Sādhanā Vol. 35, Part 4, August 2010, pp. 427–431. © Indian Academy of Sciences

Modelling traffic congestion using queuing networks

TUSHAR RAHEJA

Mechanical Engineering Department, Indian Institute of Technology Delhi,
Hauz Khas, New Delhi 110 016
e-mail: tushar@raheja.org

MS received 12 October 2009; revised 30 May 2010; accepted 7 June 2010

Abstract. Traffic studies have been carried out predominantly using simulation models which are both time and capital intensive. In this paper, an analytical model of uninterrupted single-lane traffic is proposed using queuing analysis. Well-known Traffic Flow-Density diagrams are obtained using simple Jackson queuing network analysis. Such simple analytical models can be used to capture the effect of non-homogenous traffic.

Keywords. Flow-density curves; uninterrupted traffic; Jackson networks.

1. Introduction

Traffic management has become very essential in our times where the number of vehicles in the metros are near the existing road capacity, or at some places even beyond. India has witnessed this surge only in the past decade or so, and, therefore, attention to traffic management has increased recently. In Delhi and Mumbai, Metro Rail Services have been introduced, and at many places a Bus Rapid Transit system has been initiated. Additionally, many flyovers have come up.

But, however many roads we may build, we will always fall short of the space needed to accommodate the ever increasing traffic. Hence, thrust should be on effective traffic understanding and management.

Traffic flow can be divided into two primary types (van Woensel & Vandaele 2007):

(i) The first type, uninterrupted flow, is defined as all the flows regulated by vehicle–vehicle interactions and interactions between vehicles and the roadway. For example, vehicles travelling on a highway are participating in uninterrupted flows. (ii) Interrupted flow is regulated by an external means, such as a signal.

In this paper, traffic is assumed to be uninterrupted. In the past, simulation models have been used to study traffic in India — mostly car-following ones; and little attention has been paid, world over, to analytical models. More recent additions to the development of microscopic traffic simulation are the Cellular Automaton (CA) or particle hopping models. CA-models describe the traffic system as a lattice of cells of equal size. A CA-model describes in a discrete way the movements of vehicles from cell to cell (Nagel 1996). A similar cell-based approach is used in this paper, though analytically.

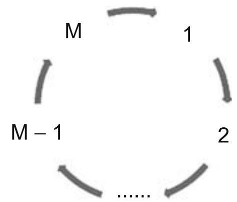

Figure 1. Closed loop with M cells.

Simulation models, apart from their general disadvantages — time and capital intensiveness - also suffer from a big handicap vis-a-vis the Indian scenario: most of these models do not consider non-homogenous traffic. In India, however, where traffic on road is composed of elements as diverse as from a cycle-rickshaw to a truck, these models have limited application.

We propose a step towards understanding non-homogenous traffic through an analytic queuing model. Analytic queuing models have been used in the past to understand traffic, albeit rarely. Jain & Macgregor Smith (1997) presented a macroscopic M/G/C/C state dependent queuing model using inverse linear and exponential velocity–density expressions to calculate the expected waiting times and number of people in system. Vandaele *et al* (2000) formulated very basic models using M/M/1, M/G/1, and G/G/1 frameworks. Their methodology is however mostly limited to single node analysis. They extended to network model in a later paper (Van Woensel & Vandaele 2007).

Our network approach, however, is a fresh one. Well-known Jackson Queuing Network models are used to study traffic on a road segment. Jackson Networks describe the simplest network of queues where service times and inter-arrival times in all queues of the network are exponentially distributed and service discipline is FIFO (Jackson 1963). In the next section, a mathematical model of road traffic is built using Jackson Networks with the help of which we construct the well-known Traffic Flow-Density diagrams. Section 3 concludes the findings and discusses future work.

2. Mathematical model

Our analytical model is defined on a one-dimensional closed system consisting of M cells (figure 1). A closed queuing network model is justified for steady state conditions. In steady state, for a single-entry and single-exit lane, the traffic flow into the system will be equal to the traffic flow out of the system. We approximate this as a closed system where the number of vehicles remains the same. Each cell can either be empty or occupied by one vehicle. To start with, we assume vehicles of identical size. Since the system is closed, the number of vehicles remains constant, say equal to N. Thus the system density can be defined as $N/M = \rho$.

Hereon, this model will be referred to as the Road Cell Network model. A vehicle moves from the first to the second and so on to the Mth cell and then back to the first cell. It is apparent that the system has attributes of a queuing system with first-in-first-out discipline. In the past, the general modelling of traffic using Queuing Theory has been macroscopic, but here instead of treating the whole closed link as a single queue, we consider it as a network of queues.

The Road Cell Network model at first appears to be a cumbersome one as each cell has limited space and, therefore, each queue in the network limited buffer space. But, our task can be made much easier by our definition of the servers.

The exact working of the model is as follows: Being a single lane model, each vehicle moves to the next cell if empty or waits, and then moves when the vehicle ahead vacates the cell.

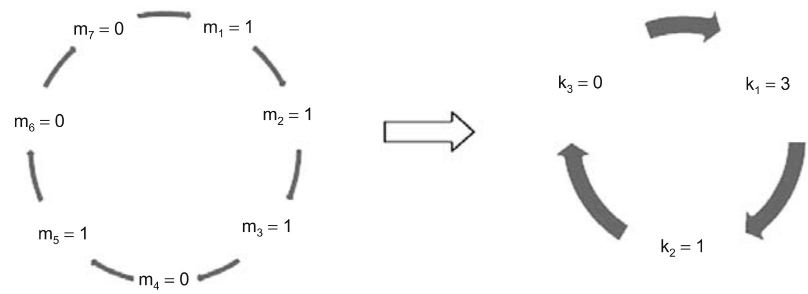

Figure 2. An example of a Road Cell Network and the corresponding Jackson Network. Here, $M = 7, N = 4, K = M - N = 3$.

Thus there can only be two configurations for a particular vehicle: either the cell ahead is empty or occupied. We say a vehicle is in service when the cell ahead is empty and 'waiting' if it is occupied. In effect, each empty cell acts as a server, and at any point in time there are always M–N servers in the system that keep changing their positions. These dynamic cells act as servers to M–N queues in the system that together form a closed network. The number of waiting units in each queue can be counted as the total number of vehicles between the empty cell and the one empty behind it. Thus if there are two consecutive empty cells, both act as servers with one of them having zero queue size.

Service in each queue is assumed to be exponential, and for the basic Road Cell Network model, assuming identical vehicles, the service rate of each vehicle is also taken to be the same. As the service is exponential, from the Poisson-in-Poisson-out property (Chen & Yao 2001), inter-arrival times at each queue are also exponential.

The Road Cell Network, we claim, can be mapped onto a cyclic Jackson network with M–N cells and N customers (figure 2).

Thus, effectively, a road segment with very limited buffer space at each queue is mapped onto a well-known cyclic queuing network with buffer space of size N.

The results of a cyclic Jackson network are well known. A state is indicated by $(k_1, k_2, \ldots, k_K)$ where $\sum k_i = N$ and k_i indicates the number of units at each stage of the closed queuing network. The probability of being in a state $(k_1, k_2, \ldots, k_K)$ is written as $P(k_1, k_2, \ldots, k_K)$. Transitions between states occur when a unit enters or leaves a stage.

Service rate μ_i at each stage is allowed to be dependent upon the number of units i in the stage. Then,

$$P(k_1, k_2, \ldots, k_K) = P(N, 0, 0, \ldots, 0)\frac{{\mu_1}^{N-k_1}}{{\mu_2}^{k_2}{\mu_3}^{k_3}\cdots{\mu_K}^{k_K}},$$

where

$$\sum P(k_1, k_2, \ldots, k_K) = 1.$$

In our case, the service rate at each node is equal, and so the probability of each state is the same, equal to the inverse of the number of ways of selecting N out of $N + K - 1$ 'places' as 'customers', the remaining $K - 1$ 'places' being the 'partitions'. It is calculated to be $P = \frac{N!(K-1)!}{(K+N-1)!}$. Here throughput, q_J of the Jackson Network is calculated to be

$$q_J = \mu P(k_1 > 0) = \mu(1 - P(k_1 = 0)),$$

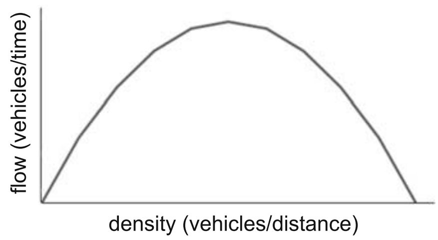

Figure 3. Traffic flow-density diagram.

where $P(n_1 = 0)/P$ is simply the number of ways of selecting N out of the last $N + K - 2$ 'places' as 'customers', the first 'place' being a 'partition'.

By (Koenigsberg 1958)

$$q_J = \mu \frac{N}{N + K - 1}.$$

For the corresponding Road Cell Network, number of queues $K = M - N$. Throughput q_R of the Road Cell Network is obtained by scaling q_J using the relation that the total rate at which service takes place in the two networks is equal, i.e.

$$Mq_R = (M - N)q_J.$$

Scaling, we obtain throughput,

$$q_R = \mu \frac{N}{M - 1} \frac{M - N}{M}.$$

For $N, M \gg 1$

$$q_R = \mu\rho(1 - \rho).$$

This relation is shown in figure 3.

This is the well-known flow-density diagram used widely by transport engineers (Daganzo 1997). Here we have shown a fresh approach to obtain it without the use of any empirical or simulation techniques.

3. Future work

The use of a queuing network model helps in extending the traffic model to incorporate non-homogenous traffic. Over the years simple product form results have been published for many queuing networks. These include multi-class networks like Kelly (1976) and BCMP (Baskett *et al* 1975). There are many more network models which include general service and arrival patterns. Thus, with help of actual road data, various combinations of routing probabilities between nodes can be used to model different vehicle types, and stationary distribution of the road network with non-homogenous traffic obtained.

This paper only gives a flavour of a fresh analytic approach towards modelling congestion. An obvious extension is to model two-lane and multi-lane segments. Also, incorporating more well-known queuing network models and onsite data, a more exact Road Cell Network model can be built. Such models, apart from quick approximations, can also help in quicker simulations using perturbation analysis.

References

Baskett F, Chandy K M, Muntz R R, Palacios F G 1975 Open closed and mixed networks of queues with different classes of customers. *J. ACM* 22: 248–260

Chen H, Yao D D 2001 *Fundamentals of queueing networks* (New York: Springer)

Daganzo C F 1997 *Fundamentals of transportation and traffic operations* (New York: Elsevier Science)

Jackson J R 1963 Jobshop-like queueing systems. *Management Sci.* 10: 131–142

Jain R, J MacGregor Smith 1997 Modelling vehicular traffic flow using M/G/C/C state dependent queueing models. *Transportation Sci.* 31: 324–336

Kelly F P 1976 Networks of queues. *Adv. Appl. Prob.* 8: 416–432

Koenigsberg E 1958 Cyclic queues. *Operational Research Quarterly* 9: 22–35

Nagel K 1996 Particle hopping models and traffic flow theory. *Physical Rev. E* 53: 4655–4672

van Woensel T, Vandaele N 2007 Modelling traffic flows with queueing models: A review. *Asia-Pacific J. Operational Res.* 24(4): 435–461

Vandaele N, van Woensel T, Verbruggen A 2000 A queueing based traffic flow model. *Transportation Res. D* 5(2): 121–135