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Effect of blow-holes on reliability of cast component

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Abstract. This paper presents a study on the effect of blow-holes on the reliability of a cast component. The most probable point (MPP) based univariate response surface approximation is used for evaluating reliability. Crack geometry, blow-hole dimensions, external loads and material properties are treated as independent random variables. The methodology involves novel function decomposition at a most probable point that facilitates the MPP-based univariate response surface approximation of the original multivariate implicit limit state/performance function in the rotated Gaussian space. Once the approximate form of the original implicit limit state/performance function is defined, the failure probability can be obtained by Monte Carlo simulation (MCS), importance sampling technique, and first- and second-order reliability methods (FORM/SORM). FORTRAN code is developed to automate calls to ABAQUS for numerically simulating responses at sample points, to construct univariate response surface approximation, and to subsequently evaluate the failure probability by MCS, importance sampling technique, and FORM/SORM.

Keywords. Cast component; blow-holes; reliability; univariate response surface approximation; failure probability.

1. Introduction

Metal casting process begins by creating a mold, which is the 'reverse' shape of the part that is needed. The mold is made from a refractory material like sand. The metal is heated in an oven until it melts, and the molten metal is poured into the mould cavity. The liquid takes the shape of cavity, which is the shape of the part. It is cooled until it solidifies. Finally, the solidified metal part is removed from the mould. Casting process inevitably produces micro-defects like blow-holes, which become more important for reliability assessment since these defects usually are larger than other micro-structural features. Blow-holes and pinholes are produced because of gas entrapped in the metal during the course of solidification. Blow-holes are smooth-walled cavities, essentially spherical in shape. The largest cavities are most often isolated; the smallest (pinholes) appear in groups of varying dimensions as shown in

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Figure 1. Blow-holes in cast component.

figure 1. In specific cases, the casting section can be strewn with blow-holes or pinholes. In the open literature the influence of porosity on the fatigue properties for cast aluminium has been investigated earlier (Stanz-Tschegg *et al* 1993; Sonsino & Ziese 1993; Skalleud 1993; Debayeh *et al* 1996; Evans *et al* 1997; Knott *et al* 2000; Mayer *et al* 2003; Linder *et al* 2006) and it is generally accepted that fatigue strength is decreased by the presence of porosity. In order to explain the role of porosity, fracture mechanics has been found to be a useful tool (Skalleud 1993; Debayeh *et al* 1996; Evans *et al* 1997; Knott *et al* 2000; Mayer *et al* 2003). By the use of fracture mechanics, blow-hole/pore size and the remote stress become crucial parameters that will affect life of cast component. Since the size and location of defects are quite random, deterministic analysis provides an incomplete picture of the safety of cast component. Moreover, the randomness in external loads and geometry also influence the reliability of cast component. From the existing literature, it is evident that in the analysis of cast components, the effect of micro-defects like blow-holes on the stress intensity factors (SIFs) is neglected. Probabilistic fracture mechanics, which combines fracture mechanics with stochastic methods, provide a useful tool to address these problems. Given the complexity of failure mechanism, the combination of finite element method with the theory of statistics and reliability has become a powerful method for safety and reliability analysis.

Fundamental problem in time-invariant component reliability analysis entails calculation of a multi-fold integral (Madsen *et al* 1986; Ditlevsen & Madsen 1996; Rackwitz 2001).

$$P_F \equiv P[g(\boldsymbol{X}) < 0] = \int_{g(\boldsymbol{x})<0} f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x}, \tag{1}$$

where $\boldsymbol{X} = \{X_1, \dots, X_N\}^T \in \mathbb{R}^N$ is a real-valued, N-dimensional random vector defined on a probability space (Ω, F, P) comprising the sample space Ω, the σ-field F, and the probability measure P; $g(\boldsymbol{x})$ is the performance function, such that $g(\boldsymbol{x}) < 0$ represents the failure domain; P_F is the probability of failure; and $f_{\boldsymbol{X}}(\boldsymbol{x})$ is the joint probability density function of $\boldsymbol{X}$, which typically represents loads, material properties, and geometry. For most practical problems, the exact evaluation of this integral, either analytically or numerically, is not possible because N is large, $f_{\boldsymbol{X}}(\boldsymbol{x})$ is generally non-Gaussian, and $g(\boldsymbol{x})$ is highly non-linear function of $\boldsymbol{x}$. The most common approach to compute the failure probability in Equation 1 involves the FORM/SORM (Breitung 1984; Madsen *et al* 1986; Hohenbichler *et al* 1987; Tvedt 1990; Cai & Elishakoff 1994; Ditlevsen & Madsen 1996; Der Kiureghian & Dakessian; 1998; Rackwitz 2001), which are based on linear (FORM) or quadratic approximation (SORM) of the limit-state surface at a most probable point (MPP). Experience has shown that FORM/SORM are sufficiently accurate for engineering purposes, provided that the limit-state surface at the MPP is close to being linear or quadratic, and no multiple MPPs exist. For highly non-linear performance functions, which exist in many structural problems, results based on FORM/SORM must be interpreted with caution. If the Rosenblatt transformation, frequently used to map non-Gaussian random input into its standard Gaussian image, yields a highly non-linear limit state, inadequate reliability estimates by FORM/SORM may result (Bjerager 1988; Nie & Ellingwood 2000). Furthermore, the existence of multiple MPPs could give rise to large errors in standard FORM/SORM approximations (Ditlevsen & Madsen 1996; Der Kiureghian & Dakessian 1998). In that case, multi-point FORM/SORM along with the system reliability concept is required for improving component reliability analysis (Der Kiureghian & Dakessian 1998).

This paper presents a study on the effect of blow-holes on the reliability of a cast component, using univariate response surface approximation (Xu & Rahman 2004, 2005; Rahman & Wei 2006) for evaluating reliability. Crack geometry, blow-hole dimensions, external loads and material properties are treated as independent random variables. In the present study, MPP-based univariate method is used to obtain the response surface approximation of the original multivariate implicit limit state/performance function. The failure probability is evaluated using the response surface approximation in conjunction with MCS, importance sampling technique, and FORM/SORM. Finite element commercial software ABAQUS, 2002 is used for numerically simulating responses at each sample point that are required for construction of univariate response surface approximation. FORTRAN code is developed which calls ABAQUS for simulating responses at each sample point, reads in the required result (SIFs in this case), deletes the resulting files after each run, and proceeds with simulating responses at next sample point. The whole process is simulated efficiently since accurate results are obtained from ABAQUS.

2. Multivariate function decomposition at MPP

Consider a continuous, differentiable, real-valued performance function $g(\boldsymbol{x})$ that depends on $\boldsymbol{x} = \{x_1, \ldots, x_N\}^T \in \mathbb{R}^N$. The transformed limit states $h(\boldsymbol{u}) = 0$ and $y(\boldsymbol{v}) = 0$ are the maps of the original limit state $g(\boldsymbol{x}) = 0$ in the standard Gaussian space ($\boldsymbol{u}$ space) and the rotated Gaussian space ($\boldsymbol{v}$ space), respectively, as shown in figure 2 for $N = 2$. The closest point on the limit-state surface to the origin, denoted by the MPP ($\boldsymbol{u}^*$ or $\boldsymbol{v}^*$) or beta point, has a distance β_{HL}, which is commonly referred to as the Hasofer–Lind reliability index (Madsen *et al* 1986; Ditlevsen & Madsen 1996; Rackwitz 2001). The determination of MPP and β_{HL} involves standard non-linear constrained optimization and is usually performed in the standard Gaussian space. Figure 2 depicts FORM and SORM approximations of the limit-state surface at MPP.

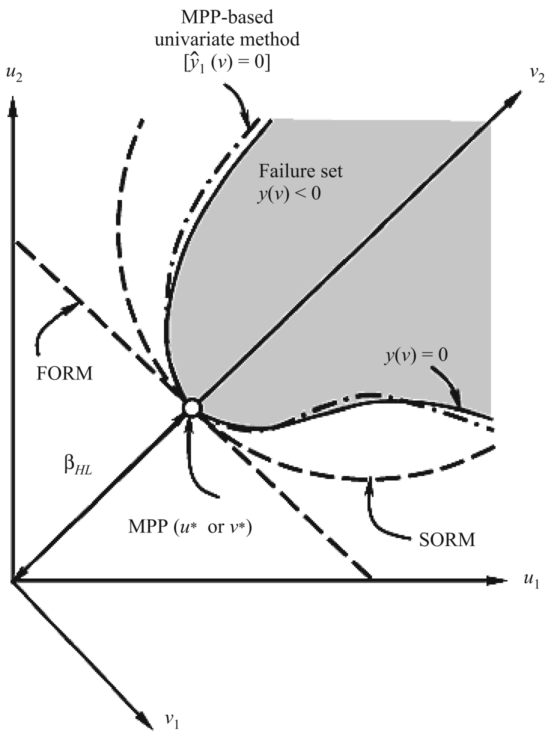

Figure 2. Performance function approximation by various methods.

Suppose that $y(\boldsymbol{v})$ has a convergent Taylor series expansion at MPP $\boldsymbol{v}^* = \{v_1^*, \ldots, \ldots, v_N^*\}^T$ and can be expressed by

$$y(\boldsymbol{v}) = y(\boldsymbol{v}^*) + \sum_{j=1}^{\infty} \frac{1}{j!} \sum_{i=1}^{N} \frac{\partial^j y}{\partial v_i^j}(\boldsymbol{v}^*)(v_i - v_i^*)^j + R_2, \tag{2}$$

where the remainder R_2 denotes all terms with dimensions two and higher.

Consider a univariate approximation of $y(\boldsymbol{v})$, denoted by

$$\begin{aligned} \hat{y}_1(\boldsymbol{v}) &\equiv \hat{y}_1(v_1, \ldots, v_N) \\ &= \sum_{i=1}^{N} y(v_1^*, \ldots, v_{i-1}^*, v_i^*, v_{i+1}^*, \ldots, v_N^*) - (N-1)y(\boldsymbol{v}^*), \end{aligned} \tag{3}$$

where each term in the summation is a function of only one variable and can be subsequently expanded in a Taylor series at $\boldsymbol{v} = \boldsymbol{v}^*$, yielding

$$\hat{y}_1(\boldsymbol{v}) \equiv \hat{y}_1(v_1, \ldots, v_N) = y(\boldsymbol{v}^*) + \sum_{j=1}^{\infty} \frac{1}{j!} \sum_{i=1}^{N} \frac{\partial^j y}{\partial v_i^j}(\boldsymbol{v}^*)(v_i - v_i^*)^j. \tag{4}$$

Comparison of Equations 2 and 4 indicates that the univariate approximation leads to the residual error $y(\boldsymbol{v}) - \hat{y}_1(\boldsymbol{v}) = R_2$, which includes contributions from terms of two-dimension and higher. For sufficiently smooth $y(\boldsymbol{v})$ with convergent Taylor series, the coefficients associated with higher-dimensional terms are usually much smaller than that with one-dimensional terms. As such, higher-dimensional terms contribute less to the function, and therefore, can be neglected (Xu & Rahman 2004, 2005; Rahman & Wei 2006).

3. Response surface generation

Consider the univariate component function $y_i(v_i) \equiv y(v_1^*, \ldots, v_{i-1}^*, v_i^*, v_{i+1}^*, \ldots, v_N^*)$ in Equation 4. If for $v_i = v_i^{(j)}$, n function values

$$y_i(v_i^{(j)}) \equiv y(v_1^*, \ldots, v_{i-1}^*, v_i^{(j)}, v_{i+1}^*, \ldots, v_N^*);\ j = 1, 2, \ldots, n, \tag{5}$$

are given, the function value for arbitrary v_i can be obtained using the Lagrange interpolation as

$$y_i(v_i) = \sum_{j=1}^{n} \phi_j(v_i) y_i(v_i^{(j)}), \tag{6}$$

where the shape function $\phi_j(v_i)$ is defined as

$$\phi_j(v_i) = \frac{\prod_{k=1,k\neq j}^{n}(v_i - v_i^{(k)})}{\prod_{k=1,k\neq j}^{n}(v_i^{(j)} - v_i^{(k)})}. \tag{7}$$

By using Equation 6, arbitrarily many values of $y_i(v_i)$ can be generated if n values of that component function are given. The same procedure is repeated for all univariate component functions, i.e. for all $y_i(v_i)$, $i = 1, \ldots, N$. Therefore, the total cost for the univariate approximation in Equation 4, in addition to that required for locating MPP, entails a maximum of $nN + 1$ function evaluations. More accurate bivariate or multivariate approximations can be developed in a similar way. However, because of much higher cost of multivariate approximations, only the univariate approximation will be examined in this paper.

4. Failure probability estimation

For component reliability analysis, MCS estimate P_F of the failure probability employing the proposed univariate approximation is

$$P_F = \frac{1}{N_S} \sum_{i=1}^{N_S} I[\hat{y}_1(\boldsymbol{v}^{(i)}) < 0], \tag{8}$$

where $\boldsymbol{v}^{(i)}$ is the ith realization of $\boldsymbol{v}$, N_S is the sample size, and $I[\bullet]$ is an indicator function such that $I = 1$ if $\boldsymbol{v}^{(i)}$ is in the failure set (i.e. when $\hat{y}_1(\boldsymbol{v}^{(i)}) < 0$ and zero otherwise.

The decomposition method involving univariate approximation (Equation 4), and n-point Lagrange interpolation (Equation 6), is defined as the MPP-based univariate response surface approximation of the original multivariate implicit limit state/performance function. Since the univariate method leads to explicit response-surface approximation of a performance function, the MCS can be conducted for any sample size. Similarly importance sampling technique, and FORM/SORM can be performed using the MPP-based univariate response surface approximation. However, the accuracy and efficiency of the resultant failure-probability calculation depend on both the univariate and response surface approximations. They will be evaluated using numerical example, as follows.

5. Numerical example

In the present study, a generic component of the dimension $20 \times 20 \times 5\,\text{mm}^3$ with a through hole of diameter 5 mm at the centre is considered. Figure 3 shows finite element mesh of the cast component obtained using PATRAN, 2005. Since it is a cast component, blow-holes are bound to occur. It is found that these blow-holes occur as a cluster and the cluster of blow-holes is modelled as a single defect (Elena 2002). In cast component, cracks are found to be initiated at the interface between the cut hole and the surface of cast component. The uncertainty in the crack geometry, blow-hole dimension, material properties and the external loads are characterized by a set of independent random variables. Statistical properties of various random parameters considered in this study are given in table 1. The critical location of the blow-hole with respect to the crack is identified by moving the blow-hole mesh is moved relative to the crack, keeping all random variables at their mean values. The failure criterion is based on a mixed-mode fracture initiation using the maximum tangential stress theory (Anderson 2004), which leads to the performance function

$$g(\boldsymbol{X}) = K_{Ic} - \left[K_I(\boldsymbol{X}) \cos^2 \frac{\Theta(\boldsymbol{X})}{2} - \frac{3}{2} K_{II}(\boldsymbol{X}) \sin \Theta(\boldsymbol{X}) \right] \cos \frac{\Theta(\boldsymbol{X})}{2}, \tag{9}$$

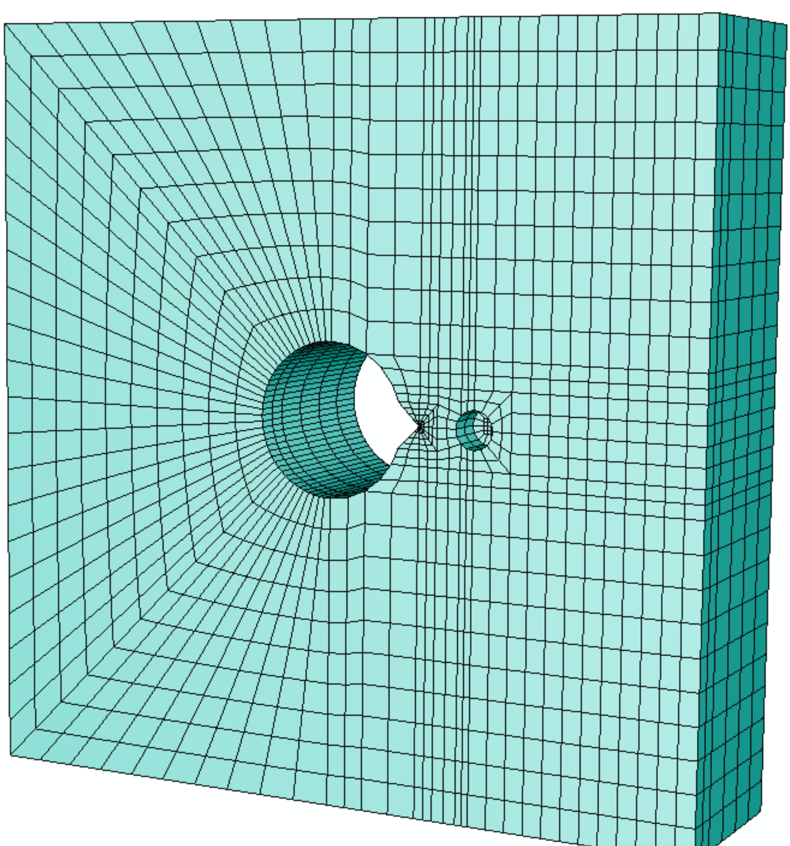

Figure 3. Finite element mesh of cast component.

Table 1. Statistical properties of various random variables.

Random variable	Distribution	Units	Mean	Standard deviation
Crack length (a)	Uniform	mm	0·55	0·0288
Blow-hole diameter (d)	Uniform	mm	0·95	0·0288
Far-field tensile stress (σ^{∞})	Normal	N/mm^2	Variable	5·458
Young's modulus (E)	Log-normal	N/mm^2	207024·71	30·81
Critical SIF (K_{Ic})	Log-normal	MPa$\sqrt{\text{mm}}$	Variable	1·673

where K_{Ic} is statistically distributed fracture toughness, typically measured from small-scale fracture experiments under mode I and plane strain conditions, and $\Theta(\boldsymbol{X})$ is the direction of crack propagation, given by

$$\Theta(\boldsymbol{X}) = \begin{cases} 2\tan^{-1}\left(\frac{1-\sqrt{1+8[K_{II}(\boldsymbol{X})/K_I(\boldsymbol{X})]^2}}{4K_{II}(\boldsymbol{X})/K_I(\boldsymbol{X})}\right), & \text{if } K_{II}(\boldsymbol{X}) > 0 \\ 2\tan^{-1}\left(\frac{1+\sqrt{1+8[K_{II}(\boldsymbol{X})/K_I(\boldsymbol{X})]^2}}{4K_{II}(\boldsymbol{X})/K_I(\boldsymbol{X})}\right), & \text{if } K_{II}(\boldsymbol{X}) < 0 \end{cases}. \tag{10}$$

For the MPP-based univariate response surface approximation of the original multivariate implicit limit state/performance function (Equation 9) in the rotated Gaussian space, $n(=\ 3, 5, 7$ or $9)$ uniformly distributed sample points $\mu_i - (n-1)\sigma_i/2$, $\mu_i - (n-3)\sigma_i/2, \ldots, \mu_i, \ldots, \mu_i + (n-3)\sigma_i/2, \mu_i + (n-1)\sigma_i/2$ are deployed along the variable axis x_i with mean μ_i and standard deviation σ_i, through the MPP. Sampling scheme for the MPP-based univariate response surface approximation of a function having one variable (v) and two variables (v_1 and v_2) is shown in figures 4a and 4b respectively. ABAQUS is used for numerically simulating responses at each sample point that are required for construction of the MPP-based univariate response surface approximation. FORTRAN code (given in Appendix 1) is developed which calls ABAQUS for simulating responses at each sample point, reads in the required result (SIFs in this case), deletes the resulting files after each run, and proceeds with simulating responses at next sample point. The whole process is simulated efficiently since accurate results are obtained from ABAQUS.

The probability of failure is estimated by MCS, importance sampling technique, and FORM/SORM using the MPP-based univariate response surface approximation. To estimate the failure probability by MCS, 10^5 samples are used. Importance sampling technique is carried by out choosing 50 sampling points around MPP. FORM/SORM is carried out on the MPP-based univariate response surface approximation to estimate the reliability of the component.

The mean value of critical SIF K_{Ic} is varied from 950 MPa$\sqrt{\text{mm}}$ to 1160 MPa$\sqrt{\text{mm}}$ by taking the mean far-field tensile stress to be 175·0234 N/mm^2 and other variables as given in table 1. The variation in the failure probability is computed by MCS, importance sampling technique, and FORM/SORM using the MPP-based univariate response surface approximation. Figure 5 shows the results of P_F vs. $E[K_{Ic}]$ the probability of failure P_F as a function of the mean critical SIF $E[K_{Ic}]$, in which $E[\bullet]$ is the expectation (mean) operator. Figure 5 demonstrates good agreement between FORM/SORM probability of failure and the results obtained by MCS, importance sampling technique. As expected, the results indicate that the failure probability decreases with increase in the mean critical SIF $E[K_{Ic}]$. The maximum

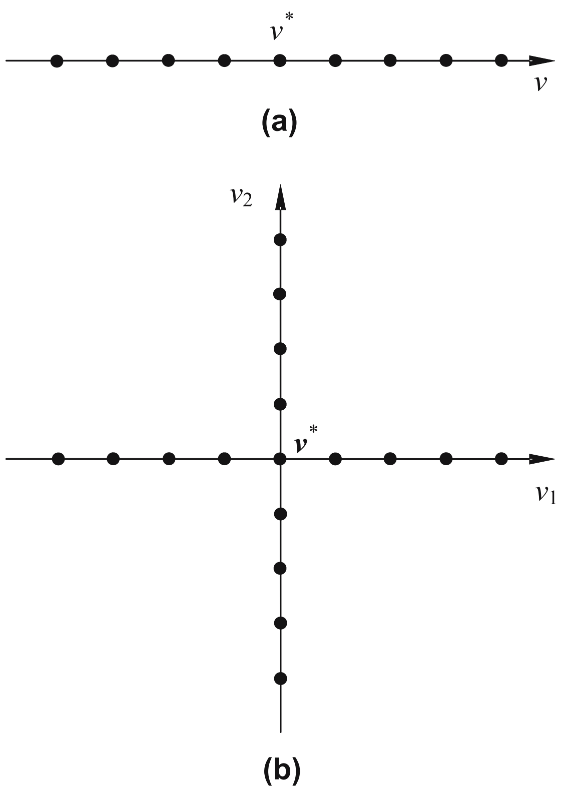

Figure 4. Sampling scheme of the MPP-based univariate response surface approximation; **(a)** For a function having one variable (v); and **(b)** For a function having two variables (v_1 and v_2).

amount of the error in the failure probability estimate using FORM when compared with MCS and importance sampling technique is 10·71%, and 7·14%, respectively. Similarly, the maximum amount of the error in the failure probability estimate using SORM when compared with MCS and importance sampling technique is 6·9%, and 3·45%, respectively.

Further, a study on the effect of a/W ratio on the failure probability is carried out. Five different values of a/W ratio are chosen and the probability of failure P_F as a function of the mean far-field tensile stress $E[\sigma^\infty]$ is evaluated using FORM, SORM and importance sampling technique by taking the mean value of critical SIF K_{Ic} to be 1161·16 MPa$\sqrt{\text{mm}}$ and other variables as given in table 1. Figures 6 shows the probability of failure P_F as a function of the mean far-field tensile stress $E[\sigma^\infty]$ obtained using importance sampling technique. As expected, the results indicate that the failure probability increases with increase in the mean far-field tensile stress $E[\sigma^\infty]$ and with increase in a/W ratio. The results obtained by FORM, SORM and importance sampling technique are observed to be overlapping when plotted graphically making it difficult to distinguish between each other, and hence the results by FORM, and SORM are not presented on the same plot.

A study on sensitivities of the random variables is carried out in order to establish a systematic framework for identifying relative importance of parameters that merit descriptions through random variables. Sensitivity index of i^{th} random variable is computed as

$$\Gamma_i = (P_F - P_{Fi})^2 \Bigg/ \sum_{j=1}^{n} (P_F - P_{Fj})^2, \tag{11}$$

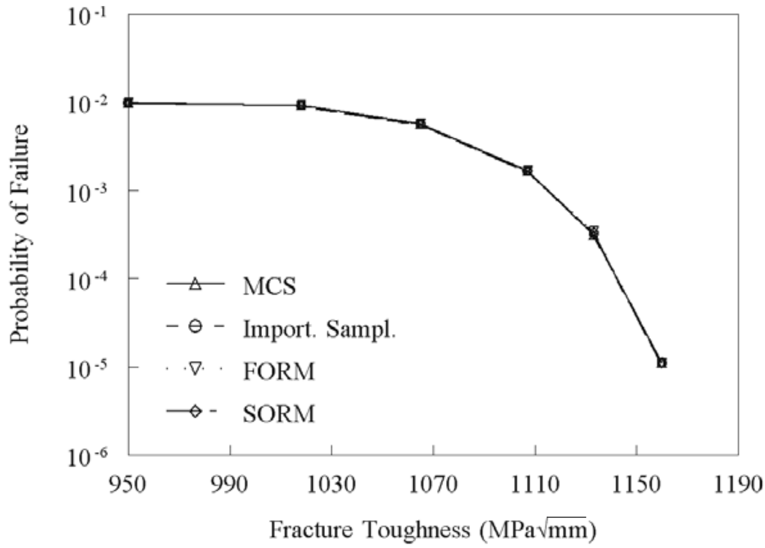

Figure 5. Failure probability of cast component by MCS, importance sampling technique, and FORM/SORM.

where, P_F is the failure probability when all the variables are considered as random and P_{Fi} is the failure probability when the i^{th} random variable is held deterministic at its mean, and the other variables are considered as random. This procedure of establishing importance measure of parameters however requires additional $n \times 10^5$ runs of MCS using the MPP-based univariate response surface approximation. Figure 7 shows the sensitivity index of each of the variable considered, and it can be observed that when compared with the other variables, blow-hole diameter has a significant effect on the failure probability.

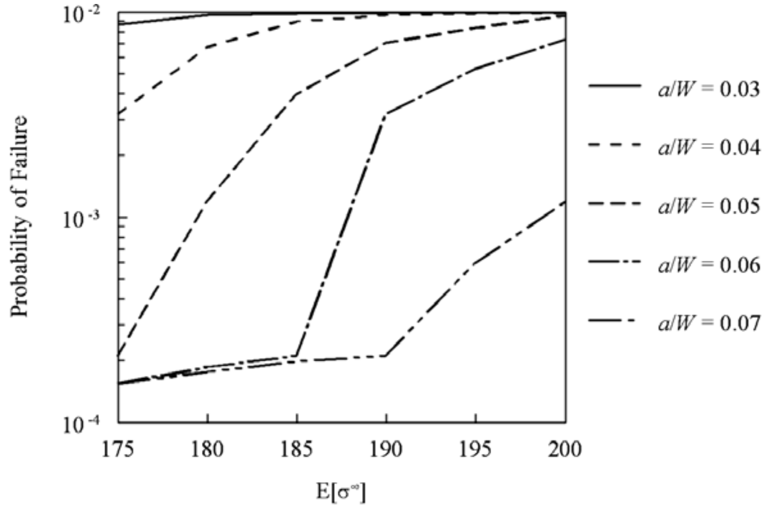

Figure 6. Effect of a/W ratio on failure probability by importance sampling technique.

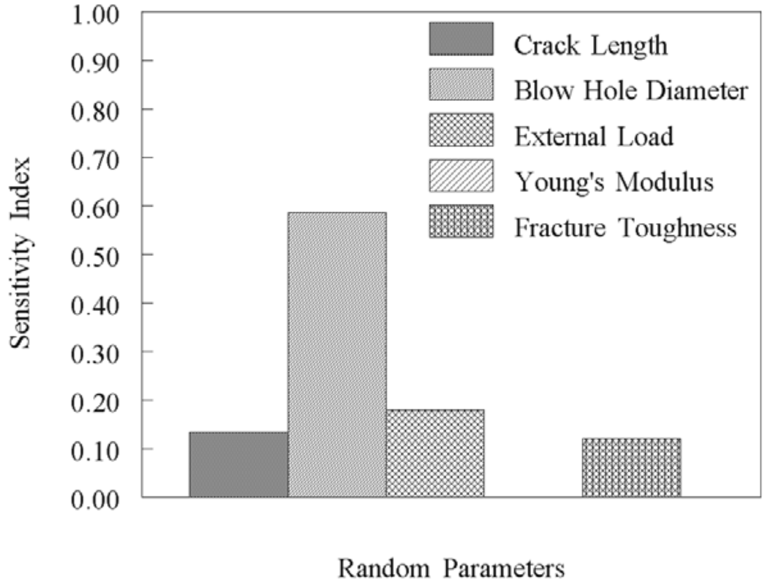

Figure 7. Sensitivity index of various random variables.

6. Conclusions

A study on the effect of blow-holes on the reliability of a cast component, using the MPP-based univariate response surface approximation is presented. Crack geometry, blow-hole dimensions, external loads and material properties are treated as independent random variables. The failure probability is evaluated using the MPP-based univariate response surface approximation in conjunction with MCS, importance sampling technique, and FORM/SORM. This study shows that micro-defects such as blow-holes have a significant effect on the failure probability of cast component.

Authors would like to express sincere gratitude to Tata Consultancy Services Limited for providing the required facilities to carry out this research at Centre for Finite Element Analysis and Design Laboratory, IIT Madras.

Appendix 1

Main Code

```
c        IMPLICIT REAL*8(A-H,O-Z)
c        IMPLICIT INTEGER (I-K,M-N)
c
      character*4 word
      character*2 word1
```

```
      character*3 word2
      character*30 cfile
      character*4 word6
      character*14 word10
      character*2 word11
      character*5 word12
      character*12 word13
      character*2 word14
      character*5 word15
      character*13 word100
      character*2 word110
      character*5 word120
      character*15 word130
      character*2 word140
      character*5 word150
      character*19 word300
        dimension inode(21692), x(3,21692)
        dimension a(50000), b(50000), c(50000), d(50000)
c
        real inc,radius,xinc,yinc,force,youngs,poisson,tmpx,
        tmpy,tmpz
          iin2 = 10
        open(iin2,file='rand_set1.dat',status='unknown',
   &   access='sequential',form='formatted')
      do ii = 1, 50000
        iin = 11
        iout = 2
        iin1 = 12
        iout1 = 3
c
        open(iin,file='nodal.inp',status='unknown',
   &   access='sequential',form='formatted')
        open(iout,file='nodal1.inp',status='unknown',
   &   access='sequential',form='formatted')
        open(iin1,file='final.inp',status='unknown',
   &   access='sequential',form='formatted')
        open(iout1,file='final1.inp',status='unknown',
   &   access='sequential',form='formatted')
c
      do i = 1, 21692
       read(iin,*) inode(i), x(1,i), x(2,i), x(3,i)
      enddo
c
      read(iin2,*) a(ii), b(ii), c(ii), d(ii)
       do i = 1, 21692
         if(x(1,i) .le. 1.5 .and. x(1,i) .ge. 1) then
         t = (x(1,i) - 1)*(0.2*c(ii))
         tmpx = x(1,i)-t
```

```
      elseif(x(1,i) .le. 2.0 .and. x(1,i) .ge. 1.5) then
        t = (2.0 - x(1,i))*(0.2*c(ii))
        tmpx = x(1,i)-t
        else
        tmpx = x(1,i)
        endif
        tmpy = x(2,i)
        tmpz = x(3,i)
        if(x(1,i).ge. -1.0 .and. x(1,i) .le. 1.0 .and. x(2,i)
     &           .ge. -1.0 .and. x(2,i) .le. 1.0) then
        radius = (x(1,i)*x(1,i)+x(2,i)*x(2,i))**(0.5)
        else
        radius = 0
        endif
        if(x(1,i) .eq. 0 .and. x(2,i) .ge. 0) then
        angle=90
        elseif(x(1,i) .eq. 0 .and. x(2,i) .le. 0) then
        angle=270
        elseif(x(2,i) .eq. 0 .and. x(1,i) .ge. 0) then
        angle = 0
        elseif(x(2,i) .eq. 0 .and. x(1,i) .le. 0) then
        angle = 180
        else
        theta = atan(x(2,i)/x(1,i))
        angle = theta*180/3.1416
        endif
        if(radius .ge. 0.5 .and. radius .le. 1.0 .and.
     &    x(1,i) .ge. 0 .and. x(2,i) .ge. 0) then
        inc=(1.0-radius)*(0.1*d(ii)/0.5)
        xinc=cosd(angle)*inc
        yinc=sind(angle)*inc
        tmpx=x(1,i)+xinc
        tmpy=x(2,i)+yinc
        elseif(radius .ge. 0.5 .and. radius .le. 1.0 .and.
     &    x(1,i) .le. 0 .and. x(2,i) .ge. 0) then
        inc=(1.0-radius)*(0.1*d(ii)/0.5)
        angle1=180+angle
        xinc=cosd(angle1)*inc
        yinc=sind(angle1)*inc
        tmpx=x(1,i)+xinc
        tmpy=x(2,i)+yinc
        elseif(radius .ge. 0.5 .and. radius .le. 1.0 .and.
     &    x(1,i) .le. 0 .and. x(2,i) .le. 0) then
        inc=(1.0-radius)*(0.1*d(ii)/0.5)
        angle1=180+angle
        xinc=cosd(angle1)*inc
        yinc=sind(angle1)*inc
        tmpx=x(1,i)-xinc
```

```
            tmpy=x(2,i)+yinc
           elseif(radius .ge. 0.5 .and. radius .le. 1.0 .and.
     &        x(1,i) .ge. 0 .and. x(2,i) .le. 0) then
            inc=(1.0-radius)*(0.1*d(ii)/0.5)
            angle1=360+angle
            xinc=cosd(angle1)*inc
            yinc=sind(angle1)*inc
            tmpx=x(1,i)-xinc
            tmpy=x(2,i)+yinc
           else
c           tmpx=x(1,i)
c           tmpy=x(2,i)
           tmpz=x(3,i)
           endif
           write(iout,91) inode(i), tmpx, tmpy, tmpz
91         FORMAT(I8,',',F14.8,',',F14.8,',',F14.8)
          enddo
          do i = 1, 25
          read(iin1,*) word6
          if(i.eq.1) write(iout1,*) '*HEADING'
          if(i.eq.2) write(iout1,*) 'ABAQUS job created on 24-Feb-06',
     &                              ' at 19:31:30'
          if(i.eq.3) write(iout1,*) '**'
          if(i.eq.4) write(iout1,*) '*RESTART, WRITE, FREQUENCY=99'
          if(i.eq.5) write(iout1,*) 'PREPRINT, HISTORY=NO, ECHO=NO, ',
     &                              'MODEL=NO'
          if(i.eq.6) write(iout1,*) '**'
          if(i.eq.7) write(iout1,*) '*NODE'
          if(i.eq.8) write(iout1,*) '*INCLUDE,INPUT=nodal1.inp'
          if(i.eq.9) write(iout1,*) '**'
          if(i.eq.10) write(iout1,*) '**'
          if(i.eq.11) write(iout1,*) '*ELEMENT, TYPE=C3D8, ELSET=PP'
          if(i.eq.12) write(iout1,*) '*INCLUDE,INPUT=elements.inp'
          if(i.eq.13) write(iout1,*) '**'
          if(i.eq.14) write(iout1,*) '**'
          if(i.eq.15) write(iout1,*) '** pp'
          if(i.eq.16) write(iout1,*) '**'
          if(i.eq.17) write(iout1,*) '*SOLID SECTION, ELSET=PP,',
     &                               ' MATERIAL=ALUM'
          if(i.eq.18) write(iout1,*) '   1.,'
          if(i.eq.19) write(iout1,*) '**'
          if(i.eq.20) write(iout1,*) '** alum'
          if(i.eq.21) write(iout1,*) '** Date: 24-Feb-06  Time: 00:41:32'
          if(i.eq.22) write(iout1,*) '**'
          if(i.eq.23) write(iout1,*) '*MATERIAL, NAME=ALUM'
          if(i.eq.24) write(iout1,*) '**'
          if(i.eq.25) write(iout1,*) '*ELASTIC, TYPE=ISO'
          enddo
```

```
      read(iin1,*) word6
      youngs=207000+1000*a(ii)
      poisson=0.33+.03*a(ii)
      force=-400-10*b(ii)
      write(iout1,100)youngs,poisson
 100  FORMAT(F17.8,',',F14.8)
      do i = 1, 41
      read(iin1,*) word6
      if(i.eq.1) write(iout1,*) '**'
      if(i.eq.2) write(iout1,*) '**'
      if(i.eq.3) write(iout1,*) '** bottom_fixed'
      if(i.eq.4) write(iout1,*) '**'
      if(i.eq.5) write(iout1,*) '*BOUNDARY, OP=NEW'
      if(i.eq.6) write(iout1,*) 'BOTTOM_FIXED, 1,,      0.'
      if(i.eq.7) write(iout1,*) 'BOTTOM_FIXED, 2,,      0.'
      if(i.eq.8) write(iout1,*) 'BOTTOM_FIXED, 3,,      0.'
      if(i.eq.9) write(iout1,*) 'BOTTOM_FIXED, 4,,      0.'
      if(i.eq.10) write(iout1,*) 'BOTTOM_FIXED, 5,,     0.'
      if(i.eq.11) write(iout1,*) 'BOTTOM_FIXED, 6,,     0.'
      if(i.eq.12) write(iout1,*) '**'
      if(i.eq.13) write(iout1,*) '** Step  1, j'
      if(i.eq.14) write(iout1,*) '** LoadCase, nc1'
      if(i.eq.15) write(iout1,*) '**'
      if(i.eq.16) write(iout1,*) '**'
      if(i.eq.17) write(iout1,*) '*STEP'
      if(i.eq.18) write(iout1,*) '*STATIC, DIRECT=NO STOP'
      if(i.eq.19) write(iout1,*) '1.0, 1.0'
      if(i.eq.20) write(iout1,*) '**'
      if(i.eq.21) write(iout1,*) '*INCLUDE,
                                   INPUT=nodal_sets.inp'
      if(i.eq.22) write(iout1,*) '*NSET, NSET=J1'
      if(i.eq.23) write(iout1,*) '21607'
      if(i.eq.24) write(iout1,*) '*NSET, NSET=J2'
      if(i.eq.25) write(iout1,*) '21617'
      if(i.eq.26) write(iout1,*) '*NSET, NSET=J3'
      if(i.eq.27) write(iout1,*) '21247'
      if(i.eq.28) write(iout1,*) '*CONTOUR INTEGRAL,
                                   CONTOURS=5,',
    &                            ' OUTPUT=BOTH'
      if(i.eq.29) write(iout1,*) 'J1, -3.0, 0.0, 0.0'
      if(i.eq.30) write(iout1,*) 'J2, -3.0, 0.0, 1.0'
      if(i.eq.31) write(iout1,*) 'J3, -3.0, 0.0, 2.0'
      if(i.eq.32) write(iout1,*) '*CONTOUR INTEGRAL,
                                   CONTOURS=5,',
    &                            ' OUTPUT=BOTH,
                                   TYPE=K FACTORS'
      if(i.eq.33) write(iout1,*) 'J1, -3.0, 0.0, 0.0'
      if(i.eq.34) write(iout1,*) 'J2, -3.0, 0.0, 1.0'
```

```
  if(i.eq.35) write(iout1,*) 'J3, -3.0, 0.0, 2.0'
  if(i.eq.36) write(iout1,*) '**'
  if(i.eq.37) write(iout1,*) '**'
  if(i.eq.38) write(iout1,*) '** top_pressure'
  if(i.eq.39) write(iout1,*) '**'
  if(i.eq.40) write(iout1,*) '*DLOAD, OP=NEW'
  if(i.eq.41) write(iout1,*) '**'
  enddo
word 300='TOP_PRESSURE_2,P5'
  write(iout1,300) word300,force
  read(iin1,*) word10,word11,word12
  write(iout1,110) word10,word11,force
  read(iin1,*) word13,word14,word15
  write(iout1,120) word13,word14,force
  read(iin1,*) word10,word11,word12
  write(iout1,110) word10,word11,force

 110  FORMAT(A14,',',A2,',',F17.1)

 120  FORMAT(A12,',',A2,',',F17.1)

 300  FORMAT(A19,',',F17.1)
    do i = 1, 4
    read(iin1,*) word6
if(i.eq.1) write(iout1,*) '**'
  if(i.eq.2) write(iout1,*) '** left_pressure'
  if(i.eq.3) write(iout1,*) '**'
  if(i.eq.4) write(iout1,*) '*DLOAD, OP=NEW'
enddo
  read(iin1,*) word100,word110,word120
  write(iout1,1110) word100,word110,force
  read(iin1,*) word130,word140,word150
  write(iout1,1120) word130,word140,force
  read(iin1,*) word130,word140,word150
  write(iout1,1120) word130,word140,force

1110  FORMAT(A13,',',A2,',',F17.1)

1120  FORMAT(A15,',',A2,',',F17.1)
   do i = 1, 4
   read(iin1,*) word6
   if(i.eq.1) write(iout1,*) '**'
   if(i.eq.2) write(iout1,*) '*NODE PRINT, FREQ=0'
   if(i.eq.3) write(iout1,*) '*EL PRINT, FREQ=0'
   if(i.eq.4) write(iout1,*) '*END STEP'
   enddo
   close(11)
   close(2)
```

```
      close(12)
      close(3)
      write(cfile, "(A19)") './run-abaqus final1'
      call system(cfile)
     enddo
     end
```

Subroutine

```
      SUBROUTINE HKSMAIN
      INCLUDE 'aba_param.inc'
      CHARACTER*6 FNAME
      DIMENSION ARRAY(513),JRRAY(NPRECD,513),LRUNIT(2,1)
      EQUIVALENCE (ARRAY(1),JRRAY(1,1))
        character*4 word
        character*5 word2
        character*1 word3
        character*2 word4
        character*3 word6
        character*5 word7
        character*5 word8
        character*5 word9
        character*5 word10
        character*5 word11
      character*3 word12
      character*9 word13
        character*6 word14
      character*6 word15
        character*6 word16
        character*6 word17
        character*6 word18
        character*6 word19

        iinn1 = 98
        iout1 = 99
        iout2 = 100
        iout3 = 110
      iout11= 120
        iout111= 130
      open(iinn1,file='final1.dat',status='unknown',
     &     access='sequential',form='formatted')
      open(iout1,file='k1.results',status='old',
     &     access='sequential',form='formatted')
      open(iout11,file='k2.results',status='old',
     &     access='sequential',form='formatted')
      open(iout111,file='angle.results',status='old',
     &     access='sequential',form='formatted')
      open(iout2,file='run1.results',status='unknown',
```

```
&     access='sequential',form='formatted')
 open(iout3,file='run2.results',status='unknown',
&     access='sequential',form='formatted')
 FNAME='final1'
 NRU=1
 LRUNIT(1,1)=8
 LRUNIT(2,1)=2
 LOUTF=0
 CALL INITPF(FNAME,NRU,LRUNIT,LOUTF)
 JUNIT=8
 CALL DBRNU(JUNIT)
   CALL DBFILE(0,ARRAY,JRCD)
 do i = 1, 450
 read(iinn1,*) word2
 if(word2 .eq. "CRACK") then
 write(iout3,*) word2
 do ii = 1, 24
 if(ii .eq. 20) then
 read(iinn1,*) word3,word4,word6,word7,word8,word9,
               word10,word11
 write(iout1,*) word11
 elseif(ii .eq. 21) then
 read(iinn1,*) word6,word7,word8,word9,word10,word11
 write(iout11,*) word11
 elseif(ii .eq. 24) then
 read(iinn1,*) word12,word13,word14,word15,word16,
               word17,word18,word19
 write(iout111,*) word19
 else
 read(iinn1,*) word3
 write(iout3,*) word3
 endif
 enddo
 else
 write(iout2,*) word2
 endif
 enddo
 CLOSE(98)
 CLOSE(99)
 CLOSE(100)
 CLOSE(110)
 CLOSE(120)
 CLOSE(130)
 STOP
 END
```

Script File

```
#!/bin/sh

#
# check to make sure an input file was specified,
  and it exists
#
if [ "X$1" = "X" ]
then
   echo "no job id specified"
   exit 1
fi

jobID=$1
if [ ! -f $jobID.inp ]
then
   echo "input file does not exist"
   exit 1
fi

#
# Remove any existing output files corresponding
  to this job id
#
for ext in com dat fil mdl msg odb prt res sta stt
do
  rm -f $jobID.$ext
done

#
# remove existing log file, if any
#
if [ -f abaqus.log ]
then
   rm -f abaqus.log
fi

#
# Run abaqus interactively with this job, output goes to
  abaqus log
#
abq644 job=$jobID interactive 2>/dev/null > abaqus.log
abq644 make job=HKSMAIN
abq644 HKSMAIN
```

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