

## Investigation of stress–strain models for confined high strength concrete

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**Abstract.** The effects of confinement reinforcement on the behaviour of high strength concrete columns are investigated for which prismatic experimental specimens were prepared. In the experiment specimens, four longitude reinforcement and confinement reinforcement were used. For each experiment, stress–strain relationship of concrete was obtained and compared with models proposed earlier. The results show that confinement reinforcement improved the ductility of high strength concrete. The ascending branch of stress–strain curves depended on the ratio of confinement reinforcement was similar to the modified Kent–Park model and the descending branch similar to the Nagashima model.

**Keywords.** High strength concrete; confined concrete; stress–strain models; ductility toughness.

### 1. Introduction

Concrete commonly used in engineering structures is defined as a composite material produced using cement, aggregate, water and chemical and mineral admixture materials when necessary. The strength and durability of concrete has undergone continuous improvement over the years and these improved materials are now commonly used. The definition of high strength concrete has changed with time, region and the production technology used. For example, in the 1950s, concrete having 34 MPa characteristic compressive strength was considered as high strength. In the 1960s, the concrete produced in USA had compressive strength between 41 and 52 MPa. In the early 1970s, the compressive strength went up further to 62 MPa. Recently, concrete having 80–100 MPa compressive strength has been used in reinforced concrete and pre-stressed concrete structures. Currently, high strength concrete with 250 MPa compressive strength is produced using high strength aggregate.

Today, high strength concrete is used in off-shore platforms, sea structures, high-rise buildings and bridges. One of its advantages is the lessening column cross-sectional areas. It was found that using high strength concrete in multi-storey, high-rise buildings is economical. However, using high strength concrete in building columns in seismic areas poses some problems. The high strength concrete has less ductility compared to ordinary concrete. It was

reported that using fibre in concrete raises ductility (Campione & Miraglia 2003; Nagarajah 1987; Eurocode 8 2003).

In seismic areas, to prevent sudden failures of building columns due to the effect of earthquake and to have ductility behaviour, seismic codes rules have been recommended (FEMA 368–369 2001; IBC 2003).

Several experimental and theoretical investigations concerned with calculating the strength and ductility of compressed ordinary concrete elements with confinement steel reinforcement and having circular or rectangular transverse cross-section have been reported. The effect of confinement reinforcement on stress–strain relationship of the concrete was first determined by Considere (1903). Modelling this effect was studied by Richart *et al* (1928, 1929). Balmer (1949) improved the analytical model proposed by Richart *et al*. In their studies, King (1946) and Blume *et al* (1961) investigated confinement reinforcement in reinforced concrete rectangular section instead of circular section. Based on small scale specimens, Roy & Sozen (1964), Soliman & Yu (1967) and Sargin (1971) proposed a different stress–strain relationship for confinement concrete. Kent & Park (1971) proposed a model depending on their previous studies. Park *et al* (1982), Saatcioglu & Razvi (1992), Sheikh & Uzumeri (1980, 1982), Sheikh & Yeh (1990), Mander *et al* (1988), Ahmad & Shah (1982) and Chung *et al* (2002) were those who proposed stress–strain models depending on test results. Studies on high strength concrete are limited compared to ordinary concrete Mugurama *et al* (1983), Nagashima *et al* (1992), Galeota *et al* (1992), Hsu & Hsu (1994), Cusson & Paultre (1995), Yong *et al* (1988) and Fafitis & Shah (1985).

In this study, the effects of confinement reinforcement on the behaviour of high strength concrete columns were investigated. For this purpose, prismatic experimental specimens were prepared. These average strength of the specimens was more than 60 MPa, their cross-sections 150 mm × 150 mm, and their heights 300 mm. In these specimens, four longitude reinforcements at 10 mm diameter and confinement reinforcement having 8 mm diameter at 50 mm, 75 mm, 100 mm, 150 mm and 300 mm spacing were used. For each experiment, stress–strain relationship of concrete was obtained. The values obtained were compared with the models proposed earlier (Kent & Park 1971), Saatcioglu & Razvi 1992), Mugurama *et al* (1983) and Nagashima *et al* (1992). The ascending branch of stress–strain curves depends on the ratio of confinement reinforcement and was similar to the modified Kent & Park model proposed by Park *et al* (1982) and the descending branch was similar to the model of Nagashima *et al* (1992). In addition, the decrease of confinement reinforcement spacing caused maximum 19% increase in compressive strength. On the other hand, it increased ductility to a large extent.

## 2. Experimental study

### 2.1 Properties of materials used

Limestone aggregate was used in producing high strength concrete. The maximum aggregate size was 16 mm. Some properties of this aggregate are given in table 1. The average compressive strength was obtained by taking core specimens having 75 mm diameter and 150 mm height from rocks used for producing aggregates.

High strength concrete was produced using PC42.5 (CEM I) cement (the number 42.5 denotes its characteristic compressive strength in MPa). Some properties of this cement are given in table 2. In the production of high strength concrete, silica fume and ASTM C-494 F type super plasticizer water-reducing admixtures were used.

**Table 1.** Some physical and mechanical properties of aggregate.

Physical properties			Mechanical properties	
Aggregate size (mm)	Density (kg/m <sup>3</sup> )	Water absorption (%)	Specimen size (mm)	Compressive strength (MPa)
Fine (< 4 mm)	2671	0.52	$\phi$ 75 mm	83
Course (< 4 mm)	2706	0.42	$h = 150$ mm	

$\phi$ : specimen diameter,  $h$  = specimen height

**Table 2.** Some physical and mechanical properties of cement.

Physical properties		Mechanical properties		
Density (g/cm <sup>3</sup> )		Age (day)	Flexural strength (MPa)	Compressive strength (MPa)
	3.10			
Specific surface (Blaine) cm <sup>2</sup> /g	3680	2	5.75	29.05
Setting time (vicat)	Initial (hours)	2.10	7	43.65
	Final (hours)	4.15	28	52.97

In the production of confined high strength concrete experiment specimens, longitude reinforcement (10 mm dia) and confinement reinforcement (8 mm dia) were used. Some mechanical properties of these reinforcements are given in table 3.

## 2.2 Properties of specimens

Confined concrete prismatic experimental specimens were produced using four longitude reinforcements (10 mm dia). In these specimens confinement reinforcement diameter was 8 mm. The specimens had 150 mm  $\times$  150 mm cross-section in area and 300 mm in height. Experimental specimens were produced using different (50, 75, 100, 150, 300 mm) confinement reinforcement spacing. Prismatic experiment specimens without confinement reinforcement were also used. A total of 36 prismatic specimens (6 for each series) were prepared. In order

**Table 3.** Some properties of reinforcements.

Reinforcement size (mm)	Yield strength (MPa)	Ultimate tension strength (MPa)	Elongation at fracture (%)
8	330	480	18.4
10	360	530	21.3

**Table 4.** The gradation of aggregate used in producing concrete and confined concrete.

Gradation class (mm)	total weight in percentage
0–0.25	4
0.25–0.50	4
0.50–1.00	5
1.00–2.00	12
2.00–4.00	20
4.00–8.00	25
8.00–16.00	30

to determine concrete strength and control reproduction, 36 standard cylinder specimens with 150 mm diameter and 300 mm height (6 for each series) were used.

### 2.3 Mixture and production

The proportion of aggregate used in producing concrete and confined concrete is given in table 4. The mix design of concrete is shown in table 5. After weighing the cement, silica fume and all types of granulometric saturated aggregate, they were placed in the concrete mixer for 3 minutes and then dried. Cement and silica fume were then added and mixed for 3 min by adding water and super plasticizer water-reducing admixtures. The concrete thus produced was placed in a standard cylinder and prismatic moulds in three stages. The concrete was placed after vibration for 15 seconds. The specimens were taken out of their moulds a day later and kept in water at  $22^{\circ}\text{C} \pm 2^{\circ}\text{C}$  for 21 days. Until the experiment, they were kept at  $23^{\circ}\text{C} \pm 2^{\circ}\text{C}$  temperature and 65% relative humidity. The specimens were 28-day-old at the time of the experiment.

## 3. Results and discussion

Before starting the experiments, the top and bottom surfaces of test specimens were smoothed. The compressive strength of the specimens thus prepared was determined. Experimental results are shown in table 6 and typical failure patterns of specimens in figure 1. In addition, during uniaxial compressive test horizontal strain under compression using strain gauges (type TML-PL90) was measured. The measurement length of these strain-gauges was 90 mm.

**Table 5.** The mix design of concrete.

W/C	Cement ( $\text{kg}/\text{m}^3$ )	Water ( $\text{kg}/\text{m}^3$ )	Total aggregate ( $\text{kg}/\text{m}^3$ )	Absorbed water ( $\text{kg}/\text{m}^3$ )	Admixtures	
					SP ( $\text{kg}/\text{m}^3$ )	SF ( $\text{kg}/\text{m}^3$ )
0.30	500	150	1785	4.66	16.5	50

SP: Superplasticizer admixture,  
SF: Silica fume

**Table 6.** The characteristic compressive strength of concrete and confined concrete.

Series	Cylinder concrete specimens	Prismatic concrete and confined concrete specimens			
	$f_{cc}$ (MPa)	$s$ (mm)	$\rho_s$	$f_{cp}$ , (MPa)	$f_{cp}(\text{conf})/f_{cp}(\text{unconf.})$
1	64.5	unconf.	0	64.2	1.00
2	64.2	50	0.0308	76.4	1.19
3	64.6	75	0.0205	73.8	1.15
4	64.4	100	0.0154	72.4	1.13
5	64.7	150	0.0103	67.3	1.05
6	64.4	300	0.0051	65.5	1.04

$f_{cc}$ : Characteristic compressive strength of standard cylinder specimens.

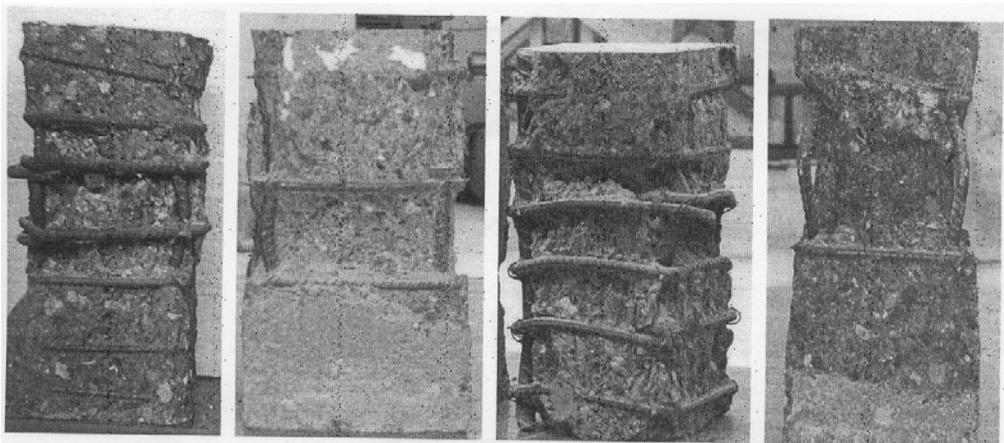
$s$ : Confinement reinforcement space.  $\rho_s$ : Volumetric ratio of confinement reinforcement.

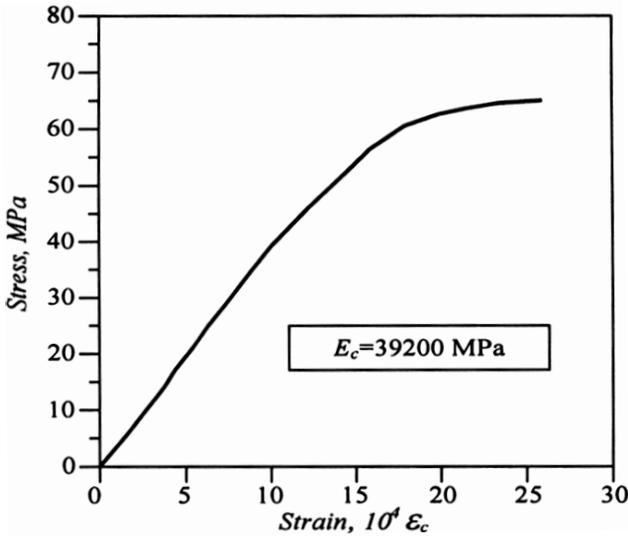
$f_{cp}$ : Characteristic compressive strength of prismatic concrete and confined concrete specimens.

The standard cylinder concrete specimens for all series produced in this study have a characteristic compressive strength of 64.5 MPa. Besides, the difference between characteristic compressive strength of prismatic experiment specimens produced without using confinement reinforcement and compressive strength of standard cylinder concrete was 0.5%.

In high strength concrete thus produced in this study, when confinement reinforcement ratio increased the compressive strength of concrete increased 19% compared to compressive strength of specimens without confinement reinforcement. However, the confinement reinforcement space decreased.

The stress–strain curve obtained from the experiments on standard cylinder concrete specimens is given in figure 2. Standard cylinder concrete experimental specimens suddenly failed when they reached their ultimate carrying capacity. Because of this, the descending branch of stress–strain curve on standard cylinder could not be determined in any of these specimens. The strain at ultimate carrying capacity was measured as 0.0026.

**Figure 1.** The typical failure patterns of confined concrete specimens.



**Figure 2.** The stress–strain curve of standard cylinder specimens.

Stress–strain curves obtained from prismatic experimental concrete and confined concrete specimens depend on confinement reinforcement spacing are given in figure 3 along with the models proposed by other researchers. The values of ductility and toughness determined from experimental stress–strain curves are given in table 7. The capacity of energy absorption is indicated in figure 4.

The ascending branch of experimental stress–strain curves for the concrete with 50 mm confinement reinforcement spacing agree with the modified Kent & Park model proposed for ordinary concrete. However, in the modified Kent & Park model more ultimate load was obtained than experimental ultimate load. The descending branch of experimental stress–strain curves agrees with that of the Nagashima *et al* (1992) model proposed for high strength concrete. The models proposed by Saatcioglu & Razvi (1992) for ordinary concrete and by Mugurama *et al* (1983) for high strength concrete had a difference of about 60–70% in ultimate strength. Experimental ultimate load was 4% smaller than the load obtained from these models (figure 3).

Experimental stress–strain curves of concrete with 75 mm confinement reinforcement space were similar to the model proposed by Nagashima *et al* (1992) for high strength concrete. The compressive strength obtained by Nagashima *et al* (1992) was 3% lower than those obtained from our experiments. The stress–strain curves obtained from the modified Kent & Park model proposed for ordinary concrete agreed with our values for ascending branch. The models proposed by Saatcioglu & Razvi (1992) and Mugurama *et al* (1983) had results different from other models (figure 3).

The ascending branch of experimental stress–strain curves for the concrete with 100 mm confinement reinforcement spacing is in harmony with the modified Kent & Park model proposed for ordinary concrete. Similarly, the descending branch of stress–strain curves obtained experimentally agree with that of Nagashima *et al* (1992) for high strength concrete. The models proposed by Saatcioglu & Razvi (1992) for ordinary concrete and Mugurama *et al* (1983) for high strength concrete had a difference of up to 50–60% of ultimate strength. The compressive strength obtained from these models was 6% lesser than experimental compressive strength (figure 3).

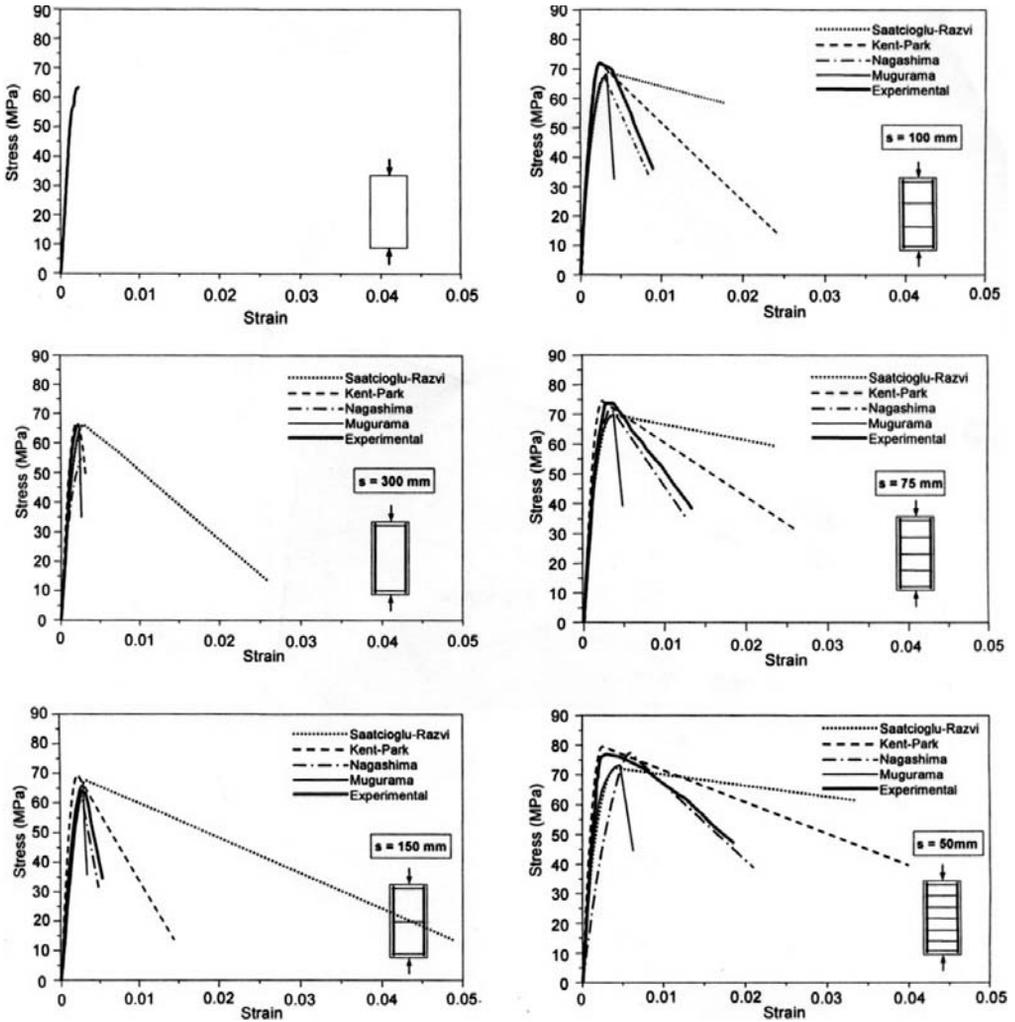
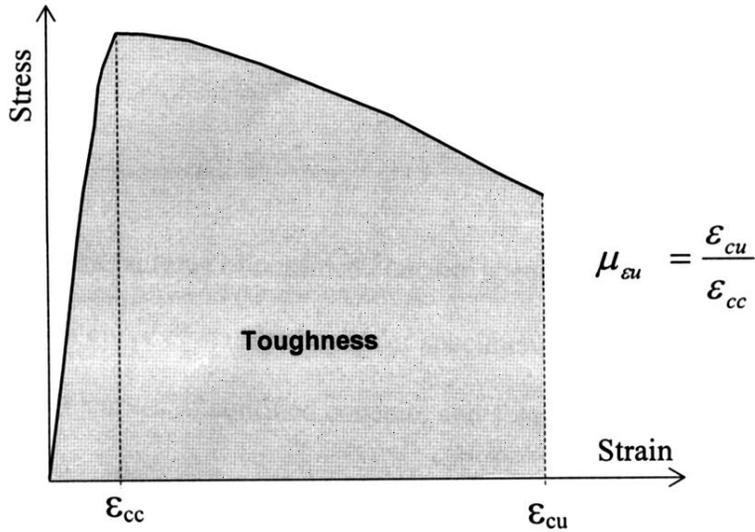


Figure 3. The stress–strain curves of confined concrete and some models.

Table 7. The ductility and toughness ratio of high strength confined concrete.

Conf. reinforcement spaces, $s$ (mm)	Ductility ( $\mu_{\epsilon_u} = \epsilon_{cu} / \epsilon_{cc}$ )	Toughness ratio $s / s_{50}$
50	8.27	1.00
75	4.90	0.64
100	4.17	0.43
150	2.15	0.21
300	-	-

$\epsilon_{cu}$ : Ultimate strain  
 $\epsilon_{cc}$ : strain at peak stress



**Figure 4.** The typical stress–strain curve for determining ductility and toughness.

The ascending branch of experimental stress–strain curves for the concrete with 150 mm confinement reinforcement spacing had a behaviour similar to the model of Saatcioglu & Razvi (1992) proposed for ordinary concrete and of Nagashima *et al* (1992) and Mugurama *et al* (1983) for high strength concrete. The descending branch of stress–strain curves obtained experimentally agrees with that of Nagashima *et al* (1992) for high strength concrete. The models Kent & Park model proposed for ordinary concrete had a difference of 50% in ultimate strength. In this model, the compressive strength was 3% lesser than experimental compressive strength (figure 3).

The ascending branch of experimental stress–strain curves for concrete with 300 mm confinement reinforcement spacing was in harmony with the modified Kent & Park model for ordinary concrete. When experimental specimens reach an ultimate strength, the concrete between confinement reinforcement exploded and scattered, because of this the descending branch of stress–strain curve could not be determined. Models proposed by Saatcioglu & Razvi (1992) for ordinary concrete and by Nagashima *et al* (1992) and Mugurama *et al* (1983) for high strength concrete had a difference approximately of 30% in ultimate strength. With this confinement reinforcement spacing, the compressive strength obtained from the model proposed by Nagashima *et al* (1992) was 20% lesser than experimental compressive strength. But, in our experiments, because of this confinement reinforcement spacing, sudden failure occurred in experimental specimens. The model proposed by Nagashima *et al* (1992) had the same breaking so the descending branch of stress–strain curve could not be determined.

Ductility ratios obtained from stress–strain curves of experimental specimens which have 50 mm confinement reinforcement spacing were 68%, 99% and 386% and higher than ductility ratios of experiment specimens which have 75 mm, 100 mm and 150 mm confinement reinforcement spacing. As a result, when confinement reinforcement spacing is decreased, high strength concrete gained ductility to a great extent. The capacity of energy absorbing (toughness) determined by calculating the area under experimental stress–strain curves increased when confinement reinforcement spacing is decreased. The results show that if high

strength concrete is used in seismic areas buildings, earthquake behaviours can be improved by lowering the confinement reinforcement spacing.

#### 4. Conclusions

The ascending branch of experimental stress–strain curves was more convenient to ascending branch of stress–strain curves of the modified Kent & Park model for ordinary concrete than other models. Descending branch was more convenient to the model proposed by Nagashima *et al* for high strength concrete.

In the high strength concrete which is more brittle than ordinary concrete, using confinement reinforcement, ductility is increased to a great extent. When confinement reinforcement spacing is decreased to half, ductility improved twice. As a result, by decreasing confinement reinforcement spacing, the earthquake behaviour of high strength concrete can be improved. Decreasing confinement reinforcement spacing has increased the compressive strength of confined concrete to a maximum of 19%.

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