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Effect of surface stress and irregularity of the interface on the propagation of SH-waves in the magneto-elastic crustal layer based on a solid semi space

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Abstract. The object of the present paper is to investigate plane SH waves through a magneto-elastic crustal layer based over an elastic, solid semi space under the influence of surface stress on the free surface of the crustal layer and irregularity of the interface. Two types of irregularities of the interface namely, rectangular and parabolic have been considered. Modulations of wave velocity due to the presence of surface stress, irregularity and the magnetic field have been studied separately. Their combined effect has also been investigated. Graphs are drawn to highlight some important peculiarities. It is observed that surface stress, irregularity and magnetic field have their respective role to play in the propagation of SH waves in the crustal layer. Further modulation of wave velocity occurs due to their combined effect.

Keywords. Surface stress; irregularity of the interface; magneto-elastic crustal layer; dispersion equation, SH-waves, wave velocity equation.

1. Introduction

From the literatures presented by Gurtin & Murdoch (1976), Chandrasekharaiah (1987a), and some other authors (Plaster 1972; Pal *et al* 1997; Gurtin 1972) it is known that surface stress plays a vital role in the propagation of waves due to the fact that the surface of a body exhibits properties quite different from those associated with the interior of the medium. For example, surface tension which is generally present in liquid may be considered as a particular case of surface stress. Presence of surface stress on the boundary of any body has been detected in some particular type of crystals where the order of magnitude agrees with the predictions made by the molecular theory (Gurtin & Murdoch 1976). Compressive surface stress is involved in the case of short peening of ductile metals (Gurtin & Murdoch 1976). Visible strain arises due to this process. This process is used in the shaping of aircraft wing panels

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(Gurtin & Murdoch 1976). A few problems of the propagation of plane waves in homogeneous and isotropic materials were considered by Gurtin & Murdoch (1976). Though the concept of surface stress is comparatively of recent origin, a few authors (Chandrasekharaiah 1987a, Plaster 1972) investigated problems which are based on the effect of surface stress from different angles. Pal *et al* (1997) investigated the effect of surface stress on the propagation of surface waves.

Knowledge of the propagation of waves in elastic media with non parallel boundaries plays its role to understand and predict the seismic behaviour at continentals margins, mountain roots, etc. Looking forward to the above concept, effect of variation of crustal thickness on Love waves were studied by different authors like De Noyer (1961), Mal (1962), Bhattacharya (1962), Sato (1952), Coulomb (1952), Stoneley (1924), Wolf (1967), Chattopadhyay (1975a), Dutta (1963a,1963b), Kar (1977).

It is noticed that seismic signals propagating through the Earth medium have to travel through various kinds of minerals which are present in the material of the Earth in the form of layers. The propagation of seismic waves is definitely effected by the elastic properties of these layered materials. Moreover, the materials of the layer might be magneto-elastic in nature. Interplay of electromagnetic field with the motion of a deformable solid has its importance owing to its theoretical and practical relevance in various branches of science and technology. Different aspects of waves in magneto-elasticity were investigated by several authors including Knopoff (1955), Dunkin & Eringen (1963), Yu & Tang (1966), Tomita & Shindo (1979). A survey of linear and nonlinear wave motion in a perfectly magneto-elastic medium has been made by Bazer (1984). Othman & Song (2006), Chattopadhyay *et al* (1998), Chattopadhyay & Chowdhury (1995) investigated a few problems related to the propagation of SH wave in magneto-elastic and thermoelastic medium. An excellent review of wave motion in magnetizable deformable media will also be seen in the research papers of Maugin (1988b), Lee & Its (1992). Hence, SH waves are always influenced by the magneto-elastic nature of media through which they have to propagate. From the literatures mentioned above it is observed that the authors did not investigate any problem where the effect of surface stress, irregularity of the surface of separation and the magneto-elastic nature of the crustal layer have been considered simultaneously, though the effect of irregular boundary and the surface stress has a prominent role in the propagation of magneto-elastic waves, specially surface waves. Motivated by the practical situation described above which the geophysicist and civil engineers have to face during the study of the seismic waves the present authors analyse the problem of the propagation of SH waves where the crustal layer is magneto-elastic with non parallel boundaries under the action of surface stress on its free surface. In our problem we have considered two types of irregularities, namely rectangular and parabolic and also comparison has been made. Variations of wave velocities due to the presence of surface stress, irregularity and the magnetic field have been studied separately. Their combined effect has also been studied. Many important peculiarities have been pointed out from the graphs. It is seen that the range of possible values of non-dimensional form of SH wave velocity depends on the magneto-elastic parameters. For real wave velocity two particular types of magnetic field, namely:

(i) magnetic field, acting in the direction of wave propagation and (ii) magnetic field acting in the vertical direction, transverse to the direction of wave propagation have been studied in detail. It is believed that the problem in its present form has not been investigated by any of the previous authors.

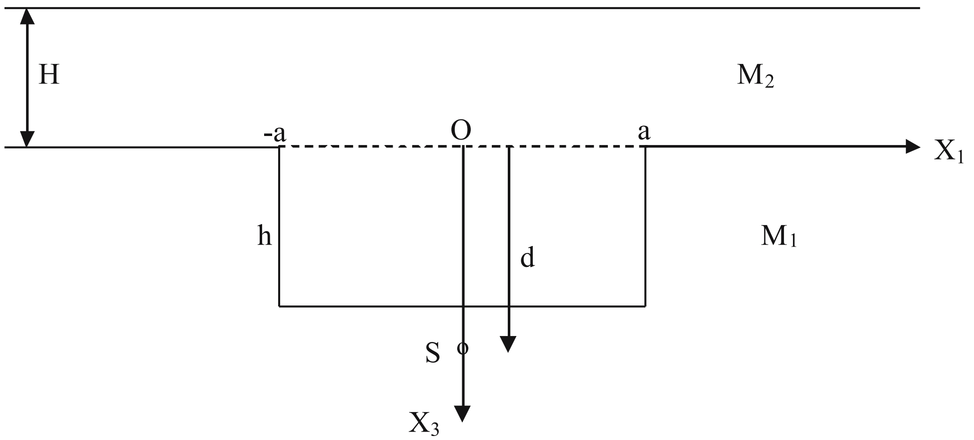

Figure 1. Schematic diagram for rectangular irregularity.

2. Formulation of the problem

Let us consider a model which consists of an isotropic perfectly conducting magneto-elastic crustal layer M_2 with one rectangular or parabolic irregularity on the interface between the layer and a semi infinite isotropic elastic medium M_1 as shown in the figures 1 and 2.

Let H be the thickness of the layer except for the irregularities. Let us consider a rectangular system of coordinate axes $OX_1X_2X_3$ with the origin O at the middle point of the span, $2a$, of the irregularities. X_1 and X_3 axis are taken as shown in the figures. The positive direction of X_3-axis points vertically downwards in the medium M_1. Let us also consider that a very thin elastic continuum of two-dimensions adheres to the free surface $x_3 = -H$ of the crustal layer. Both the media M_1 and M_2 are assumed to be homogeneous and isotropic. Let S be the position of the source on X_3-axis at a depth d below the origin where $d > h$, in which h is the depth of the rectangular irregularity held horizontally and in case of parabolic irregularity

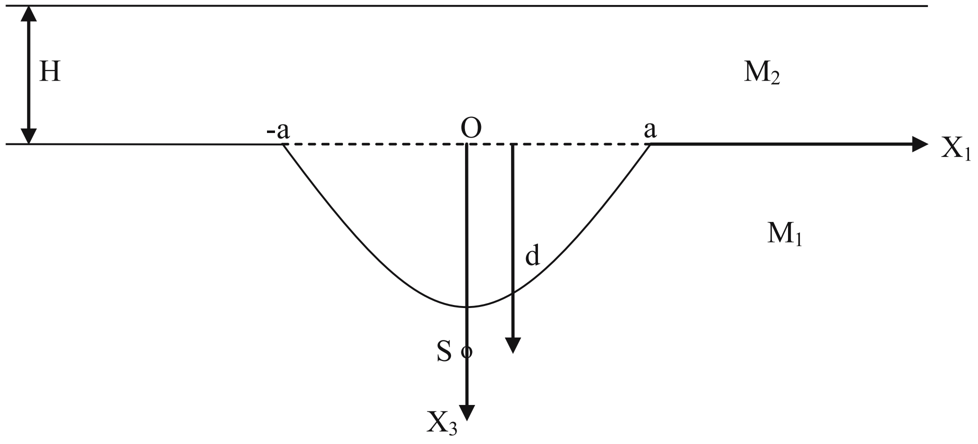

Figure 2. Schematic diagram for parabolic irregularity.

h is the depth of the vertex below $x_3 = -H$. We assume that this source produces a time harmonic disturbance.

Thus the equations of the interfaces in the cases of rectangular and parabolic irregularities may respectively be presented as

$$\begin{aligned} x_3 &= \in f(x_1) = h \quad \text{for} \quad -a \le x_1 \le a \\ &= 0 \quad \text{for } |x_1| > a \end{aligned} \tag{1}$$

and

$$\begin{aligned} x_3 &= \in f(x_1) = h\left(1 - \frac{x_1^2}{a^2}\right) \quad \text{for } |x_1| \le a \\ &= 0 \quad \text{for } |x_1| > a, \end{aligned} \tag{2}$$

where $\in = \frac{h}{2a} \ll 1$ is the perturbation parameter.

Maxwell's fundamental equations and Maxwell's stress tensor, $(\tau_{ij})^M$ (Choudhary *et al* 2004) are respectively given by

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}, \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \nabla \cdot \boldsymbol{B} = 0, \boldsymbol{B} = \mu_e \boldsymbol{H}, \boldsymbol{J} = \sigma_0 \left[\boldsymbol{E} + \frac{\partial \boldsymbol{u}}{\partial t} \times \boldsymbol{B}\right]. \tag{3}$$

and

$$(\tau_{ij})^M = \mu_e (H_i h_j + H_j h_i - H_k h_k \delta_{ij}), \tag{4}$$

where $\boldsymbol{H} = (H_{x_1}, H_{x_2}, H_{x_3})$ is the total applied field, $\boldsymbol{J}$ is the electric current vector, $\boldsymbol{E}$ is the induced electric field, $\boldsymbol{B}$ is the magnetic induction vector, μ_e is the magnetic permeability, σ_0 is the electrical conductivity, $\boldsymbol{h} = (h_1, h_2, h_3)$ is the change in the basic magnetic field $(H_1, 0, H_3)$, $\boldsymbol{u}$ is the displacement vector, δ_{ij} is the Kronecker delta, t denotes time and $i, j, k = 1, 2, 3$. Usual summation convention for the repeated index is applicable.

To suit the actual situation of the considered problem for the propagation of SH waves along X_1 axis one may take displacement vector as $\boldsymbol{u} = (0, v, 0)$ and all derivatives with respect to x_2 coordinate are equal to zero. In view of the above concept, taking the initial magnetic field as $(H_1, 0, H_3)$ and following (Choudhary *et al* 2004) one may deduce from (4) that there is no perturbation of the initial magnetic field in the x_1, x_3 direction and the total applied magnetic field may be presented as $H_{x_1} = H_1, H_{x_2} = h_2, H_{x_3} = H_3$ where the small perturbation h_2 in the direction of x_2 axis may be expressed as

$$h_2 = H_1 \frac{\partial v}{\partial x_1} + H_3 \frac{\partial v}{\partial x_3}. \tag{5}$$

Under the present situation the Lorenz force which has only one non-zero component in the direction of x_2 axis is given by

$$\boldsymbol{J} \times \boldsymbol{B} = \left(0, \mu_e \left(H_1 \frac{\partial h_2}{\partial x_1} + H_3 \frac{\partial h_2}{\partial x_3}\right), 0\right). \tag{6}$$

Insertion of (5) in (6) leads to

$$(\boldsymbol{J} \times \boldsymbol{B})_2 = \mu_e \left(H_1^2 \frac{\partial^2 v}{\partial x_1^2} + 2H_1 H_3 \frac{\partial^2 v}{\partial x_1 \partial x_3} + H_3^2 \frac{\partial^2 v}{\partial x_3^2}\right). \tag{7}$$

The dynamical equations of motion for a homogeneous isotropic perfectly conducting elastic medium M_2 having only Lorentz force may be presented as

$$\tau'_{ij,j} + (\boldsymbol{J} \times \boldsymbol{B})_i = \rho' \frac{\partial^2 u'_i}{\partial t^2} \quad i, j = 1, 2, 3, \tag{8}$$

where τ'_{ij} = components of the stress tensor $= \lambda' e'_{kk}\delta_{ij} + 2\mu' e'_{ij}$ in which $e'_{ij} = \frac{1}{2}\left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right)$ is the components of strain tensor, λ', μ' are Lame' elastic constants, ρ' is the density of M_2, $\boldsymbol{u}$ is the displacement vector such that $\boldsymbol{u} = u'_i = (u'_1, u'_2, u'_3)$. $\tau'_{ij,j} = \frac{\partial \tau'_{ij}}{\partial x_j}$, x_j denotes space coordinates of a point.

Inview of the above context the dynamical equations of motion valid for the magneto-elastic layer which is not identically satisfied may be presented as

$$P\frac{\partial^2 v_2}{\partial x_3^2} + S\frac{\partial^2 v_2}{\partial x_1 \partial x_3} + Q\frac{\partial^2 v_2}{\partial x_1^2} = \rho' \frac{\partial^2 v_2}{\partial t^2}, \tag{9}$$

where $P = \mu' + \mu_e H_3^2$, $S = 2\mu_e H_1 H_3$, $Q = \mu' + \mu_e H_1^2$, in which the displacement vector is taken as $u'_i = (0, v_2, 0)$, μ' is the rigidity of the layer M_2.

For the semi-infinite elastic medium the relevant equation of motion is taken as

$$\frac{\partial^2 v_1}{\partial x_1^2} + \frac{\partial^2 v_1}{\partial x_3^2} = \frac{1}{\beta_1^2}\frac{\partial^2 v_1}{\partial t^2} \tag{10}$$

where $\beta_1^2 = \mu/\rho$ in which μ and ρ are rigidity and density and $u_i = (0, v_1, 0)$ is the displacement vector for the medium M_1.

It is assumed that the plane $x_3 = -H$ is a material medium adhering to the layer and is a two-dimensional elastic continuum capable of supporting its own stress represented by $\Sigma_{i\alpha}$ which obeys the law (Gurtin & Murdoch 1976)

$$\begin{aligned} \Sigma_{i\alpha} &= \delta_{i\alpha}[\sigma + (\lambda_0 + \sigma)u'_{r,r}] + \mu_0 u'_{i,\alpha} + (\mu_0 - \sigma)u'_{\alpha,i} \quad \text{for } i, \alpha, r = 1, 2 \\ &= \sigma u'_{3,\alpha} \quad \text{for } i = 3, \end{aligned} \tag{11}$$

where λ_0, μ_0 are Lame' elastic constants and σ is the residual surface tension of material boundary $x_3 = -H$.

Since the wave propagating in positive x_1-direction is time harmonic, we may take the solutions of (10) and (9) as $v_j = V_j(x_1, x_3)e^i\omega t (j = 1, 2 \& i = \sqrt{-1})$ and then applying Fourier transform technique in the following form

$$\bar{V}_1(\eta, x_3) = \int_{-\infty}^{\infty} V_1(x_1, x_3)e^{i\eta x_1}dx_1$$

so that

$$V_1(x_1, x_3) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{V}_1(\eta, x_3)e^{-i\eta x_1}d\eta \tag{12}$$

the equations of motion finally transforms to

$$\frac{d^2\bar{V}_1}{dx_3^2} = p_1^2 \bar{V}_1 \tag{13}$$

for the medium M_1 and

$$P\frac{d^2\bar{V}_2}{dx_3^2} - i\eta S\frac{d\bar{V}_2}{dx_3} + (\rho'\omega^2 - Q\eta^2)\bar{V}_2 = 0, \tag{14}$$

for the medium M_2, where $p_1^2 = \eta\left(1 - \frac{c^2}{\beta_1^2}\right)$ in which $\frac{\omega}{\eta} = c$, c being the common wave velocity of the waves propagated along the surface.

The solutions of (13) and (14) are taken as

$$\bar{V}_1(\eta, x_3) = Ae^{-p_1x_3}, \qquad x_3 \geq \in f(x_1)$$

$$\bar{V}_2(\eta, x_3) = Be^{p_2x_3} + Ce^{p_3x_3}, \qquad -H \leq x_3 \leq \in f(x_1), \tag{15}$$

where A,B,C are functions of η only and $p_2 = i\eta\left(\frac{S}{2P} + \gamma_2\right)$, $p_3 = i\eta\left(\frac{S}{2P} - \gamma_2\right)$, in which $\gamma_2 = \sqrt{\frac{S^2}{4P^2} + \frac{1}{P}(\rho'c^2 - Q)}$.

Therefore, the displacements in the medium M_1 and in the layer M_2 are given by

$$V_1 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(Ae^{-p_1x_3} + \frac{2}{p_1}e^{p_1x_3}e^{-p_1d}\right)e^{-i\eta x_1}d\eta, \; x_3 \geq \in f(x_1)$$

$$V_2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}(Be^{p_2x_3} + Ce^{p_3x_3})e^{-i\eta x_1}d\eta, \; -H \leq x_3 \leq \in f(x_1), \tag{16}$$

where the second term in the expression of V_1 is due to the effect of the source at S (figures 1 and 2).

3. Boundary conditions

We consider the following boundary conditions:

(i) The continuity of the displacements on the interface gives rise to the condition

$$V_1 = V_2 \text{ on } x_3 = \in f(x_1).$$

(ii) In view of the irregular boundary surface of separation the stress continuity condition across the interface boundary $x_3 = \in f(x_1)$ may be written as

$$\mu\frac{\partial V_1}{\partial x_3} - \in f'(x_1)\mu\frac{\partial V_1}{\partial x_1} = \left\{\mu'\frac{\partial V_2}{\partial x_3} + \mu_e\left(H_1H_3\frac{\partial V_2}{\partial x_1} + H_3^2\frac{\partial V_2}{\partial x_3}\right)\right\}$$
$$- \in f'(x_1)\left\{\mu'\frac{\partial V_2}{\partial x_1} + \mu_e\left(H_1^2\frac{\partial V_2}{\partial x_1} + H_1H_3\frac{\partial V_2}{\partial x_3}\right)\right\}, \tag{17}$$

where Maxwell's stress tensor, given by (4) has also been taken into consideration.

(iii) Since the boundary $x_3 = -H$ is free from external loads and surface stress acts on this boundary we have (Verma 1986, Chattopadhyay *et al* 1998, Gurtin & Murdoch 1974–75)

$$\tau'_{23} + \tau^M_{23} + \sum_{2\alpha,\alpha} -\rho_0\frac{\partial^2 V_2}{\partial t^2} = 0\,.$$

Using (4), (5) and (11) this boundary condition transforms to

$$\mu_0 \frac{\partial^2 V_2}{\partial x_1^2} + \mu' \frac{\partial V_2}{\partial x_3} + \mu_e \left(H_1 H_3 \frac{\partial V_2}{\partial x_1} + H_3^2 \frac{\partial V_2}{\partial x_3} \right) + \rho_0 \omega^2 V_2 = 0 \quad \text{on } x_3 = -H$$

where ρ_0 is the mass per unit surface area of the geometrical surface representing the region in which surface stress is located.

4. Solution of the problem

From the first boundary condition we obtain

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \left(A e^{-p_1 \in f(x_1)} + \frac{2}{p_1} e^{p_1 \in f(x_1)} e^{-p_1 d} \right) e^{-i\eta x_1} d\eta$$
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} (B e^{p_2 \in f(x_1)} + C e^{p_3 \in f(x_1)}) e^{-i\eta x_1} d\eta$$

Expanding $A, B, C, e^{\pm p_r \in f(x_1)}$ in powers of $\in$ and since $\in$ is very small, we get

$$\in \int_{-\infty}^{\infty} (p_1 A_0 + p_2 B_0 + p_3 C_0 - 2e^{-p_1 d}) f(x_1) e^{-i\eta x_1} d\eta$$
$$= \int_{-\infty}^{\infty} \left[(A_0 - B_0 - C_0) + \in (A_1 - B_1 - C_1) + \frac{2}{p_1} e^{-p_1 d} \right] e^{-i\eta x_1} d\eta \qquad (18)$$

Similarly, from the second boundary condition, one obtains the following

$$\in \int_{-\infty}^{\infty} \Big[\{ \mu p_1^2 A_0 - \mu' p_2^2 B_0 - \mu' p_3^2 C_0 + 2\mu p_1 e^{-p_1 d} - \mu_e H_3^2 p_2^2 B_0$$
$$- \mu_e H_3^2 p_3^2 C_0 + i\eta \mu_e H_1 H_3 p_2 B_0 + i\eta \mu_e H_1 H_3 p_3 C_0 \} f(x_1)$$
$$+ \Big\{ \mu A_0 - \mu' B_0 - \mu' C_0 + \frac{2\mu}{p_1} e^{-p_1 d} - \frac{i\mu_e H_1 H_3 p_2 B_0}{\eta}$$
$$- \frac{i\mu_e H_1 H_3 p_3 C_0}{\eta} - \mu_e H_1^2 B_0 - \mu_e H_1^2 C_0 \Big\} i\eta f'(x_1) \Big] e^{-i\eta x_1} d\eta$$
$$= \int_{-\infty}^{\infty} [\mu p_1 A_0 + \mu' p_2 B_0 + \mu' p_3 C_0 - 2\mu e^{-p_1 d} + \mu_e H_3^2 p_2 B_0$$
$$+ \mu_e H_3^2 p_3 C_0 - i\eta \mu_e H_1 H_3 B_0 - i\eta \mu_e H_1 H_3 C_0 + \in \{ \mu p_1 A_1 + \mu' p_2 B_1$$
$$+ \mu' p_3 C_1 + \mu_e H_3^2 p_2 B_1 + \mu_e H_3^2 p_3 C_1 - i\eta \mu_e H_1 H_3 B_1$$
$$- i\eta \mu_e H_1 H_3 C_1 \}] e^{-i\eta x_1} d\eta \qquad (19)$$

Let us apply Fourier transform on $f(x_1)$ as

$$\bar{f}(\xi) = \int_{-\infty}^{\infty} f(x_1) e^{i\xi x_1} dx_1$$

so that

$$f(x_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{f}(\xi) e^{-i\xi x_1} d\xi \tag{20}$$

and

$$f'(x_1) = -\frac{i}{2\pi} \int_{-\infty}^{\infty} \xi \bar{f}(\xi) e^{-i\xi x_1} d\xi$$

We now apply transform (20) on (18) and (19) and get

$$\frac{\in}{2\pi} \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} (p_1 A_0 + p_2 B_0 + p_3 C_0 - 2e^{-p_1 d}) e^{-i(\eta+\xi)x_1} d\eta \right\} \bar{f}(\xi) d\xi$$
$$= \int_{-\infty}^{\infty} \left[A_0 - B_0 - C_0 + \in (A_1 - B_1 - C_1) + \frac{2}{p_1} e^{-p_1 d} \right] e^{-i\eta x_1} d\eta \tag{21}$$

and

$$\frac{\in}{2\pi} \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} (\mu p_1^2 A_0 - \mu' p_2^2 B_0 - \mu' p_3^2 C_0 + 2\mu p_1 e^{-p_1 d} - \mu_e H_3^2 p_2^2 B_0 \right.$$
$$\left. - \mu_e H_3^2 p_3^2 C_0 + i\eta \mu_e H_1 H_3 p_2 B_0 + i\eta \mu_e H_1 H_3 p_3 C_0) e^{-i(\eta+\xi)x_1} d\eta \right\} \bar{f}(\xi) d\xi$$
$$+ \frac{\in}{2\pi} \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} \left(\mu A_0 - \mu' B_0 - \mu' C_0 + \frac{2\mu}{p_1} e^{-p_1 d} - \frac{i\mu_e H_1 H_3 p_2 B_0}{\eta} \right. \right.$$
$$\left. \left. - \frac{i\mu_e H_1 H_3 p_3 C_0}{\eta} - \mu_e H_1^2 B_0 - \mu_e H_1^2 C_0 \right) e^{-i(\eta+\xi)x_1} \eta d\eta \right\} \xi \bar{f}(\xi) d\xi$$
$$= \int_{-\infty}^{\infty} \{ \mu p_1 A_0 + \mu' p_2 B_0 + \mu' p_3 C_0 - 2\mu e^{-p_1 d} + \mu_e H_3^2 p_2 B_0 + \mu_e H_3^2 p_3 C_0$$
$$- i\eta \mu_e H_1 H_3 B_0 - i\eta \mu_e H_1 H_3 C_0 + \in (\mu p_1 A_1 + \mu' p_2 B_1 + \mu' p_3 C_1$$
$$+ \mu_e H_3^2 p_2 B_1 + \mu_e H_3^2 p_3 C_1 - i\eta \mu_e H_1 H_3 B_1 - i\eta \mu_e H_1 H_3 C_1) \} e^{-i\eta x_1} d\eta \tag{22}$$

Taking $\eta + \xi = \kappa$ where ξ is constant such that $d\eta = d\kappa$ and solving the inner integral of LHS of (21) and (22) and replacing η in the RHS by κ we get

$$\in \int_{-\infty}^{\infty} R_1(\kappa) e^{-i\kappa x_1} d\kappa = \int_{-\infty}^{\infty} \left\{ A_0 - B_0 - C_0 + \in (A_1 - B_1 - C_1) + \frac{2}{p_1} e^{-p_1 d} \right\} e^{-i\kappa x_1} d\kappa$$

or,

$$A_0 - B_0 - C_0 + \in (A_1 - B_1 - C_1) + \frac{2}{p_1} e^{-p_1 d} = \in R_1(\kappa) \tag{23}$$

and

$$\begin{aligned}\in \int_{-\infty}^{\infty} R_2(\kappa) e^{-i\kappa x_1} d\kappa = \int_{-\infty}^{\infty} \{ &\mu p_1 A_0 + \mu' p_2 B_0 + \mu' p_3 C_0 - 2\mu e^{-p_1 d} \\ &+ \mu_e H_3^2 p_2 B_0 + \mu_e H_3^2 p_3 C_0 - i\eta\mu_e H_1 H_3 B_0 - i\eta\mu_e H_1 H_3 C_0 \\ &+ \in (\mu p_1 A_1 + \mu' p_2 B_1 + \mu' p_3 C_1 + \mu_e H_3^2 p_2 B_1 + \mu_e H_3^2 p_3 C_1 \\ &- i\eta\mu_e H_1 H_3 B_1 - i\eta\mu_e H_1 H_3 C_1)\} e^{-i\kappa x_1} d\kappa\end{aligned}$$

or,

$$\begin{aligned}&\mu p_1 A_0 + \mu' p_2 B_0 + \mu' p_3 C_0 + \mu_e H_3^2 p_2 B_0 + \mu_e H_3^2 p_3 C_0 - i\eta\mu_e H_1 H_3 B_0 \\ &\quad - i\eta\mu_e H_1 H_3 C_0 + \in (\mu p_1 A_1 + \mu' p_2 B_1 + \mu' p_3 C_1 + \mu_e H_3^2 p_2 B_1 \\ &\quad + \mu_e H_3^2 p_3 C_1 - i\eta\mu_e H_1 H_3 B_1 - i\eta\mu_e H_1 H_3 C_1) - 2\mu e^{-p_1 d} = \in R_2(\kappa)\end{aligned} \tag{24}$$

where

$$R_1(\kappa) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [p_1 A_0 + p_2 B_0 + p_3 C_0 - 2e^{-p_1 d}]_{\eta=\kappa-\xi} \bar{f}(\xi) d\xi \tag{25}$$

and

$$\begin{aligned}R_2(\kappa) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [&\mu p_1^2 A_0 - \mu' p_2^2 B_0 - \mu' p_3^2 C_0 + 2\mu p_1 e^{-p_1 d} - \mu_e H_3^2 p_2^2 B_0 \\ &- \mu_e H_3^2 p_3^2 C_0 + i\eta\mu_e H_1 H_3 p_2 B_0 + i\eta\mu_e H_1 H_3 p_3 C_0 \\ &+ \Bigg(\mu A_0 - \mu' B_0 - \mu' C_0 + \frac{2\mu}{p_1} e^{-p_1 d} - \frac{i\mu_e H_1 H_3 p_2 B_0}{\eta} \\ &- \frac{i\mu_e H_1 H_3 p_3 C_0}{\eta} - \mu_e H_1^2 B_0 - \mu_e H_1^2 C_0 \Bigg) \xi\eta]_{\eta=\kappa-\xi} \bar{f}(\xi) d\xi\end{aligned} \tag{26}$$

From the third boundary condition we have

$$\begin{aligned}&(-\mu' p_2 + \mu_0 \eta^2 - \rho_0 \omega^2 - \mu_e H_3^2 p_2 + i\eta\mu_e H_1 H_3) e^{-p_2 H} B_0 - (\mu' p_3 - \mu_0 \eta^2 \\ &\quad + \rho_0 \omega^2 + \mu_e H_3^2 p_3 - i\eta\mu_e H_1 H_3) e^{-p_3 H} C_0 + \in [(-\mu' p_2 + \mu_0 \eta^2 \\ &\quad - \rho_0 \omega^2 - \mu_e H_3^2 p_2 + i\eta\mu_e H_1 H_3) e^{-p_2 H} B_1 - (\mu' p_3 - \mu_0 \eta^2 + \rho_0 \omega^2 \\ &\quad + \mu_e H_3^2 p_3 - i\eta\mu_e H_1 H_3) e^{-p_3 H} C_1] = 0\end{aligned} \tag{27}$$

Equating absolute terms and coefficients of $\in$ from (23), (25) and (27) one obtains the following

$$A_0 - B_0 - C_0 + \frac{2}{p_1}e^{-p_1 d} = 0 \tag{28}$$

$$\mu p_1 A_0 + \mu' p_2 B_0 + \mu' p_3 C_0 + \mu_e H_3^2 p_2 B_0 + \mu_e H_3^2 p_3 C_0$$
$$- i\eta\mu_e H_1 H_3 B_0 - i\eta\mu_e H_1 H_3 C_0 - 2\mu e^{-p_1 d} = 0 \tag{29}$$

$$(-\mu' - \mu_e H_3^2)(p_2 e^{-p_2 H} B_0 + p_3 e^{-p_3 H} C_0)$$
$$+ (\mu_0\eta^2 - \rho_0\omega^2 + i\eta\mu_e H_1 H_3)(e^{-p_2 H} B_0 + e^{-p_3 H} C_0) = 0 \tag{30}$$

$$A_1 - B_1 - C_1 - R_1(\kappa) = 0 \tag{31}$$

$$\mu p_1 A_1 + \mu' p_2 B_1 + \mu' p_3 C_1 + \mu_e H_3^2 p_2 B_1 + \mu_e H_3^2 p_3 C_1$$
$$- i\eta\mu_e H_1 H_3 B_1 - i\eta\mu_e H_1 H_3 C_1 - R_2(\kappa) = 0 \tag{32}$$

$$(-\mu' - \mu_e H_3^2)(p_2 e^{-p_2 H} B_1 + p_3 e^{-p_3 H} C_1)$$
$$+ (\mu_0\eta^2 - \rho_0\omega^2 + i\eta\mu_e H_1 H_3)(e^{-p_2 H} B_1 + e^{-p_3 H} C_1) = 0 \tag{33}$$

Solving (28)–(33), we get

$$A_0 = 4\mu e^{-p_1 d}\frac{R_3 + (\mu_0\eta^2 - \rho_0\omega^2 + i\eta\mu_e H_1 H_3)(e^{-p_2 H} - e^{-p_3 H})}{\bar{R}(\kappa)} - \frac{2}{p_1}e^{-p_1 d}$$

$$B_0 = 4\mu e^{-p_1 d}\frac{\mu' p_3 - (\mu_0\eta^2 - \rho_0\omega^2) + \mu_e H_3^2 p_3 - i\eta\mu_e H_1 H_3}{\bar{R}(\kappa)}e^{-p_3 H}$$

$$C_0 = 4\mu e^{-p_1 d}\frac{-\mu' p_2 + (\mu_0\eta^2 - \rho_0\omega^2) - \mu_e H_3^2 p_2 + i\eta\mu_e H_1 H_3}{\bar{R}(\kappa)}e^{-p_2 H} \tag{34}$$

$$A_1 = \{R_2(\kappa) - \mu p_1 R_1(\kappa)\}\frac{R_3 + (\mu_0\eta^2 - \rho_0\omega^2 + i\eta\mu_e H_1 H_3)(e^{-p_2 H} - e^{-p_3 H})}{\bar{R}(\kappa)} + R_1(\kappa)$$

$$B_1 = \{R_2(\kappa) - \mu p_1 R_1(\kappa)\}\frac{\mu' p_3 - (\mu_0\eta^2 - \rho_0\omega^2) + \mu_e H_3^2 p_3 - i\eta\mu_e H_1 H_3}{\bar{R}(\kappa)}e^{-p_3 H}$$

$$C_1 = \{R_2(\kappa) - \mu p_1 R_1(\kappa)\}\frac{-\mu' p_2 + (\mu_0\eta^2 - \rho_0\omega^2) - \mu_e H_3^2 p_2 + i\eta\mu_e H_1 H_3}{\bar{R}(\kappa)}e^{-p_2 H} \tag{35}$$

where

$$\bar{R}(\kappa) = \{\mu' p_3 - (\mu_0\eta^2 - \rho_0\omega^2) + \mu_e H_3^2 p_3 - i\eta\mu_e H_1 H_3\}$$
$$(\mu p_1 + \mu' p_2 + \mu_e H_3^2 p_2 - i\eta\mu_e H_1 H_3)e^{-p_3 H}$$
$$+ \{-\mu' p_2 + (\mu_0\eta^2 - \rho_0\omega^2) - \mu_e H_3^2 p_2 + i\eta\mu_e H_1 H_3\}$$
$$(\mu p_1 + \mu' p_3 + \mu_e H_3^2 p_3 - i\eta\mu_e H_1 H_3)e^{-p_2 H} \tag{36}$$

and $R_3 = (-\mu' - \mu_e H_3^2)(p_2 e^{-p_2 H} - p_3 e^{-p_3 H})$.

The displacement in the layer is

$$V_2(x_1, x_3) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(Be^{p_2 x_3} + Ce^{p_3 x_3})e^{-i\eta x_1}d\eta,$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\{(e^{p_2 x_3}B_0 + e^{p_3 x_3}C_0) + \in (e^{p_2 x_3}B_1 + e^{p_3 x_3}C_1)\}e^{-i\eta x_1}d\eta$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\mu}{\bar{R}(\kappa)}e^{-p_1 d}\left[1 + \in \frac{\{R_2(\kappa) - \mu p_1 R_1(\kappa)\}}{4\mu}e^{p_1 d}\right]$$
$$\times [\{\mu' p_3 - (\mu_0\eta^2 - \rho_0\omega^2) + \mu_e H_3^2 p_3 - i\eta\mu_e H_1 H_3\}e^{p_2 x_3 - p_3 H}$$
$$+ \{-\mu' p_2 + (\mu_0\eta^2 - \rho_0\omega^2) - \mu_e H_3^2 p_2 + i\eta\mu_e H_1 H_3\}e^{p_3 x_3 - p_2 H}]e^{-i\eta x_1}d\eta$$

Now $\{R_2(\kappa) - \mu p_1 R_1(\kappa)\}$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\Bigg[(\mu p_1^2 A_0 - \mu' p_2^2 B_0 - \mu' p_3^2 C_0 + 2\mu p_1 e^{-p_1 d} - \mu_e H_3^2 p_2^2 B_0$$
$$- \mu_e H_3^2 p_3^2 C_0 + i\eta\mu_e H_1 H_3 p_2 B_0 + i\eta\mu_e H_1 H_3 p_3 C_0)$$
$$+ \Bigg(\mu A_0 - \mu' B_0 - \mu' C_0 + \frac{2\mu}{p_1}e^{-p_1 d} - \frac{i\mu_e H_1 H_3 p_2}{\eta}B_0$$
$$- \frac{i\mu_e H_1 H_3 p_3}{\eta}C_0 - \mu_e H_1^2 B_0 - \mu_e H_1^2 C_0\Bigg)\Bigg]_{\eta=\kappa-\xi}\bar{f}(\xi)d\xi$$
$$- \frac{1}{2\pi}\int_{-\infty}^{\infty}\mu p_1[p_1 A_0 + p_2 B_0 + p_3 C_0 - 2e^{-p_1 d}]_{\eta=\kappa-\xi}\bar{f}(\xi)d\xi$$
$$= \frac{2\mu}{\pi}\int_{-\infty}^{\infty}\Bigg[- p_1 p_2(\mu - \mu' - \mu_e H_3^2)Me^{-p_3 H}$$
$$- p_1 p_3(\mu - \mu' - \mu_e H_3^2)Ne^{-p_2 H} - \{\mu' p_2^2 - \mu p_1^2 + (\mu' - \mu + \mu_e H_1^2)\xi\eta$$

$$+ \mu_e H_3^2 p_2^2 - i\eta\mu_e H_1 H_3 p_2 + i\eta\mu_e H_1 H_3 p_1 + i\mu_e H_1 H_3 p_2 \xi\} M e^{-p_3 H}$$

$$- \{\mu' p_3^2 - \mu p_1^2 + (\mu' - \mu + \mu_e H_1^2)\xi\eta + \mu_e H_3^2 p_3^2 - i\eta\mu_e H_1 H_3 p_3$$

$$+ i\eta\mu_e H_1 H_3 p_1 + i\mu_e H_1 H_3 p_3 \xi\} N e^{-p_2 H}] \left.\frac{e^{-p_1 d}}{\bar{R}(\kappa)}\right]_{\eta=\kappa-\xi} \bar{f}(\xi) d\xi$$

where

$$M = \mu' p_3 - (\mu_0 \eta^2 - \rho_0 \omega^2) + \mu_e H_3^2 p_3 - i\eta\mu_e H_1 H_3$$

$$N = -\mu' p_2 + (\mu_0 \eta^2 - \rho_0 \omega^2) - \mu_e H_3^2 p_2 + i\eta\mu_e H_1 H_3$$

4.1 *Rectangular irregularity*

From (1) the interface for the rectangular irregularity is

$$\bar{f}(\xi) = \frac{4a}{\xi} \sin \xi a.$$

Hence,

$$R_2(\kappa) - \mu p_1 R_1(\kappa) = \frac{8\mu a}{\pi} \int_{-\infty}^{\infty} [g(\kappa - \xi) + g(\kappa + \xi)] \frac{\sin \xi a}{\xi} d\xi, \tag{37}$$

where

$$g(\kappa - \xi)$$

$$= \Big[[-p_1 p_2 (\mu - \mu' - \mu_e H_3^2) M e^{-p_3 H} - p_1 p_3 (\mu - \mu' - \mu_e H_3^2) N e^{-p_2 H}$$

$$- \{\mu' p_2^2 - \mu p_1^2 + (\mu' - \mu + \mu_e H_1^2)\xi\eta + \mu_e H_3^2 p_2^2 - i\eta\mu_e H_1 H_3 p_2$$

$$+ i\eta\mu_e H_1 H_3 p_1 + i\mu_e H_1 H_3 p_2 \xi\} M e^{-p_3 H} - \{\mu' p_3^2 - \mu p_1^2$$

$$+ (\mu' - \mu + \mu_e H_1^2)\xi\eta + \mu_e H_3^2 p_3^2 - i\eta\mu_e H_1 H_3 p_3 + i\eta\mu_e H_1 H_3 p_1$$

$$+ i\mu_e H_1 H_3 p_3 \xi\} N e^{-p_2 H}] \left.\frac{e^{-p_1 d}}{\bar{R}(\kappa)}\right]_{\eta=\kappa-\xi}$$

Following the asymptotic formula of Willis (1948) and Tranter (1966) we get

$$\int_{-\infty}^{\infty} [g(\kappa - \xi) + g(\kappa + \xi)] \frac{\sin \xi a}{\xi} d\xi = \pi g(\kappa).$$

Hence from (37) one obtains $\in \frac{R_2(\kappa) - \mu p_1 R_1(\kappa)}{4\mu} = h g(\kappa)$.

Now the displacement in M_2 is

$$V_2(x_1,x_3)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\mu}{\bar{R}(\kappa)}e^{-p_1d}[1+hg(\kappa)e^{p_1d}][Me^{p_2x_3-p_3H}+Ne^{p_3x_3-p_2H}]e^{-i\eta x_1}d\xi$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\mu e^{-p_1d}[Me^{p_2x_3-p_3H}+Ne^{p_3x_3-p_2H}]}{\bar{R}(\kappa)[1-hg(\kappa)e^{p_1d}]}e^{-i\eta x_1}d\eta. \tag{38}$$

The dispersion equation is

$$\bar{R}(\kappa)[1-hg(\kappa)e^{p_1d}]=0,$$

which on substitution of the expressions for $\bar{R}(\kappa)$ and $g(\kappa)$ transforms to

$$\begin{aligned}&(\mu p_1+\mu' p_2+\mu_e H_3^2 p_2-i\eta\mu_e H_1H_3)Me^{-p_3H}\\&\quad+(\mu p_1+\mu' p_3+\mu_e H_3^2p_3-i\eta\mu_eH_1H_3)Ne^{-p_2H}\\&\quad-h[-p_1p_2(\mu-\mu'-\mu_eH_3^2)Me^{-p_3H}-p_1p_3(\mu-\mu'-\mu_eH_3^2)Ne^{-p_2H}\\&\quad-(\mu' p_2^2-\mu p_1^2+\mu_eH_3^2p_2^2-i\eta\mu_eH_1H_3p_2+i\eta\mu_eH_1H_3p_1)Me^{-p_3H}\\&\quad-(\mu' p_3^2-\mu p_1^2+\mu_eH_3^2p_3^2-i\eta\mu_eH_1H_3p_3+i\eta\mu_eH_1H_3p_1)Ne^{-p_2H}]=0.\end{aligned} \tag{39}$$

After modification the equation (39) takes the following form

$$\tan\eta H\Gamma=\frac{N_0}{D_0}, \tag{40}$$

where

$$\Gamma=\frac{R_a}{R_b^2}c_b,\ N_0=\Gamma\left[R_b^2\left\{\frac{\mu c_a}{\mu'}-h\eta\left(\frac{\mu c_a^2}{\mu'}+\frac{R_a^2c_b^2}{R_b^2}\right)\right\}\right.$$
$$\left.+\frac{\eta}{k_0}\left(\eta'^2-\frac{c^2}{\beta_2^2}\right)\left\{R_b^2+h\eta c_a\left(\frac{\mu}{\mu'}-R_b^2\right)\right\}\right]$$

and

$$D_0=\frac{R_a^2c_b^2}{R_b^4}\left\{R_b^4+h\eta c_a\left(\frac{\mu R_b^2}{\mu'}-R_b^4\right)\right\}$$
$$-\frac{\eta}{k_0}\left(\eta'^2-\frac{c^2}{\beta_2^2}\right)\left\{\frac{\mu c_a}{\mu'}-h\eta\left(\frac{\mu c_a^2}{\mu'}+\frac{R_a^2c_b^2}{R_b^2}\right)\right\}$$

in which

$$R_a^2=1+\frac{\mu_eH_1^2}{\mu'}+\frac{\mu_eH_3^2}{\mu'},\ R_b^2=1+\frac{\mu_eH_3^2}{\mu'},\ c_b^2=\frac{c^2}{\beta_2^2}-1,$$

$$c_a^2=1-\frac{c^2}{\beta_1^2},\ k_0=\frac{\mu'}{\rho_0\beta_2^2},\ \beta_2^2=\frac{\mu' R_a^2}{\rho' R_b^2},\ \eta'^2=\frac{\mu_0}{\rho_0\beta_2^2}.$$

If the interface is regular (i.e. $h = 0$), but the free surface of the crustal layer is acted on by the surface stress, the corresponding dispersion equation is deduced from (40) as

$$\tan \eta H \Gamma = \frac{N_1}{D_1}. \tag{41}$$

When the surface stress is absent (i.e. $\eta'^2 = c^2/\beta_2^2$) but the interface is irregular of rectangular type, the concerned dispersion equation is obtained from (40) as

$$\tan \eta H \Gamma = \frac{N_2}{D_2}. \tag{42}$$

In the above the expressions for N_1, D_1 and N_2, D_2 are deduced from N_0, D_0 of (40) by putting $h = 0$ and $\eta'^2 = c^2/\beta_2^2$ respectively.

When the interface is regular and $x_3 = -H$ is free of surface stress then equation (40) reduces to the dispersion relation for the plane SH waves in a magneto-elastic crustal layer lying over an isotropic solid elastic half space as given below

$$\tan \eta H \Gamma = \frac{\mu c_a}{\mu' R_a c_b}.$$

4.2 *Parabolic irregularity*

Applying Fourier transform to the equation (2), the interface is

$$\bar{f}(\xi) = \frac{4ha}{\in} \times \frac{\sin \xi a - a\xi \cos \xi a}{(\xi a)^3}.$$

Now,

$$\begin{aligned} R_2(\kappa) - \mu p_1 R_1(\kappa) &= \frac{2\mu}{\pi} \int_0^\infty [g(\kappa - \xi) + g(\kappa + \xi)] \bar{f}(\xi) d\xi \\ &= \frac{8ah\mu}{\pi \in} \int_0^\infty [g(\kappa - \xi) + g(\kappa + \xi)] \sqrt{\frac{\pi}{2}} \frac{J_{3/2}(\xi a)}{(\xi a)^{\frac{3}{2}}} d\xi \\ &= \frac{16 h\mu}{3\pi \in} g(\kappa), \end{aligned}$$

in which $J_{3/2}(\xi a)$ is the Bessel function of the first kind of order $\frac{3}{2}$.

Now the displacement in M_2 is

$$\begin{aligned} V_2(x_1, x_3) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{4\mu}{\bar{R}(\kappa)} e^{-p_1 d} \left[1 + \frac{4h}{3\pi} g(\kappa) e^{p_1 d} \right] \\ &\quad \times [M e^{p_2 x_3 - p_3 H} + N e^{p_3 x_3 - p_2 H}] e^{-i\eta x_1} d\xi \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{4\mu e^{-p_1 d} [M e^{p_2 x_3 - p_3 H} + N e^{p_3 x_3 - p_2 H}]}{\bar{R}(\kappa) \left[1 - \frac{4h}{3\pi} g(\kappa) e^{p_1 d} \right]} e^{-i\eta x_1} d\eta. \end{aligned} \tag{43}$$

Since the value of the above integral depends on the contribution of the poles of the integrand, the concerned dispersion equation is given by

$$\bar{R}(\kappa)\left[1-\frac{4h}{3\pi}g(\kappa)e^{p_1 d}\right]=0.$$

or,

$$\begin{aligned}
&(\mu p_1+\mu' p_2+\mu_e H_3^2 p_2-i\eta\mu_e H_1 H_3)Me^{-p_3H}+(\mu p_1+\mu' p_3+\mu_e H_3^2 p_3\\
&\quad -i\eta\mu_e H_1H_3)Ne^{-p_2H}-\frac{4h}{3\pi}[-p_1p_2(\mu-\mu'-\mu_e H_3^2)Me^{-p_3H}\\
&\quad -p_1p_3(\mu-\mu'-\mu_e H_3^2)Ne^{-p_2H}-(\mu' p_2^2-\mu p_1^2+\mu_e H_3^2 p_2^2\\
&\quad -i\eta\mu_e H_1H_3p_2+i\eta\mu_e H_1H_3p_1)Me^{-p_3H}-(\mu' p_3^2-\mu p_1^2+\mu_e H_3^2 p_3^2\\
&\quad -i\eta\mu_e H_1H_3p_3+i\eta\mu_e H_1H_3p_1)Ne^{-p_2H}]=0.
\end{aligned} \tag{44}$$

After simplification one obtains the concerned dispersion equation as

$$\tan\eta H\Gamma=\frac{N_3}{D_3}, \tag{45}$$

where

$$\begin{aligned}
N_3=\Gamma&\left[R_b^2\left\{\frac{\mu c_a}{\mu'}-\frac{4h}{3\pi}\eta\left(\frac{\mu c_a^2}{\mu'}+\frac{R_a^2c_b^2}{R_b^2}\right)\right\}\right.\\
&\left.+\frac{\eta}{k_0}\left(\eta'^2-\frac{c^2}{\beta_2^2}\right)\left\{R_b^2+\frac{4h\eta c_a}{3\pi}\left(\frac{\mu}{\mu'}-R_b^2\right)\right\}\right]
\end{aligned}$$

and

$$\begin{aligned}
D_3=\frac{R_a^2c_b^2}{R_b^4}&\left[R_b^4+\frac{4h\eta c_a}{3\pi}\left\{\frac{\mu R_b^2}{\mu'}-R_b^4\right\}\right]\\
&-\frac{\eta}{k_0}\left(\eta'^2-\frac{c^2}{\beta_2^2}\right)\left\{\frac{\mu c_a}{\mu'}-\frac{4h\eta}{3\pi}\left(\frac{\mu c_a^2}{\mu'}+\frac{R_a^2c_b^2}{R_b^2}\right)\right\}.
\end{aligned}$$

In the absence of irregularity of the interface ($h=0$) the corresponding dispersion relation in presence of surface stress may be obtained from (45) as special case and is given by

$$\tan\eta H\Gamma=\frac{N_4}{D_4}. \tag{46}$$

In the absence of surface stress when the interface is irregular, the equation reduces

$$\tan\eta H\Gamma=\frac{N_5}{D_5}. \tag{47}$$

In the above the expressions for N_4, D_4 and N_5, D_5 are deduced from N_3, D_3 of (45) by simply putting $h = 0$ and $\eta'^2 = c^2/\beta_2^2$ respectively.

It is observed from the equations (40) and (45) that the range of possible real values of c for which the required wave may propagate i.e. the energy of plane waves be confined to the transition zone, is given by

$$\beta_2 < c < \beta_1, \tag{48}$$

where

$$\beta_2 = \sqrt{\frac{\mu'}{\rho'}}\left(1 + \frac{\frac{\mu_e H_1^2}{\mu'}}{1 + \frac{\mu_e H_3^2}{\mu'}}\right)^{\frac{1}{2}}.$$

The in equation (48) indicates that the range depends on the magnetic field but does neither depend on the surface stress parameters nor on the interface irregularity. Hence, the presence of magnetic field alters the range. From the equations (42) and (47) it is interesting to note that the surface stress has no effect when $\eta'^2 = c^2/\beta_2^2$ i.e. when $c = \sqrt{\mu_0/\rho_0}$. Hence surface stress plays its role only when $c \neq \sqrt{\mu_0/\rho_0}$. In the absence of both the surface irregularity ($h = 0$) and the magnetic field ($H_1 = 0$, $H_3 = 0$) the dispersion equation (40) and (45) reduces to the wave velocity equation for SH-waves under the influence of surface stress as

$$\tan \eta H \left(\frac{c^2}{\beta_2^2} - 1\right)^{\frac{1}{2}} = \frac{\left(\frac{c^2}{\beta_2^2} - 1\right)^{\frac{1}{2}}\left[\frac{\mu}{\mu'}\left(1 - \frac{c^2}{\beta_2^2}\frac{\beta_2^2}{\beta_1^2}\right)^{\frac{1}{2}} + \frac{\eta}{k_0}\left(\eta'^2 - \frac{c^2}{\beta_2^2}\right)\right]}{\left(\frac{c^2}{\beta_2^2} - 1\right) - \frac{\eta}{k_0}\left(\eta'^2 - \frac{c^2}{\beta_2^2}\right)\frac{\mu}{\mu'}\left(1 - \frac{c^2}{\beta_2^2}\frac{\beta_2^2}{\beta_1^2}\right)^{\frac{1}{2}}},$$

where $\beta_2^2 = \frac{\mu'}{\rho'}$, $k_0 = \frac{\mu'}{\rho_0 \beta_2^2}$, $\eta'^2 = \frac{\mu_0}{\rho_0 \beta_2^2}$, which is in conformity with the corresponding result obtained by Pal *et al* (1997).

Again if one considers the effect of irregularity only, neglecting the effect of surface stress and magnetic field the equation (45) transforms to the result obtained by Chattopadhyay & Pal (1983).

5. Numerical calculations

For the purpose of numerical calculation and graphical representation we take (Pal *et al* 1997)

$$\mu = 3{\cdot}00 \times 10^6\,\text{N/cm}^2,\ \mu' = 5{\cdot}00 \times 10^6\,\text{N/cm}^2,\ \mu_0 = 6{\cdot}47 \times 10^6\,\text{N/cm}.$$

$$\rho = 2{\cdot}72\,\text{gm/cm}^3,\ \rho' = 9{\cdot}89\,\text{gm/cm}^3,\ \rho_0 = 3{\cdot}40\,\text{gm/cm}^2.$$

To highlight the effect of magnetic field, irregular boundary and surface stress in the case of plane SH waves propagating in a homogeneous, perfectly conducting crustal layer lying over an isotropic solid half space we draw the curves (figures 3 to 10). With the help of these figures comparison may also be made among different cases. Using the frequency equations (40) and (45) numerical values of c/β_2 have been plotted against ηH for different cases as expressed in the captions of the figures. All the figures are self explanatory and contain some important peculiarities due to the effect of magnetic field, non-parallel boundary, surface

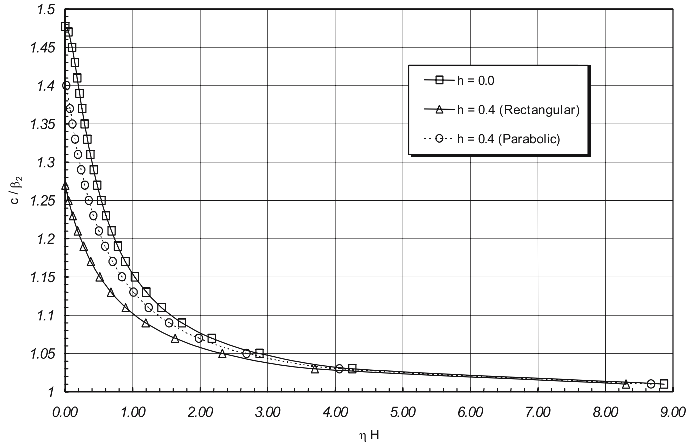

Figure 3. c/β_2 vs ηH for $H_1 = 0$, $H_3 = 0$ and surface stress $= 0{\cdot}0$.

stress on the surface $x_3 = -H$. However, for better understanding of the figures the following explanations are presented.

Figure 3 expresses the variations of c/β_2 for different values of ηH in the absence of magnetic field and surface stress. Two curves of this figure have been drawn to highlight the variation of wave velocity due to rectangular and parabolic irregularities with $h = 0{\cdot}4$ in each case. The third one which corresponds to no irregularity ($h = 0$), has been drawn for comparison. It is seen that, in each case, the values of c/β_2 start from its highest value when $\eta H = 0$ and it decreases sharply as ηH increases to a certain value. After this particular value of ηH the decrement in c/β_2 due to the increase of ηH is very small. It is marked from this figure that irregularity of any nature always reduces wave velocity (with respect to the no irregularity case) to a certain extent. For a particular value of ηH the rectangular irregularity causes more reduction of wave velocity than that of parabolic irregularity.

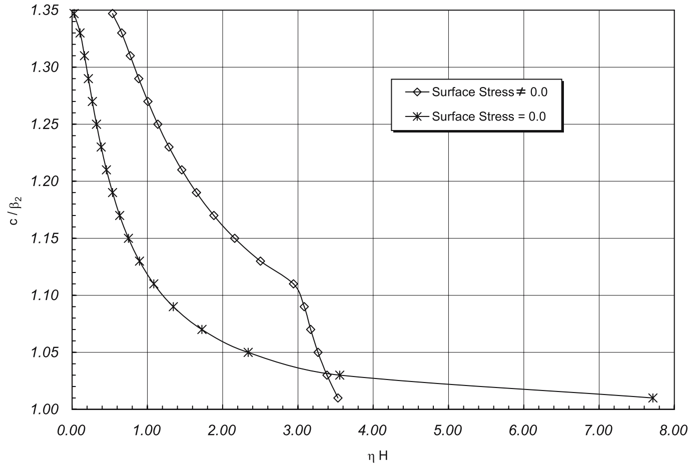

Figure 4. c/β_2 vs ηH for $h = 0{\cdot}0$ and $\mu_e H_1^2/\mu' = 0{\cdot}2$, $H_3 = 0$.

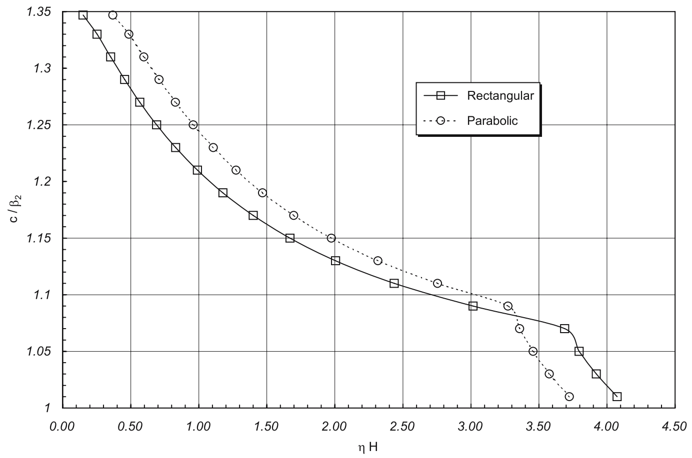

Figure 5. c/β_2 vs ηH for $h = 0{\cdot}4$, surface stress $\neq 0{\cdot}0$ and $\mu_e H_1^2/\mu' = 0{\cdot}2$, $H_3 = 0$.

In figure 4 we have considered the case of no irregularity with horizontal magnetic field ($\mu_e H_1^2/\mu' = 0{\cdot}2$) in presence of surface stress. For the sake of comparison, a curve has been plotted corresponding to the boundary which is free of surface stress. We see that there is a sudden fall of wave velocity after $\eta H = 3$. It is worthy to note that surface stress causes increment of wave velocity up to $\eta H < 3{\cdot}4$. When $\eta H = 3{\cdot}4$ surface stress has no effect on the propagation of SH-waves under the influence of magnetic field. When $\eta H > 3{\cdot}4$ surface stress causes sharp diminution of wave velocity.

Figure 5 gives the variation of c/β_2 with respect to ηH under the combined effect of horizontal magnetic field and surface stress with rectangular or parabolic irregularity. Here also it is noted that for a particular value of ηH wave velocity in the case of parabolic irregularity is greater than that of rectangular irregularity up to $\eta H = 3{\cdot}3$. When $\eta H = 3{\cdot}3$ there is no variation of wave velocity due to rectangular and parabolic irregularity. When

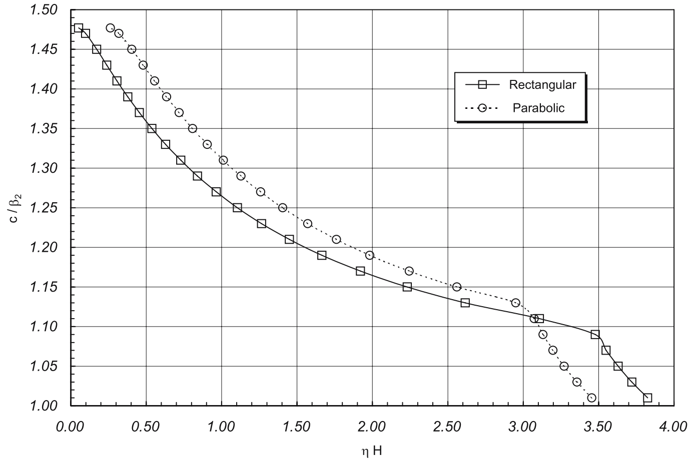

Figure 6. c/β_2 vs ηH for $h = 0{\cdot}4$, surface stress $\neq 0{\cdot}0$ and $H_1 = 0$, $H_3 = 0$.

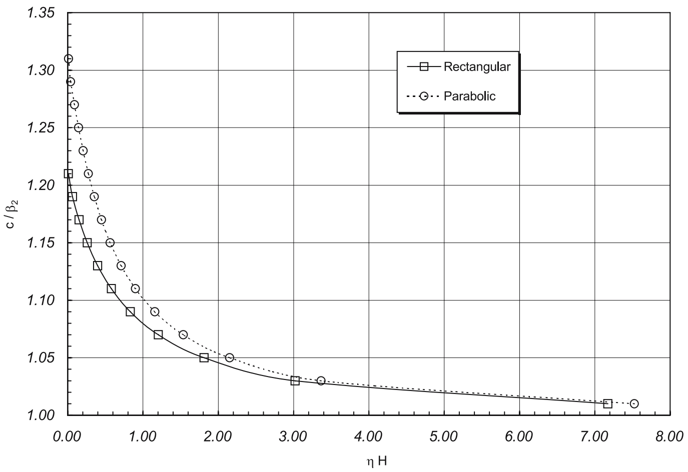

Figure 7. c/β_2 vs ηH for $h = 0{\cdot}4$, surface stress $= 0{\cdot}0$ and $\mu_e H_1^2/\mu' = 0{\cdot}2$, $H_3 = 0$.

$\eta H > 3{\cdot}3$, parabolic irregularity causes more diminution of wave velocity than that of rectangular irregularity. Sudden fall occurs just after $\eta H = 3{\cdot}3$ and $\eta H = 3{\cdot}7$ for parabolic and rectangular irregularity respectively.

From figure 6 combined effect of surface stress and irregularity of interface can be studied when there is no magnetic field. Figure 7 depicts the combined effect of horizontal magnetic field and irregular interface in the case of rectangular and parabolic irregularity.

Figure 8 shows the variation of c/β_2 with respect to ηH under the joint influence of vertical magnetic field and surface stress when there is no irregularity in the interface. The curve corresponding to surface stress equal to zero, in this case, has also been plotted. Figure 9 gives us the modulation of c/β_2 under the combined effect of surface stress, irregularity of the interface and vertical magnetic field. Figure 10 gives us the modulation of wave velocity with respect to ηH under the joint effect of irregularity and vertical magnetic field. Cases

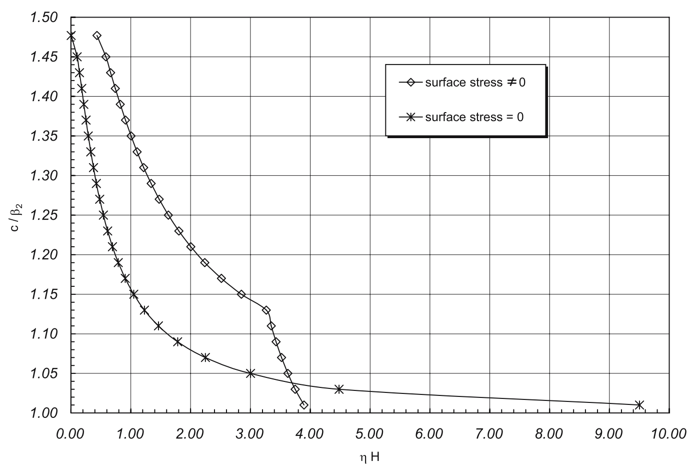

Figure 8. c/β_2 vs ηH for $h = 0{\cdot}0$ and $\mu_e H_3^2/\mu' = 0{\cdot}2$, $H_1 = 0$.

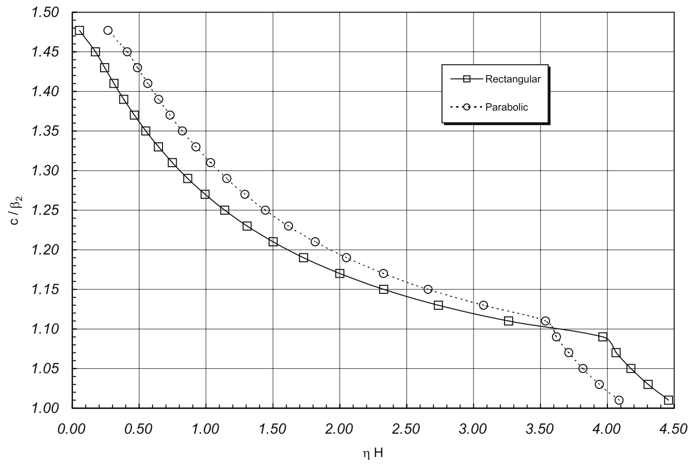

Figure 9. c/β_2 vs ηH for $h = 0{\cdot}4$, surface stress $\neq 0{\cdot}0$ and $\mu_e H_3^2/\mu' = 0{\cdot}2$, $H_1 = 0$.

of parabolic and rectangular irregularity have been plotted separately. A close observation of all the figures leads us to express that the sudden fall of wave velocity occurs only when the surface stress is present on the surface $x_3 = -H$.

6. Conclusion

The most significant outcome of this paper is that modulations of SH wave velocity takes place due to the presence of surface stress, magnetic field and irregularity near the interface. Further modulation take place due to their combined effect. In conclusion we also point out the peculiarity that a sudden fall of the wave velocity occurs after a certain value of ηH in all the cases where surface stress is present. Moreover, there exists a value (we call it critical

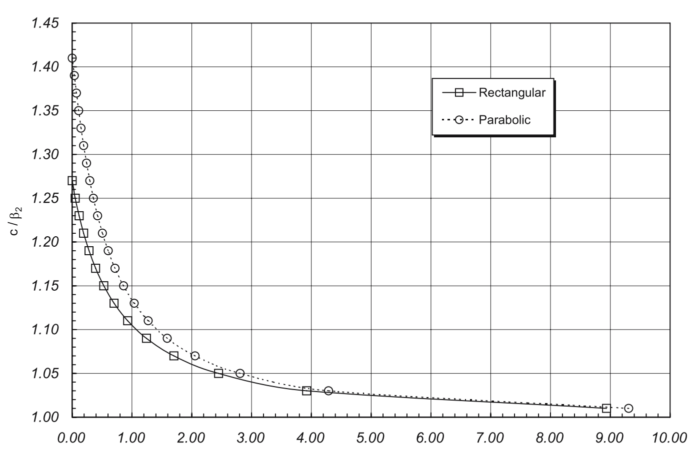

Figure 10. c/β_2 vs ηH for $h = 0{\cdot}4$, surface stress $= 0{\cdot}0$ and $\mu_e H_3^2/\mu' = 0{\cdot}2$, $H_1 = 0$.

value) of ηH for which surface stress has no role to play. When ηH is less than this critical value, surface stress causes increment of wave velocity. On the other hand, when ηH is greater than this critical value surface stress causes decrement in wave velocity. The present problem and the results obtained have their possible applications in the field of earth sciences, geophysics, seismology specially to the problems of waves and vibrations where the wave signals have to travel through different layers exhibiting surface stresses due to material properties and containing irregularities due to continental margin, mountain roots, etc.

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References

Bazer J 1984 A survey of linear and nonlinear waves in a perfect magneto-elastic medium in the mechanical behaviour of electromagnetic solid continua, (ed.) G A Maugin, (B.V. (North-Holland): Elsevier Science Publishers)

Bhattacharya J 1962 On the dispersion curve of Love waves due to irregularity in the thickness of the transversely isotropic crustal layer. *Gerl. Beitr. Geophys.* Leipzig 71: 324–333

Chandrasekharaih D S 1987a Effect of surface stresses and voids on Rayleigh waves in an elastic solid. *Int. J. Eng. Sci.* Great Britain 25: 205–211

Chattopadhyay A 1975a On the dispersion equation for Love waves due to irregularity in the thickness of the non-homogeneous crustal layer. *Acta Geophysica Polonica.* 23: 307–317

Chattopadhyay A, Choudhury S, Chattopadhyay A 1998 Diffraction of magneto-elastic shear waves in a self-reinforced medium by a rigid strip. *Ind. J. Pure Appl. Math.* 29: 81–94

Chattopadhyay A, Chowdhury S 1995 Magneto-elastic shear waves in an infinite self-reinforced plate. *Int. J. Numer. Anal. Methods Geomech.* 19: 289–304

Chattopadhyay A, Pal A K 1983 Dispersion curves of SH waves caused by irregularity in the pre-stressed internal stratum. *Acta Geophysica Polonica*. XXXI: 37–49

Chaudhary S, Kaushik V P, Tomar S K 2004 Reflection/transmission of plane SH wave through a self-reinforced elastic layer between two half-spaces. *Acta Geophysica Polonica.* 52: 219–235

Coulomb J 1952 Love waves of the Queen Charlotte Islands earthquake of August 22, 1949. *Bull. Seism. Soc. Amer.* 42: 29–36

De Noyer J 1961 The effect of variations in layer thickness of Love waves. *Bull. Seism. Soc. Amer.* 51: 227–235

Dunkin J W, Eringen A C 1963 On the propagation of waves on electromagnetic elastic solids. *Int. J. Engng. Sci.* 1: 461–495

Dutta S 1963a Love waves in a non-homogeneous internal stratum lying between two semi-infinite isotropic media. *Geophysics.* 28: 156–160

Dutta S 1963b On the propagation of Love waves in a non-homogeneous internal stratum of finite depth lying between two semi-infinite isotropic media. *Geophys. Pure Appl.* 55: 31–36

Gurtin M E 1972 The linear theory of elasticity. *Handbuch der Physik.* Springer, Berlin. VIa/3

Gurtin M E, Murdoch A I 1974–75 A continuum theory of elastic material surfaces. *Arch. Rat. Mech. Anal.* 57: 291–232

Gurtin M E, Murdoch A I 1976 Effect of surface stress on wave propagation in solids. *J. Appl. Phys.* 47: 4414–4429

Kar B K 1977 On the propagation of Love type waves in a non-homogeneous internal stratum of finite thickness lying between two semi-infinite isotropic elastic media. *Geophysik.* Leipzig. 86: 407–412

Knopoff L 1955 The interaction between elastic wave motion and a magnetic field in electrical conductors. *J. Geophys. Res.* 60: 441–456

Lee J S, Its E N 1992 Propagation of Rayleigh waves in magneto-elastic media, Trans. ASME, *J. Appl. Mech.* 59: 812–818

Mal A K 1962 On the frequency equation for Love waves due to abrupt thickening of the crustal layer. *Geophys. Pure Appl.* 52: 59–68

Maugin G A 1988b 'Shear horizontal surface acoustic waves on solids' in recent advances in surface acoustic waves. (eds) D F Parker, G A Maugin (Berlin: Springer-Verlag) 158–172

Othman M I A, Song Y 2006 The effect of rotation on the reflection of magneto-thermo-elastic waves under thermo-elasticity without energy dissipation. *Acta Mechanica.* 184: 189–204

Pal P K, Acharya D P, Sengupta P R 1997 Effect of surface stresses on surface waves in elastic solids. *Sādhanā* 22: 659–670

Plaster H J 1972 Blast cleaning and Allied Processes. *Industrial NewsPapers Ltd*. London

Sato Y 1952 Study of surface waves V, Love waves propagated upon heterogeneous medium. *Bull. Earthquake Res. Inst*. Tokyo. 30: 1

Stoneley R 1924 Elastic waves at the surface of separation of two solids. *Proc. Roy. Soc.* London, A106: 416–428

Tomita S, Shindo Y 1979 Rayleigh waves in magneto-thermo-elastic solid with thermal relaxation. *Int. J. Eng. Sci.* 17: 227–232

Tranter C J 1966 Integral transforms in mathematical physics: 63–67

Verma P D S 1986 Magneto-elastic shear waves in self-reinforced bodies. *Int. J. Eng. Sci.* 24: 1067–1073

Verma P D S, Rana O H, Verma M 1988 Magneto-elastic transverse surface waves in self–reinforced elastic solids. *Indian J. Pure Appl. Math.* 19: 713–716

Willis H F 1948 A formula for expanding an integral as a series, *Philosophical Mag.* 39: 455–459

Wolf B 1967 Propagation of Love waves in surface layers of varying thickness. *Pure Appl. Geophys.* 67: 76–82

Yu Chia P, Tang Sam 1966 Magneto-elastic waves in initially stressed conductors. *J. Appl. Math. Phys.* ZAMP. 17: 766–775