Static deformation due to a long buried dip-slip fault in an isotropic half-space welded with an orthotropic half-space

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Abstract. Closed-form analytical expressions for the displacements and the stresses at any point of a two-phase medium consisting of a homogeneous, isotropic, perfectly elastic half-space in welded contact with a homogeneous, orthotropic, perfectly elastic half-space due to a dip-slip fault of finite width located at an arbitrary distance from the interface in the isotropic half-space are obtained. The Airy stress function approach is used to obtain the expressions for the stresses and the displacements. The case of a vertical dip-slip fault is considered in detail. The variations of the displacements with the distance from the fault and with depth have been shown graphically.

Keywords. Isotropic half-space; dip-slip fault; orthotropic half-space; static deformation; welded contact.

1. Introduction

The two half-space model is suitable for computing the deformation caused by seismic sources near an internal boundary. Several researchers have studied the deformation due to shear or tensile faults in the two half-space model e.g. Ben-Zion (1989), Singh & Rani (1991), Singh *et al* (1992), Rani & Singh (1992), Bonafede & Rivalta (1999), Kumar *et al* (2002), Kumar *et al* (2005), Rani & Bala (2006) and the others. Wu & Chou (1982) applied the generalized method of images to obtain the elastic field of an in-plane line force acting in a two-phase orthotropic medium. Garg *et al* (1996) studied the antiplane strain deformation of a layered orthotropic medium. Garg *et al* (1991) studied the plane strain deformation of an orthotropic elastic half-space using transfer matrix approach.

In the present paper, we study the deformation caused by a long dip-slip fault of arbitrary dip and finite width located at an arbitrary distance from the interface in an isotropic half-space welded with an orthotropic half-space along the horizontal plane. We begin with the closed-form expression for the Airy stress function due to a dip slip line source given by Singh & Rani (1991) in a two-phase medium consisting of a homogeneous, elastic, isotropic

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half-space in welded contact with a homogeneous, elastic, orthotropic half-space. Following Rani & Bala (2006), we obtain the Airy stress function for a long buried dip-slip fault of finite width. The expressions for the displacements and the stresses follow immediately from the Airy stress function. The displacement field due to buried and interface-breaking vertical dip-slip fault has been computed numerically and is compared with the displacement field when both the half-spaces are isotropic. Orthotropic or orthorhombic symmetry is exhibited by olivine and orthopyroxenes, the principal rock forming minerals of deep crust and upper mantle. Therefore, we take the lower half-space to be orthotropic.

2. Theory

Let the Cartesian co-ordinates be denoted by (x_1, x_2, x_3) with x_3 -axis vertically upwards. Consider two homogeneous, perfectly elastic half-spaces which are welded along the plane $x_3 = 0$. The upper half-space ($x_3 > 0$) is assumed to be isotropic with stress-strain relation

$$p_{ij} = 2\mu \left[e_{ij} + \frac{\sigma}{1-2\sigma} \delta_{ij} e_{kk} \right], \quad (i, j = 1, 2, 3) \quad (1)$$

where p_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, μ is the shear modulus and σ is Poisson's ratio. The lower half-space ($x_3 < 0$) is assumed to be orthotropic with stress-strain relation

$$\begin{bmatrix} p'_{11} \\ p'_{22} \\ p'_{33} \\ p'_{23} \\ p'_{31} \\ p'_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} e'_{11} \\ e'_{22} \\ e'_{33} \\ 2e'_{23} \\ 2e'_{31} \\ 2e'_{12} \end{bmatrix}. \quad (2)$$

We consider a two-dimensional approximation in which the displacement components (u_1, u_2, u_3) are independent of x_1 so that $\partial/\partial x_1 \equiv 0$. Under this assumption the plane-strain problem ($u_1 = 0$) and the antiplane strain problem ($u_2 = u_3 = 0$) are decoupled and, therefore, can be solved separately. The plane strain problem for an isotropic medium can be solved in terms of the Airy stress function U such that

$$p_{22} = \frac{\partial^2 U}{\partial x_3^2}, \quad p_{33} = \frac{\partial^2 U}{\partial x_2^2}, \quad p_{23} = -\frac{\partial^2 U}{\partial x_2 \partial x_3}, \quad (3)$$

$$\nabla^2 \nabla^2 U = 0. \quad (4)$$

The plane strain problem for an orthotropic medium can be solved in terms of the Airy stress function U^* such that Garg *et al* (1991)

$$p'_{22} = \frac{\partial^2 U^*}{\partial x_3^2}, \quad p'_{33} = \frac{\partial^2 U^*}{\partial x_2^2}, \quad p'_{23} = -\frac{\partial^2 U^*}{\partial x_2 \partial x_3}, \quad (5)$$

$$\left(a^2 \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \right) \left(b^2 \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \right) U^* = 0, \quad (6)$$

where

$$a^2 + b^2 = \frac{(c_{22}c_{33} - c_{23}^2 - 2c_{23}c_{44})}{c_{33}c_{44}}, \quad a^2b^2 = \frac{c_{22}}{c_{33}}. \quad (7)$$

For an isotropic medium

$$\begin{aligned} c_{11} = c_{22} = c_{33} &= \frac{2\mu(1-\sigma)}{1-2\sigma}, \\ c_{12} = c_{13} = c_{23} &= \frac{2\mu\sigma}{1-2\sigma}, \\ c_{44} = c_{55} = c_{66} &= \mu. \end{aligned} \quad (8)$$

This yields $a^2 = b^2 = 1$ and equation (6) reduces to equation (4).

As given by Singh & Rani (1991), we have the Airy stress function due to a dip-slip line source parallel to the x_1 -axis and passing through the point (y_2, y_3) located in the isotropic half-space welded with the orthotropic half-space. For the isotropic half-space

$$\begin{aligned} U = \frac{\alpha\mu\bar{b}ds}{\pi} &\left[\cos 2\delta \left\{ \frac{(x_2 - y_2)(x_3 - y_3)}{R^2} - 2(D - C) \tan^{-1} \left(\frac{x_2 - y_2}{x_3 + y_3} \right) \right. \right. \\ &\quad \left. \left. - \frac{(X_1y_3 + X_3x_3)(x_2 - y_2)}{S^2} - \frac{4X_2(x_2 - y_2)y_3x_3(x_3 + y_3)}{S^4} \right\} \right. \\ &\quad \left. + \sin 2\delta \left\{ -2(D - C) \ln S + \frac{(x_3 - y_3)^2}{R^2} \right. \right. \\ &\quad \left. \left. + \frac{(X_1y_3 + X_3x_3)(x_3 + y_3) - 2X_2x_3y_3}{S^2} + \frac{4X_2x_3y_3(x_3 + y_3)^2}{S^4} \right\} \right], \end{aligned} \quad (9)$$

and for the orthotropic half-space

$$\begin{aligned} U^* = \frac{2\alpha\mu\bar{b}ds}{\pi} &\left[\cos 2\delta \left\{ C \tan^{-1} \left(\frac{x_2 - y_2}{y_3 - ax_3} \right) \right. \right. \\ &\quad \left. \left. - D \tan^{-1} \left(\frac{x_2 - y_2}{y_3 - bx_3} \right) - (x_2 - y_2)y_3 \left(\frac{A}{T^2} + \frac{B}{H^2} \right) \right\} \right. \\ &\quad \left. + \sin 2\delta \left\{ C \ln T - D \ln H + \frac{Ay_3(y_3 - ax_3)}{T^2} + \frac{By_3(y_3 - bx_3)}{H^2} \right\} \right], \end{aligned} \quad (10)$$

where

$$\alpha = \frac{1}{2(1-\sigma)},$$

$\bar{b}$ = displacement discontinuity,

ds = width of the line source,

δ = dip angle,

(x_2, x_3) = receiver location,

$$\begin{aligned}
 R^2 &= (x_2 - y_2)^2 + (x_3 - y_3)^2, & S^2 &= (x_2 - y_2)^2 + (x_3 + y_3)^2, \\
 T^2 &= (x_2 - y_2)^2 + (y_3 - ax_3)^2, & H^2 &= (x_2 - y_2)^2 + (y_3 - bx_3)^2, \\
 X_1 &= 2(A + B) - 1, & X_2 &= A(1 + a) + B(1 + b) - 1, \\
 X_3 &= 2D(1 + b) - 2C(1 + a) + 1, & A &= \frac{\alpha(1 - b + 2\mu s_2 - 2\mu r_2)}{W}, \\
 B &= \frac{\alpha(a - 1 + 2\mu r_1 - 2\mu s_1)}{W}, & C &= \frac{(1 + b - b\alpha + 2\mu\alpha s_2)}{W}, \\
 D &= \frac{(1 + a - a\alpha + 2\mu\alpha s_1)}{W}, & r_1 &= \frac{c_{33}a^2 + c_{23}}{\Delta}, \\
 r_2 &= \frac{c_{33}b^2 + c_{23}}{\Delta}, & s_1 &= \frac{c_{23}a^2 + c_{22}}{a\Delta}, \\
 s_2 &= \frac{c_{23}b^2 + c_{22}}{b\Delta}, & \Delta &= (c_{22}c_{33} - c_{23}^2), \\
 W &= (1 + a - \alpha + 2\mu\alpha r_1)(1 + b - b\alpha + 2\mu\alpha s_2) \\
 &\quad - (1 + a - a\alpha + 2\mu\alpha s_1)(1 + b - \alpha + 2\mu\alpha r_2).
 \end{aligned} \tag{11}$$

Following Rani & Bala (2006), using the polar co-ordinates (s, δ) (figure 1a)

$$y_2 = s \cos \delta, \quad y_3 = d + s \sin \delta$$

and integrating over s between the limits $(0, L)$, we obtain the following expressions for the Airy stress function for a long reverse dip-slip fault of width L and infinite length with lower edge of the fault at distance d from the interface:

$$\begin{aligned}
 U &= \frac{\alpha\mu\bar{b}}{\pi} \left[\{4(D - C) + (X_1 + 1)\}s \sin \delta \cos \delta \right. \\
 &\quad + \{2(D - C)(x_2 \sin \delta - X' \cos \delta - s \sin 2\delta) + (X_1 d + X_3 x_3) \cos \delta \\
 &\quad + X_1 x_2 \sin \delta\} \ln S + (x_2 \sin \delta - X \cos \delta) \ln R \\
 &\quad - 2(D - C)s \cos 2\delta \quad \tan^{-1} \left(\frac{x_2 - s \cos \delta}{X' + s \sin \delta} \right) \\
 &\quad - 2X_2 x_3 (x_2 \sin \delta + X' \cos \delta) (d + s \sin \delta) \frac{1}{S^2} \\
 &\quad + \{x_3 \sin \delta (-X_1 + 2X_2 + X_3) \\
 &\quad \left. - 2(D - C)(x_2 \cos \delta + X' \sin \delta)\} \tan^{-1} \left(\frac{s - x_2 \cos \delta + X' \sin \delta}{x_2 \sin \delta + X' \cos \delta} \right) \right] \Bigg|_0^L,
 \end{aligned} \tag{12}$$

$$\begin{aligned}
 U^* = \frac{2\alpha\mu\bar{b}}{\pi} & \left[\{2(D - C) + (A + B)\}s \sin \delta \cos \delta \right. \\
 & + \{A(x_2 \sin \delta + d \cos \delta) - C(x_2 \sin \delta - Y \cos \delta - s \sin 2\delta)\} \ln T \\
 & + \{B(x_2 \sin \delta + d \cos \delta) + D(x_2 \sin \delta - Y' \cos \delta - s \sin 2\delta)\} \ln H \\
 & + Cs \cos 2\delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{Y + s \sin \delta} \right) - Ds \cos 2\delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{Y' + s \sin \delta} \right) \\
 & + \{C(x_2 \cos \delta + Y \sin \delta) + Aax_3 \sin \delta\} \tan^{-1} \left(\frac{s - x_2 \cos \delta + Y \sin \delta}{x_2 \sin \delta + Y \cos \delta} \right) \\
 & \left. - \{D(x_2 \cos \delta + Y' \sin \delta) - Bbx_3 \sin \delta\} \tan^{-1} \left(\frac{s - x_2 \cos \delta + Y' \sin \delta}{x_2 \sin \delta + Y' \cos \delta} \right) \right] \Bigg|_0^L, \quad (13)
 \end{aligned}$$

Figure 1. Geometry of a reverse dip-slip fault of width L having lower edge at a distance d from the interface in the isotropic half-space in welded contact with an orthotropic half-space. **(a)** δ is the dip angle and s is the distance from the lower edge of the fault measured in the dip direction, **(b)** $\delta = 90^\circ$.

where now

$$\begin{aligned} R^2 &= (x_2 - s \cos \delta)^2 + (X - s \sin \delta)^2, \quad S^2 = (x_2 - s \cos \delta)^2 + (X' + s \sin \delta)^2, \\ T^2 &= (x_2 - s \cos \delta)^2 + (Y + s \sin \delta)^2, \quad H^2 = (x_2 - s \cos \delta)^2 + (Y' + s \sin \delta)^2, \\ X &= x_3 - d, \quad X' = x_3 + d, \quad Y = d - ax_3, \quad Y' = d - bx_3, \\ f(s)|_0^L &= f(L) - f(0). \end{aligned} \quad (14)$$

3. Stresses and displacements

Using equations (3) and (12), we obtain the following expressions for the stresses of the isotropic medium:

$$\begin{aligned} p_{22} &= \frac{\alpha \mu \bar{b}}{\pi} \left[(x_2 \sin \delta - 3X \cos \delta + s \sin 2\delta) \frac{1}{R^2} \right. \\ &\quad - 2(x_2 \sin \delta - X \cos \delta)(X - s \sin \delta)^2 \frac{1}{R^4} \\ &\quad + \left\{ 2(X_3 - D + C)(x_2 \sin \delta + X' \cos \delta) \right. \\ &\quad \left. + (4X_2 - X_1)(x_2 \sin \delta - d \cos \delta - s \sin 2\delta) + X_3 x_3 \cos \delta \right\} \frac{1}{S^2} \\ &\quad - 2 \left\{ X_1(x_2 \sin \delta + X' \cos \delta)(X' + s \sin \delta)^2 + 2X_2 x_3^2 \sin \delta (x_2 - s \cos \delta) \right. \\ &\quad - (X_1 - X_3)x_3(X' + s \sin \delta)(x_2 \sin \delta + X' \cos \delta) \\ &\quad \left. - 2X_2(d + s \sin \delta)(X' + s \sin \delta)[2(x_2 \sin \delta + X' \cos \delta) + 3x_3 \cos \delta] \right\} \frac{1}{S^4} \\ &\quad \left. - 16X_2 x_3(x_2 \sin \delta + X' \cos \delta)(d + s \sin \delta)(X' + s \sin \delta)^2 \frac{1}{S^6} \right] \Big|_0^L, \quad (15) \\ p_{23} &= \frac{\alpha \mu \bar{b}}{\pi} \left[(x_2 \cos \delta + X \sin \delta - s) \frac{1}{R^2} \right. \\ &\quad + 2(X - s \sin \delta)^2(s - x_2 \cos \delta - X \sin \delta) \frac{1}{R^4} \\ &\quad + \left\{ 2(D - C)(x_2 \cos \delta - X' \sin \delta - s) - X_1 x_3 \sin \delta + 4X_2 x_3 \sin \delta \right. \\ &\quad \left. + X_3(s + 2x_3 \sin \delta + d \sin \delta - x_2 \cos \delta) \right\} \frac{1}{S^2} \\ &\quad + \left\{ 2x_3(X_1 - 2X_2)(X' + s \sin \delta)^2 \sin \delta \right. \end{aligned}$$

$$\begin{aligned}
& -2X_1(X' + s \sin \delta)[X' \sin \delta(X' + s \sin \delta) - d \cos \delta(x_2 - s \cos \delta) \\
& + s \sin \delta(s - x_2 \cos \delta + X' \sin \delta)] + 4X_2(d + s \sin \delta) \\
& \times [(x_3 + X' + s \sin \delta)(s - x_2 \cos \delta + X' \sin \delta) + 2x_3 \sin \delta(X' + s \sin \delta)] \\
& + 2X_3x_3(X' + s \sin \delta)(x_2 \cos \delta - X' \sin \delta - s) \left\} \frac{1}{S^4} \right. \\
& \left. + 16X_2x_3(d + s \sin \delta)(X' + s \sin \delta)^2(x_2 \cos \delta - X' \sin \delta - s) \frac{1}{S^6} \right] \Bigg|_0^L, \quad (16)
\end{aligned}$$

$$\begin{aligned}
p_{33} = & \frac{\alpha \mu \bar{b}}{\pi} \left[(x_2 \sin \delta + X \cos \delta - s \sin 2\delta) \frac{1}{R^2} \right. \\
& + 2(x_2 \sin \delta - X \cos \delta)(X - s \sin \delta)^2 \frac{1}{R^4} \\
& + \left\{ 2(D - C)(x_2 \sin \delta + X' \cos \delta) \right. \\
& + X_1(x_2 \sin \delta - d \cos \delta - s \sin 2\delta) - X_3x_3 \cos \delta \left. \right\} \frac{1}{S^2} \\
& + \left\{ 2X_1(X' + s \sin \delta)[(X' + s \sin \delta)(x_2 \sin \delta + d \cos \delta) \right. \\
& - x_3 \sin \delta(x_2 - s \cos \delta)] + 4X_2x_3[x_3 \sin \delta(x_2 - s \cos \delta) \\
& - 3 \cos \delta(X' + s \sin \delta)(d + s \sin \delta)] \\
& + 2X_3x_3(X' + s \sin \delta)(x_2 \sin \delta + X' \cos \delta) \left. \right\} \frac{1}{S^4} \\
& \left. + 16X_2x_3(x_2 \sin \delta + X' \cos \delta)(d + s \sin \delta)(X' + s \sin \delta)^2 \frac{1}{S^6} \right] \Bigg|_0^L. \quad (17)
\end{aligned}$$

The stresses for the orthotropic medium are obtained from equations (5) and (13). These are:

$$\begin{aligned}
p'_{22} = & \frac{2\alpha \mu \bar{b}}{\pi} \left[a^2 \left\{ A(s \sin 2\delta - x_2 \sin \delta + d \cos \delta) + C(x_2 \sin \delta + Y \cos \delta) \right\} \frac{1}{T^2} \right. \\
& + b^2 \left\{ B(s \sin 2\delta - x_2 \sin \delta + d \cos \delta) - D(x_2 \sin \delta + Y' \cos \delta) \right\} \frac{1}{H^2} \\
& - 2a^2 A(d + s \sin \delta)(x_2 \sin \delta + Y \cos \delta)(Y + s \sin \delta) \frac{1}{T^4} \\
& \left. - 2b^2 B(d + s \sin \delta)(x_2 \sin \delta + Y' \cos \delta)(Y' + s \sin \delta) \frac{1}{H^4} \right] \Bigg|_0^L, \quad (18)
\end{aligned}$$

$$\begin{aligned}
p'_{23} = & \frac{2\alpha\mu\bar{b}}{\pi} \left[a \left\{ C(x_2 \cos \delta - Y \sin \delta - s) - aAx_3 \sin \delta \right\} \frac{1}{T^2} \right. \\
& - b \left\{ D(x_2 \cos \delta - Y' \sin \delta - s) + bBx_3 \sin \delta \right\} \frac{1}{H^2} \\
& - 2aA(d + s \sin \delta)(Y + s \sin \delta)(x_2 \cos \delta - Y \sin \delta - s) \frac{1}{T^4} \\
& \left. - 2bB(d + s \sin \delta)(Y' + s \sin \delta)(x_2 \cos \delta - Y' \sin \delta - s) \frac{1}{H^4} \right] \Bigg|_0^L, \quad (19)
\end{aligned}$$

$$\begin{aligned}
p'_{33} = & \frac{2\alpha\mu\bar{b}}{\pi} \left[\left\{ A(x_2 \sin \delta - d \cos \delta - s \sin 2\delta) - C(x_2 \sin \delta + Y \cos \delta) \right\} \frac{1}{T^2} \right. \\
& + \left\{ B(x_2 \sin \delta - d \cos \delta - s \sin 2\delta) + D(x_2 \sin \delta + Y' \cos \delta) \right\} \frac{1}{H^2} \\
& + 2A(d + s \sin \delta)(Y + s \sin \delta)(x_2 \sin \delta + Y \cos \delta) \frac{1}{T^4} \\
& \left. + 2B(d + s \sin \delta)(Y' + s \sin \delta)(x_2 \sin \delta + Y' \cos \delta) \frac{1}{H^4} \right] \Bigg|_0^L. \quad (20)
\end{aligned}$$

The displacements, for the isotropic half-space, are given by the relations (Singh & Rani 1991)

$$\begin{aligned}
2\mu u_2 = & -\frac{\partial U}{\partial x_2} + \frac{1}{2\alpha} \int (p_{22} + p_{33}) dx_2, \\
2\mu u_3 = & -\frac{\partial U}{\partial x_3} + \frac{1}{2\alpha} \int (p_{22} + p_{33}) dx_3. \quad (21)
\end{aligned}$$

The displacements, for the orthotropic half-space, are given by the expressions (Singh & Rani 1991)

$$\begin{aligned}
u'_2 = & \frac{1}{\Delta} \int (c_{33}p'_{22} - c_{23}p'_{33}) dx_2, \\
u'_3 = & \frac{1}{\Delta} \int (c_{22}p'_{33} - c_{23}p'_{22}) dx_3, \quad (22)
\end{aligned}$$

where

$$\Delta = c_{22}c_{33} - c_{23}^2. \quad (23)$$

Corresponding to the stresses (15)–(17), the displacements for the isotropic half-space are

$$\begin{aligned}
u_2 = & \frac{\alpha\bar{b}}{2\pi} \left[-\left(1 - \frac{1}{\alpha}\right) \sin \delta \ln R - \left\{ 2(D - C) + X_1 - \frac{(X_3 + 2X_2)}{\alpha} \right\} \sin \delta \ln S \right. \\
& - \frac{\cos \delta}{\alpha} \left\{ \tan^{-1} \left(\frac{x_2 - s \cos \delta}{X - s \sin \delta} \right) - X_3 \tan^{-1} \left(\frac{x_2 - s \cos \delta}{X' + s \sin \delta} \right) \right\} \\
& \left. + 2(D - C) \cos \delta \tan^{-1} \left(\frac{s - x_2 \cos \delta + X' \sin \delta}{x_2 \sin \delta + X' \cos \delta} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + (X - s \sin \delta)(x_2 \cos \delta + X \sin \delta - s) \frac{1}{R^2} \\
& + \left\{ \left(X_1 - \frac{2X_2}{\alpha} \right) (d + s \sin \delta)(s - x_2 \cos \delta + X' \sin \delta) \right. \\
& + x_3 [2X_2 x_3 \sin \delta + X_3 (s - x_2 \cos \delta + X' \sin \delta)] \left. \right\} \frac{1}{S^2} \\
& + 4X_2 x_3 (s - x_2 \cos \delta + X' \sin \delta)(d + s \sin \delta)(X' + s \sin \delta) \frac{1}{S^4} \Bigg]_0^L, \quad (24) \\
u_3 = & \frac{\alpha \bar{b}}{2\pi} \left[\left(1 - \frac{1}{\alpha} \right) \cos \delta \ln R + \left\{ 2(D - C) - X_3 \left(1 - \frac{1}{\alpha} \right) \right\} \cos \delta \ln S \right. \\
& + \frac{\sin \delta}{\alpha} \left\{ \tan^{-1} \left(\frac{X - s \sin \delta}{x_2 - s \cos \delta} \right) + (2X_2 + X_3) \tan^{-1} \left(\frac{X' + s \sin \delta}{x_2 - s \cos \delta} \right) \right\} \\
& + \{ 2(D - C) + X_1 - 2X_2 - X_3 \} \sin \delta \tan^{-1} \left(\frac{s - x_2 \cos \delta + X' \sin \delta}{x_2 \sin \delta + X' \cos \delta} \right) \\
& - (x_2 \sin \delta - X \cos \delta)(X - s \sin \delta) \frac{1}{R^2} \\
& + \left\{ \left(-X_1 + 2X_2 - \frac{2X_2}{\alpha} \right) (d + s \sin \delta)(x_2 \sin \delta + X' \cos \delta) \right. \\
& + 2X_2 x_3 (s \sin 2\delta - x_2 \sin \delta + d \cos \delta) - X_3 x_3 (x_2 \sin \delta + X' \cos \delta) \left. \right\} \frac{1}{S^2} \\
& - 4X_2 x_3 (d + s \sin \delta)(x_2 \sin \delta + X' \cos \delta)(X' + s \sin \delta) \frac{1}{S^4} \Bigg]_0^L. \quad (25)
\end{aligned}$$

Corresponding to the stresses (18)–(20), the displacements for the orthotropic half-space are

$$\begin{aligned}
u'_2 = & -\frac{2\alpha\mu\bar{b}}{\pi} \left[(A - C)r_1 \sin \delta \ln T + (B + D)r_2 \sin \delta \ln H \right. \\
& + Ar_1 (d + s \sin \delta)(x_2 \cos \delta - Y \sin \delta - s) \frac{1}{T^2} \\
& + Br_2 (d + s \sin \delta)(x_2 \cos \delta - Y' \sin \delta - s) \frac{1}{H^2} \\
& - Cr_1 \cos \delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{Y + s \sin \delta} \right) + Dr_2 \cos \delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{Y' + s \sin \delta} \right) \Bigg]_0^L. \quad (26) \\
u'_3 = & \frac{2\alpha\mu\bar{b}}{\pi} \left[Cs_1 \cos \delta \ln T - Ds_2 \cos \delta \ln H \right. \\
& + As_1 (d + s \sin \delta)(x_2 \sin \delta + Y \cos \delta) \frac{1}{T^2} \\
& + Bs_2 (d + s \sin \delta)(x_2 \sin \delta + Y' \cos \delta) \frac{1}{H^2}
\end{aligned}$$

$$\begin{aligned}
 & - (A - C)s_1 \sin \delta \tan^{-1} \left(\frac{Y + s \sin \delta}{x_2 - s \cos \delta} \right) \\
 & - (B + D)s_2 \sin \delta \tan^{-1} \left(\frac{Y' + s \sin \delta}{x_2 - s \cos \delta} \right) \Bigg] \Bigg|_0^L.
 \end{aligned} \quad (27)$$

The stresses given by (15)–(20) and the displacements given by (24)–(27) are the results for a long dip-slip fault of finite width L located in the isotropic half-space overlying the orthotropic half-space. The results obtained here satisfy the necessary continuity conditions

$$\begin{aligned}
 p_{23} &= p'_{23}, & p_{33} &= p'_{33}, \\
 u_2 &= u'_2, & u_3 &= u'_3.
 \end{aligned} \quad (28)$$

At $z = 0$ for the two half-spaces to be in welded contact along the plane $z = 0$. Moreover, when the orthotropic half-space is replaced by the isotropic one, the results of the present paper, in the limit, coincide with the corresponding results of Rani & Bala (2006) for two isotropic half-spaces in welded contact.

Figure 2. Variation of the horizontal displacement $(u'_2/\bar{b})$ with the distance from the fault due to interface-breaking vertical dip-slip fault ($d = 0$) for (a) $x_3 = -L/2$, (b) $x_3 = 0$.

Figure 3. Variation of the horizontal displacement ($u'_2/\bar{b}$) with the distance from the fault due to a long vertical dip-slip fault at distance L from the interface for (a) $x_3 = -L/2$, (b) $x_3 = 0$.

4. Numerical results

We compare the displacement field due to a long vertical dip-slip fault of width L with its edge at the distance d from the interface located in the isotropic half-space welded with orthotropic half-space along the horizontal plane with the corresponding displacement field when both the half-spaces are isotropic. Therefore, we take $\delta = 90^\circ$ (figure 1b). We assume the isotropic half-space to be Poissonian so that $\sigma = 0.25$. For the orthotropic half-space, we use the values of elastic constants given by Love (1944). For Topaz,

$$\begin{aligned} c_{11} &= 2870, c_{22} = 3560, c_{33} = 3000, \\ c_{12} &= 1280, c_{23} = 900, c_{13} = 860, \\ c_{44} &= 1100, c_{55} = 1350, c_{66} = 1330, \end{aligned}$$

in terms of a unit stress of 10^6 grammes wt/cm², this yields $a = 1.2992$ and $b = 0.8385$.

Figure 4. Variation of vertical displacement ($u'_3/\bar{b}$) with the distance from the fault due to interface breaking dip-slip fault ($d = 0$) for (a) $x_3 = -L/2$, (b) $x_3 = 0$.

For Barytes,

$$c_{11} = 907, c_{22} = 800, c_{33} = 1074,$$

$$c_{12} = 468, c_{23} = 273, c_{13} = 275,$$

$$c_{44} = 122, c_{55} = 293, c_{66} = 283,$$

in terms of 10^6 grammes wt/cm^2 , we obtain $a = 2.3118$ and $b = 0.3735$.

When the lower half-space is also isotropic,

$$c_{11} = c_{22} = c_{33} = \frac{2\mu'(1 - \sigma')}{1 - 2\sigma'},$$

$$c_{12} = c_{13} = c_{23} = \frac{2\mu'\sigma'}{1 - 2\sigma'},$$

$$c_{44} = c_{55} = c_{66} = \mu'.$$

We take $\sigma' = 0.25$ and $c_{44}/\mu = 0.5$ for numerical computations.

Figure 5. Variation of vertical displacement $(u'_3/\bar{b})$ with the distance from the fault due to a long vertical dip-slip fault at distance L from the interface for (a) $x_3 = -L/2$, (b) $x_3 = 0$.

Figures 2a–b display the horizontal displacement $(u'_2/\bar{b})$ with the distance from the fault due to interface breaking fault ($d = 0$). The horizontal displacement is symmetric about the line $x_2 = 0$. In figure 2a, the observer is in the orthotropic half-space at $x_3 = -L/2$. The horizontal displacement for Barytes varies more significantly in magnitude rather than Topaz from the corresponding one for the isotropic case. In figure 2b, the observer is at the interface, the horizontal displacement attains maximum value at origin. Figures 3a–b show the horizontal displacement $(u'_2/\bar{b})$ with the distance from the fault located at distance L from the interface. In figure 3a, the observer is in the orthotropic half-space at $x_3 = -L/2$ but in figure 3b, the observer is at the interface. We observe that magnitude of the displacement for Barytes is affected significantly, but the pattern is not affected much.

Figures 4a–b exhibit the vertical displacement $(u'_3/\bar{b})$ with the distance from the fault due to interface breaking fault ($d = 0$). The displacement is antisymmetric about the line $x_2 = 0$. The effect of anisotropy on the magnitude of the displacement is more pronounced when the observer is in orthotropic half-space. When the observer is at the interface $x_3 = 0$, the displacement is singular at the lower edge point. Figures 5a–b display the vertical

Figure 6. Variation of the horizontal displacement $(u'_2/\bar{b})$ with depth at $x_2 = L/10$ for interface breaking fault for different values of ratio of rigidities $c_{44}/\mu = 0.1, 0.5, 2$ and 10 taking the orthotropic material as (a) Topaz (b) Barytes. The magnitude of the displacement is affected.

displacement $(u'_3/\bar{b})$ for a buried fault at a distance L from the interface. We note that there is a strong variation in the vertical displacement for Barytes as well as Topaz from the vertical displacement for the isotropic case when the observer is in the orthotropic half-space.

Next, we compute the effect of ratio of rigidities of the two media on the displacement field for two different orthotropic materials, namely Topaz and Barytes. Figures 6a–b show the variation of the horizontal displacement in unit of $\bar{b}$ with depth for four values of $c_{44}/\mu = 0.1, 0.5, 2$ and 10 for Topaz and Barytes, respectively, for interface breaking fault. For all values of ratio of rigidities, the horizontal displacement attains maximum value at the lower end of the fault and minimum value at the upper end of the fault. Figures 7a–b are for the vertical displacement. The values of the ratio of rigidities affect the magnitude of the horizontal and the vertical displacement in a reverse way. The horizontal displacement varies more significantly in the orthotropic half-space as well as in the isotropic half-space. But the vertical displacement varies in the orthotropic half-space and up to fault depth in the isotropic half-space significantly.

Figure 7. Variation of the vertical displacement ($u'_3/\bar{b}$) with depth at $x_2 = L/10$ for interface breaking fault for different values of ratio of rigidities $c_{44}/\mu = 0.1, 0.5, 2$ and 10 taking the orthotropic material as (a) Topaz (b) Barytes.

5. Conclusions

We have obtained the closed-form expressions for the elastic residual field caused by a long dip-slip fault of finite width located in an isotropic half-space overlying an orthotropic half-space. The displacement field caused by a long dip-slip fault of width L has been compared with the displacement field when both the half-spaces are isotropic. Two orthotropic materials namely Topaz and Barytes have been considered for numerical computations. Numerical results show that the effect of anisotropy on the displacement field is more pronounced when the observer is in the orthotropic half-space. In case of horizontal displacement, there is a strong variation in the behaviour of Barytes from the corresponding isotropic case, but the vertical displacement for Barytes as well as for Topaz varies significantly from the corresponding isotropic case. The rock forming minerals of deep crust and upper mantle of the earth exhibit orthotropic symmetry, the results obtained may find applications in geophysical engineering.

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