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RBF neural network based $\mathcal{H}_\infty$ synchronization for unknown chaotic systems

CHOON KI AHN

Department of Automative Engineering, Seoul National University of Technology, 172 Gongneung 2-dong, Nowon-gu, Seoul 139–743
e-mail: hironaka@hanmail.net

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Abstract. In this paper, we propose a new $\mathcal{H}_\infty$ synchronization strategy, called a Radial Basis Function Neural Network $\mathcal{H}_\infty$ synchronization (RBFNNHS) strategy, for unknown chaotic systems in the presence of external disturbance. In the proposed framework, a radial basis function neural network (RBFNN) is constructed as an alternative to approximate the unknown nonlinear function of the chaotic system. Based on this neural network and linear matrix inequality (LMI) formulation, the RBFNNHS controller and the learning laws are presented to reduce the effect of disturbance to an $\mathcal{H}_\infty$ norm constraint. It is shown that finding the RBFNNHS controller and the learning laws can be transformed into the LMI problem and solved using the convex optimization method. A numerical example is presented to demonstrate the validity of the proposed RBFNNHS scheme.

Keywords. $\mathcal{H}_\infty$ synchronization; radial basis function neural network (RBFNN); unknown chaotic systems; linear matrix inequality (LMI); learning law.

1. Introduction

Since the discovery of chaos synchronization by Pecora and Carroll (1996), Interest in studying the synchronization of various chaotic systems has been increased significantly (Chen & Dong 1998). The idea of synchronization is to use the output of the drive system to control the response system so that the output of the response system follows the output of the drive system asymptotically. In the literature, various synchronization schemes, such as variable structure control (Wang & Su 2004), OGY method (Ott *et al* 1990), parameters adaptive control (Wang *et al* 2003, Park 2005), observer-based control (Yang & Chen 2002), active control (Bai & Lonngen 1997, Bai & Lonngren 2000), time-delay feedback approach (Park 2005, Ahn 2010), backstepping design technique (Wu & Lu 2003, Hu *et al* 2005), complete synchronization (Zhan *et al* 2003), and so on, have been successfully applied to the chaos synchronization.

In recent years, neural networks have attracted considerable attention as they proved to be essential in applications such as pattern recognition, associative memories, signal processing, fixed-point computations, and so on (Gupta *et al* 2003). Due to the universal approximation ability of neural networks, they have been widely used to approximate unknown systems, and design robust controllers based on the outputs of neural networks. Recently, neural networks have been successfully used in the synchronization controller design for uncertain chaotic systems, and some research results have been presented (Chen *et al* 2006, Chen *et al* 2009, Chen & Chen 2009). In Chen *et al* (2006), a synchronization scheme was studied for uncertain chaotic systems via radial basis function neural network (RBFNN). A synchronization control scheme was presented with a RBFNN disturbance observer for two chaotic systems in Chen *et al* (2009). A RBFNN based chaos synchronization method for a class of time-delayed chaotic systems was proposed in (Chen & Chen 2009).

In real physical systems, one is faced with model uncertainties and a lack of statistical information on the signals. This had led in recent years to an interest in min–max control, with the belief that $\mathcal{H}_\infty$ control is more robust and less sensitive to disturbance variances and model uncertainties (Stoorvogel 1992). In order to reduce the effect of the disturbance, (Hou *et al* 2007) firstly adopted the $\mathcal{H}_\infty$ control concept (Stoorvogel 1992) for chaotic synchronization problem of a class of chaotic systems. Recently, a controller for the $\mathcal{H}_\infty$ anti-synchronization was proposed by Ahn (2009). Despite these advances in $\mathcal{H}_\infty$ synchronization, most research results were restricted to known chaotic systems. Can we obtain a RBFNN based $\mathcal{H}_\infty$ synchronization method for unknown chaotic systems? This paper gives an answer for it. To the best of our knowledge, however, for the RBFNN based $\mathcal{H}_\infty$ synchronization of unknown chaotic systems, there is no result in the literature so far, which still remains open and challenging.

In this paper, a new $\mathcal{H}_\infty$ synchronization method based on RBFNN model for unknown chaotic systems with external disturbance is proposed. This method is called a radial basis function neural network $\mathcal{H}_\infty$ synchronization (RBFNNHS) method. In this scheme, a RBFNN is constructed to precisely approximate the unknown nonlinear function of the chaotic system. Based on this neural network model, the RBFNNHS controller with the learning laws is developed to ensure that the $\mathcal{H}_\infty$ norm from the disturbance to the synchronization error is reduced to a disturbance attenuation level. By virtue of Lyapunov method and linear matrix inequality (LMI) formulation, an existence criterion for the proposed scheme is represented in terms of the LMI. The LMI problem can be solved efficiently by using recently developed convex optimization algorithms (Boyd *et al* 1994).

This paper is organized as follows. In section 2, we formulate the problem. In section 3, an LMI problem for the RBFNNHS of unknown chaotic systems is proposed. In section 4, a numerical example is given, and finally, conclusions are presented in section 5.

2. Problem formulation

Consider the chaotic system in the form of

$$\dot{x}(t) = Ax(t) + f(x(t)), \tag{1}$$

where $x(t) \in R^n$ is the state vector, $A \in R^{n\times n}$ is a known constant matrix, and $f(x(t)) : R^n \to R^n$ is the unknown nonlinear function vector satisfying the local Lipschitz condition. The RBFNN is used to approximate $f(x(t))$ and the synchronization controller is designed based on the output of RBFNN. The approximation of $f(x(t))$ using RBFNN is expressed as

$$\hat{f}(x(t)) = \hat{W}(t)\phi(x(t)), \tag{2}$$

where $\hat{W}(t) \in R^{n\times n}$ is the weight matrix and $\phi(x(t)) = [\phi_1(x(t))\ldots\phi_n(x(t))]^T \in R^n$ is a set of base functions of the corresponding RBFNN. The element functions $\phi_i(x(t))$

$(i = 1, \ldots, n)$ are given by

$$\phi_i(x(t)) = \exp(-\|x(t) - c_i\|^2/\delta_i^2), \tag{3}$$

where c_i and δ_i are the center and width of the neural cell of the i-th hidden layer. The optimal weight value of RBFNN is defined as

$$W^* = \arg \min_{\hat{W}(t)\in S_W} \left\{ \sup_{x(t)\in S_x} \|\hat{f}(x(t)) - f(x(t))\| \right\}, \tag{4}$$

where $S_W \subset R^{n\times n}$ is a compact set of weight matrices and $S_x \subset R^n$ is a compact set of state vectors. Under the optimal weight value, the unknown nonlinear function vector $f(x(t))$ can be written as

$$f(x(t)) = W^*\phi(x(t)) + \varepsilon(x(t)), \tag{5}$$

where $\varepsilon(x(t)) = [\varepsilon_1(x(t)) \ldots \varepsilon_n(x(t))]^T \in R^n$ is the smallest approximation error of the RBFNN. We assume that

$$\|\varepsilon(x(t))\| \leq \eta, \tag{6}$$

where $\eta > 0$ is the unknown upper bound of the approximation error. By using (5), the unknown chaotic system (1) can be represented by

$$\dot{x}(t) = Ax(t) + W^*\phi(x(t)) + \varepsilon(x(t)). \tag{7}$$

The synchronization problem of the system (7) is considered by using the drive-response configuration. The system (7) is considered as a drive system. According to the drive-response concept, the controlled response system is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + u(t) + d(t), \tag{8}$$

where $\hat{x}(t) \in R^n$ is the state vector of the response system (8), $u(t) \in R^n$ is the control input, and $d(t) \in R^n$ is the external disturbance. Define the synchronization error $e(t) = \hat{x}(t) - x(t)$. Then we obtain the synchronization error system

$$\dot{e}(t) = Ae(t) - W^*\phi(x(t)) - \varepsilon(x(t)) + u(t) + d(t). \tag{9}$$

Throughout this paper, $\hat{\eta}(t)$ is defined as the estimate of η.

DEFINITION 1 (RBFNN $\mathcal{H}_\infty$ synchronization).

With zero initial condition and a given level $\gamma > 0$, the error system (9) is RBFNN $\mathcal{H}_\infty$ synchronized if the synchronization error $e(t)$ satisfies

$$\int_0^\infty e^T(t)Se(t)dt < \gamma^2 \int_0^\infty d^T(t)d(t)dt, \tag{10}$$

under the learning law $\hat{W}(t)$ and the controller $u(t)$, where S is a positive symmetric matrix. The parameter γ is called the $\mathcal{H}_\infty$ norm bound or the disturbance attenuation level.

DEFINITION 2 (RBFNN asymptotical synchronization).

The error system (9) is RBFNN asymptotically synchronized if the synchronization error $e(t)$ satisfies

$$\lim_{t\to\infty} e(t) = 0, \tag{11}$$

under the learning law $\hat{W}(t)$ and the controller $u(t)$.

Remark 1. *The $\mathcal{H}_\infty$ norm (Stoorvogel 1992) is defined as*

$$\|T_{ed}\|_\infty = \frac{\sqrt{\int_0^\infty e^T(t)Se(t)dt}}{\sqrt{\int_0^\infty d^T(t)d(t)dt}}$$

where T_{ed} is a transfer function matrix from $d(t)$ to $e(t)$. For a given level $\gamma > 0$, $\|T_{ed}\|_\infty < \gamma$ can be restated in the equivalent form (10).

The purpose of this paper is to design the controller $u(t)$ and the learning law $\hat{W}(t)$ guaranteeing the RBFNN $\mathcal{H}_\infty$ synchronization for unknown chaotic systems. In addition, the controller $u(t)$ and the learning law $\hat{W}(t)$ will be shown to guarantee the RBFNN asymptotical synchronization when the external disturbance $d(t)$ disappears.

3. RBFNN $\mathcal{H}_\infty$ synchronization

This section designs the RBFNN $\mathcal{H}_\infty$ synchronization controller for unknown chaotic systems. The existence condition for the RBFNN $\mathcal{H}_\infty$ synchronization is proposed in the following theorem.

Theorem 1. *For given $\gamma > 0$ and $S = S^T > 0$, assume that there exist $X = X^T > 0$ and Y such that*

$$\begin{bmatrix} AX + Y + (AX + Y)^T & I & X \\ I & -\gamma^2 I & 0 \\ X & 0 & -S^{-1} \end{bmatrix} < 0. \tag{12}$$

Construct the following controller

$$u(t) = YX^{-1}(\hat{x}(t) - x(t)) + \hat{W}(t)\phi(x(t)) + u_f(t), \tag{13}$$

where $u_f(t)$ is designed as

$$u_f(t) = \begin{cases} -\frac{X^{-1}(\hat{x}(t)-x(t))}{\|(\hat{x}(t)-x(t))^T X^{-1}\|}\hat{\eta}(t), & \|(\hat{x}(t) - x(t))^T X^{-1}\| \neq 0, \\ 0, & \|(\hat{x}(t) - x(t))^T X^{-1}\| = 0. \end{cases} \tag{14}$$

If the weight $\hat{W}(t)$ and the parameter estimation value $\hat{\eta}(t)$ of the RBFNN are updated as

$$\dot{\hat{W}}(t) = -\Phi X^{-1}(\hat{x}(t) - x(t))\phi^T(x(t)), \tag{15}$$

$$\dot{\hat{\eta}}(t) = \rho\|(\hat{x}(t) - x(t))^T X^{-1}\|, \tag{16}$$

where Φ is a symmetric positive definite matrix and ρ is a positive constant, then the RBFNNHS with the disturbance attenuation level γ is achieved.

Proof. The closed-loop synchronization error system with the control input $u(t) = Ke(t) + \hat{W}(t)\phi(x(t)) + u_f(t)$ can be written as

$$\dot{e}(t) = (A + K)e(t) + \tilde{W}(t)\phi(x(t)) - \varepsilon(x(t)) + u_f(t) + d(t), \tag{17}$$

where $\tilde{W}(t) = \hat{W}(t) - W^*$. Define $\tilde{\eta}(t) = \hat{\eta}(t) - \eta$ and consider the following Lyapunov function:

$$V(t) = e^T(t)Pe(t) + \text{trace}\,\{\tilde{W}^T(t)\Phi^{-1}\tilde{W}(t)\} + \frac{1}{\rho}\tilde{\eta}^2(t), \tag{18}$$

where trace $\{\cdot\}$ stands for the trace and is defined as the sum of all the diagonal elements of a matrix. The time derivative of $V(t)$ along the trajectory of (17) is

$$\begin{aligned}
\dot{V}(t) &= \dot{e}(t)^T Pe(t) + e^T(t)P\dot{e}(t) + 2\,\text{trace}\,\{\dot{\hat{W}}^T(t)\Phi^{-1}\tilde{W}(t)\} + \frac{2}{\rho}\tilde{\eta}(t)\dot{\hat{\eta}}(t) \\
&= e^T(t)[A^TP + PA + PK + K^TP]e(t) + e^T(t)Pd(t) \\
&\quad + d^T(t)Pe(t) + 2e^T(t)Pu_f(t) + 2e^T(t)P\tilde{W}(t)\phi(x(t)) \\
&\quad + 2\,\text{trace}\,\{\dot{\hat{W}}^T(t)\Phi^{-1}\tilde{W}(t)\} + \frac{2}{\rho}\tilde{\eta}(t)\dot{\hat{\eta}}(t) - 2e^T(t)P\varepsilon(x(t)).
\end{aligned}$$

If we use the inequality $X^TY + Y^TX \le X^T\Lambda X + Y^T\Lambda^{-1}Y$, which is valid for any matrices $X \in R^{n\times m}$, $Y \in R^{n\times m}$, $\Lambda = \Lambda^T > 0$, $\Lambda \in R^{n\times n}$, we have

$$e(t)^TPd(t) + d^T(t)Pe(t) \le \gamma^2 d^T(t)d(t) + \frac{1}{\gamma^2}e(t)^TPPe(t). \tag{19}$$

Using (19), we obtain

$$\begin{aligned}
\dot{V}(t) &\le e^T(t)\left[A^TP + PA + PK + K^TP + \frac{1}{\gamma^2}PP\right]e(t) + \gamma^2 d^T(t)d(t) \\
&\quad + 2e^T(t)Pu_f(t) + 2e^T(t)P\tilde{W}(t)\phi(x(t)) + 2\,\text{trace}\,\{\dot{\hat{W}}^T(t)\Phi^{-1}\tilde{W}(t)\} \\
&\quad + \frac{2}{\rho}\tilde{\eta}(t)\dot{\hat{\eta}}(t) + 2\|e^T(t)P\|\eta \\
&= e^T(t)\left[A^TP + PA + PK + K^TP + \frac{1}{\gamma^2}PP\right]e(t) + \gamma^2 d^T(t)d(t) \\
&\quad + 2e^T(t)Pu_f(t) + 2\,\text{trace}\,\{\phi(x(t))e^T(t)P\tilde{W}(t)\} \\
&\quad + 2\,\text{trace}\,\{\dot{\hat{W}}^T(t)\Phi^{-1}\tilde{W}(t)\} + \frac{2}{\rho}\tilde{\eta}(t)\dot{\hat{\eta}}(t) + 2\|e^T(t)P\|\eta \\
&= e^T(t)\left[A^TP + PA + PK + K^TP + \frac{1}{\gamma^2}PP\right]e(t) + \gamma^2 d^T(t)d(t) \\
&\quad + 2e^T(t)Pu_f(t) + 2\,\text{trace}\,\{[\phi(x(t))e^T(t)P + \dot{\hat{W}}^T(t)\Phi^{-1}]\tilde{W}(t)\} \\
&\quad + \frac{2}{\rho}\tilde{\eta}(t)\dot{\hat{\eta}}(t) + 2\|e^T(t)P\|\eta.
\end{aligned} \tag{20}$$

By considering $\tilde{\eta}(t) = \hat{\eta}(t) - \eta$, with $u_f(t)$ defined by

$$u_f(t) = \begin{cases} -\frac{Pe(t)}{\|e^T(t)P\|}\hat{\eta}(t), & \|e^T(t)P\| \neq 0, \\ 0, & \|e^T(t)P\| = 0, \end{cases} \tag{21}$$

(20) becomes

$$\begin{aligned} \dot{V}(t) &\leq e^T(t)\left[A^T P + PA + PK + K^T P + \frac{1}{\gamma^2}PP\right]e(t) + \gamma^2 d^T(t)d(t) \\ &\quad + 2\,\text{trace}\,\{[\phi(x(t))e^T(t)P + \dot{\hat{W}}^T(t)\Phi^{-1}]\tilde{W}(t)\} \\ &\quad + \frac{2}{\rho}\tilde{\eta}(t)\dot{\hat{\eta}}(t) - 2\|e^T(t)P\|\tilde{\eta}(t) \\ &= e^T(t)\left[A^T P + PA + PK + K^T P + \frac{1}{\gamma^2}PP\right]e(t) + \gamma^2 d^T(t)d(t) \\ &\quad + 2\,\text{trace}\,\{[\phi(x(t))e^T(t)P + \dot{\hat{W}}^T(t)\Phi^{-1}]\tilde{W}(t)\} \\ &\quad + \frac{2}{\rho}\tilde{\eta}(t)[\dot{\hat{\eta}}(t) - \rho\|e^T(t)P\|]. \end{aligned} \tag{22}$$

If we use the following learning laws

$$\dot{\hat{W}}(t) = -\Phi Pe(t)\phi^T(x(t)), \tag{23}$$

$$\dot{\hat{\eta}}(t) = \rho\|e^T(t)P\|, \tag{24}$$

and the following matrix inequality is satisfied

$$A^T P + PA + PK + K^T P + \frac{1}{\gamma^2}PP + S < 0, \tag{25}$$

we have

$$\dot{V}(t) < -e^T(t)Se(t) + \gamma^2 d^T(t)d(t). \tag{26}$$

Integrating both sides of (26) from 0 to ∞ gives

$$V(\infty) - V(0) < -\int_0^\infty e^T(t)Se(t)dt + \gamma^2\int_0^\infty d^T(t)d(t)dt.$$

Since $V(\infty) \geq 0$ and $V(0) = 0$, we have the relation (10). From Schur complement, the matrix inequality (25) is equivalent to

$$\begin{bmatrix} A^T P + PA + PK + K^T P & P & I \\ P & -\gamma^2 I & 0 \\ I & 0 & -S^{-1} \end{bmatrix} < 0. \tag{27}$$

Pre- and post-multiplying (27) by $diag(P^{-1}, I, I)$ and introducing change of variables such as $X = P^{-1}$ and $Y = KP^{-1}$, (27) is equivalently changed into the LMI (12). Then the gain matrix of the control input $u(t)$ is given by $K = YX^{-1}$. Also, the learning laws (23) and (24) are equivalently changed into (15) and (16), respectively. By considering $e(t) = \hat{x}(t) - x(t)$ and $P = X^{-1}$, the signal $u_f(t)$ (21) becomes (14). This completes the proof. □

Remark 2. *Various efficient convex optimization algorithms can be used to check whether the LMI (12) is feasible. In this paper, in order to solve the LMI, we utilize MATLAB LMI Control Toolbox (Gahinet et al 1995), which implements state-of-the-art interior-point algorithms.*

COROLLARY 1.

Without the external disturbance, if we use the control input (13) and the learning laws (15)–(16), the RBFNN asymptotical synchronization is obtained.

Proof. When $d(t) = 0$, we obtain

$$\dot{V}(t) < -e^T(t)Se(t) \leq 0 \tag{28}$$

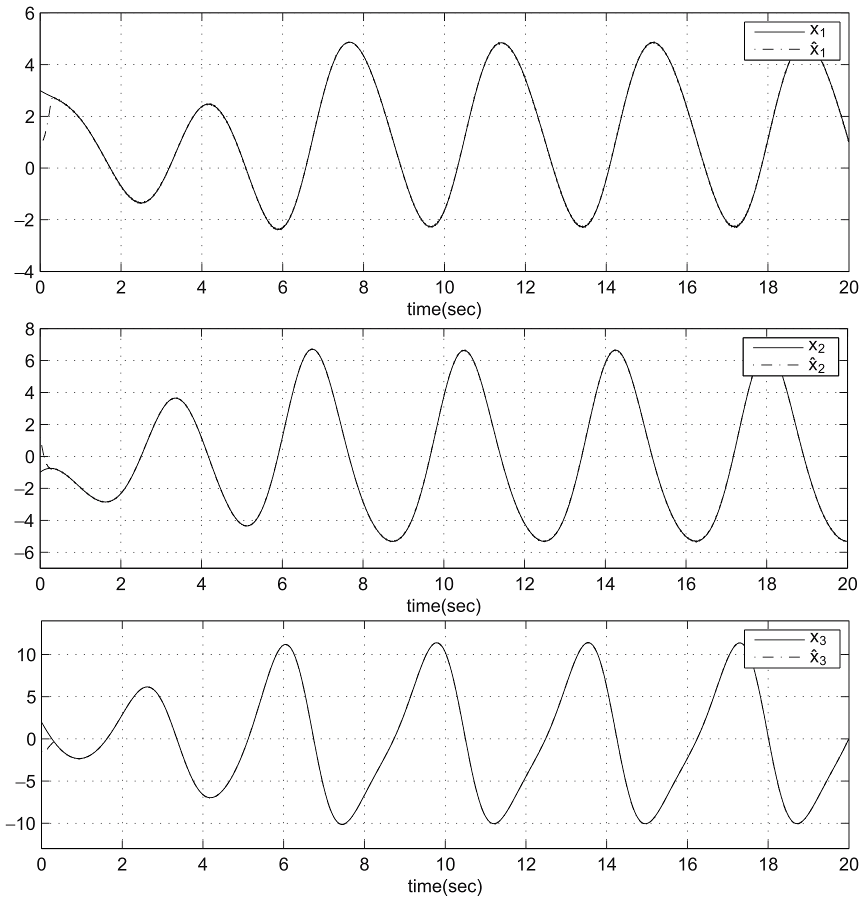

Figure 1. State trajectories ($\gamma = 0{\cdot}4$).

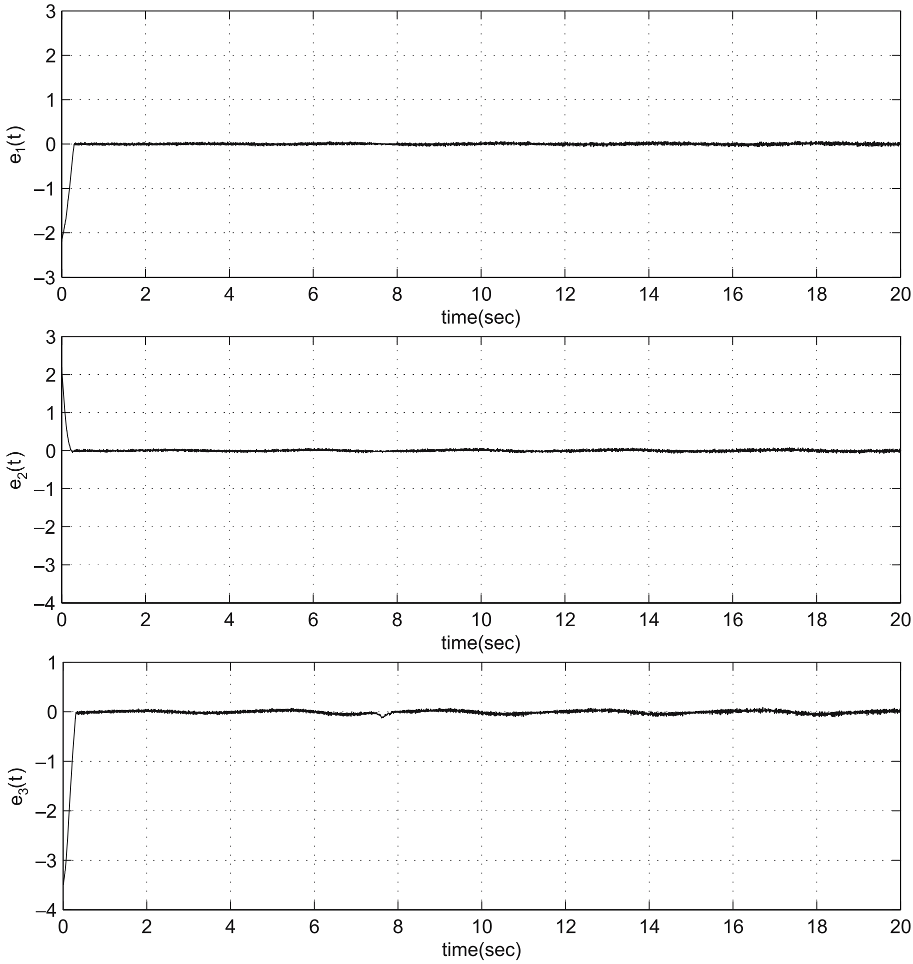

Figure 2. Synchronization errors ($\gamma = 0{\cdot}4$).

from (26). This guarantees

$$\lim_{t\to\infty} e(t) = 0 \tag{29}$$

from Lyapunov theory. This completes the proof. □

4. Numerical example

Consider the following Genesio–Tesi chaotic system (Genesio & Tesi 1992):

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -c & -b & -a \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ x_1^2(t) \end{bmatrix}. \tag{30}$$

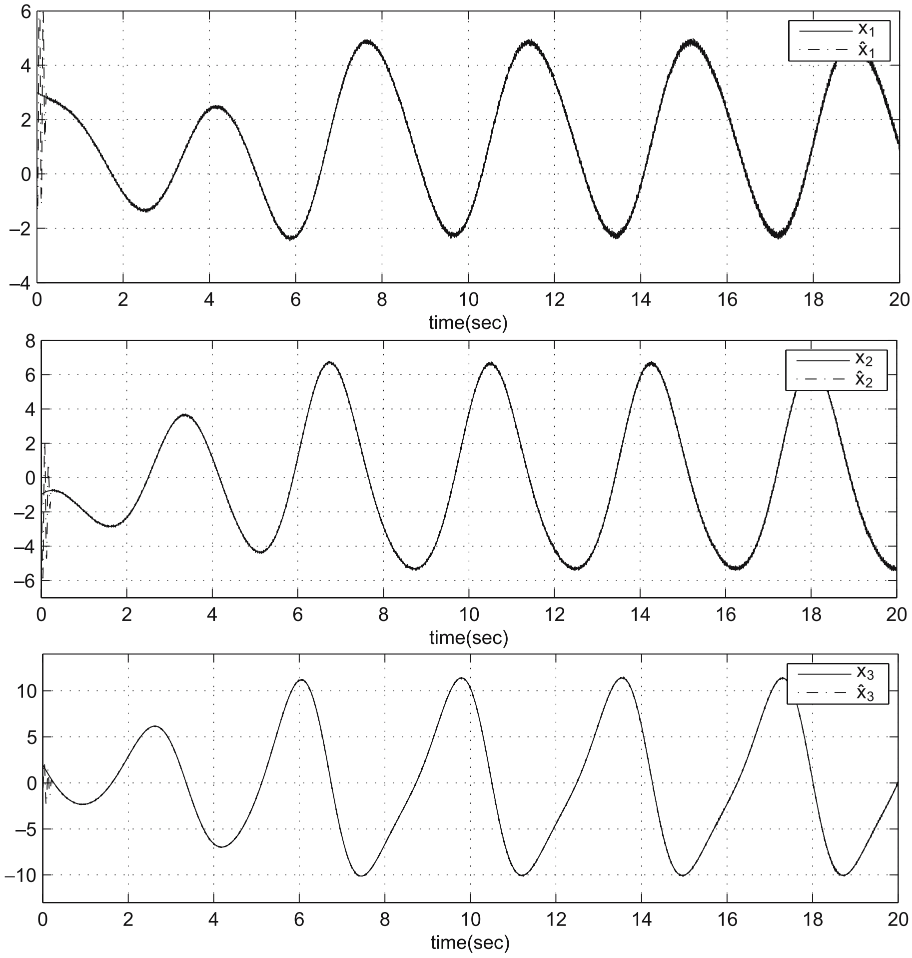

Figure 3. State trajectories ($\gamma = 0{\cdot}88$).

When $a = 1{\cdot}2$, $b = 2{\cdot}92$, and $c = 6$, the Genesio–Tesi chaotic system exhibits a chaotic behaviour. The parameters of base functions are taken as $c_i = 0$ and $\delta_i = 1 (i = 1, 2, 3)$. For the numerical simulation, we use the following parameters:

$$\Phi = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \qquad S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \rho = 50.$$

For the design objective (10), let the $\mathcal{H}_\infty$ performance be specified by $\gamma = 0{\cdot}4$. Solving the LMI (12) by the convex optimization technique of MATLAB software gives

$$X = \begin{bmatrix} 1{\cdot}9200 & 0{\cdot}0000 & 0{\cdot}0000 \\ 0{\cdot}0000 & 1{\cdot}9200 & 0{\cdot}0000 \\ 0{\cdot}0000 & 0{\cdot}0000 & 1{\cdot}9200 \end{bmatrix}, \quad Y = \begin{bmatrix} -7{\cdot}4433 & 5{\cdot}0788 & 6{\cdot}6251 \\ -6{\cdot}9988 & -7{\cdot}4433 & 6{\cdot}2918 \\ 4{\cdot}8949 & -2{\cdot}6054 & -5{\cdot}1393 \end{bmatrix}.$$

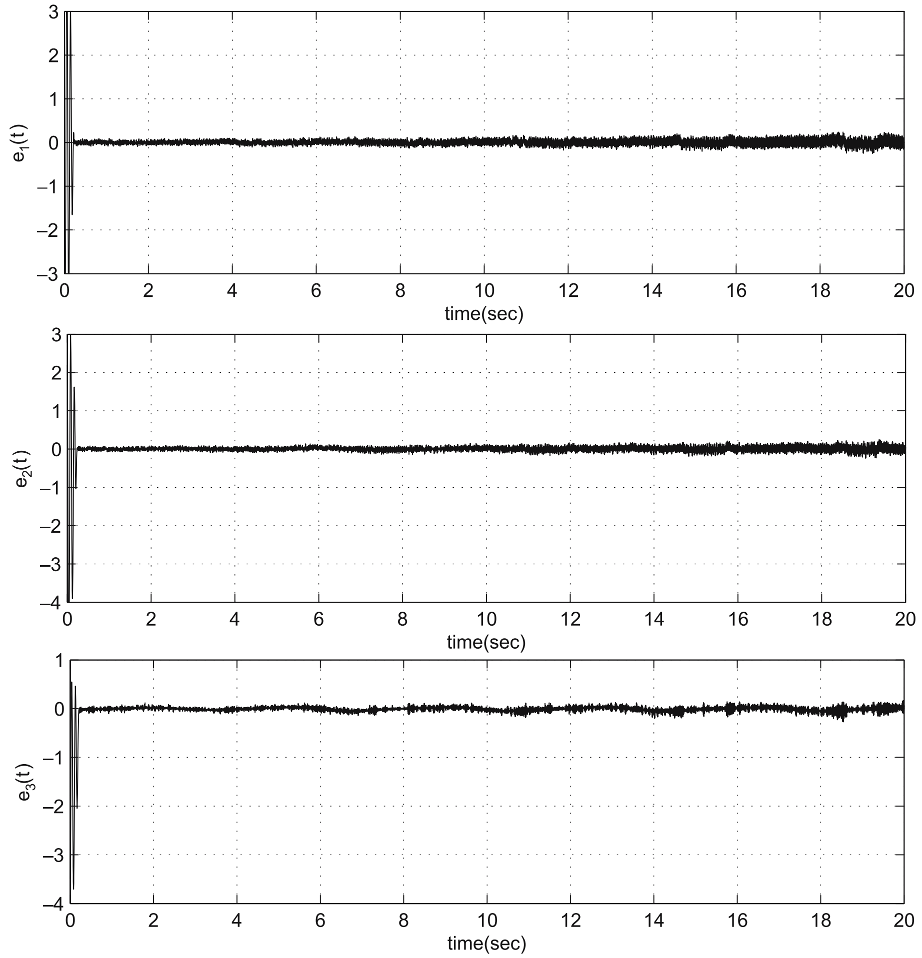

Figure 4. Synchronization errors ($\gamma = 0{\cdot}88$).

Figure 1 shows state trajectories when the initial conditions are given by

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} \hat{x}_1(0) \\ \hat{x}_2(0) \\ \hat{x}_3(0) \end{bmatrix} = \begin{bmatrix} 0{\cdot}8 \\ 1{\cdot}2 \\ -1{\cdot}5 \end{bmatrix},$$

$$\hat{W}(0) = \begin{bmatrix} 0{\cdot}1 & -0{\cdot}25 & 0{\cdot}5 \\ 0{\cdot}25 & -0{\cdot}1 & 0{\cdot}2 \\ 0{\cdot}1 & -0{\cdot}1 & 0{\cdot}2 \end{bmatrix}, \quad \hat{\eta}(0) = 0, \tag{31}$$

and the external disturbance $d_i(t)(i = 1, 2, 3)$ is given by $w(t)$, where $w(t)$ means a Gaussian noise with mean 0 and variance 10. Figure 2 shows, by the proposed RBFNNHS method, that the synchronization error $e(t)$ is bounded around the origin. Next, we increase the disturbance attenuation level γ to 0·88 with the matrix S remained invariant. Solving for the LMI (12) gives

$$X = \begin{bmatrix} 0{\cdot}8388 & 0{\cdot}0000 & 0{\cdot}0000 \\ 0{\cdot}0000 & 0{\cdot}8388 & 0{\cdot}0000 \\ 0{\cdot}0000 & 0{\cdot}0000 & 0{\cdot}8388 \end{bmatrix}, \quad Y = \begin{bmatrix} -1{\cdot}8831 & -52{\cdot}3742 & 28{\cdot}6016 \\ 51{\cdot}5354 & -1{\cdot}8831 & 2{\cdot}7105 \\ -23{\cdot}5686 & -1{\cdot}1000 & -0{\cdot}8765 \end{bmatrix}.$$

State trajectories and error responses for the closed-loop chaotic system with the disturbance attenuation level $\gamma = 0{\cdot}88$ are illustrated in figure 3 and figure 4, respectively. From the simulation results, it can be seen that the resulting disturbance attenuation performance is relatively poor for higher attenuation level.

5. Conclusion

In this paper, we have proposed the RBFNNHS controller, which is a new $\mathcal{H}_\infty$ synchronization controller, for unknown chaotic systems with external disturbance. In the presented design framework, the RBFNN was built and trained to emulate the unknown nonlinear function of the chaotic system. The RBFNNHS controller and the learning laws were derived to achieve the $\mathcal{H}_\infty$ performance, with a prespecified attenuation for the external disturbance. Furthermore, the synchronization for the Genesio–Tesi chaotic system is given to illustrate the effectiveness of the proposed RBFNNHS scheme.

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