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Determination of bounds on failure probability in the presence of hybrid uncertainties

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Abstract. A fundamental component of safety assessment is the appropriate representation and incorporation of uncertainty. A procedure for handling hybrid uncertainties in stochastic mechanics problems is presented. The procedure can be used for determining the bounds on failure probability for cases where failure probability is a monotonic function of the fuzzy variables. The procedure is illustrated through an example problem of safety assessment of a nuclear power plant piping component against stress corrosion cracking, considering the stochastic evolution of stress corrosion cracks with time. It is found that the bounds obtained enclose the values of failure probability obtained from probabilistic analyses.

Keywords. Safety assessment; probability of failure; fuzzy sets; possibility measure; necessity measure.

1. Introduction

A fundamental component of safety assessment of complex engineering facilities, such as nuclear power plants, is the appropriate representation and incorporation of uncertainty in different variables (Helton & Oberkampf 2004). Uncertainty can be classified into two main types, namely, aleatory (random or irreducible or Type A) uncertainty and epistemic (reducible or Type B) uncertainty (Nikolaidis & Haftka 2001). Aleatory uncertainty arises due to inherent randomness in physical phenomenon or processes, and epistemic uncertainty arises due to lack of knowledge about the quantities. While probability theory has been traditionally used to represent both types of uncertainties, various researchers have pointed out that it may not be proper to use probability theory to represent epistemic uncertainty in the presence of limited knowledge (Helton & Oberkampf 2004, Chen *et al* 1999). A number of alternative theories, such as fuzzy set theory, evidence theory, convex modelling, and interval analysis, for modelling epistemic uncertainties have been proposed by various researchers (Helton & Oberkampf 2004). In particular, fuzzy set theory provides a more rational framework for handling uncertainties arising from *vagueness*, namely, imprecision of definition or use of

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linguistic terms in a natural or artificial language (Ross 1995). The benefits of fuzzifying uncertain variables include greater generality, higher expressive power, an enhanced ability to model real world problems, and a methodology for exploiting tolerance for imprecision (Ross 1995).

Savoia (2002) proposed a procedure for structural reliability analysis using the framework of possibility theory, and has shown that the proposed procedure gives conservative values for small or large fractiles of output variables when compared to that obtained using probability theory. But Savoia (2002) considered only the fuzzy uncertainties, while in real world problem both probabilistic and fuzzy uncertainties co-exist. Thus, there is a need to develop special techniques, which can handle hybrid uncertainties (i.e. fuzzy and random), for carrying out safety assessment. Different methods have been proposed by various researchers for handling fuzzy and random uncertainties together (Haldar & Reddy 1992, Dubois *et al* 1993, Anoop *et al* 2006, Anoop *et al* 2008). One method is to determine equivalent probability distributions for the fuzzy sets (or vice versa), and carry out the analysis in the framework of probability theory (or fuzzy set theory) (Dubois *et al* 1993, Anoop *et al* 2006). But, according to possibility theory, if the membership function of the fuzzy set is the only available information, there is a class of probability measures which are equally valid (Savoia 2002). Hence, it is not adequate to represent the fuzzy set using a single equivalent probability distribution in the reliability analysis. In the present study, a procedure to determine the bounds on failure probability compatible with the available data, considering both probabilistic and fuzzy uncertainties, is proposed. The procedure is based on possibility theory, and can be used for calculating the bounds on failure probability for stochastic systems.

The paper is organized as follows. A brief introduction to possibility theory is given in the next section. The proposed procedure for determination of bounds on failure probability in the presence of probabilistic and fuzzy uncertainties is presented in § 3. The procedure is illustrated through an example problem of safety assessment of a nuclear power plant piping component subjected to stress corrosion cracking in § 4. The results and discussions are given in § 5, and the conclusions are given in § 6.

2. Possibility theory

Theory of possibility is based on two measures of confidence, namely, possibility measure (Π) and necessity measure (N). A confidence measure, $g(A)$, is a number $0 \leq g(A) \leq 1$, which represents the confidence one has on the occurrence of event $A \subseteq \Omega$, where Ω is the sure event (Savoia 2002). While several kinds of confidence measures can be defined, depending on the amount and the type of information available, all confidence measures must satisfy at least the axiom of monotonicity with respect to set inclusion. That is,

$$A \subseteq B \Rightarrow g(A) \leq g(B). \quad (1)$$

If the membership function of a fuzzy set X, defined over Ω, is the available information, the possibility and necessity measures, associated with the occurrence of an event $A \subseteq \Omega$, are defined as (Savoia 2002, Ferrari & Savoia 1998, Dubois *et al* 1999):

$$\Pi(A) = \sup_{\omega \in A} \pi_x(\omega) \quad (2)$$

$$N(A) = \inf_{\omega \in \bar{A}} (1 - \pi_x(\omega)) = 1 - \Pi(\bar{A}), \quad (3)$$

where π_x is the possibility distribution of X. Since it is assumed that membership function of the fuzzy set X is available, the same can be considered as the possibility distribution of X. In equation (3), $\bar{A}$ denotes complement of event A. It has been shown by Dubois & Prade (1980) and Savoia (2002) that possibility and necessity are limit cases of an equivalence class of probability distributions compatible with available data. This is in line with the consistency principle proposed by Zadeh which has been translated by Dubois & Prade (1980) as: 'the degree of possibility of an event is greater than or equal to its degree of probability, which must be itself greater than or equal to its degree of necessity'. Thus, an equivalence class $\mathcal{P}$ of probability measures P compatible with available data can be defined as:

$$\{P|\forall A, N(A) \leq P(A) \leq \Pi(A)\}. \tag{4}$$

Without any additional information, all probability measures defined by equation (4) are equally valid (Savoia 2002, Ferrari & Savoia 1998).

Considering a fuzzy variable Q with membership function $\mu_Q(x)$ and defining an event A as $A = (-\infty, x]$, the following cumulative density functions (CDF) can be defined:

$$F_*(x) = N((-\infty, x])$$

$$F^*(x) = \Pi((-\infty, x]). \tag{5}$$

From equation (4), it is noted that the CDFs defined by equation (5) represent lower- and upper-bound for all probability measures P belonging to class $\mathcal{P}$, compatible with available data. Thus,

$$F_*(x) \leq F(x) \leq F^*(x);\ F(x) = P((-\infty, x]). \tag{6}$$

Using equations (2) and (3), $F_*(x)$ and $F^*(x)$ can be rewritten in terms of membership function of fuzzy set Q as:

$$F_*(x) = \inf\{1 - \mu_Q(\omega)/\omega > x\}$$

$$F^*(x) = \sup\{\mu_Q(\omega)/\omega \leq x\}. \tag{7}$$

Thus, from the membership function of the fuzzy variable, one can determine the CDFs corresponding to the lower-bound ($F_*(x)$) and upper-bound ($F^*(x)$) of X, representing the bounds for all probability measures compatible with available information on X (figure 1). Each of these CDFs can be used in reliability analysis to compute the respective values of probability of failure (P_F) for a specified value of X. For cases where P_F is a monotonic function of variable X, the values of failure probability obtained using the CDFs $F_*(x)$ and $F^*(x)$ will give the bounds on P_F. The proposed procedure for determining the bounds on P_F in the presence of probabilistic and fuzzy uncertainties is given in the next section.

3. Proposed procedure

(i) Identify the random variables ($X_r^1, X_r^2, \ldots, X_r^n$) and fuzzy variables ($X_f^1, X_f^2, \ldots, X_f^m$) based on source and type of uncertainties.

(ii) For each random variable, X_r^i, identify the type of probability distribution and the values of the associated parameters.

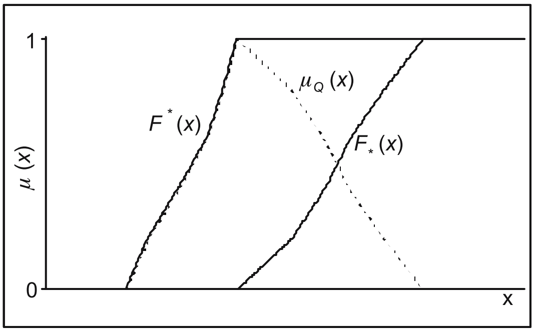

Figure 1. Bounds for probability measures compatible with the available information.

(iii) For each fuzzy variable, X_f^i, identify the suitable membership function, $\mu_{X_f^i}$. For engineering applications, to reduce computational complexity, fuzzy sets with triangular or trapezoidal form are most commonly used (Anoop *et al* 2006).
(iv) For each fuzzy variable, X_f^i, determine the lower-bound CDF, $F_*(x_f^i)$, and the upper-bound CDF, $F^*(x_f^i)$, using equation (7).
(v) Carry out the reliability analysis considering each combination of $F_*(x_f^i)$ and $F^*(x_f^i)$ for the m fuzzy variables along with the random variables $(X_r^1, X_r^2, \ldots, X_r^n)$ together (total number of combinations to be considered[1] $= 2^m$), and compute the respective values of P_F.
(vi) Determine the bounds for failure probability as the minimum ($P_{F,\min}$) and maximum ($P_{F,\max}$) values from the 2^m values of P_F obtained.

Computation of values of P_F in Step 5 involves repeated application of reliability analysis techniques such as FORM, SORM. It is known that application of FORM/SORM for time variant reliability analysis problems with resistance degradation would involve restrictive assumptions regarding resistance and loading process types so that the problem can be solved within analytical framework. However, such difficulties can be overcome by Monte Carlo simulation techniques, and hence in this study, Monte Carlo simulation technique is used for computing the values of P_F. It is realised that Monte Carlo simulation technique is computationally intensive. An example problem of safety assessment of a nuclear power plant piping component against stress corrosion cracking, considering stochastic evolution of stress corrosion cracks with time, is provided in the next section to illustrate the proposed procedure.

4. Example

Stress corrosion cracking (SCC) is an important degradation mechanism to be considered for safety assessment of nuclear piping components made of austenitic steels. Safety assessment of a typical piping component made of AISI 304 stainless steel against SCC is carried out using proposed procedure. The sample problem, given as SCC baseline case in PRAISE manual (Harris *et al* 1992), is considered for this purpose. The piping component has an inner radius

[1]For each fuzzy variable, two combinations need to be considered; one corresponding to the upper-bound CDF and the other corresponding to the lower-bound CDF. Hence the total number of combinations to be considered is $2 \times 2 \times \cdots \times 2$ (m times) $= 2^m$.

Table 1. Pipe geometry, material properties and environmental conditions.

Pipe wall thickness (m)	0·021
Inner diameter (m)	0·364
Operating temperature (°C)	288·0
Oxygen concentration (ppm)	0·20
Water conductivity (μs/cm)	0·20
Pipe material	AISI 304 stainless steel
Modulus of elasticity, E (MPa)	$1{\cdot}83 \times 10^5$

of 181·86 mm and a thickness of 21·34 mm. Stress due to dead weight and thermal expansion is 70·38 MPa and the operating pressure is 8·63 MPa. Total tensile stress in the pipe due to dead weight, thermal expansion and operating pressure is 105·09 MPa. It is assumed that there are no residual stresses. The *approximate* water temperature (T) and *average* oxygen concentration (O_2) at steady state condition are 288°C and 0·20 ppm, respectively. The inputs related to geometry and material properties of the pipe and operating conditions are given in table 1.

4.1 *Modelling stress corrosion cracking*

The methodology recommended in PRAISE (Harris *et al* 1992) for modelling SCC in pipe is followed in this study. In PRAISE, occurrence of SCC is modelled by considering it as a two-stage process, namely, (1) crack initiation and (2) crack propagation. Methodology recommended in PRAISE for modelling SCC is briefly described below.

4.1a *Time to initiation:* Time to initiation of stress corrosion crack is considered as a function of damage parameter, D, which represents effects of loading, environment and material variables on SCC. The damage parameter is given by

$$D = f_1(\text{material}) \cdot f_2(\text{environment}) \cdot f_3(\text{loading}), \tag{8}$$

where f_1, f_2 and f_3 are given by

$$f_1 = C_1(Pa)^{C_2}, \tag{9}$$

where Pa is a measure of degree of sensitization, given by EPR (Electrochemical Potentiokinetic Reactivation) (in C/cm^2).

$$f_2 = O_2^{C_3} \exp[C_4/(T + 273)] \log(C_5\gamma^{C_6}), \tag{10}$$

where O_2 is oxygen concentration in ppm, T is temperature in degrees centigrade and γ is water conductivity in μs/cm.

The loading term f_3 is considered to be a function of stress. For constant applied load case, f_3 is given by

$$f_3 = (C_8\sigma^{C_9})^{C_7}, \tag{11}$$

where σ is stress in ksi.

C_1 to C_9 are constants whose values depend on type of material, and are evaluated by applying curve-fitting procedures to laboratory and field data. For AISI 304 austenitic stainless steel, values of these constants are given by $C_1 = 23{\cdot}0, C_2 = 0{\cdot}51, C_3 = 0{\cdot}18, C_4 = -1123{\cdot}0, C_5 = 8{\cdot}7096, C_6 = 0{\cdot}35, C_7 = 0{\cdot}55, C_8 = 2{\cdot}21 \times 10^{-15}$ and $C_9 = 6{\cdot}0$ (Harris *et al* 1992).

In order to cater to observed scatter in experimental data of initiation time, time to initiation (t_I) for a given D is considered as a random variable following lognormal distribution. The mean and standard deviation of $\log(t_I)$ are given by:

$$\left.\begin{array}{l}\text{Mean value of } \log(t_I) = B_0 + B_1 \log(D) \\ \text{Standard deviation of } \log(t_I) = B_2 + B_3 \log(D)\end{array}\right\}. \tag{12}$$

B_0, B_1, B_2 and B_3 are constants whose values depend on type of material and loading conditions (i.e. constant load or changing load), and are evaluated by applying curve-fitting procedures to laboratory and field data. For AISI 304 austenitic stainless steel under constant load, $B_0 = -3{\cdot}10$, $B_1 = -4{\cdot}21$, $B_2 = 0{\cdot}3081$ and $B_3 = 0{\cdot}0$ (Harris *et al* 1992).

4.1b *Crack size at initiation:* In PRAISE, shape of surface crack initiated due to SCC is considered to be semi-elliptical, which is also consistent with shapes of stress corrosion cracks reported by Helie *et al* (1996) and Lu *et al* (2005). Surface length of initiated cracks, ($l = 2b$), is assumed to be lognormally distributed with a median value of 1/8 inch and standard deviation of $\log(b)$ as 0·85 (Harris *et al* 1992). Depth of initiated crack is taken to be 0·0254 mm (0·001 inch).

4.1c *Multiple cracks:* In materials subjected to SCC, many cracks would initiate successively and propagate simultaneously (Lu *et al* 2005), and hence multiple cracks can be present in a given weld. The expressions, given in PRAISE, for determining statistical properties of t_I, are mainly based on data from laboratory experiments on specimens about 50 mm long. Hence, these expressions are applicable to specimens of about 50 mm only. This is taken into account in PRAISE, by considering a given weld in the pipe to be composed of 50 mm segments adding up to length of weld. Initiation time for each segment is assumed to be independent and identically distributed.

4.1d *Crack propagation due to SCC:* It is assumed that initiated cracks grow at a constant velocity (initiation velocity, v_1) until conditions are appropriate for treating crack growth by fracture mechanics. The statistical properties of v_1 are determined using expressions given in PRAISE, which is obtained by correlating results of laboratory experiments with values of damage parameter (D). In order to take into account considerable scatter in v_1 observed during experiments, v_1 is considered as a lognormally distributed random variable for a given D. While the standard deviation of v_1 is independent of D, mean of $\log(v_1)$ varies linearly with $\log(D)$, and is given by:

$$\log(v_1) = F + G \log(D), \tag{13}$$

where F is normally distributed and G is a constant. It can be noted that equation (13) is similar in form (power law) as that proposed by Helie *et al* (1996), based on experimental observations. For AISI 304 austenitic stainless steel, F has a mean of 2·551 and standard deviation of 0·4269, and $G = 1{\cdot}3447$ (Harris *et al* 1992). From parametric studies carried out, a value of 5% coefficient of variation of $\log(v_1)$ was found to be critical to life of piping component (Priya *et al* 2005), and hence this value is used in this study.

The procedure followed for transition from initiation to fracture mechanics crack growth rate in the present study is (Anoop *et al* 2008):

(i) Pre-existing cracks always grow at fracture mechanics velocity.
(ii) Initiation velocity is always assigned to initiated cracks.
(iii) At any given time, if fracture mechanics velocity (v_2) is greater than initiation velocity (i.e. $v_2 > v_1$) and depth of crack is greater than 2·54 mm, that particular crack grows at fracture mechanics velocity thereafter.
(iv) If the stress intensity factor for a crack is negative, the crack will not grow.

Fracture mechanics based crack growth velocity, v_2 (inches/year), is given by (Harris *et al* 1992):

$$\log(v_2) = C_{14} + C_{15} D_K, \tag{14}$$

where D_K is stress intensity factor (K)—related damage parameter given by

$$D_K = C_{12} \log[f_2(\text{environment})] + C_{13} K, \tag{15}$$

where C_{12}, C_{13} and C_{15} are constants and C_{14} is normally distributed. For AISI 304 austenitic stainless steel, $C_{12} = 0{\cdot}8192$, $C_{13} = 0{\cdot}03621$ and $C_{15} = 1{\cdot}7935$, mean value of $C_{14} = -3{\cdot}1671$ and standard deviation of $C_{14} = 0{\cdot}7260$ (Harris *et al* 1992).

From a probabilistic failure analysis of austenitic nuclear pipe against SCC, Priya *et al* (2005) inferred that expressions given in PRAISE for computation of stress intensity factors for modelling crack propagation need modification. In Priya *et al* (2005), this modification has been introduced by using well-accepted expressions given in ASM, and with modified PRAISE approach, stochastic propagation of stress corrosion cracks with time has been studied. It has been noted that trend of distribution of crack depths at initial stages is in satisfactory agreement with relevant experimental observations reported in literature (Priya *et al* 2005). Hence, modified PRAISE approach proposed by Priya *et al* (2005) is followed in the present study.

4.1e *Coalescence of cracks:* The multiple cracks that may be present can coalesce as they grow. Linking of two cracks takes place if spacing between them is less than the sum of their depths. After coalescence of two cracks, dimensions of modified crack are given by (Harris *et al* 1992)

$$\left.\begin{array}{l} \text{length, } l = l_1 + S + l_2 \\ \text{depth, } a = a_1 \text{ or } a_2\text{, whichever is greater} \end{array}\right\}, \tag{16}$$

where l_1 and l_2 are lengths of two cracks under consideration, a_1 and a_2 are crack depths and S is spacing between them.

The initiation and propagation stages of stress corrosion cracks are modelled using modified PRAISE approach (Anoop *et al* 2008, Harris *et al* 1992, Priya *et al* 2005). The random variables, together with their distributions and their statistical properties, namely, mean and coefficient of variation (or standard deviation), are presented in table 2. To take into consideration the uncertainties arising due to the linguistic specifications of T and O_2, these variables are represented by fuzzy sets with symmetric triangular membership functions. For engineering applications, to reduce computational complexity, fuzzy sets with triangular or trapezoidal form are most commonly used (Anoop *et al* 2006). Also, it has been shown by Dubois *et al* (1999) that symmetric triangular fuzzy sets are natural fuzzy counterparts to uniform probability distributions on bounded intervals, and hence, are the best probabilistic

Table 2. Details of random variables considered.[a]

Variable	Distribution	Parameters	Reference
Applied stress (σ)	Normal	Mean = 105·09 COV[b] = 0·20	Priya *et al* (2005)
Yield strength (σ_y)	Lognormal	Mean = 151·53 MPa SD[c] = 14·86 MPa	Rahman (1997)
Ultimate strength (σ_u)	Lognormal	Mean = 450·63 KJm^{-2} SD = 25·53 KJm^{-2}	Rahman (1997)
Material fracture toughness[d] (J_{IC})	Lognormal	Mean = 1059·56 KJm^{-2} SD = 450·0 KJm^{-2}	Rahman (1997)

(Notes: [a]-In addition, degree of sensitization, time to crack initiation, initial crack length, crack initiation velocity and fracture mechanics based crack growth velocity are considered as random variables with parameters given in PRAISE manual (Harris *et al* 1992, Priya *et al* 2005).
[b]-COV - Coefficient of variation
[c]-SD - Standard deviation
[d]-K_{IC} is computed from $J_{IC} = K_{IC}^2/E'$ where $E' = E/(1-\nu^2)$ for plane strain, ν is Poisson's ratio, taken as 0·3)

approximation. The specified values for the fuzzy variables, given in table 1, are taken as the corresponding prototypical elements (one and only element with grade of membership equal to 1·0). Since authors could not get information on variation of T, support of fuzzy set of operating temperature is determined assuming a variation of $\pm 10\%$ about the specified value. A higher variation of $\pm 20\%$ about the specified value is chosen for determining support of fuzzy set of O_2 since it is found to vary over a significant range (Lin 1996). The membership functions of different fuzzy variables are shown in figure 2.

The rate of reduction in pipe wall thickness due to SCC increases with increase in operating temperature (T) and oxygen concentration (O_2). Therefore, P_F can be considered as a monotonic function of T and O_2, and hence the proposed procedure can be used for determining the bounds on P_F. The R6 procedure with option 1 curve (Ainsworth 1996) is used for safety assessment. Reference stress values (σ_{ref}), required in R6 procedure, are determined using the expressions given in API (2000). Flow stress (σ_f) is computed as the average value of yield strength (σ_y) and ultimate strength (σ_u). Following assumptions are made in the analysis:

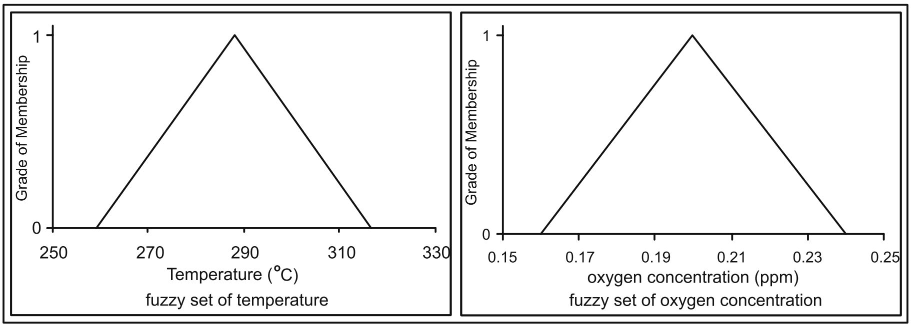

Figure 2. Fuzzy sets of operating temperature and oxygen concentration.

- Variation in material properties (namely, material fracture toughness, yield strength, ultimate strength and modulus of elasticity) with change in temperature, for the range of operating temperature considered (figure 2), is neglected.
- From the results of statistical analysis reported by Rahman (1997), it is noted that the degree of correlation between yield strength and ultimate strength of AISI 304 stainless steel is small (i.e. correlation coefficient = 0·31). Hence, yield strength and ultimate strength are assumed to be statistically independent.
- Modulus of elasticity of AISI 304 stainless steel is considered to be deterministic.

In the present study, Monte Carlo simulation method is used for determining the failure probability. Ten thousand simulation cycles are used for each combination of CDF bounds of the fuzzy variables (total number of combinations $= 2^m = 4$, since there are two fuzzy variables in this case), and inverse transformation technique is used for the generation of values of random variables corresponding to the CDFs considered.

For the purpose of comparison, probabilistic analyses are also carried out by: (i) without considering fuzzy uncertainties, taking values of T and O_2 as the specified values (hereafter this case is referred to as Probabilistic Analysis I), and, (ii) by considering T and O_2 as random variables (instead of fuzzy variables) following symmetric triangular probability distributions with same range as that of corresponding fuzzy set (hereafter this case is referred to as Probabilistic Analysis II).

5. Results and discussion

The aspect ratio of cracks (defined as the ratio of crack depth to half crack length) that corresponds to failure at different times, for the four combinations of the CDFs considered, are shown in figure 3. From the dimensions of the cracks corresponding to failure, it is noted that the coalesced cracks are the ones which contribute to failure, i.e. the initiated cracks coalesce as they grow, and further growth of these coalesced cracks lead to failure.

The variation in P_F with time corresponding to four combinations of the CDFs are shown in figure 4. From this figure, it is noted that the combination corresponding to upper-bound CDFs of T and O_2 gives lower values of P_F while that corresponding to lower-bound CDFs gives higher values of P_F. This is expected because use of lower-bound CDFs result in generation of variables with higher values, and since rate of reduction in pipe wall thickness due to SCC increases with increase in T and O_2, combination of lower-bound CDFs of T and O_2 gives higher values of P_F.

The effect of coalescence of cracks on failure probability is shown in figure 5. It is noted from this figure that, as expected, an increase (or decrease) in the number of coalescing cracks result in an increase (or decrease) in the rate of failure probability in the near future (within 3 to 4 years). Thus, the plateau observed in the time versus failure probability curves (as seen in figures 4 and 5) can be attributed to the decrease in the number of coalescing cracks.

The bounds on P_F obtained from the proposed procedure along with the P_F values obtained using Probabilistic Analyses I and II are shown in figure 6. It is noted that values of P_F obtained using Probabilistic Analyses I and II are more or less in agreement with each other, indicating that the random variations in T and O_2 do not affect failure probability significantly. It is also noted that, as expected, bounds on P_F obtained using the proposed procedure encloses the P_F values obtained from Probabilistic Analyses I and II. While it is possible to determine the bounds on P_F by carrying out a probabilistic analysis, the bounds obtained using the proposed procedure are more rational since appropriate representations of uncertainty

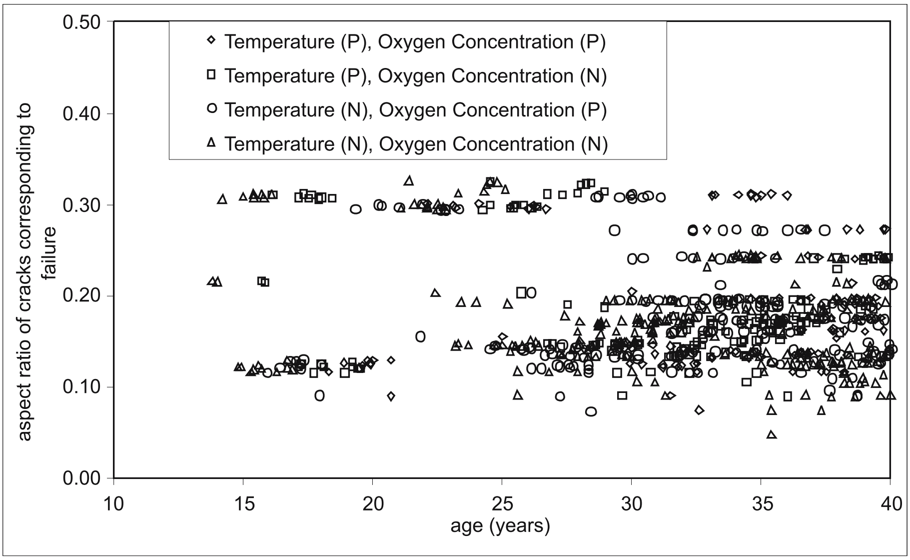

Figure 3. Aspect ratio of cracks corresponding to failure at different times for the four combinations of lower- and upper-bound CDFs considered ((P) - upper-bound CDF corresponding to possibility measure; (N) - lower-bound CDF corresponding to necessity measure).

are used for the different variables, and hence are consistent with the available information. The upper-bound for P_F shown in figure 6 can be used in decision-making. For instance, at the design stage, the upper-bound curve can be used along with the specified target P_F

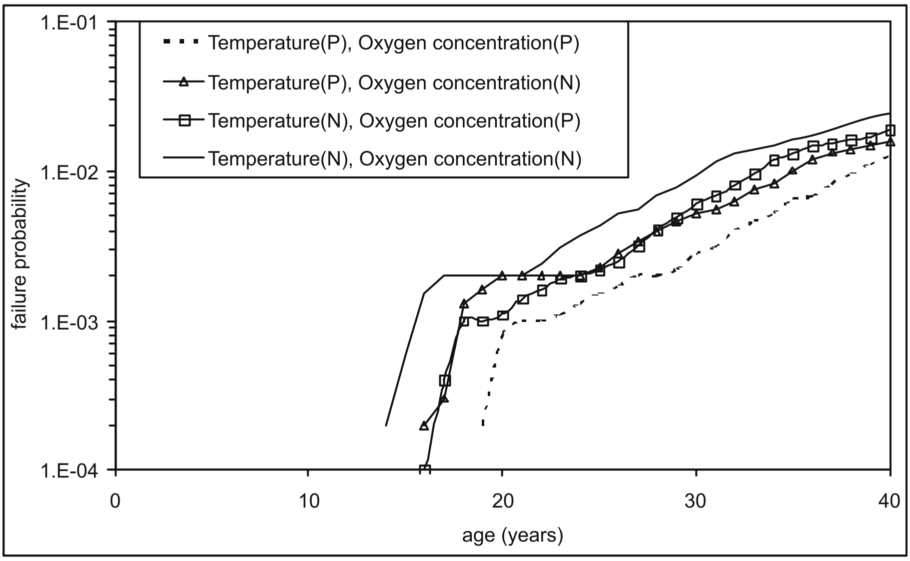

Figure 4. Variation in P_F with time for the four combinations lower- and upper-bound CDFs considered for the piping component subjected to stress corrosion cracking ((P) - upper-bound CDF corresponding to possibility measure; (N) - lower-bound CDF corresponding to necessity measure).

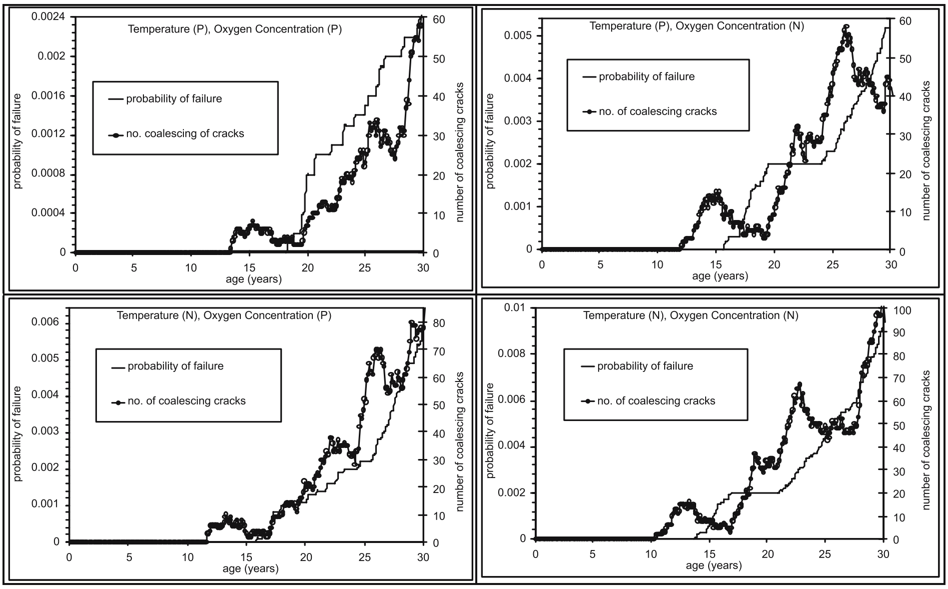

Figure 5. Variation in P_F and number of coalescing cracks with time for the four combinations of lower- and upper-bound CDFs considered ((P) - upper-bound CDF corresponding to possibility measure; (N) - lower-bound CDF corresponding to necessity measure).

values for design decision-making. For existing power plants, the P_F value can be combined with conditional core damage probability associated with SCC failure of piping component to determine contribution towards plant core damage probability. Carrying out this type of

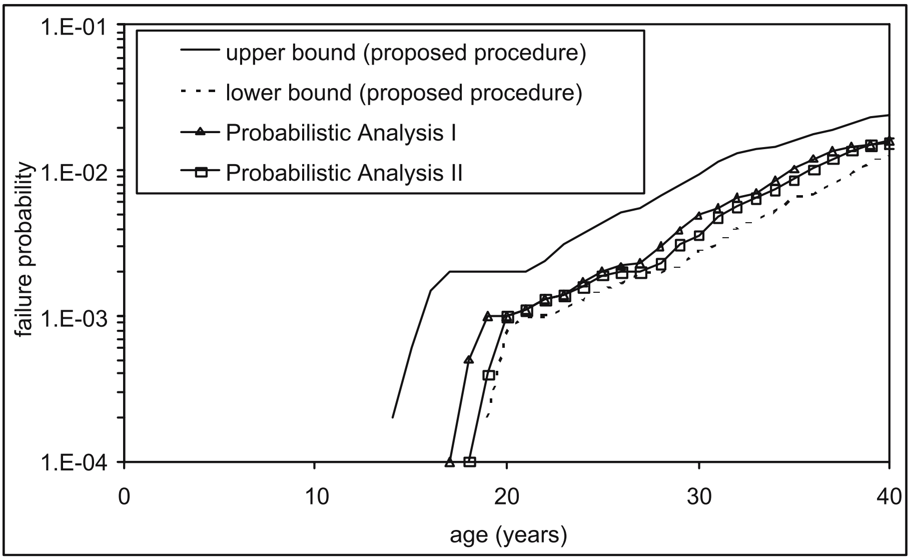

Figure 6. Comparison of bounds for P_F obtained using proposed procedure with the P_F values obtained using probabilistic approaches for the piping component subjected to stress corrosion cracking.

analysis for different types of piping components susceptible to SCC will help in ranking them for risk-informed, in-service inspection and maintenance activities.

6. Conclusions

A procedure for safety assessment in the presence of probabilistic and fuzzy uncertainties is presented. The procedure can be used for determining the bounds on failure probability for cases where the failure probability is a monotonic function of the variables considered as fuzzy. The usefulness of the proposed procedure is illustrated through an example problem of safety assessment of a nuclear power plant piping component against stress corrosion cracking, considering stochastic evolution of stress corrosion cracks with time. It is noted that the bounds on failure probability obtained using the proposed procedure encloses the values of failure probability obtained from probabilistic analysis. From the results obtained, it is noted that the approach presented shows promise for safety assessment of engineering facilities, such as nuclear power plants.

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