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An approximate method for lateral stability analysis of wall-frame buildings including shear deformations of walls

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Abstract. This study presents an approximate method based on the continuum approach and transfer matrix method for lateral stability analysis of buildings. In this method, the whole structure is idealized as an equivalent sandwich beam which includes all deformations. The effect of shear deformations of walls has been taken into consideration and incorporated in the formulation of the governing equations. Initially the stability differential equation of this equivalent sandwich beam is presented, and then shape functions for each storey is obtained by the solution of the differential equations. By using boundary conditions and stability storey transfer matrices obtained by shape functions, system buckling load can be calculated. To verify the presented method, four numerical examples have been solved. The results of the samples demonstrate the agreement between the presented method and the other methods given in the literature.

Keywords. Stability; transfer matrix; continuum model; shear deformation.

1. Introduction

The stability analysis of a building can and should be assessed by looking at its individual elements as well as examining its stability as a whole (Zalka 2002). A number of methods including finite element method have been developed for stability analysis of the buildings. In the literature there are numerous studies (Rutenberg *et al* 1988; Syngellakis & Kameshki 1994; Aristizabal-Ochoa 1997; Aristizabal-Ochoa 2002; Hoenderkamp 2002; Zalka 2002; Aristizabal-Ochoa 2003; Potzta & Kollar 2003; Zalka 2003; Girgin *et al* 2006; Kaveh & Salimbahrami 2006; Mageirou & Gantes 2006; Tong & Ji 2007; Gomes *et al* 2007; Girgin & Ozmen 2007; Xu & Wang 2007) concerning the stability analysis.

Rutenberg (1988) proposed a simple approximate lower bound formula for the gravity buckling loads of coupled shear-wall structures using continuous medium assumption. Hoenderkamp (2002) on the other hand, presented a simplified hand method for the calculation of the overall critical load of planar lateral resisting structures commonly used to provide stability in tall buildings. Zalka (2002) derived simplified analytical expressions for

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the stability of wall-frame buildings. It has been assumed that the structures are regular (i.e. their characteristics do not vary over the height). Aristazabal Ochoa's papers (Aristazabal Ochoa 1997, 2002, 2003), storey-buckling approach was used for the stability of the unbraced frame. Also, Potzta & Kollar (2003) developed a hand method for stability and dynamic analysis of regular buildings. In their paper, the stiffened building structure was replaced by a sandwich beam. Additionally, Girgin & Ozmen (2007) proposed a simplified procedure for determining buckling loads of three-dimensional framed structures.

In this study, an approximate method based on continuum system model and transfer matrix approach has been suggested for the lateral stability analysis of the buildings. The effect of shear deformations of walls has been taken into consideration and incorporated in the formulation of the governing equations. The following assumptions are made in this study; the behaviour of the material is linear elastic, the floor slabs of the building have great in-plane and small out-of-plane stiffness, the vertical load acts on storey level and the critical loads of the structures define the bifurcation point.

2. Analyses

2.1 *Transfer matrix method*

In various engineering problems, as the number of constants to be determined by the use of boundary conditions increases, the computations become more tedious and the possibility of making errors also increases. For this reason, ways of reducing the number of constants to a minimum are sought. The transfer matrix method makes this possible. The main principle of this methodology, which is applied to problems with one variable, is to convert boundary value problems into problems of initials values. Thus, new constants that may result from the use of intermediate condition are eliminated. Therefore, it is a method of expressing the equations in terms of the initial constants and it makes no distinction between the so called determinate and indeterminate problems of elastomechanics (Inan 1968). Transfer matrix method is an efficient and easily computerized method which also provides a fast and practical solution since the dimension of the matrix for elements and system never changes (Pestel & Leckie 1963).

2.2 *Physical model*

High rise buildings demonstrate neither Timoshenko beam behaviour nor Euler-Bernouilli beam behaviour under horizontal loads (Potzta & Kollar 2003). The behaviour of the high rise buildings may be presented by the sandwich beam which consists of two Timoshenko cantilever beams (A and B) and demonstrates both of the mentioned behaviours (figure 1).

The flexural rigidity (EI) of beam A is the sum of the flexural rigidities of shear walls and columns. The shear rigidity (GA_w) of beam A is the sum of the shear rigidities of walls. Meanwhile, the shear rigidity $(GA)_f$ of the beam B is equal to the sum of shear rigidities of frames and sum of shear rigidities of the connecting beams. The global flexural rigidity (D) of beam B can be calculated with the help of axial deformation of shear walls and columns.

2.3 *Stability transfer matrices*

Stability equations for high rise buildings under the horizontal loads are shown in the equations (1)–(3), (Lee *et al* 2008). Equation (1) presents the distributed load for i-th storey.

$$\frac{d}{dz_i}\left[(GA)_{wi}\left(\frac{dy_i}{dz_i}-\psi_{wi}\right)\right]+\frac{d}{dz_i}\left[(GA)_{fi}\left(\frac{dy_i}{dz_i}-\psi_{fi}\right)\right]-N_i\frac{d^2y_i}{dz_i^2}=0. \tag{1}$$

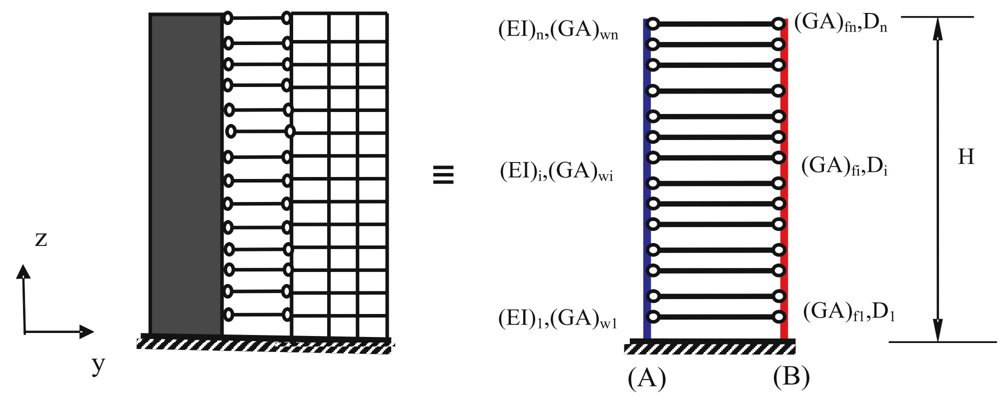

Figure 1. Physical model of equivalent sandwich beam.

Equations (2) and (3) present the shear force equilibrium for beam A and beam B, respectively, which are shown in figure 1.

$$\frac{d}{dz_i}\left[(EI)_{wi}\frac{d\psi_{wi}}{dz_i}\right] + (GA)_{wi}\left(\frac{dy_i}{dz_i} - \psi_{wi}\right) = 0 \tag{2}$$

$$\frac{d}{dz_i}\left[(D)_i\frac{d\psi_{fi}}{dz_i}\right] + (GA)_{fi}\left(\frac{dy_i}{dz_i} - \psi_{fi}\right) = 0. \tag{3}$$

In the above equations, y_i are the total displacement functions; z_i are the vertical axis of i-th storey; N_i are the axial forces; ψ_{wi} denote rotations of a transverse normal of shear wall; ψ_{fi} denote rotations of a transverse normal of frame; EI_i are the total bending rigidities of shear walls and columns; and D_i are the global bending rigidities of frame and can be calculated using the equation below,

$$D_i = \sum_{j=1}^{n} EA_j r_j^2, \tag{4}$$

where A_j are the cross sectional areas of j-th shear wall/column; n is the number of columns; and r are the distances of the j-th shear wall/column from the center of the cross sections. (GA_{wi}) are the equivalent shear rigidities of walls and (GA_{fi}) are the equivalent shear rigidities of the framework. For frame elements which consist of n columns and $n-1$ beams, GA_{fi} can be calculated by equation (5) (Murashev *et al* 1972; Stafford Smith & Crowe 1986).

$$GA_{fi} = \frac{12E}{h_i\left[1/\sum_1^n I_c/h_i + 1/\sum_1^{n-1} I_g/l\right]}. \tag{5}$$

Here, $\sum I_c/h_i$ represents the sum of moments of inertia of the columns per unit height in i-th storey of frame j, and $\sum I_g/l$ represents the sum of moments of inertia of each beam per unit span across one floor of frame j.

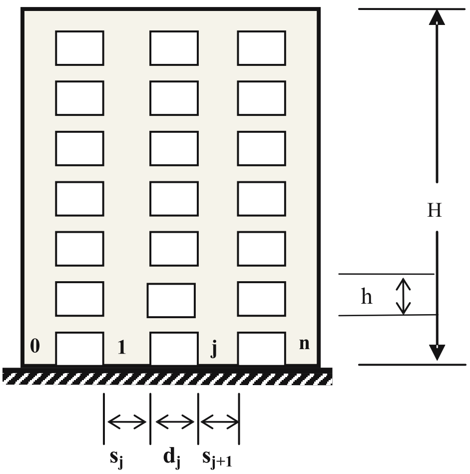

Figure 2. Coupled shear wall.

For coupled shear wall which consists of n walls and $n-1$ connecting beams, ignoring the wall effects GA_{fi} can be calculated by equation (6) (Rosman 1964; Murashev *et al* 1972),

$$GA_{fi} = \sum_{j=1}^{n-1} \frac{6EI_{bj}[(d_j + s_j)^2 + (d_j + s_{j+1})^2]}{d_j^3 \left(h_i + \frac{12\rho EI_{bj}}{GA_{bj} d_j^2}\right)}, \tag{6}$$

where, h_i are heights of storeys; d_j are the clear span lengths of coupling beam; s_j are the wall lengths (figure 2); EI_{bj} and GA_{bj} represent the flexural rigidities and the shear rigidities of connecting beams, respectively. ρ is the constant depending on the shape of cross-sections of the beams ($\rho = 1{\cdot}2$ for rectangular cross-sections).

With the solution of the equations (1), (2) and (3), total displacement functions (y_i) and rotation angles (ψ_{wi}, ψ_{fi}) can be obtained as follows:

$$y_i(z_i) = c_1 + c_2 z_i + c_3 \cosh(a_i z_i) + c_4 \sinh(a_i z_i) + c_5 \cos(b_i z_i) + c_6 \sin(b_i z_i), \tag{7}$$

$$\psi_{wi}(z_i) = c_2 + R_{wi} c_3 \sinh(a_i z_i) + R_{wi} c_4 \cosh(a_i z_i) - F_{wi} c_5 \sin(b_i z_i) + F_{wi} c_6 \cos(b_i z_i), \tag{8}$$

$$\psi_{fi}(z_i) = c_2 + R_{fi} c_3 \sinh(a_i z_i) + R_{fi} c_4 \cosh(a_i z_i) - F_{fi} c_5 \sin(b_i z_i) + F_{fi} c_6 \cos(b_i z_i), \tag{9}$$

where $c_1, c_2, c_3, c_4, c_5, c_6$ are integral constants; a_i and b_i are the representative of the long terms of equations which can be calculated by using formulas given below.

$$a_i = \sqrt{\frac{g_i/e_i + \sqrt{(g_i/e_i)^2 + 4N_i GA_{wi} GA_{fi}/e_i}}{2}} \tag{10}$$

$$b_i = \sqrt{\frac{-g_i/e_i + \sqrt{(g_i/e_i)^2 + 4N_i GA_{wi} GA_{fi}/e_i}}{2}}, \tag{11}$$

e_i, g_i can be calculated from equations (12) and (13) as shown below:

$$e_i = EI_{wi}D_i(GA_{wi} + GA_{fi} - N_i) \tag{12}$$

$$g_i = [GA_{wi}GA_{fi}(D_i + EI_i) - N_i(GA_f EI_w + GA_w D_i)]. \tag{13}$$

R_{wi}, R_{fi}, F_{wi} and F_{fi} can be calculated from equations (14), (15), (16) and (17) as shown below:

$$R_{wi} = \frac{(GA)_{wi}a_i}{[(GA)_{wi} - a_i^2(EI)_{wi}]} \tag{14}$$

$$R_{fi} = \frac{(GA)_{fi}a_i}{[(GA)_{fi} - a_i^2(D)_i]} \tag{15}$$

$$F_{wi} = \frac{(GA)_{wi}b_i}{[(GA)_{wi} + b_i^2(EI)_{wi}]} \tag{16}$$

$$F_{fi} = \frac{(GA)_{fi}b_i}{[(GA)_{fi} - b_i^2(D)_i]}. \tag{17}$$

With the help of equations (7), (8) and (9), bending moment of the shear wall and bending moment of the frames due to axial deformation along with the total shear force can be obtained as follows:

$$\begin{aligned} M_{wi}(z_i) = (EI)_{wi}\psi'_{wi} = (EI)_{wi}[&a_i R_{wi}c_3\cosh(a_i z_i) + a_i R_{wi}c_4\sinh(a_i z_i) \\ &- b_i F_{wi}c_5\cos(b_i z_i) - b_i F_{wi}c_6\sin(b_i z_i)] \end{aligned} \tag{18}$$

$$\begin{aligned} M_{fi}(z_i) = D_i\psi'_{fi} = D_i[&a_i R_{fi}c_3\cosh(a_i z_i) + a_i R_{fi}c_4\sinh(a_i z_i) \\ &- b_i F_{fi}c_5\cos(b_i z_i) - b_i F_{fi}c_6\sin(b_i z_i)] \end{aligned} \tag{19}$$

$$\begin{aligned} V_i(z_i) &= (EI_{wi})\frac{d^2\psi_{wi}}{dz_i^2} + D_i\frac{d^2\psi_{fi}}{dz_i^2} + N_i\frac{dy_i}{dz_i} \\ &= c_2N_i + c_3[EI_iR_{wi}a_i^2\sinh(a_iz_i) + D_iR_{fi}a_i^2\sinh(a_iz_i) + N_ia_i\sinh(a_iz_i)] \\ &\quad + c_4[EI_iR_{wi}a_i^2\cosh(a_iz_i) + D_iR_{fi}a_i^2\cosh(a_iz_i) + N_ia_i\cosh(a_iz_i)] \\ &\quad + c_5[EI_iF_{wi}b_i^2\sin(b_iz_i) + D_iF_{fi}b_i^2\sin(b_iz_i) - N_ib_i\sin(b_iz_i)] \\ &\quad + c_6[-EI_iF_{wi}b_i^2\cos(b_iz_i) - D_iF_{fi}b_i^2\cos(b_iz_i) + N_ib_i\cos(b_iz_i)] \end{aligned} \tag{20}$$

Equation (21) shows the matrix form of equations (7), (8), (9), (18), (19) and (20):

$$\begin{bmatrix} y_i(z_i) \\ \psi_{wi}(z_i) \\ \psi_{fi}(z_i) \\ M_{wi}(z_i) \\ M_{fi}(z_i) \\ V_i(z_i) \end{bmatrix} = B_i(z_i)\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix}. \tag{21}$$

At the initial point of the storey for $z_i = 0$, equation (21) can be written as:

$$\begin{bmatrix} y_i(0) \\ \psi_{wi}(0) \\ \psi_{fi}(0) \\ M_{wi}(0) \\ M_{fi}(0) \\ V_i(0) \end{bmatrix} = B_i(0) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix}. \tag{22}$$

The vector in right-hand side of equation (22) can be shown as follows:

$$c = [c_1\ c_2\ c_3\ c_4\ c_5\ c_6]^t. \tag{23}$$

When vector c is solved from equation (22) and is substituted to the equations (21), then equation (24) is obtained.

$$\begin{bmatrix} y_i(z) \\ \psi_{wi}(z) \\ \psi_{fi}(z) \\ M_{wi}(z) \\ M_{fi}(z) \\ V_i(z) \end{bmatrix} = B_i(z)B_i(0)^{-1} \begin{bmatrix} y_i(0) \\ \psi_{wi}(0) \\ \psi_{fi}(0) \\ M_{wi}(0) \\ M_{fi}(0) \\ V_i(0) \end{bmatrix}, \tag{24}$$

where $S_i = B_i(h_i)B_i(0)^{-1}$ is the storey transfer matrix for $z_i = h_i$.

3. Determination of critical buckling load

The storey stability transfer matrices in equation (24) can be used for stability analysis of high rise buildings.

The displacements and internal forces relationship between the base and the top of the structure can be found as follows:

$$\begin{bmatrix} y_{\text{top}} \\ \psi_{\text{wtop}} \\ \psi_{\text{ftop}} \\ M_{\text{wtop}} \\ M_{\text{ftop}} \\ V_{\text{top}} \end{bmatrix} = S_n S_{(n-1)} \ldots\ldots\ldots S_1 \begin{bmatrix} y_{\text{base}} \\ \psi_{\text{wbase}} \\ \psi_{\text{fbase}} \\ M_{\text{wbase}} \\ M_{\text{fbase}} \\ V_{\text{base}} \end{bmatrix}. \tag{25}$$

The boundary conditions of the shear wall-frame system are:

1) $y_{\text{base}} = 0$, 2) $\psi_{\text{wbase}} = 0$, 3) $\psi_{\text{fbase}} = 0$, 4) $M_{\text{wtop}} = 0$, 5) $M_{\text{ftop}} = 0$, 6) $V_{\text{top}} = 0$.

When the boundary conditions are considered in equation (25), for non-trivial solution of system transfer matrix ($S = S_n S_{n-1} S_{n-2} \ldots\ldots\ldots S_1$) equation (26) is obtained:

$$f = \begin{bmatrix} s_{44} & s_{45} & s_{46} \\ s_{54} & s_{55} & s_{56} \\ s_{64} & s_{65} & s_{66} \end{bmatrix}. \tag{26}$$

The values of N which set the determinant to zero are the critical buckling load of the building.

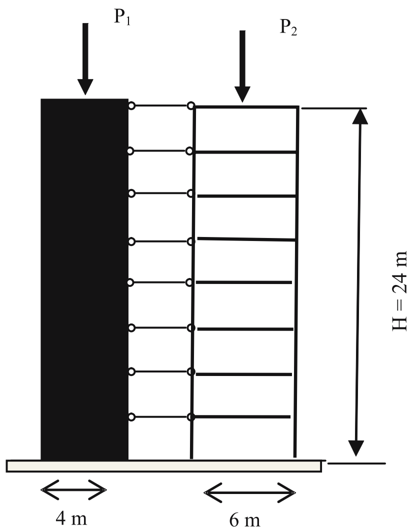

Figure 3. Wall-frame structure.

4. Procedure of computation

The step-by-step procedure of the computation of transfer matrix method is presented below:

(i) Calculation of the structural properties (GA_i, EI_i, D_i, N_i) of each storey using equations (4), (5) and (6).
(ii) Computation of storey transfer matrices as determined in equation (24) for each storey using the structural properties obtained in step 1.
(iii) Computation of system stability transfer matrix as in equation (25) with the help of the storey transfer matrices.
(iv) Applying the boundary conditions in equation (25) and obtaining the non-trivial equation (equation 26).
(v) Determination of the buckling load by using numerical method.

5. Numerical examples

In this part of the study, to verify the presented method, four numerical examples have been solved by a program written in MATLAB. The results have been compared with those given in the literature.

Example 1. Consider the wall-frame dual system which is vertically loaded at top as shown in figure 3. The total height of building is 24 m. The equivalent rigidities of the shear wall are $EI_i = 32 * 10^6$kNm^2, $GA_{wi} = 10{\cdot}3 * 10^6$kN, and the equivalent rigidities of the frames are: $D_i = 39{\cdot}2 * 10^6$kNm^2, $GA_{fi} = 24{\cdot}5 * 10^6$kN. The critical buckling load factor has been calculated with the presented method and has been compared (table 1) with that found in the literature (Gengshu *et al* 2008).

To investigate the shear deformation effects on the critical buckling load, the structure has been analysed with, and without, considering the shear deformation effects. Besides, the same structure with different heights has been investigated (tables 2 and table 3) to present the shear

Table 1. Comparison of critical buckling load factor in example 1.

	Gengshu *et al* (2008)	Presented method
Critical load factor	302053·931 kN	302050 kN

Table 2. Comparison of shear effect in the structure with different number of storeys ($l = 4$ m).

Total storey	Without shear deformation	With shear deformation
2	4614500 kN	4229400 kN
4	1202100 kN	1174400 kN
6	538630 kN	532990 kN
8	303850 kN	302050 kN

Table 3. Comparison of shear effect in the structure with different number of storeys ($l = 6$ m).

Total storey	Without shear deformation	With shear deformation
2	9823400 kN	7425700 kN
4	2510200 kN	2228300 kN
6	1117400 kN	1075800 kN
8	629410 kN	615960 kN

deformation effects. Further, the length of the shear wall (l) is taken as 4 m (table 2) and 6 m (table 3).

As can be seen from the tables 2 and 3, the shear deformation effect becomes more important when the length of the shear wall is increased and the height of the structure is decreased.

Example 2. The frame structure with two spans eleven storeys in figure 4 has been considered for Example 2. The section properties of columns and heights of the storeys are given in table 4. All girders have the same cross section, and the modulus of elasticity of the structure is $E = 25000$ MN/m^2. The equivalent rigidities of each storey have been calculated and are presented in table 5.

The critical buckling load factor has been calculated using the present method and compared with the obtained results of Syngellakis & Kameshki 1994 (table 6).

Example 3. In this example, the coupled shear wall structure as in figure 5 has been analysed by the proposed method. The shear walls have 4 m length and 0·4 m width. The height of connecting beams between the shear walls are 0·8 m and the thickness of the beams are 0·4 m. The modulus of elasticity and the Poisson's ratio of the system are equal to 20 kN/mm^2 and 0·25, respectively.

Equivalent rigidities have been calculated and the storey transfer matrices have been obtained by using equation (24). The system stability matrix has been obtained by using equation (25). Finally, by applying the boundary conditions, equation (26) is gained and the critical buckling load is found. It has been compared with the result found by SAP 2000 (table 7).

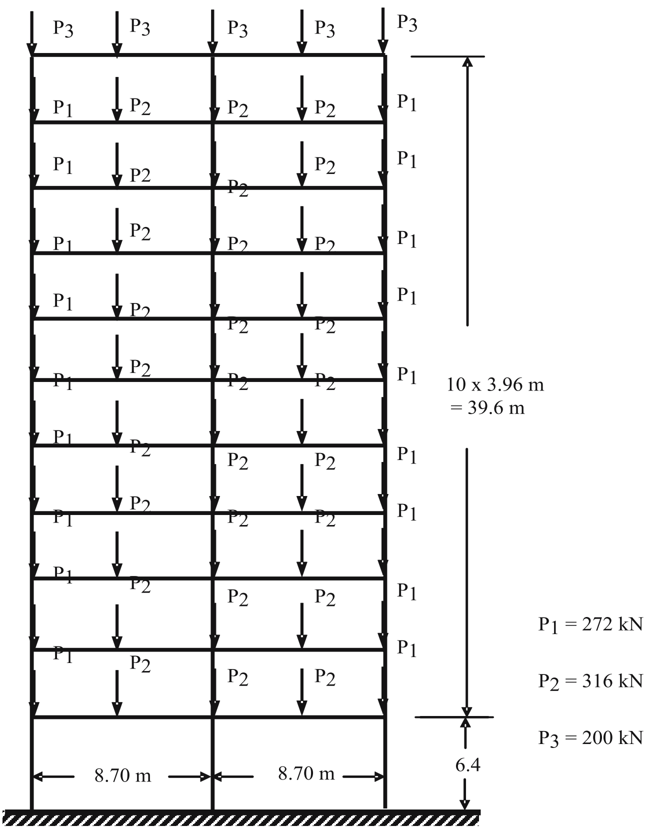

Figure 4. Two bay frame (Example 1).

Example 4. A 15-storey tube in tube structure has been analysed as an example. The plan of structure is shown in figure 6. The modulus of elasticity of the structure is $E = 2{\cdot}1 * 10^6\,\text{Mp/m}^2$, the height of each storey is $h = 3{\cdot}20\,\text{m}$, and the total height of the building is $H = 48\,\text{m}$. The cross-sections of the rectangular columns and lintels of the outer tube are 0·35 m*0·55 m and 0·30 m*0·80 m, respectively. The thickness of shear wall is 0·20 m. The equivalent rigidities of each storey have been calculated by using equations (4), (5) and are presented in tables 8 and 9. The critical buckling loads have been calculated in the two directions by this method and compared with those found in the literature (Rosman 1974). The results are shown in table 10.

Table 4. Moment of inertia of columns in Example 2.

Storey	Storey height	Exterior column	Interior column
1	6·4 m	204370 cm^4	250160 cm^4
2	3·96 m	204370 cm^4	250160 cm^4
3–4	3·96 m	146929 cm^4	183140 cm^4
5–6	3·96 m	111130 cm^4	146929 cm^4
7–8	3·96 m	84079 cm^4	146929 cm^4
9–11	3·96 m	57024 cm^4	84079 cm^4

Table 5. Equivalent rigidities in Example 2.

Storey	(EI)	$(GA)_{fi}$	D	$(GA)_w$
1	135074·5 kNm^2	5612·119 kN	Infinite	Infinite
2	135074·5 kNm^2	9502·238 kN	Infinite	Infinite
3–4	97785 kNm^2	9179·119 kN	Infinite	Infinite
5–6	75684·048 kNm^2	8861·857 kN	Infinite	Infinite
7–8	64592·57 kNm^2	8635·381 kN	Infinite	Infinite
9–11	40616·36 kNm^2	7828·071 kN	Infinite	Infinite

Table 6. Comparison of critical buckling load factor in Example 2.

	Syngellakis & Kameshki (1994)	Presented method
Critical load factor	7·7000 kN	7·8142 kN

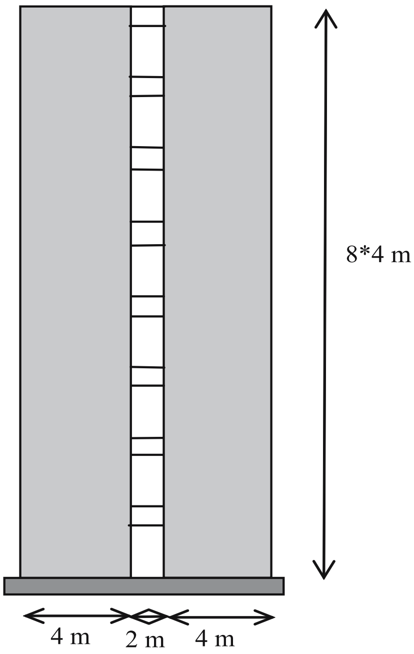

Figure 5. Coupled shear wall.

Table 7. Comparison of critical buckling load factor in Example 2.

Critical buckling load	
SAP 2000 (Wide column)	Presented method
3103975·345 kN	3302240 kN

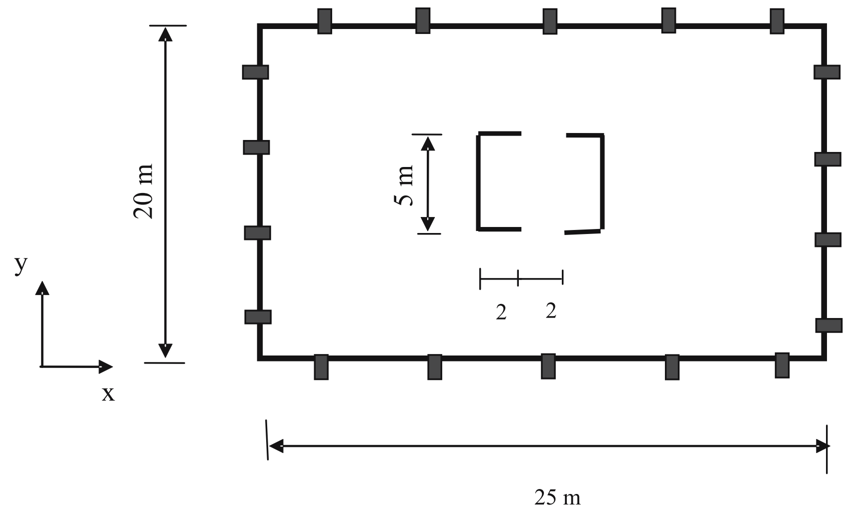

Figure 6. Tube in tube structure.

Table 8. Equivalent rigidities of the system in Example 4 in xz plane.

Storey	$(GA)_{wi}$	$(EI)_{wi}$	$(GA)_{fi}$	$(D)_i$
1–15	infinite	$2{\cdot}987 * 10^6$ Mpm^2	53140 Mp	infinite

Table 9. Equivalent rigidities of the system in Example 4 in yz plane.

Storey	$(GA)_{wi}$	$(EI)_{wi}$	$(GA)_{fi}$	$(D)_i$
1–15	infinite	$2{\cdot}975 * 10^7$ Mpm^2	41868·182 Mp	infinite

Table 10. Comparison of critical buckling load in Example 4.

	Critical buckling loads (Mp)		
Direction	Rosman (1974)	Presented method	Difference (%)
y	$1{\cdot}044 * 10^5$	$1{\cdot}123 * 10^5$	7·57
z	$2{\cdot}195 * 10^5$	$2{\cdot}182 * 10^5$	0·59

6. Conclusions

In this study, an approximate method based on the continuum approach and transfer matrix method for lateral stability analysis of buildings has been presented. In this method, the whole structure is idealized as an equivalent sandwich beam which includes all deformations. The effect of shear deformations of walls has been taken into consideration and incorporated in the formulation of the governing equations. Examples have shown that the results obtained from the proposed method are in good agreement with Finite Element Method and the analytical solution which has been developed by Rosman. The proposed method is not only simple and accurate enough to be used both at the concept design stage and for final analyses, but at the same time takes less computational time than the Finite Element Method.

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