Natural frequencies of a flat viaduct road part simply supported on two ends

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MS received 7 December 2004; revised 29 March 2005

Abstract. Viaduct roads have wide application in big cities with high traffic loads, in order to decrease traffic density and to connect subways to highways. Viaduct roads are constructed using steel structures instead of concrete ones in areas of earthquake risks. The low weight of steel structures however causes problems such as vibration and noise. There is increasing demand especially in populated areas to suppress vibration and noise on highway roads for reducing noise-related environmental pollution.

In this study, bending vibrations of rectangular plate viaduct roads, which are supported by six fixed elements of rectangular cross-sectional elements are considered. Natural frequencies are obtained using the Rayleigh–Ritz technique, finite elements analysis, experimentally and neural networks (NN).

Keywords. Bending vibration; natural frequency; rectangular plate; Rayleigh–Ritz technique; neural networks.

1. Introduction

Stiffened plates are important structural elements that are used commonly in airplanes, ships, railway vehicles, floor systems and bridges. Vehicles passing on these plates create vibrations, as do earthquakes. If the natural frequency of these forces coincides with the natural frequency of the system, resonance occurs and thus causes the failure of the system (bridge, building etc.). For this reason, determinations of the natural frequencies are very important (Long 1971; Ney & Kulkarni 1972; Srinivasan & Munaswamy 1978; Balendra & Shanmugan 1982; Bhat 1982; Bert & Newberry 1986; Heyliger & Reddy 1988; Mukherjee & Mukhopadhyay 1988; Mukhopadhyay 1989).

Many researchers have tested the dynamic behaviour of structural elements such as beams and plates. Analyses on these systems were conducted on the basis of some methods such

A list of symbols is given at the end of the paper.

as those by Leissa (1969, 1973), Basilly & Dickinson (1975), Kim & Dickinson (1987), Sakata & Hosokawa (1988) and Kim *et al* (1990). These methods are suitable approaches for the solution of the system with appropriate mathematical models. Leissa (1973) tested the free lateral vibrations of thin rectangular plates and also used series type functions approaches for beams and plates. In free edge plate problems, some degenerated beam functions were proposed by Basilly & Dickinson (1975) for the condition for which the Rayleigh–Ritz method was applied. Kim & Dickinson (1987) examined the bending vibrations of linearly supported plate systems with Rayleigh–Ritz method. Sakata & Hosokawa (1988) used even trigonometric series and found more accurate results for natural frequencies of rectangular cantilevered skew plates. Kim *et al* (1990) tested the lateral vibrations of rectangular plates by using simple polynomials in the Rayleigh–Ritz method.

During the preliminary design stage of natural frequency, the designers should have reliable and preferably practical design tools for defining these kinds of parameters of the viaducts. As an inevitable alternative, Neural Networks (NN) have been increasingly utilized in many disciplines such as control, early detection of machine faults, robotics, mechatronic systems etc. as well as design and dynamic system problems. The usefulness of NN process is the capability of solving nonlinear problems in which the convergence cannot be provided using linear approaches. Thus, the identification of high precision nonlinear relations becomes easier with the benefit of the efficient algorithms presented by Neural Networks (NN).

In this study, natural frequency values of a viaduct road model that is simply supported at two ends are determined with Rayleigh–Ritz method. Comparing the experimental values found by Rayleigh–Ritz method is used for the basis of neural networks in training phase. Thus, after the NN model is trained once, other required frequency values are easily determined in the test phase of the algorithm.

2. Mathematical formulation

Viaduct roads are generally constructed by connecting straight and inclined plates. A schematic of the straight part of a viaduct road model is given in figure 1, composed of one plate with dimensions of $1.8 \times 0.9 \times 0.006$ m and 6 beams with dimensions of $1.8 \times 0.2 \times 0.004$ m supporting the plate at the bottom (Yukseket al 2000). This model is constructed with the example of viaduct road structures mostly used in Tokyo. In the experimental work part, a real viaduct road model is given.

2.1 Rayleigh–Ritz method

Solutions of differential equations are known for simply supported rectangular plates (Leissa 1969) whose one side is a cantilever and other sides have different connection properties. For

Figure 1. Flat viaduct road model.

the solutions of plates that have various boundary conditions, numerical methods must be taken into account. For such problems, the method by Ritz (Young 1950) was very useful. For the stiffened plates that are composed of beams and plates together, approximate solutions for equations can be determined by Rayleigh–Ritz method.

In this study, the Rayleigh–Ritz method used in the vibrational analysis of stiffened rectangular plate, beam functions are used for displacement function (Szilard 1974). We will deal with a rectangular plate problem, simply supported along the short face-to-face sides a , free along long sides b , and with 6 beams of rectangular cross-section supporting the rectangular plate. The potential energy expression, for a rectangular plate that is supported by 6 prismatic beams parallel from bottom part to side b , is;

$$\begin{aligned} V = \int_0^a \int_0^b \frac{1}{2} D \left[\left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 2\nu \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right. \\ \left. + 2(1 - \nu) \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dx dy \\ + \frac{1}{2} \sum_{k=1}^6 \left[E_k I_k \int_0^b \left(\frac{\partial^2 w}{\partial y^2} \right)_{x_k}^2 dy + G_k J_k \int_0^b \left(\frac{\partial^2 w}{\partial x \partial y} \right)_{x_k}^2 dy \right]. \end{aligned} \quad (1)$$

And kinetic energy expression is;

$$T = \frac{1}{2} \rho h \int_0^a \int_0^b \left(\frac{\partial w}{\partial t} \right)^2 dx dy + \frac{1}{2} \sum_{k=1}^6 \left[\rho_k A_k \int_0^b \left(\frac{\partial w}{\partial t} \right)_{x_k}^2 dy \right] \quad (2)$$

$$D = \frac{E h^3}{12(1 - \nu^2)}.$$

Here, D is plate rigidity, I_k beam rigidity, ρ_k beam density, A_k lateral cross-section of beam. Due to assumptions that vibrations are harmonic, the expression for small oscillations of the stiffened rectangular plate's middle surface, can be written as,

$$w = W(x, y) e^{i\omega_n t}. \quad (3)$$

In the expressions of potential and kinetic energy, substituting the necessary derivatives of w , for maximum energy expressions we find that

$$\begin{aligned} V_{\max} = \frac{1}{2} D \int_0^a \int_0^b \left[\left(\frac{\partial^2 W}{\partial x^2} \right)^2 + \left(\frac{\partial^2 W}{\partial y^2} \right)^2 + 2\nu \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2} \right. \\ \left. + 2(1 - \nu) \left(\frac{\partial^2 W}{\partial x \partial y} \right)^2 \right] dx dy \\ \times \frac{1}{2} \sum_{k=1}^6 \left[E_k I_k \int_0^b \left(\frac{\partial^2 W}{\partial y^2} \right)_{x_k}^2 dy + G_k J_k \int_0^b \left(\frac{\partial^2 W}{\partial x \partial y} \right)_{x_k}^2 dy \right], \end{aligned} \quad (4)$$

$$T_{\max} = \frac{1}{2} \rho h \omega_n^2 \int_0^a \int_0^b W^2 dx dy + \frac{1}{2} \omega_n^2 \left[\sum_{k=1}^6 \rho_k A_k \int_0^b (W)_{x_k}^2 dy \right]. \quad (5)$$

Displacement function in energy expressions is assumed as,

$$W(x, y) = \sum_m \sum_n a_{mn} \phi_m(x) \sin \frac{n\pi y}{b}, \quad m = 1, 2, 3, \dots, 8, \quad n = 1, 2, 3, \dots, 8, \quad (6)$$

and only first eight terms are considered.

Here,

$$m = 1, \text{ then } \phi_1 = 1; m = 2, \text{ then } \phi_2 = 2 \left(\frac{x}{a} \right) - 1;$$

$$m > 2, \text{ then } \phi_m(x) = \cosh(\lambda_m x/a) + \cos(\lambda_m x/a) - \frac{\cosh \lambda_m - \cos \lambda_m}{\sinh \lambda_m - \sin \lambda_m} [\sinh(\lambda_m x/a) + \sin(\lambda_m x/a)], \quad (7)$$

$$\lambda_m = (2m - 3)\pi/2, \quad m = 3, 4, 5, \dots \quad (8)$$

Using the energy expressions in Rayleigh–Ritz method and minimizing the Rayleigh ratio according to unknown coefficients a_i , in a way,

$$(\partial/\partial a_i)(V_{\max} - T_{\max}) = 0. \quad (9)$$

Basic value expression is determined. In the solution of (9) the following matrix form is used and natural frequencies of basic value problem are determined.

$$\sum [A_{ij} - \omega_n^2 B_{ij}] a_{ij} = 0. \quad (10)$$

3. Experimental settings

A real viaduct road model is seen in figure 2. The setting of this model and the sensor and magnetic force actuator are given in figures 3 and 4 respectively.

The experiment set up performs modal analysis of viaduct road model's straight part. By means of this analysis, it becomes possible to test the natural frequency values, mode shapes and damping effects.

Experiment is conducted in two different ways. In the first case, impulse hammer is used, in the second case, by placing a magnetic damper under the road as seen in figures 3 and 4. The exit of signal generator is connected to a power amplifier. In both systems a sensor is placed on the road. The exit of sensor is connected to a computer that is capable of fast Fourier (FFT) analysis. In both cases, by means of frequency response curves that are generated by FFT,

Figure 2. A real viaduct road model.

Figure 3. Experimental settings for viaduct road model.

critical frequencies are determined. Also synoise package program is used for finding mode shapes. Finally, it is observed that experimental mode shapes and the modes determined by finite element package program are the same.

4. Neural networks (NN)

Neural networks have been successfully used in many areas such as control and early detection of machine faults. The feed-forward neural network is usually trained by a back-propagation

Figure 4. The sensor and magnetic force actuator.

training algorithm first proposed by Rumelhart *et al* (1986). This was the effective usage of it only after 1980s. With the advantage of high speed computational technology, NNs are more realistic, easily updateable and implementable today. The distributed weights in the network contribute to the distributed intelligence or associative memory property of the network. With the network initially untrained, i.e., with the weights selected at random, the output signal patterns totally mismatch the desired output patterns for a given input pattern. The actual output pattern is compared with the desired output pattern and the weights are adjusted by the supervised back-propagation training algorithm until the pattern matching occurs, i.e., the pattern errors become acceptably small.

The impressive advantages of NNs are the capability of solving highly nonlinear and complex problems and the efficiency of processing imprecise and noisy data. In the following sections the Classic and the Fast Back-propagation NN algorithms are summarized.

4.1 Classic back-propagation algorithm (CBA)

Following equations show the basic steps of classic error back-propagation algorithm (Haykin 1999):

$$\text{if } o_j = f(\text{net}_j) = f(x), \text{ net}_j = \sum_j^i w_{ji} o_i + \theta_j, \quad (11)$$

$$E_p = \frac{1}{2} \sum_{j-\text{output}} (t_{pj} - o_{pj})^2, \quad (12)$$

$$\delta_{pj} = (t_{pj} - o_{pj}),$$

$$\Delta_p w_{ji} = -\alpha \left(\frac{\partial E_p}{\partial w_{ji}} \right), \quad (13)$$

$$\Delta_p \theta_j = -\alpha \left(\frac{\partial E_p}{\partial \theta_j} \right).$$

In the operation element, if, as transfer (threshold) function we use “sigmoid” one,

$$o_{pj} = \frac{1}{\sum_i 1 + e^{-w_{ji} o_{pi} + \theta_j}}, \quad (14)$$

$$(\text{net } p_j) = \sum_i w_{ji} o_{pi} + \theta_j.$$

Equation (14) is derived and simplified;

$$\frac{\partial o_{pj}}{\partial \text{net}_{pj}} = o_{pj}(1 - o_{pj}). \quad (15)$$

Substituting in (14), for output element,

$$\delta_{pj} = o_{pj}(t_{pj} - o_{pj})(1 - o_{pj}). \quad (16)$$

For hidden layer element,

$$\delta_{pj} = o_{pj}(1 - o_{pj}) \sum_k \delta_{pk} w_{kj}. \quad (17)$$

If momentum term (ε) is added to the general equation set to speed up the computation of the algorithm, in the most general condition, we get output and hidden layer equations as follows:

$$\begin{aligned}\Delta_p w_{ji}(t+1) &= \alpha \delta_{pj} o_{pi} + \varepsilon \Delta_p w_{ji}(t), \\ \Delta_p \theta_j(t+1) &= \alpha \delta_{pj} + \varepsilon \Delta_p \theta_j(t).\end{aligned}\quad (18)$$

Here, t is the number of learning cycles, (α) learning rate, $0.01 < \alpha < 10$, (ε) momentum rate, $0 < \varepsilon < 1$.

4.2 Fast back-propagation algorithm (FBA)

The fast version of the back-propagation algorithm (Karayiannis & Venetsanopoulos 1991, 1992, 1993) is derived by sequentially minimizing the objective function $G_k(\lambda)$, defined by (19), for $k = 1, 2, \dots, m$. The update equation for the synaptic weights w_{pq} is obtained as (20).

$$G_k(\lambda) = \lambda \sum_{i=1}^{n_0} \phi_2(e_{i,k}) + (1 - \lambda) \sum_{i=1}^{n_0} \phi_1(e_{i,k}) \quad \forall k = 1, 2, \dots, m, \quad (19)$$

$$w_{p,k} = w_{p,k-1} + \alpha \varepsilon_{p,k}^0(\lambda) \hat{h}_k. \quad (20)$$

If the output of the network is analog,

$$\varepsilon_{p,k}^0(\lambda) = \lambda(y_{p,k} - \hat{y}_{p,k}) + (1 - \lambda) \tanh[\beta(y_{p,k} - \hat{y}_{p,k})]. \quad (21)$$

On the other hand, if the network has binary outputs,

$$\varepsilon_{p,k}^0(\lambda) = (1 - \hat{y}_{p,k}^2)(y_{p,k} - \lambda \hat{y}_{p,k}^2). \quad (22)$$

Because of its simplicity and fast nature, this algorithm provides an ideal basis for investigating the role of λ during training. Here, $\varepsilon_{p,k}$ and α denote the output error and the learning rate, respectively. λ is given as,

$$\lambda = \exp(-\mu/E^2), \quad (23)$$

where μ is controlled intuitively by the user with respect to the problem.

5. Results

After performing the simulation by using the above formulas, the system's natural frequency values are given for comparison in table 1. In this study, frequency values of under 200 Hz are calculated. The experimental results by Nastran FEM analysis program are shown separately in this table. It can be seen that for low frequencies, critical frequency values of three different methods are very close to each other.

In neural networks application, for training purpose, 11 frequency values are taken as basis and other 5 natural frequencies are determined in test phase of the algorithm shown in table 2. These determined values are very close to real ones. Thus, in this study, neural network method gives the designer not only an easy way to determine natural frequencies of the viaduct in order to detect the effects of vibration and noise, but also adaptive, practical and dynamic approach according to the other methods – Energy Method and FEM. In addition to this,

Table 1. Natural frequency of viaduct road model under 200 Hz.

Mod number	Mode	Experimental results (Hz)	Energy method (Hz)	FEM (Hz)
1	(1,1)	10.25	10.10	10.21
2	(2,1)	16.00	15.73	15.64
3	(1,2)	40.50	40.43	39.15
4	(3,1)	48.00	43.90	44.29
5	(2,2)	49.50	50.66	47.56
6	(3,2)	74.75	75.96	73.12
7	(1,3)	86.50	90.98	85.27
8	(2,3)	94.50	108.11	93.27
9	(4,1)	112.80	103.99	105.00
10	(3,3)	122.30	135.15	123.05
11	(4,2)	135.3	129.89	129.79
12	(1,4)	145.3	161.75	145.32
13	(2,4)	156.00	188.41	152.70
14	(4,3)	177.00	182.84	174.87
15	(3,4)	184.50	220.33	182.62
16	(5,1)	201.02	195.34	198.78

dynamic structure of NNs ensures an important advantage. The advantage is that, when the experimental results (for a defined scale of mode shape) are obtained once, there is no need to make measurements for in-scale values again. The architecture basis of the neural network tool and the comparison analysis of the results for both CBA and FBA are given in figures 5, 6 and 7 respectively. The numerical results of energy method is compared with NN results in table 2. It is easily shown that NNs allow the possibility of obtaining the desired in-scale value with its dynamic and adaptive structure in test phase.

Table 2. Neural network results compared with energy method ones.

Mod number	Energy method (Hz)	Neural network (CBA) results (Hz)	Neural network (FBA) results (Hz)
1	10.10		
2	15.73		
3	40.43	38.95	41.19
4	43.90		
5	50.66		
6	75.96		
7	90.98	89.87	89.91
8	108.11		
9	103.99	103.406	103.447
10	135.15		
11	129.89	131.116	128.292
12	161.75		
13	188.41		
14	182.84	183.18	182.53
15	220.33		
16	195.34

Figure 5. Neural network architecture of proposed system.

Figure 6. CBA results of energy method for 5 test values.

Figure 7. FBA results of energy method for 5 test values.

6. Conclusions

The first aim of this study is to obtain an artificial neural network based tool to suppress vibration and noise on highways, which has a wide application in big cities with high traffic loads in order to decrease the traffic density and to connect the subways to highways. The second one is to ensure a robust comparison between the methods of NNs, energy method and FEM. The proposed tool is able to predict the natural frequency values of the system with a high precision even with a few experimental values. The application range and reliability rate of the proposed model can be improved by deriving the NN dynamic structure enriched by additional or specific data. Furthermore, depending on the convenience of the design data structure, the proposed model can easily be generalized and reconstructed for other different mode applications of viaducts.

List of symbols

Viaduct properties

a	side of the plate parallel to x axis;
$a_1, a_2, \dots$	constant coefficients in displacement function w for stiffened plates;
A_k	lateral cross-section of beam;
b	side of the plate parallel to y axis;
b_1	width of beam;
D	bending rigidity of the plate;
E	elastic modulus of plate;
E_k	elastic modulus of beams;
ϕ_m	figure function for the free boundary conditioned beam;
G_k	shear modulus of beams;
H	height of plate;
h_1	height of beams;
I_k	moment of Inertia;
I_0	polar moment of inertia of each beam;
x_k	location of beams under plate;

k	number of beams;
λ_m	coefficient in beam function dependent on m ;
m, n	mode numbers of stiffened plate;
ν	Poisson's ratio;
ω_n	natural frequency of stiffened plate;
ρ	density of plate material;
ρ_k	density of beam material;
t	time;
$T_{\max}$	maximum kinetic energy of stiffened plate;
$V_{\max}$	maximum potential energy of stiffened plate;
w	displacement function of stiffened plates during vibration;
W	assumed displacement function of stiffened plates.

Neural networks

$G(\lambda)$	generalized objective function;
i, j	learning set matrix indices;
t	the number of learning cycles;
w	synaptic weights;
α	learning rate;
ε	momentum coefficient;
λ	input variable;
o	output variable.

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