent the exciter. The model of the proposed exciter placed at the tip of a cantilever is shown in figure 5. In figure 5, an appropriate finite element model replaces the actual cantilever. The term $\{T_c\} \alpha \{T_c\}^T$ is the damping term, which gets coupled to the structure due to the presence of the exciter. The interesting observation is that the excitation force

Figure 5. (a) Model of proposed exciter along with a beam, (b) the displacement of the stack.

proportionally increases with frequency of excitation, provided the displacement of the stack remains unchanged.

3. Case study

A conducting plate and a couple of cylindrical magnets of following dimensions are considered.

- Diameter of conducting plate $D_1 = 0.050$ m
- Thickness of plate $t = 0.002$ m
- Gap between the pole faces of the magnet = 0.004 m
- Diameter of permanent magnet $D_2 = 0.030$ m
- Length of permanent magnet $L = 0.050$ m.

The permanent magnet has a magnetization (magnetic dipole moment per unit volume) of $\bar{M} = 8 \times 10^5 \hat{k}$ A/m. The copper conducting disk has a resistivity of $1.72 \times 10^{-8} \Omega\text{-m}$ (the conductivity, σ is its inverse) at room temperature. Since the conductor is symmetrically placed in the gap between the pole faces of the magnet, the x-component of the magnetic flux density B_x is assumed to be zero at the location of the conductor. The z-component of flux density B_z is evaluated using equation (11) for individual magnets and then superposing the results. The maximum value of flux density B_z at the conducting disk (B_{dz}) is 0.83 Wb/m^2 . The flux lines and the flux density values are shown in figures 6a and b, respectively. The radial distributions of the flux density computed using equation (11), in-house finite element code and an ANSYS simulation are compared in figure 6c. In figure 7 a three-dimensional distribution of B_{dz} over the conducting disk is displayed. A value of 1 mm/s is considered as the relative velocity of the conductor. The Lorentz force generated due to this velocity can be taken as coefficient connecting the force and relative velocity (between the conducting disk and the stack) and expressed in

Figure 6. (a). The magnetic flux lines obtained from an axisymmetric finite element analysis using ANSYS. (b). The magnetic flux density vector $\vec{B}$ computed using ANSYS. (c). Comparison of flux density values B_{dz} at the location of the conducting disk computed using equation (11) and the axisymmetric finite element simulation using code developed in-house and ANSYS.

$N/(\text{mm/s})$. This coefficient is denoted using the symbol α . The right hand side of equation (30) is shown in figure 8. Figure 9 shows the plot of the current density vector expressed by equation (25). The distribution of Lorentz force is plotted in figure 10. The total Lorentz force for a relative velocity of 1 mm/s is computed using relation (35). The value of the coefficient α turns out to be $\alpha = 0.01172 \text{ Ns/mm}$.

The piezoelectric stack can vibrate harmonically with a maximum amplitude of 0.1 mm. Therefore, the force in the right hand side of equation (46) becomes $\alpha \omega U_p \cos \omega t = 0.001172 \omega \cos \omega t N$. If the stack displacement amplitude is kept constant, the amplitude of force for the proposed exciter is proportional to the excitation frequency. For an excitation frequency of 10 Hz = $20 \pi = 62.8 \text{ rad/s}$, the force becomes $0.074 \cos 2\pi(10)t N$.

Figure 7. The distribution of z-component of magnetic flux density at the location of the disk.

The model of the exciter along with a beam (actually finite element model of the beam) is shown in figure 5. This model is now analysed for specific typical numerical values. At the point of application of the exciter, in addition to the excitation force of $0.001172\omega \cos \omega t N$, the exciter also adds a discrete viscous damper of damping coefficient $\alpha = 0.01172 \text{ Ns/mm}$.

The beam is 500 mm long and has a circular cross-section of diameter 15 mm. The conducting copper disk attached to its tip has a mass of 0.175 kg. From a single degree of freedom approximation for the first mode of the beam it is computed that the damping due to the exciter is 12% of the critical damping of the beam. The response of the beam due to the exciter is plotted against frequency in figure 11 up to the second mode. If the mass of the conducting disk and the damping due to the exciter is neglected, the difference in the response is also shown in the same figure. However, since the amount of mass and damping coupling is known from the exciter characteristics, one can make appropriate corrections by performing a modal analysis. One can

Figure 8. The distribution of $\nabla \cdot (\bar{v} \times \bar{B}_d)$ at the location of the disk.

Figure 9. Eddy current distribution over the disk.

Figure 10. Distribution of Lorentz force over the disk.

Figure 11. Amplitude versus frequency plot for the proposed exciter with and without mass and damping couplings.

easily note that the amplitude of the excitation force at frequencies 260 rad/s and 1600 rad/s are approximately 0.3 N and 1.8 N respectively.

For a stiffer system with similar values of natural frequencies as above higher exciter forces may be obtained by increasing the flux density at the conducting disk. In the above case the flux density is $(B_{dz})_{\max} = 0.83 \text{ Wb/m}^2$. For a flux density of $\bar{M} = 2.5 \text{ W/m}^2$ the forces at frequencies 260 rad/s and 1600 rad/s increases by approximately 10 times and will be 3 N and 18 N respectively. The force also increases with decrease in the gap between the pole faces of the magnet.

The excitation force of the proposed exciter does not depend on the amplitude of vibration unless the latter is exceptionally large. Initially, the pole faces of the magnets are placed coaxially with the conducting plate. Due to vibration an offset in the y -direction is developed between the

Figure 12. The flux density B_z (Wb/m^2) for a 4 mm displacement of the structure.

axis of the magnets and the disk centre. This offset is equal to the relative displacement between the magnets and the disk during vibration. Let the maximum offset here be 4 mm (amplitude of vibration) in either direction. At these extreme positions the values of coefficient α are computed. For the dimensions of the pole faces and the disk considered here, the values of the coefficient α are found to be very close. This confirms that the excitation force is almost independent of the amplitude of vibration. The distribution of flux for an offset of 4 mm is shown in figure 12.

4. Qualitative experimentation

The objective of the experiment is just to show that a vibrating magnetic field indeed generates a non-contact excitation force on a structure (beam here) via eddy currents in a conductor fixed to the structure. Piezoelectric stacks and NdFeB permanent magnets are not among the available facilities in the authors' laboratory. In place of permanent magnets an electromagnet is employed in this qualitative experiment. In order to impart a vibratory motion to the magnet, the electromagnet is placed on an electrodynamic shaker. However, it is very difficult to precisely control the displacement of the electrodynamic shaker near its own natural frequency, which is low and may be close to the natural frequency of the structure being excited. Therefore, it is never possible to substitute a piezoelectric stack, which can be very precisely controlled over a wide frequency range.

Figure 13a shows the experimental set-up. An electromagnet is placed on an electrodynamic shaker. A signal generator generates a harmonic signal with a specific frequency. This signal is an input to a power amplifier. The power amplifier supplies sufficient current to the electrodynamic shaker (SYSCON 200 N) so that it vibrates harmonically with same frequency as the signal generated by the signal generator. Since the magnet is placed on the shaker, it also vibrates with the same frequency. There is a 4 mm gap between the pole faces of the magnet. A square copper plate can freely move within the gap of the pole faces of the electromagnet without any contact with the latter. This copper plate is bolted to a cantilevered beam. When the coil of the electromagnet is energized, due to the vibrating field of the electromagnet, eddy current is generated in the copper plate. An ammeter shows the current in the coil of the electromagnet. A switch mode power supply (SMPS) is used as a 12V direct current supply to the electromagnet. A rheostat is connected in series to the coil of the electromagnet. Due to interaction between the magnetic fields of the electromagnet and the eddy current in the copper disk, a non-contact harmonic force will be generated and start acting on the beam. A non-contact inductive displacement sensor (Micro-Epsilon IWS-8-M-CA3-U) is used to measure the displacement of the beam. A voltage proportional to this displacement is then displayed in the oscilloscope screen (TEKTRONIX TDS 1002). Figure 13b shows the vibration of the beam when the current in the coil of the electromagnet is turned on. When the power supply is switched off, the vibration, as shown in the oscilloscope screen of figure 13c, becomes zero.

5. Conclusion

Analysis of a proposed non-contact Lorentz force based vibration exciter is performed in this work. The main advantage of this exciter is that the excitation force does not depend on the amplitude of vibration. The force generated by this exciter is small to moderate. An increase

Figure 13. (a). The experimental set-up. (b). Oscilloscope showing output of displacement sensor when the electromagnet is energized (ammeter in figure showing current through the coil of the magnet). (c). Oscilloscope showing zero output of displacement sensor when the electromagnet is switched off (ammeter in figure showing zero current through the coil of the magnet).

in the magnetization of the magnets and a decrease in the gap between them can sufficiently increase the force. However, the increased force also introduces larger damping in the system. Adjusting the displacement of the stack can precisely control the excitation force. Since the force increases proportionally with higher frequency, this exciter is suitable to be used with light and flexible structures at comparatively higher frequencies.

List of symbols

$\bar{A}$	Magnetic vector potential
$\bar{M}$	Magnetization vector (dipole moment per unit volume)
μ_0	Permeability of free space
V_0	Volume of magnetized body
$\bar{r}, \bar{r}', \bar{r}_0$	Position vectors of a point with respect to origin, of the same point with respect to an infinitesimal volume, of the infinitesimal volume with respect to origin respectively
$\bar{J}_b$	Bound volume current vector
$\bar{K}_b$	Bound surface current vector
ρ_0, θ_0, z_0	Coordinates of a point inside the magnetized body in cylindrical polar coordinate system
$\hat{e}$	Unit vector, the suffix denotes the direction
x, z	Cartesian coordinate directions shown in figure 4a
R	Radius of the cylindrical permanent magnet
$\bar{B}$	Magnetic flux density vector
$\bar{j}$	Current density vector
$\bar{v}$	Relative velocity of the conducting plate with respect to the magnet
$\bar{B}_d$	Magnetic flux density vector at the location of the disk
$\hat{n}$	Unit normal vector to the conducting disk
Θ	$\Theta = r A_\theta$
N_j	Shape functions
$\bar{B}, \bar{H}, \bar{H}_c$	Magnetic flux density, auxiliary field and coercive force respectively
H_{cr}, H_{cz}	Components of coercive force along radial and z directions
$\bar{M}$	Magnetic dipole moment for unit volume
μ_0, μ_r	Permeability of vacuum and relative permeability respectively, $\mu = \mu_0 \mu_r$
Φ	Scalar potential function of the induced electric field, the electrical field is expressed as $\bar{E} = -\nabla \Phi$
$\bar{F}_L$	Lorentz force, F_{Ly} stands for Lorentz force in y -direction
$\dot{u}_p, \dot{u}_d, \dot{u}_m$	Velocities (in the y -direction) of the piezoelectric stack, the conducting disk and the m^{th} degree of freedom of the structure respectively, $\dot{u}_d = \dot{u}_m$
$[M], [K], [C]$	The standard mass, stiffness and damping matrices in finite elements or modal methods
$\{T_C\}$	This vector actually places the force (or velocity) at a specified degree of freedom appropriately in the force vector (or velocity vector)
U_p	Amplitude of displacement of piezoelectric stack $u_p = U_p \sin \omega t$
α	Proportionality constant connecting the force and relative velocity
ω	Frequency in radians/s of displacement of piezoelectric stack
σ	Conductivity of the conducting disk

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