Identification of resonant earthquake ground motion

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Abstract. Resonant ground motion has been observed in earthquake records measured at several parts of the world. This class of ground motion is characterized by its energy being contained in a narrow frequency band. This paper develops measures to quantify the frequency content of the ground motion using the entropy principle and the dispersion index. These measures are based on the geometric properties of the power spectral density function of the ground acceleration. The application of these measures to quantify the frequency content of random earthquake models is demonstrated first. Subsequently, these measures are used to quantify the frequency content of the ground acceleration for near-field records measured at rock and soil sites, short-duration and long-duration earthquakes.

Keywords. Disorder; dispersion index; entropy rate; frequency content; resonant ground motion, uncertainty.

1. Introduction

Earthquake ground motion is highly uncertain and difficult to be predicted (Geller *et al* 1997). Earthquake uncertainties include time, location, magnitude, intensity, and duration. These uncertainties can be divided into two main categories, referred to as aleatory and epistemic uncertainties (Wen *et al* 2003; Sigbjornsson & Ambraseys 2004; Bommer & Scherbaum 2005). Epistemic uncertainties result from the lack of knowledge to accurately describe the phenomenon. Obtaining new data and refining the modelling can reduce these uncertainties. Aleatory uncertainties are related to the inherent unpredictability of earthquake process which cannot be reduced. Uncertainty quantification of ground motion is necessary in reliability analysis and performance and optimal design of structures (Sasani & Bertero 2000; Wen & Song 2003). The increasing availability of recorded ground motions at many parts of the world and the ease in accessing this data from the Internet or in the form of CD-Rom pave the way for better understanding of earthquake ground motions. Table 1 lists some sources for strong ground motion records (Bozorgnia & Campbell 2004).

A few studies have attempted to identify resonant or critical ground accelerations. For instance, the effective peak velocity of the ground motion has been used to define resonant records (Makris & Black 2004; Amiri & Dana 2005). Takewaki (2001) and Zhai & Xie (2007) identified critical ground motion for nonlinear structures using inelastic time-history analysis. The effect of energy concentration of earthquake accelerations on the nonlinear response

Table 1. List of some sources for strong ground motion records (Bozorgnia & Campbell 2004).

Ground motion data source	Internet website
(1) California Strong Motion Instrumentation Program (SCMIP)	http://www.consrv.ca.gov/cgs/smip
(2) California Integrated Seismic Network (CISN)	http://docinet3.consrv.ca.gov/csmip/cisn-edc/default.htm
(3) U.S. Geological Survey (USGS)	http://nsmrp.wr.usgs.gov
(4) U.S. Geological Survey (USGS)*	http://nsmrp.wr.usgs.gov/cdrom.html
(5) Consortium of Organizations for Strong-Motion Observation system (COSMOS)	http://db.cosmos-eq.org
(6) TriNet and ShakeMap	http://www.trinet.org/ and http://earthquake.usgs.gov/shakemap
(7) University of Southern California (USC)	http://www.usc.edu/dept/civil_eng/Earthquake_eng
(8) US National Geophysical Data Center (NGDC/NOAA)	http://www.ngdc.noaa.gov/seg/hazard/strong.html
(9) Pacific Earthquake Engineering Research (PEER) Center	http://peer.berkeley.edu/smcat/search.html
(10) SAC steel project, strong-motion database	http://nisee.berkeley.edu/data/strong_motion/sacsteel/ground_motions.html
(11) Caltech strong motion database (SMARTS)**	http://www.eerl.caltech.edu/smarts/smarts.html
(12) Pacific Northwest Seismograph Network (PNSN)	http://www.geophys.washington.edu/SEIS/PNSN/SMO
(13) European Strong-Motion Database (ESD)	http://www.isesd.hi.is/ESD_Local/frameset.htm
(14) Japan, Kyoshin Net (K-Net); National Research Institute for Earth Science and Disaster Prevention (NIED)	http://www.k-net.bosai.go.jp
(15) Japan, KiK-Net; National Research Institute for Earth Science and Disaster Prevention (NIED)	http://www.kik.bosai.go.jp/kik/index_en.shtml
(16) Mexico, Guerrero Accelerograph Network	http://www.seismo.unr.edu/ftp/zeng/GUERRERO/guerrero.html
(17) Switzerland, National Strong Motion Network	http://seispc2.ethz.ch/strong_motion/home.jsp
(18) Taiwan, Central Weather Bureau (CWB)***	http://www.cwb.gov.tw/V4e/index.htm
(19) Turkey, General Directorate of Disaster Affairs	http://angora.deprem.gov.tr/indexen.htm

*CD ROM of digitized strong-motion accelerograms of North and Central American Earthquakes, 1933–1986 (Seekins *et al* 1992)

**Diskettes of selected records (<http://www.eerl.caltech.edu/smarts/smarts.html>)

***For 1999 Chi-Chi earthquake, a CD of strong motion records was also produced by Lee *et al* (2001)

of RC structures has been recently studied (Cao & Friswell 2009; Moustafa 2009). This paper develops quantitative measures to quantify the uncertainty or the amount of disorder in earthquake ground motions using the entropy principle and the dispersion index. These measures are based on the geometric properties of the power spectral density function (PSDF) of the ground acceleration, and are used for quantifying the frequency content of the ground motion.

The idea of using the entropy to measure the amount of information in random signals sent along a transmission line was proposed by Shannon (1948). The literature on the use of entropy in engineering is vast (Kapur 1993; Papoulis & Pillai 2002). The entropy principle is often associated with uncertainty or incomplete information. In the context of earthquake engineering, Manohar & Sarkar (1995) recognized the importance of entropy in modelling critical earthquake loads. They derived the critical random earthquake by maximizing the structure's response and the entropy at the same time. This author placed explicit constraints on the entropy of the PSDF of the critical ground motion models (Abbas & Manohar 2002; 2007). This constraint was quantified from past recorded accelerograms and was shown to produce realistic ground motion. Readers can refer Iyengar & Manohar (1987); Iyengar (1989); Manohar & Sarkar 1995; Abbas & Manohar 2002 for more details on the critical excitation method. The next section demonstrates the quantification of uncertainty for stationary random models of ground accelerations.

2. Quantification of uncertainty in probabilistic earthquake models using entropy

Entropy in its basic form is a measure of uncertainty rather than a measure of information. For instance, the entropy of a random variable x is a measure of the uncertainty associated with that random variable. Thus, the uncertainty in a dice experiment is larger than that in a coin experiment. When entropy of a random variable is large, the uncertainty as to the value of the random variable is also large. An example is a normal random variable with large coefficient of variation. On the other hand, the entropy or uncertainty of a deterministic quantity is zero. The same concept applies to random processes. The entropy rate or the amount of disorder of a stationary Gaussian random process $\ddot{u}_g(t)$ is given as (Papoulis & Pillai 2002):

$$H = \ln \sqrt{2\pi e} + \frac{1}{2(\omega_u - \omega_0)} \int_{\omega_0}^{\omega_u} \ln S(\omega) d\omega, \quad (1)$$

where (ω_0, ω_u) , $S(\omega)$ are the frequency range and the PSDF of the ground acceleration, e is a constant ($e = 2.72$) and the term $\ln \sqrt{2\pi e}$ represents a reference level from which entropy is measured. Equation (1) reveals that the entropy rate of a stationary Gaussian process is constant. Thus, when the entropy of $\ddot{u}_g(t)$ is large, the uncertainty as to the value of $\ddot{u}_g(t)$ at discrete time instants is also large. Conversely, the uncertainty of a pulse-like signal of random amplitude and a single frequency (e.g. $\ddot{u}_g(t) = A \sin(0.50\pi t)$) is almost zero. In other words, the energy of resonant or pulse-like time series of a single or a few frequencies is well ordered in the frequency domain, while that of a time signal composed of several frequencies will be disordered. Note that two stationary random processes with the same energy (i.e. same variance) need not have the same entropy. This is because, entropy depends on the distribution of the spectral amplitude of the PSDF at different frequencies (see equation 1). Hence, entropy can reflect the frequency range of the ground acceleration (Abbas & Manohar 2002; 2007).

In this context, it may be emphasized that the time history and the Fourier amplitude spectra of the ground acceleration may reflect the frequency content of the ground motion

qualitatively. The entropy rate and the dispersion index introduced in this paper, as will be shown in the next sections, provide quantitative measures for the frequency content of the ground motion. Furthermore, the time history or the Fourier amplitude spectra of the ground acceleration does not provide an estimate for comparing the frequency content of a set of acceleration records. On the other hand, the proposed measures can be effectively used to rank the records based on their frequency contents. This aspect is of relevance in the selection of earthquake records for time-history analysis of structures (Moustafa *et al* 2010).

To gain more insights into the concept of entropy in quantifying uncertainty of ground motions we derive the entropy rate for probabilistic models of stationary Gaussian ground motions below.

2.1 Band-limited white noise model

The band-limited white noise process has been used as an approximation for earthquake and wind loads. This process has constant spectral amplitude in the specified frequency range. Considering a band-limited stationary random process of intensity s_0 in the frequency range (ω_0, ω_u) , the entropy of this process is given as (see equation 1):

$$\begin{aligned} H &= \ln \sqrt{2\pi e} + \frac{1}{2(\omega_u - \omega_0)} \int_{\omega_0}^{\omega_u} \ln s_0 d\omega \\ &= \ln \sqrt{2\pi e} + \frac{1}{2(\omega_u - \omega_0)} \times (\omega_u - \omega_0) \ln s_0 = \ln \sqrt{2\pi e} + \frac{\ln s_0}{2}. \end{aligned} \quad (2)$$

Therefore, the spectral parameter s_0 defines the entropy or the amount of disorder for the band-limited process.

2.2 Exponential PSD model

Consider a stationary model of earthquake acceleration defined in terms of the auto-correlation function and the associated PSDF as (Lin 1967):

$$R(\tau) = s_0 \exp(-a\tau^2); S(\omega) = \frac{s_0}{2\sqrt{\pi a}} \exp\left(\frac{-\omega^2}{2a}\right), \quad (3)$$

where a is a constant and τ is the time-lag. Accordingly, equation (1) leads to:

$$H = \ln \sqrt{2\pi e} + \frac{1}{2(\omega_u - \omega_0)} \int_{\omega_0}^{\omega_u} \ln \left[\frac{s_0}{2\sqrt{\pi a}} \exp\left(\frac{-\omega^2}{2a}\right) \right] d\omega. \quad (4)$$

Carrying out the integration and performing necessary reductions, we get:

$$H = \ln \sqrt{2\pi e} + \frac{1}{2} \left[\ln \left(\frac{s_0}{2\sqrt{a\pi}} \right) - \frac{1}{6a} (\omega_u^2 + \omega_u\omega_0 + \omega_0^2) \right]. \quad (5)$$

Thus, the parameters $a, s_0, \omega_0, \omega_u$ define the entropy of this random process.

2.3 Kanai-Tajimi model

The Kanai-Tajimi model has been widely used in modelling earthquake ground motions due to its physical meaning and simplicity. The PSDF of the ground acceleration is obtained by

passing a band-limited white noise through a filter that represents the soil layer above the rock level, given as (Kanai 1957; Tajimi 1960):

$$S(\omega) = s_0 \frac{\omega_g^4 + 4\eta_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\eta_g^2 \omega_g^2 \omega^2}, \quad (6)$$

where, s_0 , η_g , ω_g are the intensity of the PSDF at the rock level and the damping and natural frequency of the soil layer, respectively. The entropy rate of the Kanai–Tajimi model can be estimated numerically instead of carrying out the integration of equation (1) analytically since it leads to a lengthy expression.

This section demonstrated the quantification of the entropy rate as a measure of uncertainty of probabilistic stationary ground motion models. It is shown that the entropy rate of a stationary random process is constant. Note that the entropy of a non-stationary random process is a time-dependent function. The next subsection explains the quantification of relative entropy of two random processes which is used later in identifying resonant accelerograms.

2.4 Relative entropy rate

To compare uncertainty or amount of disorder of different earthquake acceleration models, we measure the entropy rate of the stationary process $\ddot{u}_g(t)$ with reference to a band-limited white noise process $\ddot{z}_g(t)$ of spectral intensity s_0 . This is known as the relative entropy rate of the two random processes $\ddot{u}_g(t)$ and $\ddot{z}_g(t)$. Thus, assuming that $\ddot{u}_g(t)$ is independent of $\ddot{z}_g(t)$, the increase in the entropy rate when $\ddot{u}_g(t)$ is added to $\ddot{z}_g(t)$ is given as (Abbas & Manohar 2002; 2007):

$$\Delta H = \frac{1}{2(\omega_u - \omega_0)} \int_{\omega_0}^{\omega_u} \ln \left[1 + \frac{S(\omega)}{s_0} \right] d\omega. \quad (7)$$

Hence, equation (7) can be used to estimate the entropy rate ΔH of the random process $\ddot{u}_g(t)$ measured from a band-limited random process $\ddot{z}_g(t)$ as a reference level.

We now calculate the entropy rate for the narrow-band, the Kanai–Tajimi and the band-limited models, described in the previous sub-sections, from a reference band-limited random process of intensity $0.02 \text{ m}^2/\text{s}^3$. The PSDF for each of these three models is normalized such that it possesses unit area (see figure 1). This normalization implies equality of the energy

Figure 1. (a) Power spectral density function for alternative earthquake models; (b) Kanai–Tajimi power spectral density function for different soil types.

Table 2. Entropy and dispersion indices for alternative earthquake models and different soil site conditions.

Earthquake model	Soil type					
	Soft soil		Medium soil		Stiff soil	
	Entropy	Dispersion	Entropy	Dispersion	Entropy	Dispersion
Narrow-band*	0.0319	0.0785	0.0319	0.0785	0.0319	0.0785
Kanai–Tajimi**	0.2600	4.0869	0.5353	14.7227	0.7199	26.2513
Band-limited	0.8673	45.2997	0.8673	45.2997	0.8673	45.2997
1940 Elecnro	–	–	0.5112	8.6923	–	–

* $\omega_g = \pi, 3\pi, 6\pi$ for soft, medium and stiff soil, respectively

** $\omega_g = \pi, 3\pi, 6\pi$ and $\eta_g = \pi, 3\pi, 6\pi$ for soft, medium and stiff soil, respectively

from the three models (Arias 1970). The parameters of the Kanai–Tajimi model ω_g, η_g are taken as π rad/s, 0.20 for soft soil, 3π rad/s, 0.40 for medium soil and 6π rad/s, 0.60 for stiff soil, see table 2 (Der Kiureghian 1996). Additionally, the spectral intensity at the rock level is taken as $s_0 = 0.02 \text{ m}^2/\text{s}^3$ and the central frequency of the narrow-band process is taken as $\omega_g = \pi, 3\pi, 6\pi$ rad/s for soft, medium and stiff soil, respectively. The numerical results are shown in table 2. Based on a careful investigation of these results, the following observations are made:

- The narrow-band signal possesses the smallest entropy rate, and, thus the associated amount of disorder is the lowest among the three models. In other words, the PSDF of this model is well ordered in the frequency range (ω_0, ω_u) since the energy of the signal is concentrated at a single frequency. This process represents a pulse-like or resonant signal (Moustafa 2010). Note also that the central frequency of the narrow-band signal has no effect on the value of the entropy rate (see table 2). Thus, the entropy rate of narrow-band processes with equal intensity is invariant regardless of the resonant frequency.
- The band-limited signal possesses the highest entropy rate among the three models. Thus, the associated amount of disorder is large. This is because the energy of the process is well distributed across the frequency range (ω_0, ω_u). This scenario represents a random process composed of several harmonic signals with different frequencies (e.g. $\ddot{u}_g(t) = \sum_{i=1}^N A_i \cos(\omega_i t + \phi_i)$). Thus, $\ddot{u}_g(t)$ possesses a large amount of disorder since the energy is well represented at all frequencies.
- The Kanai–Tajimi acceleration is significantly disordered. This result is consistent since the PSDF is reasonably distributed across a significant frequency range (see figure 1a). Additionally, the entropy for the case of soft soil is small while that for stiff soil is large. This is expected since the PSDF for soft soil is narrow-banded compared to that for stiff soil that is distributed across a wider frequency range (see figure 1b).
- The entropy rate for the Kanai–Tajimi model is bounded between the entropy rate of the narrow-band white noise and that for the band-limited noise for all soil types (see table 2). This result is important since it provides lower and upper bounds on the amount of disorder for the Kanai–Tajimi acceleration model. These bounds can be used in validating the mathematical models of earthquake ground motions.

In general, real earthquake accelerations have energies distributed across a significant wide frequency range. This is because the earthquake energy released at the source gets amplified and filtered due to path and local soil effects. Such accelerograms possess significant amount of disorder or uncertainty. However, some real accelerograms show the resonance nature, and thus the associated entropy rate is small. An example of this scenario is near-fault earthquakes with directivity effects (Housner & Hudson 1958; Anderson & Bertero 1987; Sasani and Bertero 2000; Kalkan & Kunnath 2008, Moustafa 2010). This is because, the ground acceleration measured near the source is influenced, mainly, by the source mechanism.

This section explained the quantification of the uncertainty or the amount of disorder for probabilistic earthquake acceleration models. It is shown that the narrow-band white noise and the band-limited white noise processes provide lower and upper bounds on the entropy rate of the Kanai–Tajimi acceleration model. The entropy rate of recorded accelerations is estimated below.

2.5 Quantification of uncertainty in recorded accelerograms using entropy principle

This section demonstrates the quantification of the entropy index of recorded ground motions. Consider a recorded earthquake acceleration $\ddot{x}_g(t)$ that is represented as:

$$\ddot{x}_g(t) = e(t)\ddot{u}_g(t) = A_0[\exp(-\alpha_1 t) - \exp(-\alpha_2 t)]\ddot{u}_g(t). \quad (8)$$

Here $\ddot{u}_g(t)$ represents a steady-state function and $e(t)$ is an envelope function that controls the non-stationarity of $\ddot{x}_g(t)$. The envelope parameters A_0, α_1, α_2 can be estimated by matching the transient trend of the earthquake acceleration. Subsequently, the stationary part of the ground acceleration $\ddot{u}_g(t)$ is obtained by dividing $\ddot{x}_g(t)$ by the estimated envelope. The PSDF of $\ddot{u}_g(t)$ is then determined. This is followed by the estimation of the entropy rate using equation (7).

Figure 2 shows the PSDF for four ground accelerations. The first acceleration represents a sample narrow-band signal $\ddot{x}_g(t) = e(t)A \cos \omega_g t$, with random amplitude A and central frequency $\omega_g = 3\pi$. The second acceleration represents a sample band-limited model $\ddot{x}_g(t) = e(t) \sum_{i=1}^N A_i \cos(\omega_i t + \phi_i)$. The third signal is a simulated acceleration from the Kanai–Tajimi model of equation (6) ($\omega_g = 3\pi$ rad/s; $\eta_g = 0.40$). The fourth acceleration represents the popular 1940 El Centro NS accelerogram that is usually adopted by engineers for time history analysis of structures (PEER 2005). The stationary components $\ddot{u}_g(t)$ for the first three models are modulated by the envelope function $e(t)$ that matches the transient trend of El Centro record. All accelerations are normalized to unit intensity $\int_0^\infty [\ddot{x}_g(t)]^2 dt = 1.0$ (Arias 1970).

The relative entropy of each of these accelerations, from a band-limited acceleration of spectral intensity $0.02 \text{ m}^2/\text{s}^3$, is determined. The numerical values are found to be 0.03 for the narrow-band acceleration, 0.87 for the band-limited acceleration, 0.54 for the simulated Kanai–Tajimi acceleration and 0.51 for the El Centro record (see table 2). These results confirm that the amount of disorder from the Kanai–Tajimi model is bounded between the entropy of the narrow-band acceleration and the entropy of the band-limited acceleration. Interestingly, the amount of disorder of the actual ground acceleration is also bounded by the narrow-band and the band-limited signals. Note that the narrow-band signal represents resonant or pulse-like acceleration that is poor in frequency content. The band-limited signal, on the other hand, represents an acceleration that is rich in the frequency content. The next section explains the quantification of the dispersion index of recorded ground accelerations.

Figure 2. Power spectral density function for ground acceleration (a) narrow-band, (b) band-limited, (c) Kanai–Tajimi, (d) 1940 El Centro NS.

3. Quantification of uncertainty in ground motion using dispersion index

Consider a zero mean stationary Gaussian random process $\ddot{u}_g(t)$ defined in terms of the one-sided PSDF $S(\omega)$ (see figure 3). The i th frequency is defined as:

$$\omega_i = \left(\frac{\int_0^\infty S(\omega) d\omega}{\int_0^\infty \omega^i S(\omega) d\omega} \right)^i; i = 1, 2, \dots, N. \quad (9)$$

Here $\omega_1 = \omega_c$ is the central or dominant frequency of the ground acceleration that reflects the influence of the soil condition. ω_2 indicates where the spectral mass of the PSDF is located along the frequency range. The frequency ω_s can be defined as the radius of gyration of $S(\omega)$ about the frequency origin ω_c , given by (Vanmarcke 1972; 1976):

$$\omega_s = \sqrt{\omega_2^2 - \omega_1^2}. \quad (10)$$

The radius of gyration ω_s is a measure of dispersion of the PSDF about the central frequency (Vanmarcke 1972; 1976). The application of equation (10) to earthquake records requires the determination of the PSDF of the stationary acceleration which has been demonstrated in the previous section. We estimate the dispersion index for the four acceleration models discussed in the previous section (table 2 and figure 2). The numerical results listed in table 2 reveal that the dispersion index for the narrow-band and the band-limited signals provide lower and upper bounds on the dispersion index of the Kanai–Tajimi model and the El Centro record. These results agree with the results obtained on the entropy rate in the previous section. The next section provides practical applications of these measures in identifying resonant accelerograms.

Figure 3. Power spectral density function for the stationary acceleration $\ddot{u}_g(t)$.

4. Applications and numerical results

To examine the applicability of using the entropy rate and the dispersion index in identifying resonant accelerograms we consider the four earthquake groups shown in table 3. These records include near-fault accelerograms (site-source distance less than about 40 km) measured at rock site (shear velocity > 750 m/s), near-fault accelerograms measured at soil site (shear velocity < 750 m/s), short-duration and long-duration earthquakes (PEER 2005). Note that the short- and the long-duration records have different soil conditions. Note also that the classification of these earthquake records has been carried out by Abrahamson *et al* (1998). The selection of records for different soil conditions and for different earthquake durations is meant to examine the robustness of the proposed measures to different earthquake characteristics. Ground accelerations of a minimum of 0.05 g peak ground acceleration are only considered. The two horizontal accelerations are considered in the numerical analyses. Table 3 provides information on magnitude, epicentral distance, PGA, Arias intensity, total duration and recording station for each of these records. Further details on these records can be found in Abrahamson *et al* (1998). In the numerical calculations, all accelerations are scaled to the same Arias intensity (Arias 1970).

The numerical results on entropy rate and dispersion index for each of these records are provided in table 3. Figures 4–7 show the PSDF for the stationary accelerations for the first four earthquakes from each group. The results reveal clearly that the entropy and the dispersion indices correlate well with each other and that they both successfully identify resonant records. It is seen that the narrow-band records possess the smallest entropy and dispersion indices (e.g. 1995 Kobe at OSAJ and 1994 Northridge at Sylmar). The ground accelerations that are rich in frequency content (e.g. 1992 Landers and 1986 N. Palm spring earthquakes) possess the largest entropy and dispersion. It is also evident that the two acceleration components of the same earthquake have different entropy and dispersion indices. This observation is consistent with the findings of Anderson & Bertero (1987). This can be seen in the PSDF for the 1992 Landers earthquake which has a larger epicentral distance compared to other near-fault records measured at rock site (see figure 4). The short-duration earthquakes are seen to possess higher entropy and dispersion indices compared to the long-duration earthquakes. It is also remarkable that near-fault earthquakes measured at soil and rock sites have similar entropy and dispersion indices (see figures 4 and 5). In this context, it may be recalled that very close to the fault, the seismic source dominates the ground motion rather than site condition.

Table 3. Information on earthquake records and entropy and dispersion indices for different soil conditions (PEER 2005).

Group type	Earthquake (station)	M_w	Ep. distance (km)	PGA (g)	I_A^* (m ² /s ³)	Total duration (s)	Entropy index**	Dispersion index *** (rad/s)
Near-fault (rock)	1992 Landers (Lucerne)	7.2	44.02	0.79/0.72	41.10/43.46	48.13	0.68/0.71	27.90/34.85
	1992 Cape Mendocino (CM)	7.0	10.36	1.50/1.04	37.19/14.91	30.00	0.63/0.58	21.34/15.93
	1995 Kobe (JMA)	6.9	18.27	0.82/0.60	52.40/33.94	48.00	0.46/0.46	7.73/8.30
	1989 Loma Prieta (LG pres. center)	6.9	18.46	0.61/0.56	49.12/19.28	24.97	0.48/0.56	17.79/22.29
Near-fault (soil)	1978 Tabas (Tabas)	7.4	55.24	0.84/0.85	46.03/24.74	32.84	0.67/0.60	27.64/17.30
	1992 Cape Mendocino (Petroliia)	7.0	4.51	0.59/0.66	21.33/23.86	36.00	0.58/0.53	17.51/14.82
	1994 Northridge (Rinaldi)	6.7	10.91	0.84/0.85	72.09/72.01	14.95	0.59/0.64	23.01/28.15
	1994 Northridge (Sylmar converter)	6.7	13.11	0.61/0.90	36.42/32.99	40.00	0.48/0.41	9.88/9.71
	1979 Imperial Valley (Meloland)	6.5	19.44	0.31/0.30	5.36/6.74	39.99	0.33/0.37	5.27/9.23
Short duration	1983 Caolinga (Anticline Ridge)	5.0	3.38	0.67/0.58	4.42/3.90	9.84	0.55/0.54	18.09/24.87
	1970 Lytle Creek (Devils canyon)	5.4	20.43	0.15/0.15	0.65/0.57	13.61	0.40/0.46	20.33/24.56
	1986 N. Palm Spring (SR Mountain)	6.0	48.98	0.10/0.10	0.29/0.39	10.40	0.79/0.82	70.27/70.42
	1975 Northern Calif. (CM Petroliia)	5.2	60.38	0.18/0.12	0.74/0.55	14.61	0.58/0.53	15.26/12.53
	1971 San Fernando (Gormon-Oso)	6.6	49.83	0.11/0.08	0.37/0.28	9.23	0.64/0.70	17.68/13.76
	1984 Morgan Hill (Lick Observatory)	6.2	46.90	0.05/0.08	0.15/0.50	16.00	0.58/0.53	15.93/17.05
Long duration	1992 Big Bear (SB-E & hospitality)	6.4	45.51	0.10/0.10	1.70/1.83	100.00	0.45/0.53	11.26/14.12
	1995 Kobe (OSAJ)	6.9	100.15	0.08/0.06	1.44/1.22	120.00	0.19/0.17	1.89/1.40
	1999 Kocaeli (Ambarli)	7.4	112.26	0.25/0.19	6.28/7.74	150.41	0.37/0.37	5.68/5.10
	1999 Chichi (CHY034)	7.6	46.13	0.31/0.25	11.35/9.12	250.00	0.38/0.35	4.38/4.16

* $I_A = \int_0^\infty [\ddot{x}_g(t)]^2 dt$ (Arias intensity)
** Entropy index calculated from equation (7)
*** Dispersion index calculated from equation (11)

Figure 4. Power spectral density function: Near-fault (rock site).

In other words, the frequency content and duration of the earthquake records depend on the site-source distance and the source properties.

It may be noted that a limited number of records has been studied in this paper. However, for this limited number of records, the entropy rate and the dispersion index successfully identify the resonant records (i.e. the records with narrow frequency content), such as, the 1979 Imperial Valley and the 1995 Kobe earthquakes. Accordingly, these measures provide useful quantitative measures of the frequency content of the ground motion.

Figure 5. Power spectral density function: Near-fault (soil site).

To examine the effect of the frequency bandwidth of the ground acceleration on the structural damage, we consider the response analysis of three SDOF building structures of fundamental natural frequencies of 0.13, 0.21, and 0.41 Hz. The yield displacement and strength are taken as $u_y = 0.10$ m, $f_y = 10^4$ N for the three structures and a viscous damping of 0.05 damping ratio is adopted. The material nonlinearity is modelled using elastic–plastic force–deformation relation. Nonlinear dynamic analysis is carried out for each structure subjected to a single acceleration. All records are normalized to the same Arias intensity of $6.00 \text{ m}^2/\text{s}^3$. The Park and Ang damage index for each of these structures driven by the ground acceleration

Figure 6. Power spectral density function: Short-duration records.

is calculated as follows (Park & Ang 1985, Park *et al* 1985):

$$DI_{PA} = \frac{u_{\max}}{u_u} + \beta \frac{E_H}{f_y u_u} = \frac{\mu_{\max}}{\mu_u} + \beta \frac{E_H}{f_y \mu_u}. \quad (11)$$

Here, $u_{\max}$, E_H are the maximum absolute displacement and dissipated hysteretic energy (excluding elastic energy) under the earthquake (Park & Ang 1985; Park *et al* 1985, 1987; Abbas 2006). μ_u is the ultimate yield ductility capacity under monotonic loading and β is a

Figure 7. Power spectral density function: Long-duration records.

positive constant that weights the effect of cyclic loading on structural damage (taken as 6 and 0.15, respectively). The numerical results on DI_{PA} are provided in table 4. It is evident from these numerical results that the Cape Mendocino record, which has lower entropy and dispersion measures, produces more damage than the Landers acceleration. In fact, the three structures are damaged beyond repair ($DI_{PA} > 0.40$) by the first acceleration while they sustain the second acceleration with repairable damage ($DI_{PA} < 0.40$).

Table 4. Ductility factor and damage index for SDOF in elastic structures subjected to ground acceleration.

Earthquake (station)	Structure fundamental frequency					
	$f = 0.13 \text{ Hz}$		$f = 0.21 \text{ Hz}$		$f = 0.41 \text{ Hz}$	
	Ductility factor	Damage index	Ductility factor	Damage index	Ductility factor	Damage index
1992 Cape Mendocino (CM)	4.39	0.74	7.09	1.05	1.41	0.19
1992 Landers (Lurcene)	0.93	0.12	0.60	0.08	0.43	0.05

5. Concluding remarks

A measure of the uncertainty or the amount of disorder in earthquake ground motions using the entropy principle is developed in this paper. The uncertainty quantification of the ground motions is important in reliability analysis and performance and optimal design of structures. The entropy rate provides a useful measure of the frequency content of the ground acceleration. Resonant ground motions are shown to possess small entropy while ground motions that are rich in frequency content possess high entropy. An additional measure of the dispersion in the power spectral density function developed by Vanmarcke (1972; 1976) is also proposed for characterizing the frequency bandwidth of the ground motion. Bounds on the dispersion and entropy indices of probabilistic and deterministic earthquake accelerations are also estimated. These measures are used for identifying resonant acceleration records. For the limited number of earthquake records considered, the numerical results reveal that the entropy rate and the dispersion indices successfully identify resonant accelerograms.

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