Reverse stream flow routing by using Muskingum models

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Abstract. Reverse stream flow routing is a procedure that determines the upstream hydrograph given the downstream hydrograph. This paper presents the development of methodology for Muskingum models parameter estimation for reverse stream flow routing. The standard application of the Muskingum models involves calibration and prediction steps. The calibration step must be performed before the prediction step. The calibration step in a reverse stream flow routing system uses the outflow hydrograph and the inflow at the end period of the inflow hydrograph as the known inputs and Muskingum model parameters are determined by minimizing the error between the remaining portion of the predicted and observed inflow hydrographs. In the present study, methodology for parameter estimation is developed which is based on the concept of minimizing the sum of squares of normalized difference between observed and computed inflows subject to the satisfaction of the routing equation. The parameter estimation problems are formulated as constrained nonlinear optimization problem, and a computational scheme is developed to solve the resulting nonlinear problem. The performance evaluation tests indicate that a fresh calibration is necessary to use the Muskingum models for reverse stream flow routing.

Keywords. Reverse stream flow routing; Muskingum model; parameter estimation; unconstrained minimization; linear model; nonlinear model.

1. Introduction

The flow resistance properties of natural streams cause the attenuation of flood hydrographs in its passage. Flood routing traces the attenuated outflow hydrographs along the path of the river given the inflow hydrograph. The flood routing is accomplished by using either hydraulic or hydrologic procedures. To trace the outflow hydrographs, the hydraulic flood routing procedures are used if the resistance properties are known in a detailed manner and an elaborate stream gauging arrangement is available, else, the hydrologic flood routing procedures are used. The flood waves can be (i) forces of momentum and acceleration dominated, and (ii) forces of friction dominated. The forces of friction are due to the roughness effects of the riverbed and are generally assessed by using empirical procedures. Most of the floods in natural channels belong to the second category. A flood wave undergoes translation and reservoir action during its passage through natural streams. The translation action dominates

over the reservoir action for rivers with steep bed slope, and the reverse for rivers with mild bed slope. There are also some rivers that belong to an intermediate category in between these two extreme cases. When the translation action dominates over the reservoir action, the flood routing must be accomplished by using hydraulic methods. For rivers where the storage effects dominate, the hydrologic methods are popularly used.

The stream flow routing procedure originally developed by McCarthy (1938) for flood control studies in the Muskingum River in Ohio, USA is a widely used hydrologic method for routing flood waves in natural streams. This method is popularly called the Muskingum method. A large group of water resources engineers consider the Muskingum method of flow routing as a very useful tool for practical stream flow routing and flood forecasting. As a result, the study of Muskingum method has remained included in the academic programs in many universities and engineering software like HEC-1 (USACE 1998) that use Muskingum method are very much found in use as on today. The traditional Muskingum procedure determines the outflow hydrograph for a given inflow hydrograph. For further description, it is called 'forward routing'. To perform forward routing, the Muskingum procedure requires the calibration of the model coefficients by using a set of observed inflow and outflow hydrograph data. The Muskingum model assumes that the model coefficients capture the overall flood attenuating properties of the river, which remain invariant and independent of the flooding event. In other words, the flood attenuating properties determined by using a historic set of observed inflow and outflow data remains usable and are used for routing of any future inflow hydrograph. The accuracy of flood forecasting by application of Muskingum routing procedure highly depends on the numerical values of model parameters. The art of such parameter identification commonly appears as an analytical-cum-theoretical exercise in which, the observed information on the inflow and outflow hydrographs is given the first priority and the development of the art receives a second priority. Considering this particular view, the earlier researchers attempted to evolve numerical procedures for accurate determination of the Muskingum model parameters for forward routing.

There can be situations that may demand, 'reverse routing' i.e. determination of the inflow hydrograph given the outflow hydrograph. The following applications fit to reverse routing. (i) To sanction emergency rehabilitation projects immediately after the passage of devastating floods, the administrative authorities may require estimating the expenditure that under normal conditions should be proportional to the flood magnitude. If the flood hydrographs at a down stream location only is available, the information about the flood devastations in the upstream reaches of the river can be traced back by using reverse routing. (ii) Human activities such as commercial open cast sand mining on the river bed for use of sand as a building construction material, undertaking of additional river training works, construction of flow retarding structures and bridges, etc. may alter the local flow characteristics in the concerned reach of the river. Any depth gauge existing in the close proximity to such reaches of the river may also fall inadequate to give accurate hydrograph data. To recalibrate such gauges with respect to an unaffected downstream gauge, the reverse routing may prove useful. (iii) The discharge records at an upstream gauging site can be verified with respect to the discharge records at immediate downstream gauging site by using reverse routing. (iv) The upstream gauge may be recalibrated by using reverse routing when a new gauge is installed in between two existing gauges. (v) The disputed reservoir releases (if exists) in inter-state rivers can be verified by using reverse routing. The reverse stream flow routing by using the traditional Muskingum model parameters may lead to serious errors. This paper attempts to prove the need for separate determination of Muskingum model parameters for reverse routing cases.

To make the flood forecasts reliable, the water resources engineers attempt to determine the parameter values of Muskingum models in a manner that the flood predictions are as close to the observed values as possible. For ideal sets of inflow and outflow hydrograph data, in which the outflow hydrograph is obtained by applying the forward Muskingum routing procedure to the inflow hydrograph, one set of Muskingum model parameters is applicable for forward as well as reverse routing of flood hydrographs. But, the real life inflow and outflow hydrograph data are never ideal. Therefore, one set of Muskingum model parameter values may not remain applicable for forward and reverse routing of flood hydrographs. Several methods of parameter estimation for forward routing are available in the literature to accomplish forward routing. The numerical procedure of parameter estimation in forward routing determines a set of parameter values such that the differences between the computed and observed outflow hydrographs are minimum. The existing procedures in the literature use either regression analysis and/or least square error minimization. This study intends to develop a procedure of determining the Muskingum models parameter estimation for reverse routing of flood hydrographs.

The overall Muskingum model is made of (i) the basic Hydrologic continuity equation, and (ii) any one of the three available forms (one linear form and two nonlinear forms) of Muskingum channel storage models. The hydrologic continuity equation, which is written as:

$$I - O = \frac{dS}{dt}, \quad (1)$$

where I = the inflow; O = the outflow; S = the channel storage; and t = the time. The physical channel storage is generally computed by using (1) as:

$$S_{i+1} - S_i = (\Delta t/2)\{(I_{i+1} + I_i) - (O_{i+1} + O_i)\}. \quad (2)$$

For routing of floods by using the Muskingum method, the channel storages in (2) are commonly substituted by any one of the following three forms of Muskingum channel storage models which relate storage, inflow, outflow, and the model parameters (Cunge 1969; Chow *et al* 1988; Linsley *et al* 1982; Wilson 1974; Gill 1978; Tung 1985; Yoon & Padmanabhan 1993; and Mohan 1997).

The linear model is:

$$S = K[XI + (1 - X)O]. \quad (3)$$

One nonlinear model is:

$$S = K[XI + (1 - X)O]^m. \quad (4)$$

Second nonlinear model is:

$$S = K[XI^m + (1 - X)O^m], \quad (5)$$

in which, K = the storage constant; X = the dimensionless weighting factor; and m = exponent power. The routing equations for each of the three models are obtained by substituting (3), (4), and (5) in (2) respectively and are expressed as:

Linear model:

$$\begin{aligned} & K[XI_{i+1} + (1 - X)O_{i+1}] - K[XI_i + (1 - X)O_i] \\ & - (\Delta t/2)\{(I_{i+1} + I_i) - (O_{i+1} + O_i)\} = 0. \end{aligned} \quad (6)$$

Nonlinear model 1:

$$K[XI_{i+1} + (1 - X)O_{i+1}]^m - K[XI_i + (1 - X)O_i]^m - (\Delta t/2)\{(I_{i+1} + I_i) - (O_{i+1} + O_i)\} = 0. \quad (7)$$

Nonlinear model 2:

$$K[XI_{i+1}^m + (1 - X)O_{i+1}^m] - K[XI_i^m + (1 - X)O_i^m] - (\Delta t/2)\{(I_{i+1} + I_i) - (O_{i+1} + O_i)\} = 0. \quad (8)$$

where, subscript i = the time level; and Δt = the time interval. In the forward stream flow routing problem, typically, the numerical values of I_i , I_{i+1} , O_i for any routing period are known. The parameter values K , X , and m are also known (for the linear model $m = 1$). The O_{i+1} is easily determined by solving (6), (7) and (8) for the linear and two nonlinear models respectively. For the reverse stream flow routing problem, the numerical values of I_{i+1} , O_{i+1} , O_i for any routing period should be known. The parameter values K , X , and m should also be known. The I_i is to be determined by solving (6), (7) and (8) for the linear and two nonlinear models respectively. The parameter estimation procedures are used to determine the parameter values and the routing procedures are used to determine the output hydrographs.

Cunge (1969), Gill (1978), Gavilan & Houck (1985), Tung (1985), Yoon and Padmanabhan (1993), Mohan (1997), and Das (2004) reported the application of various numerical techniques for estimation of Muskingum model parameter values for forward stream flow routing. The graphical procedure for determination of routing parameters K and X for the linear model is described in many text references (e.g. Mutreja 1986, Wilson 1974). The graphical procedure assumes that $XI + (1 - X)O$ and S are linearly related. Accordingly the observed S as given by eq. (2) is repeatedly plotted against $XI + (1 - X)O$ for various assumed values of X until a linear relationship is identified. The slope of the linear relationship gives the value of K . The graphical procedure is prone to subjective interpretation of the user regarding the establishment of the said linear relationship.

Gill (1978) converted the graphical procedure to least-square regression analysis based procedure for the linear model. A procedure that solved routing equations for three points within the time horizon of the hydrographs was also suggested for the second nonlinear model. The success of the method depends on the selection of these three points in the time domain of the hydrograph. Tung (1985) used Hook and Jeeves (HJ) pattern search technique in conjunction with (i) linear programming (LP), (2) conjugate gradient method (CG), and (3) Davidon–Fletcher–Powell method (DFP) to determine the parameter values for the first nonlinear model. Tung (1985) used least square regression between the observed storage (2) and computed storage (4) to determine the parameter values. Yoon & Padmanabhan (1993) minimized the residual sum of squares between the actual and flow weighted storages by using nonlinear programming (NONLR) technique. Mohan (1997) used minimization of residual sum of squares between actual and routed outflows to determine the parameter values of the Muskingum models. Mohan (1997) used genetic algorithm of optimization technique to solve the parameter estimation problems. The above methodologies use optimization routines for numerical computation. Das (2004) proposed dual formulations for the parameter estimation procedure, which minimized the sum of square of normalized differences between routed and actual outflows. The routing equations for each time interval were the constraints of the model. However, the above suites of studies are applicable for forward stream flow routing.

The applicability of the Muskingum model parameter values that were suggested in the above mentioned research works for forward routing to reverse stream flow routing case, and if a separate parameter estimation for reverse stream flow routing is needed or not remains as matters of research and are taken up in the present study. A particular form of the common parameter estimation problem attempts to determine the model parameters as well as the model predicted output responses for which the error between the observed and model predicted output responses are minimum (Willis & Yeh 1987). To estimate the parameters of Muskingum models for reverse stream flow routing, the inflow hydrograph information constitute the aforesaid output responses. The consideration of reverse stream flow routing is a major departure of the present methodology compared to the existing studies on forward stream flow routing by using Muskingum models. The objectives of the present study are as follows:

- (i) To develop a methodology for parameter estimation for the reverse stream flow routing.
- (ii) To establish the need for development of parameter estimation models for reverse stream flow routing by using Muskingum method of stream flow routing.
- (iii) To evaluate the performance of the developed model.

2. The parameter estimation models

The proposed parameter estimation models use the concept of minimizing the sum of squares of difference between observed and computed inflows subject to the satisfaction of the routing equation. In these models, the basic concept of regressing between the observed and predicted output responses is utilized. The parameter estimation problem is formulated as constrained nonlinear optimization problem, and a computational scheme is developed that iteratively solves the governing simultaneous equations to identify the nonlinear/linear Muskingum model parameters for reverse stream flow routing.

The parameter estimation methodology seeks to determine the values of K , X , and m (for nonlinear models $m \neq 1$) for which the difference between observed and model-predicted inflows are minimum. The difference can occur in both negative and positive directions from zero. The number of such difference terms appearing in the optimization model will be equal to the number of unknown ordinates of the inflow hydrograph. The magnitude of these differences may vary within a wide range at any iteration and may affect the optimal solutions. To restrain the variation of the difference magnitudes during any iteration within a uniform range, the difference magnitudes are normalized by using the respective observed values of the inflow and the sum of squares of normalized differences between observed and computed inflows are minimized. One can take the respective absolute values of the differences instead of taking the squares of the difference magnitudes, however, at the expense of high degree of nonlinearity, discontinuity and possibility of non-convergence to an optimal solution for the final mathematical programming problem. The objective function of present study is formally expressed as:

$$C1 = \sum_{i=1}^{n+1} \left\{ \frac{(I_{i_o} - I_i)}{I_{i_o}} \right\}^2, \quad (9)$$

in which, $C1$ = the objective function when the difference magnitudes are normalized, I_{i_o} = the observed inflow at time level i , I_i = the computed inflow at time level i , n = the total number of time intervals in the routing problem. By using the above objective function and

any one of the routing equations (6), (7), and (8), three optimization problems denoted as P1, P2, and P3 are formulated. These three models are formally expressed as:

For P1, Minimize: (9) Subject to: (6)

For P2, Minimize: (9) Subject to: (7)

For P3, Minimize: (9) Subject to: (8).

The equations (6), (7) and (8) are to be applied for $i = n, n - 1, n - 2, \dots, 2, 1$. The restrictions that the parameter values and the inflows cannot be negative quantities are also imposed in these optimization problems. One nonlinear objective function and n (= number of routing periods) number of equality type routing equation constraints make the nonlinear optimization problems. There are n constraints and $n + 1$ inflows, of which, n inflows are to be determined because I_{n+1} i.e. inflow at end time is known. The parameter values K , X , and m (for linear Muskingum model $m = 1$) are also to be determined. The overall dimension of the problem (i.e. total number of variables) of the problem depends upon the value of n . Here, K , X , and m are the state variables and the I_i 's are the decision variables of the optimization problems. These constrained optimization problems are transformed into unconstrained problems by using Lagrange multipliers (Reklaitis *et al* 1983). The transformed unconstrained Lagrangian objective functions are written as:

For P1,

Minimize:

$$L = \sum_{i=1}^{n+1} \left\{ \frac{(I_{i_o} - I_i)^2}{I_{i_o}} \right\} + \sum_{i=1}^n \left[K \{ X I_{i+1} + (1 - X) O_{i+1} \} - K \{ X I_i + (1 - X) O_i \} - \frac{\Delta t}{2} \{ (I_{i+1} + I_i) - (O_{i+1} + O_i) \} \right] \lambda_i. \quad (10)$$

For P2,

Minimize:

$$L = \sum_{i=1}^{n+1} \left\{ \frac{(I_{i_o} - I_i)^2}{I_{i_o}} \right\} + \sum_{i=1}^n \left[K \{ X I_{i+1} + (1 - X) O_{i+1} \}^m - K \{ X I_i + (1 - X) O_i \}^m - \frac{\Delta t}{2} \{ (I_{i+1} + I_i) - (O_{i+1} + O_i) \} \right] \lambda_i. \quad (11)$$

For P3,

Minimize:

$$L = \sum_{i=1}^{n+1} \left\{ \frac{(I_{i_o} - I_i)^2}{I_{i_o}} \right\} + \sum_{i=1}^n \left[K \{ X I_{i+1}^m + (1 - X) O_{i+1}^m \} - K \{ X I_i^m + (1 - X) O_i^m \} - \frac{\Delta t}{2} \{ (I_{i+1} + I_i) - (O_{i+1} + O_i) \} \right] \lambda_i, \quad (12)$$

in which λ_i = the Lagrange multipliers corresponding to the i 'th constraint. The Lagrange multipliers indicate the effect of a unit change in the right hand side vector of the constraints on the objective function. If the value of any particular λ_i becomes equal to zero, it indicates

that the particular constraint is redundant. For the present problems, all λ_i 's must be nonzero, because the constraints represent the routing equations.

To obtain optimal solutions, the first order necessary conditions for optima (Reklaitis *et al* 1983) are used. The values of λ_i 's are determined from the following necessary condition.

$$\frac{\partial L}{\partial I_i} = 0 \text{ for } i = 1, 2, 3, \dots, n. \quad (13)$$

Here (13) is not applicable for $i = n + 1$ because I_{n+1} is a known quantity for the routing problems. By applying (13) for the three optimization problems the following expressions are obtained.

For P1,

$$\left\{ -KX - \frac{\Delta t}{2} \right\} \lambda_i = \frac{2(I_{i_o} - I_i)}{I_{i_o}^2} \text{ for } i = 1 \quad (14)$$

and,

$$\left\{ KX - \frac{\Delta t}{2} \right\} \lambda_{i-1} + \left\{ -KX - \frac{\Delta t}{2} \right\} \lambda_i = \frac{2(I_{i_o} - I_i)}{I_{i_o}^2} \text{ for } i = 2, 3, \dots, n. \quad (15)$$

For P2,

$$\left[-KXm\{XI_i + (1 - X)O_i\}^{m-1} - \frac{\Delta t}{2} \right] \lambda_i = \frac{2(I_{i_o} - I_i)}{I_{i_o}^2} \text{ for } i = 1 \quad (16)$$

and,

$$\begin{aligned} & \left[KXm\{XI_i + (1 - X)O_i\}^{m-1} - \frac{\Delta t}{2} \right] \lambda_{i-1} \\ & + \left[-KXm\{XI_i + (1 - X)O_i\}^{m-1} - \frac{\Delta t}{2} \right] \lambda_i = \frac{2(I_{i_o} - I_i)}{I_{i_o}^2} \text{ for } i = 2, 3, \dots, n. \end{aligned} \quad (17)$$

For P3,

$$\left[-KXmI_i^{m-1} - \frac{\Delta t}{2} \right] \lambda_i = \frac{2(I_{i_o} - I_i)}{I_{i_o}^2} \text{ for } i = 1 \quad (18)$$

and,

$$\left[KXmI_i^{m-1} - \frac{\Delta t}{2} \right] \lambda_{i-1} + \left[-KXmI_i^{m-1} - \frac{\Delta t}{2} \right] \lambda_i = \frac{2(I_{i_o} - I_i)}{I_{i_o}^2} \text{ for } i = 2, 3, \dots, n. \quad (19)$$

For given values of I_i , O_i , K , X , and m , the set of expressions (14) and (15); (16) and (17); and (18) and (19) represent linear simultaneous equations to determine the values of λ_i 's for

P1, P2, and P3 respectively which can be expressed in generalized matrix form as:

$$\begin{pmatrix} a_{1,1} & 0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{2,1} & a_{2,2} & 0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & a_{i,i-1} & a_{i,i} & 0 & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{n-1,n-2} & a_{n-1,n-1} & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{n,n-1} & a_{n,n} & \dots \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \dots \\ \dots \\ \dots \\ \lambda_i \\ \dots \\ \dots \\ \dots \\ \lambda_{n-1} \\ \lambda_n \end{pmatrix} = \begin{pmatrix} B_1 \\ B_2 \\ \dots \\ \dots \\ \dots \\ B_i \\ \dots \\ \dots \\ \dots \\ B_{n-1} \\ B_n \end{pmatrix} \quad (20)$$

in which, a = the coefficient terms corresponding to the λ values; and B = the right hand side vector. The λ_i 's are determined by using forward-substitution. The first order necessary condition for optima that gives the routing equations is expressed as:

$$\frac{\partial L}{\partial \lambda_i} = 0 \text{ for } i = 1, 2, \dots, n. \quad (21)$$

To determine the K values, the required necessary condition is expressed as:

$$\frac{\partial L}{\partial K} = 0, \quad (22)$$

which is applied to P1, P2, and P3 separately to obtain the following expressions.

For P1,

$$\sum_{i=1}^n [\{XI_{i+1} + (1-X)O_{i+1}\} - \{XI_i + (1-X)O_i\}]\lambda_i = 0 \quad (23)$$

For P2,

$$\sum_{i=1}^n [\{XI_{i+1} + (1-X)O_{i+1}\}^m - \{XI_i + (1-X)O_i\}^m]\lambda_i = 0. \quad (24)$$

For P3,

$$\sum_{i=1}^n [\{XI_{i+1}^m + (1-X)O_{i+1}^m\} - \{XI_i^m + (1-X)O_i^m\}]\lambda_i = 0. \quad (25)$$

To determine the X values, the required necessary condition is expressed as:

$$\frac{\partial L}{\partial X} = 0. \quad (26)$$

For P1 (26) gives:

$$\sum_{i=1}^n [K\{I_{i+1} - O_{i+1}\} - \{I_i - O_i\}]\lambda_i = 0. \quad (27)$$

For P2 (26) gives:

$$\sum_{i=1}^n [Km(I_{i+1} - O_{i+1})\{XI_{i+1} + (1 - X)O_{i+1}\}^{m-1} - Km(I_i - O_i)\{XI_i + (1 - X)O_i\}^{m-1}] \lambda_i = 0. \quad (28)$$

For P3 (26) gives:

$$\sum_{i=1}^n [K(I_{i+1}^m - O_{i+1}^m) - K(I_i^m - O_i^m)] \lambda_i = 0. \quad (29)$$

To determine the m values for P2 and P3, the required necessary condition is given as:

$$\frac{\partial L}{\partial m} = 0. \quad (30)$$

For P2 (30) gives:

$$\sum_{i=1}^n [K\{XI_{i+1} + (1 - X)O_{i+1}\}^m \log\{XI_{i+1} + (1 - X)O_{i+1}\} - K\{XI_i + (1 - X)O_i\}^m \log\{XI_i + (1 - X)O_i\}] \lambda_i = 0. \quad (31)$$

For P3 (30) gives:

$$\sum_{i=1}^n [K\{XI_{i+1}^m \log(I_{i+1}) + (1 - X)O_{i+1}^m \log(O_{i+1})\} - K\{XI_i^m \log(I_i) + (1 - X)O_i^m \log(O_i)\}] \lambda_i = 0. \quad (32)$$

To estimate the Muskingum model parameters, (23) and (27); (24), (28), and (31); and (25), (29), and (32) must be iteratively solved for P1, P2, and P3 respectively. Bi-section method is used to solve each of these equations. To solve these equations, the λ_i values are determined by using (20) and to use (20) the most updated values of K , X , and m and I_i 's are used. For the most updated values of K , X , and m , the updated values of I_i 's are computed again by using the method of bi-section. The following steps are used in the developed iterative procedure.

- Step 1. Initialize: Observed inflow and outflow hydrograph data, desired type of model, convergence parameters, K , X , m_{old} , and $\Delta = 1.0$.
- Step 2. For nonlinear models go to step 3 and for linear model go to step 4.
- Step 3. $m = m_{\text{old}}$.
- Step 4. $X_{\text{old}} = X$, $K_{\text{old}} = K$.
- Step 5. Compute K from $\frac{\partial L}{\partial K} = 0$.
- Step 6. Compute X from $\frac{\partial L}{\partial X} = 0$.
- Step 7. If $X \approx X_{\text{old}}$, and $K \approx K_{\text{old}}$ go to next step, else go to step 4.
- Step 8. For linear model stop and for nonlinear model go to next step.
- Step 9. Compute m from $\frac{\partial L}{\partial m} = 0$.
- Step 10. If $m \approx m_{\text{old}}$ stop else go to next step.
- Step 11. If $m > m_{\text{old}}$ then $m_{\text{old}} = m_{\text{old}} + \Delta$.
- Step 12. If $m < m_{\text{old}}$ then $m_{\text{old}} = m_{\text{old}} - \Delta$, $\Delta = \Delta/10.0$, and $m_{\text{old}} = m_{\text{old}} + \Delta$.
- Step 13. Repeat step 3 onwards.

Here, the subscript ‘old’ refers to old values of the variables K , X , and m , respectively, and Δ is an incremental parameter. In this algorithm, only one variable value is updated at a given step while the other two variable values are kept unchanged at their immediate old values for that given step only. To compute the value of K from $\frac{\partial L}{\partial K} = 0$, the values of X and m are kept fixed at their immediate previous values. Similarly, when X is computed from $\frac{\partial L}{\partial X} = 0$, the values of K and m are kept fixed at their immediate previous values. Likewise, when m is computed from $\frac{\partial L}{\partial m} = 0$, the value of K and X are kept fixed at their immediate previous values. The initial values of K , X , and m , however, have no influence on the final estimate of the parameter values. They are initiated with any arbitrary positive non-zero values only to start the computation. The procedure determines the final values through iterations.

3. Evaluation of the developed methodology

As a common practice in research, the earlier researchers adopted Wilson’s (1974) single set of inflow–outflow hydrograph data for Muskingum model parameter estimation techniques for forward stream flow routing, and suggested a number of sets of parameter values some of which are given in table 1. Identical to previous researcher’s approach, the present study also uses 100% of the inflow and outflow hydrograph data of a single flood event given by Wilson (1974) but for reverse stream flow routing. The principle of Muskingum model parameter estimation by using the Wilson (1974) data that is used in the present study and also earlier studies reported in the literature are far away from the principles of traditional calibration and validation in other fields of water resources systems modelling. In traditional modelling, parts of the observed data (usually 50 to 75%) are used for calibration and the remaining data are used for validation. But, in Muskingum model parameter estimation research, the complete inflow/outflow hydrograph data are used for calibration. The calibration procedure generates computed inflow/outflow hydrograph data. The traditional validation step is not performed in research because; the Wilson (1974) hydrograph data is for a single flood event for a river reach and Wilson (1974) did not provide another flood event data for the same river reach. This limitation is well recognized in the literature.

The developed parameter estimation methodology is applied to determine the K , X , and m values for reverse stream flow routing corresponding to the Wilson’s (1974) inflow-outflow

Table 1. Parameter values of earlier studies for forward stream flow routing.

Method	K	X	M	Model
GA	0.1033	0.2813	1.8282	Nonlinear 1
HJ + CG	0.0699	0.2685	1.9291	Nonlinear 1
HJ + DFP	0.0764	0.2677	1.8978	Nonlinear 1
NONLR	0.0600	0.2700	2.3600	Nonlinear 1
Gill	0.0100	0.2500	2.3470	Nonlinear 1
Das	0.0753	0.2769	2.2932	Nonlinear 1
GA	0.1773	0.2305	1.7105	Nonlinear 2
NONLR	0.0500	0.2200	2.3500	Nonlinear 2
Das	0.0437	0.1702	2.3932	Nonlinear 2
Wilson	36.0000	0.2500	1.0000	Linear
Das	24.3804	0.2010	1.0000	Linear

Table 2. Estimated parameter values in the present study for reverse stream flow routing.

K	X	m	Model
0.0852	0.2917	2.2626	Nonlinear 1
0.0642	0.2751	2.2914	Nonlinear 2
22.0011	0.3396	1.0000	Linear

data. Table 2 gives the estimated parameter values. The parameter estimation model also predicts the inflow hydrograph corresponding to Wilson's (1974) inflow–outflow hydrograph data and the model estimated parameter values. Three inflow hydrograph corresponding to three parameter estimation models P1, P2, and P3 corresponding to the three forms Muskingum models respectively are predicted and are presented in tabular form tables 3–5 and figures 1–3 respectively to facilitate easy retrieval of the data by the future researchers.

To investigate the necessity for developing a methodology for parameter estimation for reverse stream flow routing, the forward stream flow routing parameter values of earlier

Table 3. Comparison of hydrographs for nonlinear model 1.

[illegible]

Table 4. Comparison of hydrographs for nonlinear model 2.

Time (hr)	Wilson's observed hydrographs (m ³ /s)		Computed inflow hydrographs (m ³ /s)			
	Outflow	Inflow	GA	NONLR	Das	Present study
0.0	22.0	22.0	22.0	21.1	21.8	20.2
6.0	21.0	23.0	19.9	19.0	17.6	22.8
12.0	21.0	35.0	22.6	36.0	32.0	41.3
18.0	26.0	71.0	29.7	59.5	58.0	62.1
24.0	34.0	103.0	40.3	80.4	82.3	79.9
30.0	44.0	111.0	52.5	95.6	100.0	92.5
36.0	55.0	109.0	65.8	103.7	109.7	99.2
42.0	66.0	100.0	75.8	104.0	110.2	98.9
48.0	75.0	86.0	83.9	98.4	104.1	93.5
54.0	82.0	71.0	86.7	85.1	88.9	81.7
60.0	85.0	59.0	84.6	69.1	70.3	67.6
66.0	84.0	47.0	80.2	54.4	53.5	54.3
72.0	80.0	39.0	72.9	39.2	36.0	40.9
78.0	73.0	32.0	64.1	28.1	23.1	31.0
84.0	64.0	28.0	54.9	22.2	17.7	24.9
90.0	54.0	24.0	45.5	19.0	14.6	21.7
96.0	44.0	22.0	37.6	21.9	19.3	23.3
102.0	36.0	21.0	32.3	23.3	22.5	23.6
108.0	30.0	20.0	26.6	20.7	19.7	21.1
114.0	25.0	19.0	23.6	21.2	21.2	20.9
120.0	22.0	19.0	20.5	17.7	17.4	17.8
126.0	19.0	18.0	18.0	18.0	18.0	18.0

researchers given in table 1, Wilson's (1974) full outflow hydrograph data and inflow data at the end time (i.e. at $t = 126$ hours) are used to obtain respective inflow hydrograph by running the reverse routing process. These inflow hydrographs are also presented in tables 3, 4, and 5 and figures 1, 2, and 3 according to the use of respective Muskingum models. To further assess the effectiveness of the developed methodology, the sums of squares of normalized errors as represented by Eq. (9) were computed for each of the computed inflow hydrographs of tables 3, 4, and 5. Table 6 gives this comparative analysis of prediction error. It may be observed that the presently developed model gives the lowest prediction error for all three Muskingum models.

A common reader may analyse the results in terms of normalized deviations of the predicted results with respect to their observed values where the normalizing factor is the observed value. Here, due importance is given to the observed values by taking it as normalizing factor. If such an analysis is allowed any importance, it would indicate a range of variation from -36.5% to $+31.0\%$ and 15 out of 22 values with more than $+16\%$ error for the linear model. From the above statistics, a common reader may infer the imperfect performance of the linear model. Here, the performance indicator function must be selected with care. Simple normalized deviations of individual values indicate their respective biases, when the square of the normalized deviations indicates their accuracies. An imperfection can be either negative or positively biased within a wide range. But accuracy is always positive and hence sum of the accuracies is used as the final performance indicator in parameter estimation modelling as done in the present study. Here three types of storage models are used. For a given data set, the

Table 5. Comparison of hydrographs for linear model.

Time (hr)	Wilson's observed hydrographs (m ³ /s)		Computed inflow hydrographs (m ³ /s)		
	Outflow	Inflow	Wilson	Das	Present study
0-0	22.0	22.0	29.5	20.3	24.4
6-0	21.0	23.0	41.9	26.7	31.4
12-0	21.0	35.0	62.8	44.8	45.4
18-0	26.0	71.0	79.7	60.7	58.6
24-0	34.0	103.0	93.4	75.5	71.1
30-0	44.0	111.0	102.8	88.0	81.6
36-0	55.0	109.0	106.6	97.0	89.0
42-0	66.0	100.0	103.1	99.4	91.5
48-0	75.0	86.0	95.2	98.4	90.4
54-0	82.0	71.0	80.5	89.4	83.6
60-0	85.0	59.0	64.0	77.2	74.0
66-0	84.0	47.0	47.9	64.3	63.2
72-0	80.0	39.0	31.8	49.4	50.9
78-0	73.0	32.0	18.7	35.6	39.4
84-0	64.0	28.0	9.4	24.0	29.5
90-0	54.0	24.0	4.8	15.9	22.4
96-0	44.0	22.0	5.5	13.8	19.2
102-0	36.0	21.0	7.1	13.1	17.2
108-0	30.0	20.0	8.1	11.8	15.6
114-0	25.0	19.0	11.2	13.5	15.8
120-0	22.0	19.0	12.5	12.5	15.3
126-0	19.0	18.0	18.0	18.0	18.0

Figure 1. Inflow hydrographs for nonlinear model 1.

Figure 2. Inflow hydrographs for nonlinear model 2.

storage model that gives the minimum value of the performance indicator should be selected for use. For the present application, the first nonlinear model should be chosen for use.

A further observation of inflow hydrograph ordinates reveals that although the use of forward routing parameter of NONLR and Das (2004) resulted in good peak values of inflow

Figure 3. Inflow hydrographs for linear model.

Table 6. Statistical comparison.

Value of Eq. (9)	Methodology	Model
6.5640	GA	Nonlinear 1
6.3445	HJ + CG	Nonlinear 1
6.5200	HJ + DFP	Nonlinear 1
0.1483	NONLR	Nonlinear 1
6.6007	Gill	Nonlinear 1
0.0967	Das	Nonlinear 1
0.0881	Present Methodology	Nonlinear 1
6.5344	GA	Nonlinear 2
0.3760	NONLR	Nonlinear 2
0.7336	Das	Nonlinear 2
0.2779	Present Methodology	Nonlinear 2
4.4203	Wilson	Linear
1.4453	Das	Linear
1.0075	Present Methodology	Linear

hydrograph, their overall performance is inferior to the results of present study. Further, an observation of parameter values in tables 1 and 2 reveals that completely new sets of parameter values are obtained for reverse stream flow routing. This particular trend of result certainly proves the need for developing a separate parameter estimation model for Muskingum models for reverse stream flow routing and the necessity is fulfilled through the presently developed methodology. Also, the above analysis certainly proves the satisfactory performance of the present models in totality. The models can be easily applied for actual reverse flood routing problems. To apply the model, the user does not need to use the standard optimization packages.

A basic assumption for application of Muskingum models for traditional stream flow routing is that the model parameters capture the combined forward flood propagating characteristics of a river reach. Another interpretation of the above trend of result is that the Muskingum models also capture the combined backward propagation characteristics for reverse routing of flood waves in a river reach. The non-uniqueness of the parameter values in the two cases of stream flow routing i.e. forward routing and reverse routing supports the above statement.

Actual field data are not used in the present study. This limitation exists in all earlier studies reported in the literature and this study is no exception. Checking of robustness of the proposed algorithm by using real-life data remains as an issue of future research. Real-life data is however prone to measurement inaccuracies and effects of other sources of uncertainties, treating of which may warrant developing a chance constrained model and also remains as future research topic.

The physical flow measurements in rivers are subject to many uncertainties and the hydrograph data generally contains measurement errors. For an ideal case with no measurement errors the two sets of parameter values for forward stream flow routing and backward stream flow routing may converge to a single set of parameter values. However, such a situation is rare. From the physical standpoint also the Muskingum models should have two sets of parameter values for forward and reverse stream flow cases. The presently developed methodology determines the parameter values for reverse stream flow routing of channels.

4. Conclusions

The concept of reverse stream flow routing by using Muskingum models is introduced. Parameter estimation models for reverse stream flow routing are developed that emphasize minimizing the normalized differences between observed and predicted inflows subject to satisfaction of the Muskingum stream flow routing equations. Three forms of Muskingum models are used for calibration. The Muskingum storage models are combined within the routing equation. In the developed methodology, unconstrained objective functions for each of the three forms of Muskingum models are formulated by using Lagrange multipliers. The necessary condition equations for minimization are used to derive the governing equations for parameter estimation. A generalized solution technique for solving the governing equations for parameter estimation is developed and applied to a standard inflow–outflow data set. The evaluation test yields parameter values for the three types of Muskingum channel storage models and corresponding inflow hydrographs fit the observed inflows to satisfactory level. The performance evaluation tests indicate that separate calibration of the Muskingum models for reverse stream flow routing must be performed before actual determination of the inflow hydrographs.

List of symbols

I = inflow (L^3/T);
 K = storage constant;
 m = exponent power;
 n = total number of time stages in the stream flow routing process;
 O = outflow (L^3/T);
 S = channel storage (L^3);
 t = time (T);
 X = dimensionless weighting factor;
 Δt = time interval (T);
 λ_i = Lagrange multiplier for i 'th constraint.

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