Sādhanā Vol. 30, Parts 2 & 3, April/June 2005, pp. 403–429. © Printed in India

e-Commerce and supply chains: Modelling of dynamics through fuzzy enhanced high level petri net

VIPUL JAIN, S WADHWA and S G DESHMUKH*

Department of Mechanical Engineering, Indian Institute of Technology Delhi, Hauz Khas, New Delhi 110 016, India
e-mail: deshmukh@mech.iitd.ernet.in

Abstract. Although information plays a major role in effective functioning of supply chain networks (SCNs), studies that deal specifically with the dynamics of supply chains are few. This problem is relatively new since fast communications and the means to employ it for effective management of supply chains did not exist till recently. In order to provide a vehicle for dynamic modelling and analysis of supply chain operations in vague and uncertain environments, we propose a fuzzy enhanced high level petri net (FEHLPN) model. The proposed model captures the capability of petri nets for graphical and analytical representation of dynamic SCNs with the management of uncertain information provided by fuzzy logic. The dynamics associated with two production planning and control policies are modelled, viz. make-to-stock and assemble-to-order in vague and ambiguous situations in electronic commerce environment. A fuzzy set and fuzzy truth-values are attached to an uncertain fuzzy token to model imprecision and uncertainty. The proposed FEHLPN incorporates essential aspects of rule-based systems, such as conservation of facts, refraction, and closed-world assumption.

Keywords. Supply chains; modelling of dynamics; fuzzy enhanced petri nets; uncertainty.

1. Introduction

Information technology (IT) and globalization are thoroughly changing the face of business and organizations the world over. In the last decade, communication and information between firms, business-to-business electronic commerce (EC) applications have become increasingly universal vehicles (Gavireni *et al* 1999; Cachon & Fisher 2000; Kulp 2002; Mahadevan 2003). The influence of EC as a dynamic force of alteration in commercial processes and logistics is undeniable not only within companies but also within supply chains and consumer markets. For instance, the authority of EC on grocery retail and other goods market is the focus of several publications, whereas its impact on markets of transport services receives

*For correspondence

comparatively little attention. However, the understanding of EC is not constrained to just the electronic realization of transactions, but it takes into account the following.

- Marketplace information or forecasts, that is pre-transaction support
- Transaction support
- Monitoring, tracing, tracking etc that is post-transaction support

EC not only blurs traditional company boundaries but also constructs entirely new functions, therefore, EC induces an absolutely original context for transport business and on the management of logistics. Complex, time-consuming and high priced paper-based interactions have now been superseded by virtual online interactions, negotiations and communications. Electronic market places (EMPs) are the centre point of all such activities. By smoothing out information flows and transaction between companies, EMPs have revolutionized inter-enterprise business processes and interfaces increasing the overall efficiency of the supply chain (SC). EMPs have emerged as invaluable intermediaries with SCs. The main influences of EC on supply chain management (SCM) are as below.

- Vertical integration among trading partners that is both logistic service providers and shippers, which pertains to sharing of information, switch over of existing functions and common planning
- The manifestation of entirely new functions and companies, principally for logistics services, the most essential new functions is that of infomediaries. Also, the new functions are centreed around information services
- Optimization of operations planning by proper sharing of production schedules and providing real-time order status information

Operations of different entities in a supply chain are restricted by different sets of objectives and constraints. Performance improvement of the supply chain considering the main objectives of on time delivery, quality assurance and cost minimization are highly interdependent. This affects the performance of any entity in a supply chain, which depends on the performance of others, and their willingness and ability to coordinate activities within a supply chain. Although information plays a major role in effective functioning of supply chain networks, there is paucity of studies that deal specifically with the dynamics of supply chains. This problem is relatively new as faster communications over the internet or by some other means and the willingness to employ it for effective management of supply chains did not exist a few decades ago. Thus, the issue of dynamic configuration of supply chains needs serious research attention.

In addition, uncertainties associated with supply chain networks result in inefficient manufacturing enterprises. This is principally due to their imprecise interfaces and real-world character, where uncertainties in various activities right from raw material procurement to the end user may make supply chains imprecise. The true nature of the problem involves data, which are often vague and imprecise. In the production scenario, various elements like setup time (ST), processing time (PT), mean time between failure (MTBF), mean time to repair (MTTR) etc. are better expressed as fuzzy variables, as they are often expressed in imprecise, vague terms like 'Processing time is high' or 'Set up time is low'. Therefore, real-world production planning, inventory control and scheduling are usually imprecise. However, managers have to interact in an intelligent way with this environment. Thus, they have to reach out for new kind of reasoning based on whatever imprecise knowledge is available.

The traditional way of coping with uncertainties has been to build inventories or to provide excess capacity, which is both expensive and inefficient. Often, objectives at the shop floor level keeps changing and may often be in conflict with one another. This warrants a decision framework wherein the above dynamics are captured. Further, the improvement in an individual organization does not necessarily contribute to the improvement of the whole supply chain. Therefore, competition between individual organizations is being replaced by competition between supply chains. In order to create lean and responsive supply chains, uncertainties that restrict operational performance on the chain level should be systematically and jointly tackled at all stages in the supply chain (Lee *et al* 1997). Advances in technology in EC support SCM as it provides better information exchange, appreciably in terms of lead time, transparency and completeness as below.

- Time-to-market new products diminishes when sharing of data is by means of product data interchange
- The bullwhip effect is prevented, when information transfer regarding orders and sales decreases safety stocks

In classical scheduling methods, to reduce problem complexity, researchers often consider only a small subset of the goals or only one criterion. However, in real applications it is reasonable to find viable compromise between conflicting objectives and goals, by determining a degree of satisfaction among the degrees of importance associated with goals, objectives and constraints. The fuzzy approach provides tools to satisfy goals, objectives and constraints to certain degrees and to take into account the relative importance in a format easily understood by human experts. The gradual satisfaction can help schedulers to find approximately good solutions and to simplify the complexity of the production planning scheduling problems (Turksen & Zarandi 1999).

Moreover, traditional analysis has assumed that the various subsystems work almost independently i.e. the enterprise is weakly coupled. Since today's manufacturing enterprises are more strongly coupled in terms of material, information and service flows, hence there exists a strong urge for process-oriented approach to address the issues of integrated modelling and analysis. All previous studies neglected significant impacts of such integration issues because of dramatic increase in modelling complexity required. Therefore, models resulting from previous studies are confined in their capability and applicability to analyse real supply chain process.

It is to be noted that the operations of SCs are often complex, but what complexity is, and how it can be measured and controlled, are more controversial topics. Numerous investigative routes have been followed to evaluate and simplify the complex nature of information in supply chains (Cachon & Fisher 2000; Kulp 2002). However, with existing information technology (such as EDI, internet, data mining) still in an emerging data-rich environment, lack of primary knowledge regarding supply chain operations in a fuzzy and uncertain environment is even now a key problem faced by industry. The interacting network formation of a supply chain is inherently complex (Wilding 1998), with the majority of firms operating simultaneously in multiple SCs. The operational complexity of supply chains is further complicates the structural complexity, which shows itself in SCs as consistent and unpredictable materials and information flow. The information flow of the distributed manufacturing system includes centralized management of information and distributed control. Since there exist problems of uncertainties and management of information flow in the sub-systems, it makes the modelling and analysis of a distributed manufacturing system very complicated (Lin & Huang 1996).

Expert systems are fuzzy and typically knowledge in expert system is often vague and modified frequently. Hence, there is a strong need to design a dynamic knowledge inference system, which is adaptable according to knowledge variation as human cognition and thinking. An important role has been played by petri nets (PN) in the modelling field (Peterson 1981; Desrochers *et al* 1994). The advantages of petri nets have been identified and comparisons have been made with other models in recent past by several researchers like (Vishwanadham & Narahari 1994; Murata 1989; Zhou & Jeng 1998; Jeng *et al* 1998) etc. Decades of innovative research have enhanced its capability to handle intricacies of modelling system. The various extensions of PNs include predicate/transition nets (P/T nets) (Generich & Lautenbach 1979), stochastic nets (Vishwanadham & Raghavan 2000; Jain *et al* 2003), decision petri nets (Wadhwa & Browne 1989), coloured stochastic PNs (Zenie 1985; Jeng *et al* 2000). However, coloured PNs with fuzzy attributes need enrichment in complex discrete event dynamic systems like supply chain networks. In addition, PNs have an inherent quality in representing logic in intuitive and visual way, and fuzzy petri nets take all the advantage of PNs (Scarpelli & Gomide 1993; Looney 1994; Yeung & Tsang 1994). However, in several situations, great difficulty is involved in capturing data in a precise form. Although the petri net is a powerful modelling tool, it is unable to model system uncertainty and network flow in complex supply chain networks. Hence, a fuzzy expert enhanced high-level petri net is developed to model the network performance and system uncertainty for supply chains.

In this paper, we combine expert enhanced high-level petri net models with fuzzy logic to model the uncertainty associated with the complex SCNs by fuzzy enhanced high level petri net (FEHLPN).

- The proposed model captures faithfully the capability of petri nets for graphical and analytical representation of dynamic SCNs with the management of uncertain information provided by fuzzy logic.
- The proposed FEHLPN model is also able to include mechanisms for dealing with uncertainty or imprecision within the representation tool, thus permitting a better and more realistic modelling of the processes. Also, the FEHLPN model provides immediate synchronization capabilities among concurrent stages in a direct way.
- In the proposed model, fuzzy places are finite sets of places that model the fuzzy production rules. It carries information to describe fuzzy variable and the fuzzy set of fuzzy conditions. The strength of connections between places and transitions is represented by an arc labeled associated with fuzzy weights. The marking function of the FEHLPN indicates the uncertainty for each corresponding situations that has occurred or is occurring. A fuzzy set and a fuzzy truth values are attached to an uncertain fuzzy token to model imprecision and uncertainty in SCNs with greater flexibility and ease of development.
- The proposed FEHLPN serves as a bridge that brings the fuzzy logic and petri nets together into a hybrid approach to model vague and uncertain parameters in SCNs embedded in upstream and downstream electronic marketplace. The better exchange of information in vague and uncertain environment and the interoperability of planning systems is one of the essential features of the proposed model.

The rest of the paper is arranged as follows: Section 2 deals with the fuzzy system modelling of supply chain dynamics. In §3, we discuss in detail the performance analysis of supply chain by the proposed fuzzy enhanced high level petri net model and finally conclude the paper in §4.

2. Supply chain systems and fuzzy systems modelling

Petrovic *et al* (1999) highlighted the uncertainties in a supply chain system as follows: "A real supply chain operates in an uncertain environment. Different sources and types of uncertainty exist along the supply chain. They are random events such as uncertainty in judgment, lack of evidence, lack of certainty in judgment, lack of evidence, lack of certainty of evidence that appear in customer demand, production and supply. Each facility in the supply chain must deal with uncertainty demand imposed by succeeding facilities and uncertain delivery of the preceding facilities in the supply chain". Generally, supply chain networks (SCNs) include several subsystems with unlimited interfaces and relations. Every subsystem usually contains uncertainties. Obviously, uncertainties associated with each subsystem or components make the whole system vague. Also, the nature of interfaces in SCNs causes SCNs to function in completely imprecise and uncertain environment. These interfaces are rooted in the information flows, material flows and supplier-buyer relations (Jackson & Browne 1992; Goyal & Gopalakrishnan 1996). Moreover, relations among entities of SCNs critically depend on human activities. This fact forms the main reason why emergent SCNs necessitates fuzzy system modelling. Sugeno & Yasukawa (1999) state "Fuzzy algorithms are nothing but qualitative descriptions of human actions in decision making".

Supply chain decision making is a complex process due to the following.

(1) Hierarchical structure of decisions
(2) Large scale nature of the SCNs
(3) Randomness of several inputs and operations
(4) Dynamic nature of interactions among supply chain entities

The dynamics of complexity transfer depend on the extent to which organizations are able to absorb variability and uncertainty through internal flexibility, high inventories or excess capacity. The method by which the external and internal transmission of complexities between organizations exists is summarized in figures 1 and 2.

The weakness associated with quantitative models as stated by Turksen (1992) are

(1) complexities in understanding them, and
(2) intricacies in expressing them in the natural language of managers.

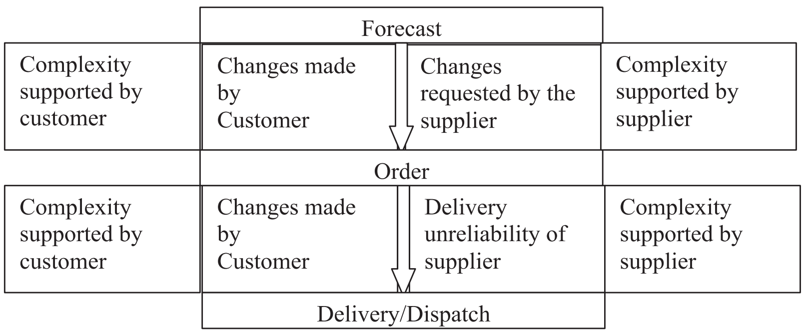


Figure 1. External methods of complexity transmissions in organizations.

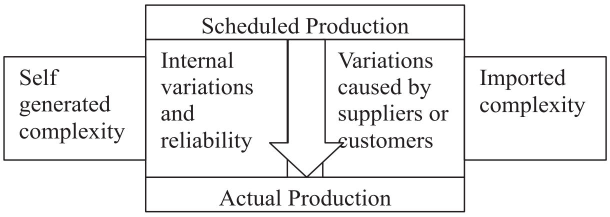


Figure 2. Internal methods of complexity transmissions in organizations.

In the present information age, companies capture great amounts of data in data warehouses, hence the complexity of analysing a real system increases. Zimmerman (1996) argued that real situations cannot be portrayed specifically and humans are unable to analyse real systems and alongside understand them. Supply chains are not only a theoretical and mathematical approach but also real-world problem analysers and solvers.

Zadeh (1973) also states: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics. It is in this sense that precise quantitative analyses of the behavior of humanistic systems are not likely to have much relevance to the real-world societal, political, economic, and other types of problems which involve humans either as individuals or in groups". Sun (1999) develops a distribution constraint satisfaction problem formulation in the modelling of the supply chain as a total system using the fuzzy technology. Petrovic *et al* (1999) examine uncertainties in supply chains by focusing on decentralized control of each inventory and partial coordination in the inventories. Turksen & Zarandi (1999) discuss many advantages of fuzzy system approach in real-world applications and that motivate the authors to apply fuzzy modelling in complex SCNs, which are listed below.

(1) Fuzzy system models are flexible, and any given system like SCNs is easy to handle with fuzzy system models.
(2) Nearly all nonlinear functions of arbitrary complexity can be captured by fuzzy system models. Also fuzzy models are conceptually simple to understand.
(3) Superior communication between experts and managers is provided by fuzzy system models. Moreover, these are based on natural languages and are tolerant of imprecise and vague data.
(4) Fuzzy system models can be constructed on the top of the experience of experts and can be mingled with conventional control techniques.

There are several reasons behind the computational pattern for rule-based reasoning on petri net theory (Chen 1994; Looney 1994; Scarpelli *et al* 1996; Cardoso *et al* 1999).

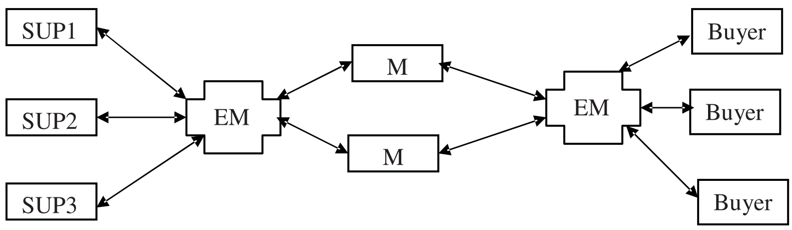
(1) The graphic nature of petri nets provides visualization of dynamic behaviour of rule-based reasoning
(2) The structuring of knowledge within rule bases is accomplished by petri nets that convey the relationships between rules, and also assist experts to construct and modify rules bases
(3) The analytic capability of petri nets provides a foundation for developing knowledge verification techniques, thus an efficient reasoning algorithms can be designed
(4) The underlying relationship of concurrency among rules activation is modelled by petri nets, an essential aspect where real-time performance is crucial.

In classical scheduling methods, to reduce the problem complexity researchers often consider only a small subset of the goals, or only one criterion. However, in real applications it is reasonable to find viable compromises between conflicting objectives and goals, by determining a degree of satisfaction among the degrees of importance associated with goals, objectives and constraints. Classical supply chain networks cannot come up with the imprecision and uncertainty born out of human components in these systems. Fuzzy set theory by assigning membership functions to the inputs and outputs of supply chain networks models has a great potential to resolve main aspects of the real-world SCNs, which are generally in conflict with one another. Thus tapping the features of fuzzy logic and petri nets, we model complex and imprecisely described SCNs by a fuzzy enhanced high level petri net [FEHLPN] model. The proposed model analyses faithfully the imprecise parameters and conditions associated with complex SCNs.

3. Problem environment

We consider supply chain networks (SCNs) comprising sub-assembly manufacturers and a large number of component suppliers. Original equipment manufacturers (OEMs) and logistics service suppliers in various geographical locations, work together through web-based electronic market place. e-Commerce (EC) linked with better planning tools and flexible production techniques, facilitates retailers and producers to minimize inventories and lead times (ECR) and ascertain responsive production, simultaneously in quantity and in design (mass customization). This leads to highly fragmented freight shipments, characteristics by common shipments of less-than–full-truckloads to and from distribution centres (DCs). We assume an electronic market place for components thorough which the component suppliers sell a variety of components to the subassembly manufacturer. The subassembly manufacturer uses these components in the production of a variety of subassemblies, which are then sold to OEMs through the marketplaces for subassemblies. For the movement of goods between the various geographical locations of the sub-assembly manufacture can procure the services of warehousing, transportation and third party logistics companies through a logistics exchange (figure 3).

Direct trade between suppliers and actual users is enhanced as both can access each other's information more easily both in transport service markets and goods markets. This outcome is described as dis-intermediation: certain intermediating roles in the chain may become redundant. Still, in large scale markets like transport, the abundance of information may lead to overload, parties in the chain cannot find information they are looking for or simply cannot



EM: Electronic marketplaces

Figure 3. Electronic marketplace.

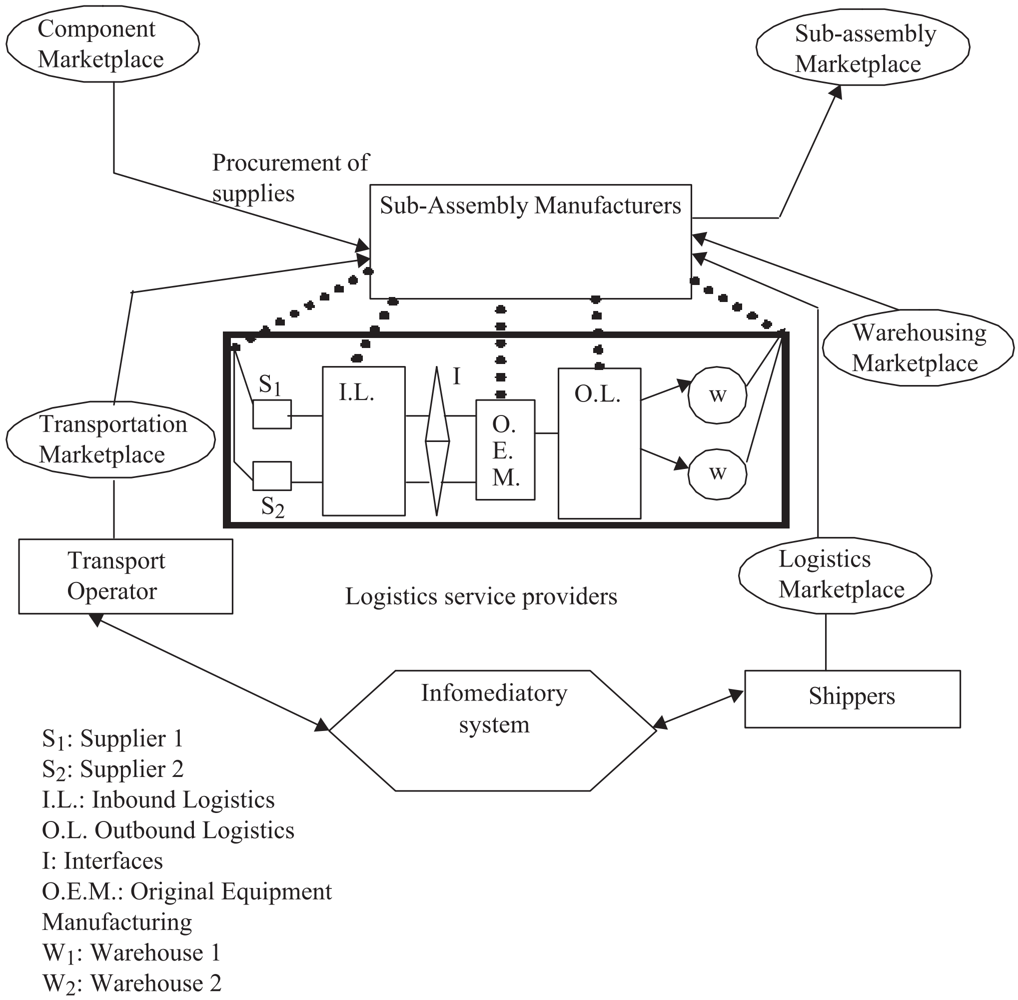


Figure 4. Connectivity between the manufacturer and different electronic marketplaces with infomediation.

handle the information available. This opens up opportunities for new roles within the supply chain, generally referred to as information brokers or infomediaries. Further, the concept of infomediation as a separate function makes it possible to optimize physical flows independent of information flows, therefore, several transport movements, actually skip intermediate storage points. Transport and groupage can be freely optimized as infomediaries in business-to-business trade channels quit handling all their products themselves. The participants of market place find information on each other's supply/demand functions and their capacities through negotiations prior to transactions. The supply chain configuration as seen by sub-assembly manufacturer as well as information shortcut through infomediation is shown in figure 4.

In this paper, we desire to study the supply chain dynamics in vague and uncertain environments. Transport activities are coordinated in several interconnected markets, (particularly in logistics services by logistics service providers, forwarders and third parties) and trans-

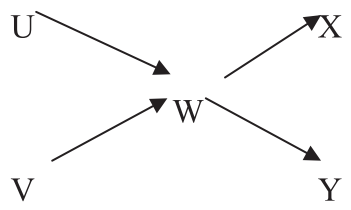


Figure 5. Product structure.

port services (presented by carriers and operators). Transactions are supported by electronic commerce (EC) applications in all of these markets. By tapping the properties of fuzzy logic and petri net, we model the supply chain network (SCN) as Fuzzy Enhanced High Level Petri Net (FEHLPN). The principal idea of the proposed FEHLPN model is to clearly capture all process-related information together with resources, actions and organizational units as well as their interdependencies accounting for both the procurement process and delivery logistics that exist among members of supply chain. Figure 5 shows the product structure considered for study. The two end products X and Y are available at warehouses WH_1 and WH_2 respectively. Further, the demand for X and Y are stochastic and W is the common base sub-assembly for X and Y. Also, W is assembled from raw materials U and V, provided by Supplier 1 (SUP_1) and Supplier 2 (SUP_2) respectively. The inventories pertaining to U, V, W, X, Y are maintained at the respective facilities. In terms of quick access to end products X and Y, the Make-to-Stock (MTS) kind of system presents superior serviceability in terms of faster access to end products X and Y, thus decreasing the probability of back ordering them. This naturally signifies holding excess finished goods inventory that may get outdated, if customers demands are not firm and dense in nature. Therefore, we incorporate an appealing dimension to the proposed model, in place of marking end products X and Y to stock, what would turn up if they were assembled-to-order (ATO) from the common base component W. However, in terms of holding inventory for supply chain, the ATO case offers lower costs, but at the expense of substantial customer lead times. This implies that the customers orders at WH_1 and WH_2, which cannot wait until the product is assembled to order will be lost. Under two different policies for material flow, viz. make-to-stock (MTS) and assemble-to-order (ATO), we propose the FEHLPN model. We model the ATO case, when the production of X and Y are triggered directly by customer order. Owing to various sources of uncertainty (i.e. suppliers, manufacturers, and customers) and imprecision (i.e. capacity, due date etc) besides the interdependence amongst the elements in the network, the modelling and analysis of SC is fairly challenging.

3.1 *Fuzzy enhanced high level petri net (FEHLPN) model*

Normally, ordinary PNs are not always sufficient. Therefore, PN approach can easily be combined with other techniques and theories such as object-oriented programming, fuzzy theory, neural networks etc.

The proposed fuzzy enhanced high level petri net (FEHLPN) is a graphical and mathematical tool having the capability to perform an in-depth analysis and tackle the complex modelling process related to systems with more intricate problems like the SCNs. The proposed FEHLPN model is a 15-tuple $\{P_{LC}, T_{SN}, A_{RC}, Z, M_{RKG}, ARC_{LBL}, \beta, C_{LR}, \Lambda, F, B_{JM}, I_D, G_{DF}, N_D, W_T)$ where

- P_{LC} is a set of places satisfying the relation

$$P_{LC} = P_{NMLC} \cup P_{IHLC} \cup P_{CTLC} \cup P_{TNLC},$$

where,

P_{NMLC} is the set of normal places; P_{IHLC} is the set of inhibitor places; P_{CTLC} is the set of control places; P_{TNLC} is the set of transition places; P_{FZ} is the set of fuzzy places.

Normal place (P_{NMLC}): It depicts the feasible steps involved in complex SCNs,

$$P_{NMLC} = \{P_{NMLC1}, P_{NMLC2}, P_{NMLC3}, \ldots\}.$$

Inhibitor place (P_{IHLC}): It represents the unsuccessful processing of the operations pertaining to SCNs,

$$P_{IHLC} = \{P_{IHLC1}, P_{IHLC2}, P_{IHLC3}, \ldots\}.$$

Control place (P_{CTLC}): Control places find its application when there is more than one feasible path. It selects the optimal path among the various feasible paths

$$P_{CTLC} = \{P_{CTLC1}, P_{CTLC2}, P_{CTLC3}, \ldots\}.$$

Transition place (P_{TNLC}): These are places associated with transitions having atleast one incoming arc. Their function is to avoid refraction (phenomenon of uncontrolled looping between a transition and a place). The arcs associated with a transition and places are always directed from transition place to the respective transition

$$P_{TNLC} = \{P_{TNLC1}, P_{TNLC2}, P_{TNLC3}, \ldots\}.$$

Fuzzy place (P_{FZ}): These are finite sets of places that model the fuzzy production rules,

$$P_{FZ} = \{p_{fz1}, p_{fz2}, \ldots, p_{fzk}\}, k \geq 0.$$

- T_{SN} is a finite set of transitions such that,

$$T_{SN} = \{T_{CN}, T_{FN}\},$$

$T_{CN} = \{t_{cn1}, t_{cn2}, \ldots, t_{cnj}\}$, denotes a finite set of transition called control transition that are connected to and from control places, $j \geq 0$,
$T_{FN} = \{t_{fn1}, t_{fn1}, \ldots, t_{fni}\}$, denotes a finite set of transition called fuzzy transition that are connected to and from fuzzy places, $i \geq 0$,

$$T_{CN} \cap T_{FN} \neq \emptyset,$$

- $Z = \{z_1, z_2, \ldots, z_h\}$, a finite set of prepositions, such that $|P_F| = |Z|$.
- A_{RC} is a set of arcs, such that,

$$A_{RC} = I_{PT} \cup O_{PT}$$

$I_{PT} \cap O_{PT} = \Phi$, $I_{PT} \cup O_{PT} \neq \Phi$, where I_{PT} is a set of input arcs, $I_{PT} = I_{CTLC} \cup I_{TNLC}$,
I_{EIA} is a set of excitant input arcs and I_{IIA} is a set of inhibitor input arcs,

$$I_{EIA} \cup I_{IIA} \neq \Phi, I_{EIA} \cap I_{IIA} = \Phi.$$

O_{PT} is a set of output arcs,

$$O_{PT} = O_{EOA} \cup O_{IOA},$$

O_{EOA} is a set of excitant output arcs,

O_{IOA} is a set of inhibitor output arcs,

$$O_{EOA} \cap O_{IOA} = \Phi, O_{EOA} \cup O_{IOA} \neq \Phi.$$

Excitant arcs: These are the arcs connecting places and transitions, where feasible operations take place,

$$E_{ARC} = I_{EIA} \cup O_{EOA},$$

$$I_{EIA} \cup O_{EOA} \neq \Phi, I_{EIA} \cap O_{EOA} = \Phi,$$

where $E_{ARC}$is the set of excitant arcs.

Inhibitor arcs: These are the arcs connecting places and transitions, where operations taking place are not fruitful. In other words, these are arcs connecting transitions to inhibitor places,

$$INH_{ARC} = I_{IIA} \cup O_{EOA},$$

$$I_{IIA} \cup O_{EOA} \neq \Phi, I_{IIA} \cap O_{EOA} \neq \Phi,$$

where INH_{ARC} is the set of inhibitor arcs. Arc set A_{CS} can also be classified on the basis of tokens present in the place linked with the arc,

$$A_{CS} = A_{GRN} \cup A_{RD},$$

where, $A_{GRN}$is a set of arcs associated with the places containing green tokens; and A_{RD} is a set of arcs associated with places having red tokens.

- M_{RKG} is the set of marking,

$$M_{RKG} : P_{LC} \longrightarrow \{0, 1, 2, \dots\},$$

$$M_{RKG} = \{N_T(P_{LC1}), N_T(P_{LC2}), \dots\}.$$

Also, $M_{RKG} = M_{GRN} \cup M_{RD}$,

where M_{GRN} is the marking of places containing green tokens,

M_{RD} is the Marking of places containing red tokens,

marking M_{RKG} for a set of ξ places is represented by a $\xi \times 1$ matrix.

State equations: Let M^{*}_{RKG} be the marking after ϕ firing and M^{**}_{RKG} be the marking after $\phi - 1$ firing. Let ψ be the incidence matrix denoting the change of marking then,

$$M^{*}_{RKG} = M^{**}_{RKG} + \psi^{T}\Gamma_{\phi}, \forall\phi = 1, 2, 3, \dots, \tag{1}$$

where Γ_{ϕ} is a $\vartheta \times 1$ column matrix, and is known as control vector. It consists of $\vartheta - 1$, 0's and a 1 in the T^{th} position. Equation (1) indicates that transition 'T_{SN}' can fire at the ϕ^{th} firing.

Reachability condition: If M^*_{RKG} is the reachable marking from M_{RKGO}, through a firing sequence $\{\Gamma_1, \Gamma_2, \Gamma_3\text{- - -}\}$, then, the condition for reachability is

$$M^*_{RKG} = M_{RKGO} + \psi\ {}^T\sum_{\varphi=1}^{\varphi} \Gamma_\varphi, \tag{2}$$

$$M^*_{RKG} - M_{RKGO} = {}^T\sum_{\varphi=1}^{\varphi} \Gamma_\varphi, \tag{3}$$

$\Delta M_{RKG} = M^*_{RKG} - M_{RKGO} \equiv$ change in marking,

$$\Delta M_{RKG} = \psi\ {}^T\sum_{\varphi=1}^{\varphi} \Gamma_\varphi. \tag{4}$$

- ARC_{LBL} is the set of arc labels used for designating arcs. Further, labels can also be used to depict relationship between components, at any stage of operation.
  Also,

  $$ARC_{LBL} = ARC_{LBL(GRN)} \cup ARC_{LBL(RD)},$$

  where, $ARC_{LBL(GRN)}$ is the label of the arc associated with places containing green tokens, and $ARC_{LBL(RD)}$ is the label of the arc associated with places containing red tokens.
  Also,

  $$ARC_{LBL(GRN)} \cup ARC_{LBL(RD)} \neq \Phi,\ ARC_{LBL(GRN)} \cup ARC_{LBL(RD)} = \Phi.$$

- β is a set of constants and variables,

  $$\beta = \prod \cup \coprod,$$

  where $\prod$ is a set of constants and $\coprod$ is a set of variables.
  Also, $\prod$ and $\coprod$ are sets of ϕ-tuple

  $$\prod = \langle \prod_1, \prod_2, \prod_3 \ldots \rangle,$$
  $$\coprod = \langle \coprod_1, \coprod_2, \coprod_3 \ldots \rangle.$$

- C_{LR} is a set of colours associated with tokens,

  $$C_{LR} = \{GRN, RD\},$$

  where 'GRN' stands for green colour and RD stands for red colour. Moreover, green tokens are associated with occurrence of positive events, i.e. desirable events. Here, the associated arcs belong to the set of excitant arcs. Red colour shows the occurrence of negative events, i.e. events that are not desired. Here the associated arcs belong to the set of inhibitor arcs. Red tokens are further classified into two groups, i.e. default red and deduced red. Default red marking indicates that the occurrence of the event is assumed to be false under a closed-world assumption (CWA), while deduced red indicates that the occurrence of event is proved to be false. A transition $T \varepsilon T_{SN}$ is enabled under marking $M_{RKG} = M_{GRN} \cup M_{RD}$ iff,

$$\text{Colour } (ARC_{LBL(GRN)}(P_{LC}, T_{SN})\ \varepsilon\ M_{GRN}(P_{LC}),\ \forall P_{LC}\ \varepsilon\ I_{PT},\ (P_{LC}, T_{SN}) \varepsilon A_{GRN},$$

$$\text{Colour } (ARC_{LBL(RD)}(P_{LC}, T_{SN})\ \varepsilon\ M_{RD}(P_{LC}),\ \forall P_{LC}\ \varepsilon\ I_{PT},\ (P_{LC}, T_{SN})\ \varepsilon\ A_{RD}.$$

After firing of a transition, tokens leave the input places resulting a change in the marking state. Suppose transition T_{SN} fires under initial marking M_{GRN}, it loses green tokens represented by the term colour $(ARC_{LBL(GRN)}(P_{LC}, T_{SN}))$. At the same time, if there is an arc from T_{SN} to P_{LC}, some green tokens get added. This is given by the term colour $(ARC_{LBL(GRN)}\ (T_{SN}, P_{LC}))$. The new marking $M^{\tau}_{GRN}(P_{LC})$ is given by,

$$\begin{aligned} M^{\tau}_{GRN}(P_{LC}) &= M_{GRN}(P_{LC}) - \text{colour } ((ARC_{LBL(GRN)}(P_{LC}, T_{SN}) \\ &\quad + \text{colour } ARC_{LBL(GRN)}(T_{SN}, P_{LC}), \end{aligned} \tag{5}$$

$$\begin{aligned} M^{\tau}_{RD}(P_{LC}) &= M_{RD}(P_{LC}) - \text{colour } ((ARC_{LBL(RD)}(P_{LC}, T_{SN}) \\ &\quad + \text{colour } ARC_{LBL(RD)}(T_{SN}, P_{LC}). \end{aligned} \tag{6}$$

- Λ represents the knowledge set, such that $\Lambda = \Lambda(P_{LC}) \cup (T_{SN})$,

  where Λ is a mapping from $P_{LC} \times T_{SN}$ to non-empty sets. $\Lambda(P_{LC})$ represents the place knowledge whereas $\Lambda(T_{SN})$ represents the transition knowledge,

  $$\Lambda(P_{LC}) = \cup\{\Lambda(P_{TNLC}), \Lambda(P_{CTLC}), \Lambda(P_{NMLC}), \Lambda(P_{IHLC})\},$$

  $$\Lambda(T_{SN}) = \cup\{\Lambda(T_{SN})\},$$

  $\Lambda^{P}_{LC} : P_{LC}\Lambda(P_{LC})$, Λ^{P}_{LC} maps set P_{LC} to place knowledge set $K(P_{LC})$,

  $\Lambda^{T}_{SN} : T_{SN}\Lambda(T_{SN})$, Λ^{T}_{SN} maps transition set T_{SN} to the transition knowledge set Λ^{T}_{SN}.
- F: $T_{SN} \rightarrow [0, 1]$, an association function that assigns a certainty value to every colour utilized in each fuzzy transition.
- $B_{JM} : P_{FZ} \rightarrow Z$ corresponds to a bijective mapping from fuzzy places to prepositions.
- I_D: an initialization double (I_F, A_F),
  $I_F : P_{LC} \rightarrow$ expression, an initialization function,
  $\forall p \in P_{LC}$: [Type $(I_F(p)) = C_{LR}(p)_{MLST}$], where $_{MLST}$ stands for multi set,
  A_F: represents an association function, which assigns a certainty value in the range [0, 1] to every token in fuzzy places.
- $G_{DF} : T_{SN} \rightarrow$ expression, a guard function, $\forall t_{sn} \in T_{SN}$: Type (Var $(G_{DF}))) \subset C_{LR})$, where type (Vars) to represent the set types {Type (v)/v ε Vars}, Vars is a set of variables, $Var(G_{DF}(t_{sn}))$ denotes the variables used in $G_{DF}(t_{sn})$
- $N_D : A_{RC} \rightarrow P_{LC} \times T_{SN} \cup T_{SN} \times P_{LC}$, a node function, which maps each arc into a pair where the first element is the source node and the second is the destination node, however the two nodes have to be different types.

  IN: an input function that maps each node, Y, to the set of nodes that are connected to Y by an input arc of Y.

OUT: an output function that maps each node Y, to the set of its nodes that are connected to Y by an output arc of Y.

- W_T: corresponds to set of weights, i.e.

$$W_T = \{W_I \cup W_O\}.$$

$W_I : I \to [0, 1]$ and $W_O : O \to [0, 1]$, are sets of input weights and output weights, which assign weights to all the arcs of a net.

Satisfaction of rule is necessary for firing a transition connected to it. Firing rules are mapped into places and transitions of petri nets. Incoming arcs to transition from rule places represent enabling of preconditions and the outgoing arcs represent the post conditions of the events. After a rule is fired, the state associated with changes located in the precondition of the rule becomes false. Hence for conservation of facts, each inplace of the transition is treated as an outplace of the transition. Let "T_{SN}" be a transition and "P_{LC}" be any inplace associated with it. If (P_{LC}, T_{SN}) is an excitant arc, then an excitant arc is added from "T_{SN}" to "P_{LC}" labelled by $ARC_{LBL(GRN)}$ (T_{SN}, P_{LC}). If (P_{LC}, T_{SN}) is an inhibitor arc, then add an inhibitor arc from "T_{SN}" to "P_{LC}", labelled by $ARC^{\tau}_{LBL(RD)}(T_{SN}, P_{LC})$ which is similar to $ARC_{LBL(RD)}(T_{SN}, P_{LC})$.

However, this may also lead to refraction (phenomenon of uncontrolled looping between the same set of places P_{LC}" and transition "T_{SN}"), which is controlled by means of a transition place. Transition place is associated with particular rule(s). Only after these rules are fired and enabled, the refraction is checked. A better approach to this situation is to first model isolated entities followed by imposition of additional constraints for activity synchronization.

3.2 *Modelling rule-based expert system with FEHLPN model*

Suppose, a rule-based system (RBS) consists of ∂ transition places (P_{TNLC1}, P_{TNLC2}, $P_{TNLC3} \ldots P_{TNLC\partial}$). The set contains 'Y' positive conditions, 'δ' negative condition, 'β' positive conclusion, and 'σ' negative conclusions. When an operation takes place, its input, that is, conditions are known in advance, whereas conclusions are reformed as the outputs. Inplaces consist of the outputs, which comprises transition places, positive conditions, and negative conditions.

i.e.

$$I_{PTi} = \{p_{ci}, P_{TNLCi}, cd^+_{i1}, cd^+_{i2}, \ldots cd^+_{iY}, cd^-_{i1}, cd^-_{i2}, \ldots cd^-_{i\delta}\},$$

where,

I_{PTi} = set of input places transition'I_{PT}',

P_{TNLCi} = transition place associated with transition'P_{TNLC}'.

$cd^+_{i1}, cd^+_{i2}, \ldots, cd^+_{iY}$ are the'Y' positive conditions.

$cd^-_{i1}, cd^-_{i2}, \ldots, cd^-_{i\delta}$ are the'δ' negative conditions.

Similarly, output places consist of positive and negative conditions along with positive and negative conclusions.

i.e.

$$O_{PTi} = \{cl^+_{i1}, cl^+_{i2}, \ldots cl^+_{i\beta}, cl^-_{i1}, cl^-_{i2}, \ldots, cl^-_{i\sigma}\}.$$

where, O_{PTi} is the set of output to transition 'O_{PT}'.

$cl^+_{i1}, cl^+_{i2}, \ldots, cl^+_{i\beta}$ are the 'β' positive conclusions.

$cl^-_{i1}, cl^-_{i2}, \ldots, cl^-_{i\sigma}$ are the 'σ' negative conclusions.

Obviously,

$$P_{LC} = \bigcup_{i=1}^{\partial} I_{PTi} \cup O_{PTi}, \tag{7}$$

$$T_{SN} = \{t_{sn1}, t_{sn2}, t_{sn3} \ldots t_{sn\partial}\},$$

Arc, A_{RC} is associated with positive occurrences and hence they are linked with positive conditions and positive conclusions.

$$A_{RCGRN} = \bigcup_{i=1}^{\partial} \{(cd^+_{ij}, t_{sni}), (t_{sni}, cd^+_{ij})|_{1\le j\le Y}\} \bigcup \bigcup_{i=1}^{\partial} \{(t_{sni}, cl^+_{ij})|_{1\le j\le \beta}\}$$
$$\times \bigcup \bigcup_{i=1}^{\partial} \{(p_{Ti}, t_{sni})\}. \tag{8}$$

Arc, A_{RC} is associated with negative occurrences and therefore are linked with negative conditions and negative conclusions.

$$A_{RCRD} = \bigcup_{i=1}^{\partial} \{(cd^-_{ij}, t_{sni}), (t_{sni}, cd^-_{ij})|_{1\le j\le \delta}\} \bigcup \bigcup_{i=1}^{\partial} \{(t_{sni}, cl^-_{ij})|_{1\le j\le \sigma}\} \tag{9}$$

Also,

$$ARC_{LBL(GRN)}(cd^+_{ij}, t_{sni}) = ARC_{LBL(GRN)}(tsn_i, cd^+_{ij}) = \{\textstyle\prod^+_{ij}\},$$

where, $\prod^+_{ij}$ denotes a positive constant.

$ARC_{LBL(GRN)}(t_{sni}, cl^+_{ij}) = \{\coprod^+_{ij}\}$, where, $\coprod^+_{ij}$ denotes a positive variable.

$ARC_{LBL(RD)}(cd^-_{ij}, t_{sni}) = ARC_{LBL(RD)}(t_{sni}, cd^-_{ij}) = \{\prod^-_{ij}\}$, where, $\prod^-_{ij}$ is a negative constant.

$ARC_{LBL(RD)}(t_{sni}, cl^-_{ij}) = \{\coprod^-_{ij}\}$, where, $\coprod^-_{ij}$ denotes a negative variable.

$ARC_{LBL(GRN)}(p_{Ti}, t_{sni}) = \{\coprod_1, \coprod_2, \coprod_3, \ldots, \coprod_k\} \equiv$ set of variables.

The firing rules pertaining to the fuzzy transitions are as follows:

- A transition fires if and only if, all the certainty values (A_F) of the input token are greater than a threshold value where threshold value $\varepsilon[0, 1]$
- Similar to the firing rules for the control transition, the firing process of the transition removes the input tokens from their input places and deposits tokens into each of its output places.
- All the firing rules are for a fuzzy transition.
- The degree of truth of the output token will be the product of the certainty value of the input proposition and the strength of the belief in the rule (that is the certainty value pertaining to the fuzzy transition of the corresponding colour)

Firing rules for control transition are as follows:

- The firing of the transition is permitted if and only if the colour token in the input places of that transition are members of the colour sets affiliated with the transition.
- With the firing of the transition the coloured tokens are eliminated from the input places. Moreover, a set of colour tokens will be composed in the output places as determined by the expression of the output arc.
- Colours associated with the tokens are permitted to change across transitions.

3.3 *Colouring of FEHLPN model*

In order to delineate the procedures that are inherently embedded in the fuzzy enhanced high level petri net (FEHLPN) model, the steps involved in the colouring are as given below.

(1) To begin with, every transition is assigned a green coloured token.
(2) At the start, a green token as they are associated with positive events, is assigned to every control place.
(3) Each place (other than transition place) is assigned a default red token.
(4) A green token is assigned, through every positive event.
(5) With every negative event, which is proved to be false, is assigned a default red token.
(6) With every negative event, which is assumed to be false, a default red token is assigned.

The icons incorporated in the proposed FEHLPN petri net model are

☆ Green ▲ Default red ▯ Fuzzy transitions

Let M_{RKGO} be the initial marking,

M_{GRN0} the initial green marking,

M_{DRD0} the initial deduced red marking, and

M_{DFRD0} the initial default red marking.

Clearly,

(i) $M_{GRN0}(p_{Ti}) = C(t_i)$
(ii) $M_{DRD0}(p_{Ti}) = M_{DFRFO}(p_{Ti}) = \phi, \forall p_{Ti} \in P_T$
(iii) $M_{GRN0}(p_{LC}) = M_{DRD0}(p_{LC}) = \phi$
(iv) $M_{DFRD0}(p_{LC}) = C(p_{LC}), \forall p_{LC} \in P_{LC} - P_T$.

3.4 *Change of marking*

When a transition fires, the marking of the network gets changed. For instance, let us suppose a transition 't_{sn}' has 'm' negative inplaces, some of them being enabled by default red colours and the rest by deduced red colours. When 't_{sn}' fires, the marking 'M_{RKG}' changes to marking M_{RKG}'.

Clearly,

$$
\left.
\begin{aligned}
&\text{(i) } M'_{RKG} = M'_{GRN} \cap (M'_{DRDO} \cap M'_{DFRDO})\\
&\text{(ii) } M'_{GRN}(p_{LC}) = M_{GRN}(p_{LC}) - \text{ Colour }(ARC_{LBL(GRN)}(p_{LC}, t_{sn}))\\
&\qquad\qquad + \text{Colour }(ARC_{LBL(GRN)}(t_{sn}, p_{LC})\\
&\text{(iii) } M'_{DRDO}(p_{LC}) = M_{DRDO}(p_{LC}) - \text{ Colour }(ARC_{LBL(RD)}(p_{LC}, t_{sn}))\\
&\qquad\qquad + \text{Colour }(ARC_{LBL'(RD)}(p_{LC}, t_{sn}) + \text{ Colour}\\
&\qquad\qquad (ARC_{LBL(RD)}(t_{sn}, p_{LC}))\\
&\text{(iv) } M'_{DFRDO}(p_{LC}) = M_{DFRDO}(p_{LC}) - \text{ Colour }(ARC_{LBL(RD)}(p_{LC}, t_{sn}))\\
&\qquad\qquad - \text{Colour }(ARC_{LBL(GRN)}(p_{LC}, t_{sn}))\\
&\qquad\qquad + \text{Colour }(ARC_{LBL'(RD)}(t_{sn}, p_{LC}))\\
&\qquad\qquad - \text{Colour }(ARC_{LBL'(GRN)}((t_{sn}, p_{LC}))\\
&\qquad\qquad - \text{Colour }(ARC_{LBL(GRN)}(p_{LC}, t_{sn})
\end{aligned}
\right]
\tag{10}
$$

It is worthwhile noting that colour $(ARC_{LBL(RD)}(p_{LC}, t_{sn}))$ signifies those red colours flowing through the input arcs and the colour $(ARC_{LBL'(GRN}(p_{LC}, t_{sn}))$ denotes the feedback red colours corresponding to these. Similarly, $M'_{DFRDO}(p_{LC}, t_{sn})$, colour $(ARC_{LBL(GRN)}(p_{LC}, t_{sn}))$ signifies those default red colours which had flown through the input arcs and colour $(ARC_{LBL'\ (RD)}(t_{sn}, p_{LC}))$ denotes the feedback red colours corresponding to these. Therefore, colour $(ARC_{LBL(RD)}(p_{LC}, t_{sn}))$ = colour $(ARC_{LBL'(RD)}(p_{LC}, t_{sn}))$ for both $M'_{DRD0}(p_{LC})$ and $M'_{DFRDO}(p_{LC})$, where, colour $(ARC_{LBL(GRN)}(p_{LC}, t_{sn})) \subseteq$ colour $(ARC_{LBL(GRN)}(t_{sn}, p_{LC}))$.

Hence the set of equations in (10) can be simplified to:

$$
\left.
\begin{aligned}
&M'_{GRN}(p_{LC}) = M_{GRN}(p_{LC}) - \text{ colour }(ARC_{LBL(GRN)}(P_{LC}, t_{sn}))\\
&\qquad\qquad + \text{colour }(ARC_{LBL(GRN)}(t_{sn}, p_{LC})),\\
&M'_{DRD0}(p_{LC}) = M_{DRD0}(p_{LC}) + \text{ colour }(ARC_{LBL(RD)}(t_{sn}, p_{LC})),\\
&M'_{DFRD0}(p_{LC}) = M_{DFRD0}(p_{LC}) - \text{ colour }(ARC_{LBL(GRN)}(t_{sn}, p_{LC}))\\
&\qquad\qquad - \text{colour }(ARC_{LBL(RD)}(t_{sn}, p_{LC})),\\
&M_{DFRD0}(p_{LC}) \cap M_{DRD0}(p_{LC}) = \phi,\\
&M_{DFRD0}(p_{LC}) \cap M_{GRN}(p_{LC}) = \phi.
\end{aligned}
\right]
\tag{11}
$$

Thus, not only any intermediate marking but also the intermediate stage involved in the operation can be determined by using the aforementioned equations. The proposed FEHLPN model for figure 4 is shown in figure 6. The place transition interpretation and the colours associated with the proposed FEHLPN model are given in table 1 and 2 respectively. The algorithm for the proposed FEHLPN is shown in figure 7. Also, the proposed FEHLPN model for assembled to order (ATO) is shown in figure 8. The initial marking as shown in the proposed FEHLPN model, consists of token in places PIN_U, PIN_V, PIN_W, PFI_X and PFI_Y equaling in number to the respective targeted finished goods inventory of U, V, W, X, and Y. In case of modelling fuzzy rule-based systems, a fuzzy preposition is associated with

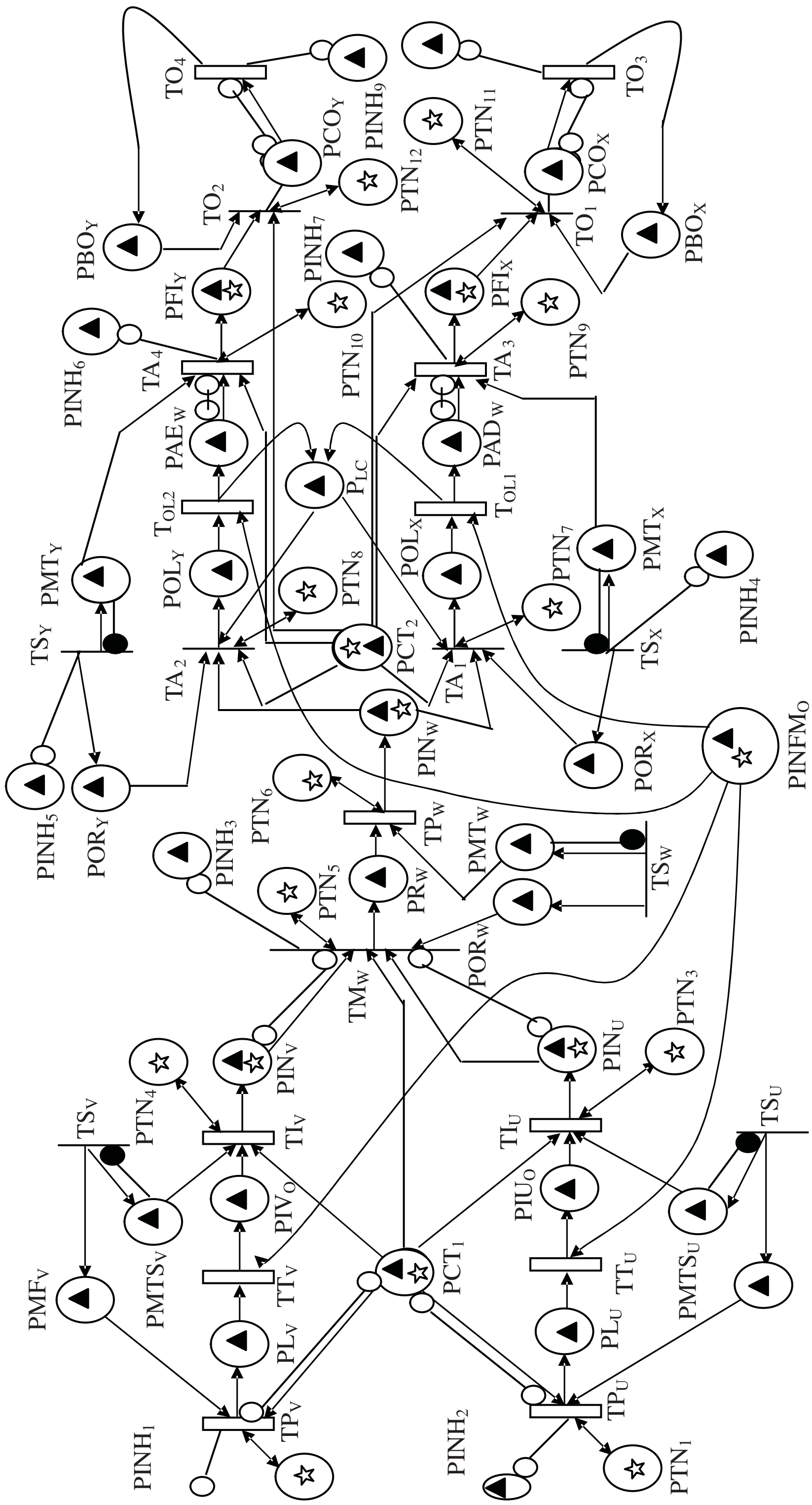


Figure 6. The proposed FEHLPN model with infomediation for MTS policy.

Table 1. Places and transitions interpretation for figures 6 and 8.

Symbol	Description	Symbol	Description
$PMTS_U$	On order material to supplier of U	TR_W	Processing of W
PMF_U	Manufacturing at supplier of U	T_X	Stimulate for assembling of X
TP_U	Processing by supplier of U	T_Y	Stimulate for assembling of Y
PL_U	Logistics from ‘U’ supplier	TA_1	End of assembly of X from W
TT_U	Transportation from supplier of U	TA_2	End of assembly of Y from W
PIU_O	Interface between supplier U logistics and OEM	T_{OL1}	Outbound logistics of X
TI_U	Interfaces or paper work with supplier of U	T_{OL2}	Outbound logistics of Y
PIN_U	Available inventory of U	TA_3	Assembling of X
$PMTS_V$	On order material to supplier of V	TA_4	Assembling of Y
PMF_V	Manufacturing at supplier of V	TO_1	Customer order for X served
TP_V	Processing by supplier of V	TO_2	Customer order for Y served
PL_V	Logistics from ‘V’ supplier	TO_3	Arrival order for X
TT_V	Transportation from supplier of V	TO_4	Arrival order for Y
PIV_O	Interface between supplier U logistics and OEM	PTN_1	Transition place associated with transition TP_U
TI_V	Interfaces or paper work with supplier of V	PTN_2	Transition place associated with transition TP_V
PIN_V	Available Inventory of V	PTN_3	Transition place associated with transition TI_U
POR_W	Order receipt for production of W	PTN_4	Transition place associated with transition TI_V
PMT_W	Material on order for production of W	PTN_5	Transition place associated with transition TM_W
PR_W	‘W Production	PTN_6	Transition place associated with transition TP_W
PIN_W	Available inventory of W	PTN_7	Transition place associated with transition TA_1
POR_X	Order receipt for production of X	PTN_8	Transition place associated with transition TA_2
PMT_X	Material on order for production of X	PTN_9	Transition place associated with transition TA_3
POR_Y	Receipt of order for production of Y	PTN_{10}	Transition place associated with transition TA_4
PMT_Y	Material on order for production of Y	PTN_{11}	Transition place associated with transition TO_1
POL_X	Outbound logistics of X from plant to WH	PTN_{12}	Transition place associated with transition TO_2
POL_Y	Outbound logistics of Y from plant to WH	PCT_1	Control place associated with transitions TP_U, TP_V, TI_U, TI_V, TM_W

(*Continued on next page*)

Table 1. (*Continued*)

Symbol	Description	Symbol	Description
P_{LC}	Logistic carriers availability	PCT_2	Control place associated with transitions TA_1, TA_2, TA_3, TA_4, TO_1, TO_2
PA_{XW}	Assembling of X from Inventory of W	$PINH_1$	Inhibitor place for transition TP_V
PA_{YW}	Assembling of Y from Inventory of W	$PINH_2$	Inhibitor place for transition TP_U
PF_{IX}	Finished goods inventory of X at WH	$PINH_3$	Inhibitor place for transition TM_W
PF_{IY}	Finished goods inventory of Y at WH	$PINH_4$	Inhibitor place for transition TS_X
PBO_X	Back order for X ready	$PINH_5$	Inhibitor place for transition TS_Y
PBO_Y	Back order for Y ready	$PINH_6$	Inhibitor place for transition TA_4
PCO_X	Customer order for X ready	$PINH_7$	Inhibitor place for transition TA_3
PCO_Y	Customer order for Y ready	$PINH_8$	Inhibitor place for transition TO_3
TS_A	Begin of manufacturing of A	$PINH_9$	Inhibitor place for transition TO_4
TS_V	Begin of manufacturing of V	PINFMo	Infomediaries operating infomediation
TW_W	Trigger for production of W		
TM_W	Manufacturing of W starts production		

Table 2. Sets of colours incorporated in the proposed FEHLPN model for SCN.

Places	Set of colours		
	Green	Red	
		Default ref	Deduced red
$PMTS_U$	0	1	0
PMF_U	0	1	0
PL_U	0	1	0
PIU_O	0	1	0
PIN_U	1	1	0
$PMTS_V$	0	1	0
PMF_V	0	1	0
PL_V	0	1	0
PIV_O	0	1	0
PIN_V	1	1	0
POR_W	0	1	0
PMT_W	0	1	0
PR_W	0	1	0
PIN_W	1	1	0
POR_X	0	1	0
PMT_X	0	1	0
POR_Y	0	1	0
PMT_Y	0	1	0
POL_X	0	1	0
POL_Y	0	1	0
P_{LC}	0	1	0
PA_{XW}	0	1	0
PA_{YW}	0	1	0
PF_{IX}	1	1	0
PF_{IY}	1	1	0
PBO_X	0	1	0
PBO_Y	0	1	0
PCO_X	0	1	0
PCO_Y	0	1	0
PIN_V	0	1	0
PTN_1	1	0	0
PTN_2	1	0	0
PTN_3	1	0	0
PTN_4	1	0	0
PTN_5	1	0	0
PTN_6	1	0	0

(*Continued on next page*)

Table 2. (*Continued*).

Places	Set of colours		
	Green	Red	
		Default ref	Deduced red
PTN_7	1	0	0
PTN_8	1	0	0
PTN_9	1	0	0
PTN_{10}	1	0	0
PTN_{11}	1	0	0
PTN_{12}	1	0	0
PCT_1	1	1	0
PCT_2	1	1	0
$PINH_1$	0	1	0
$PINH_2$	0	1	0
$PINH_3$	0	1	0
$PINH_4$	0	1	0
$PINH_5$	0	1	0
$PINH_6$	0	1	0
$PINH_7$	0	1	0
$PINH_8$	0	1	0
$PINH_9$	0	1	0
PINFMo	1	1	0

every place. Usually, supply chains systems contain several subsystems with limited relations and interfaces. Each subsystem usually contains uncertainties. The immediate transitions TS_U, TS_V, TM_W, TS_X and TS_Y are enabled only when the tokens in the places showing inventory U, V, W, X, and Y reach their respective order points. Note that once this condition is satisfied, the transitions can keep on firing indefinitely. To avoid this we define inhibitor arcs from places $PMTS_U$, $PMTS_V$, PMT_W, PMT_X and PMT_Y to the above transitions. The place connected with the inhibitor arc is called an inhibitor place. If there exists a colour marking in the transition connected to an inhibitor place, the transition is inhibited from firing. Thus, these places signify that material is already on order. The fuzzification procedure is performed in a fuzzy transition. A fuzzy transition is fired whenever it receives the desired token, usually a token with the input physical value. In a marked FEHLPN, the fuzzification procedure contains the information of membership value in a colour token. In a fuzzy transition, each token experiences the fuzzification procedure, which owns the membership value of the fuzzy set. A transition is associated with every rule in the fuzzy knowledge base and places with propositions. This is done in such a way that the places associated with propositions in the antecedent part of a rule are the input places for the corresponding transition and analogously for the propositions in the consequent part and output places. Thus, the proposed FEHLPN enhances the modelling ability of the standard petri net for modelling ambiguous and uncertain situations in complex supply chain networks.

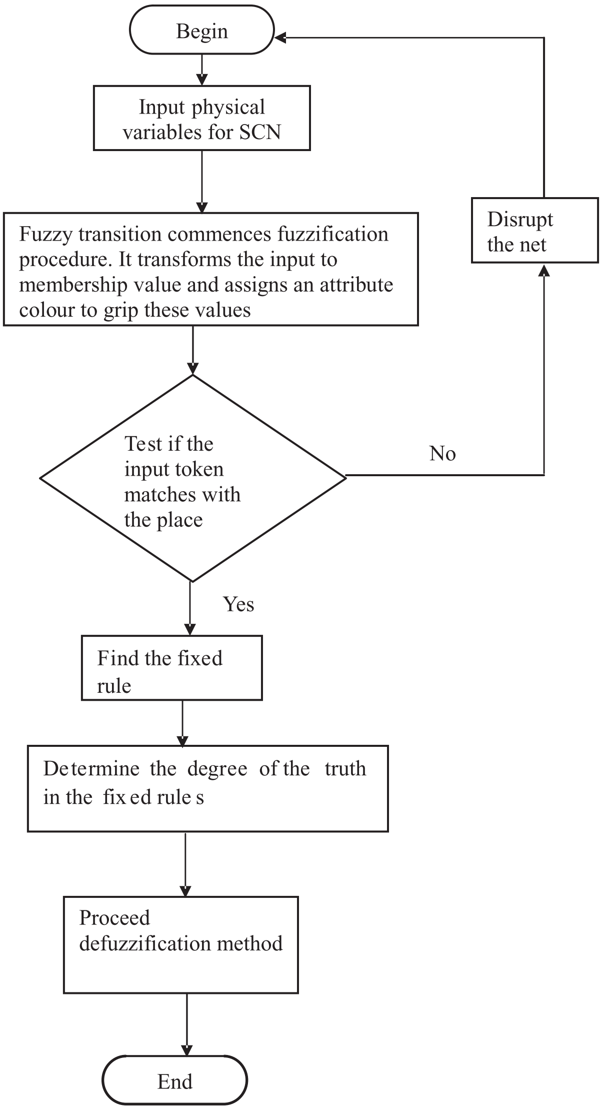


Figure 7. Flowchart for FEHLPN algorithm.

4. Concluding remarks

A single supply chain configuration is neither optimal nor efficient under dynamic and uncertain conditions, where objectives may conflict. For instance, the number of stages involved essentially multiplies the number of combinations of these stages to get any given product through the supply chain from start to finish. Similarly, the choices in products including the price, quality, delivery time, and quantity available add to the complexity of the process. The preference of the ultimate customer adds yet another layer of complexity to managing these supply chains. Although information plays a major role in effective functioning of supply chain networks, there is a paucity of studies that deal with the dynamics of supply chains. One of the least studied of these facets is adaptive or dynamic configuration of supply chains. This problem is relatively new since faster communications over the internet or by any other means

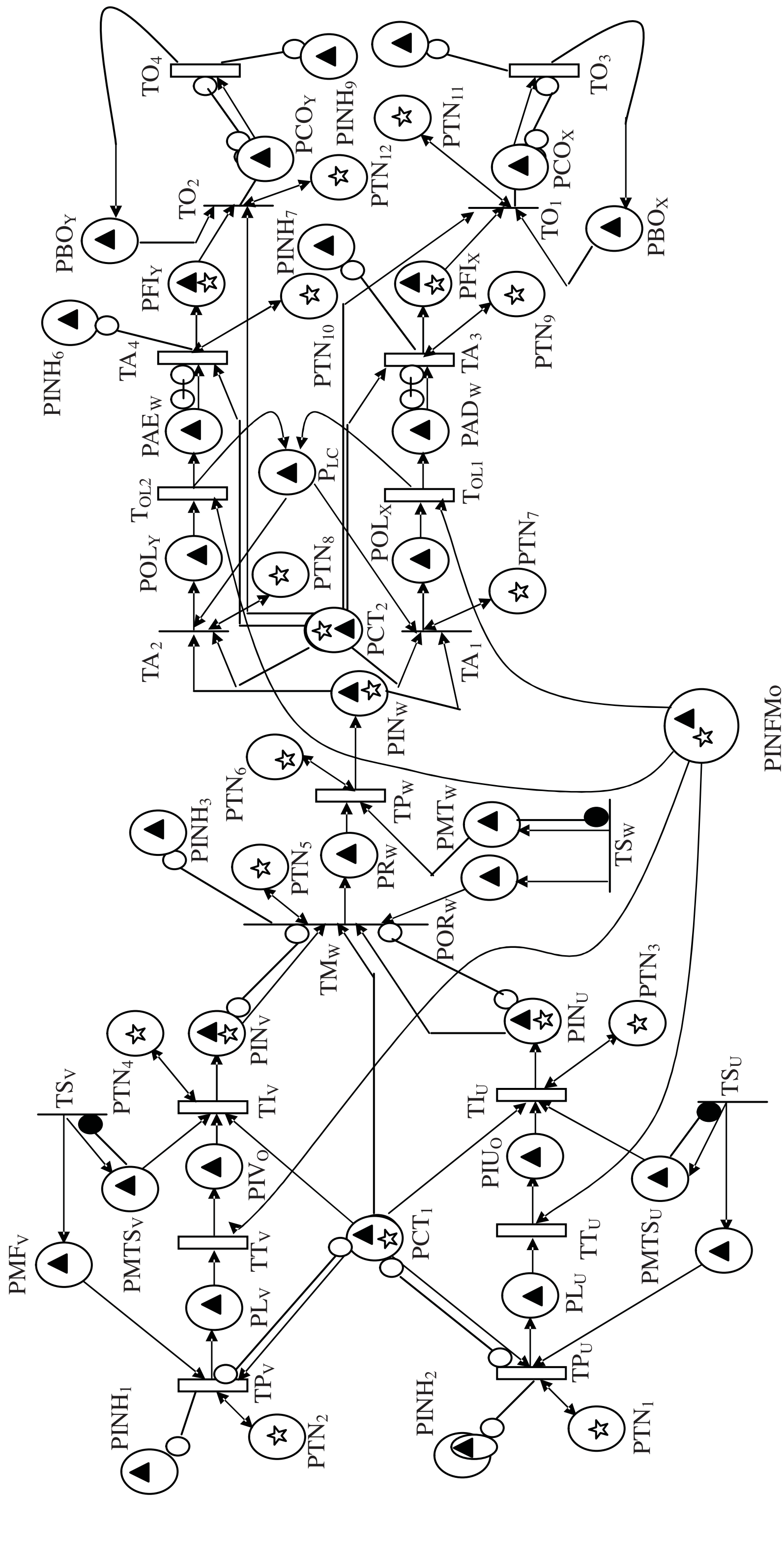


Figure 8. The proposed FEHLPN model with infomediation for ATO policy.

and the willingness to utilize it for effective management of supply chains did not exist a few decades ago. Lack of systems compatibility, non-integrated managerial structures and the loss of superior local information are the three key barriers to effective data base implementation.

Owing to the extremely multifaceted nature of large supply chains, designing, analysing and re-engineering of supply chain networks using formal and quantitative approaches are quite difficult. In an electronic market place environment, supply chain partners are selected online, based on negotiation regarding prices, quantities and delivery schedules. The dynamics of complexity transfer depends on the degree to which organizations are able to absorb variability and uncertainty through interval flexibility, towering inventories or surplus capacity. However, it is found that there is presently no formal and quantitative model that is capable of capturing some essential issues including the validation and verification of the overall supply chain processes with imprecise parameters and conditions.

In this paper, we integrate expert enhanced high-level petri net models with fuzzy logic to reap the benefits of both formalisms, in modelling complex and imprecisely described systems like SCNs. The proposed fuzzy enhanced high level petri net (FEHLPN) model combines strong mathematical foundation with an intuitive graphical representation of dynamic SCNs in the age of electronic commerce (EC). EC seems to guide the way to a superior utilization of intermediation by providing new management tools and precision of transport service markets.

- EC in SCM separates information flow with goods flows, thus assisting separate, flexible optimisation of transport.
- EC in SCM leads to fragmentation and less-than-truck loads (JIT), which leads to coordination among shippers and further outsourcing to professional carriers at the same time, which is frequently a foremost step to intermodal transport.
- EC in SCM facilitates globalization.

In this paper, we combine expert enhanced high-level petri net models with fuzzy logic to model the uncertainty associated with complex SCNs by fuzzy enhanced high level petri net (FEHLPN).

- The proposed model captures faithfully the capability of petri nets for graphical and analytical representation of dynamic SCNs with the management of uncertain information provided by fuzzy logic. The proposed model combines a strong mathematical foundation, which effortlessly models essential aspects of rule-based systems, such as conservation of facts, refraction, and closed-world assumption the application of the model can be seen in the Business-to-Business service markets where transport movement is coordinated in various interlinked markets, specially in logistics services (by logistics service providers, third parties, forwarders) and transport services (by operators and carriers). Transactions are supported by EC applications in all these markets.
- One of the benefits of proposed FEHLPN model is the inclusion of mechanisms for dealing with uncertainty or imprecision within the representation tool, thus permitting a better and more realistic modelling of the processes. Also, the FEHLPN model provides immediate synchronization capabilities among concurrent stages in a direct way.
- In the proposed model, fuzzy places are finite sets of places that model the fuzzy production rules. It carries information to describe fuzzy variable and the fuzzy set of fuzzy conditions. The strength of connections between places and transitions is represented by an arc labeled associated with fuzzy weights. The marking function of the FEHLPN indicates the uncertainty for each corresponding situations that has occurred or is occurring. A fuzzy set and fuzzy truth-values are attached to an uncertain fuzzy token to model imprecision and uncertainty in SCNs with greater flexibility and ease of development.

- FEHLPN serves as a bridge that brings the fuzzy logic and petri nets together into a hybrid approach to model vague and uncertain parameters in SCNs embedded in upstream and downstream electronic marketplace.
- The better exchange of information in vague and uncertain environment and the interoperability of planning systems is one of the essential features of the proposed model. The better exchange of information and the interoperability of planning systems is one of the important drivers in this functional integration. Moreover, knowledge representation through FEHLPN make experts easy to build up and modify rule bases, which can achieve the structuring of knowledge within rule bases. Also, it makes easy the users to realize the knowledge by identifying the relationships among rules.
- FEHLPN's graphic nature provides the visualization of the dynamic behaviour of SCNs. Further, we follow petri net's original semantics of markings and incidence matrixes and stick to the petri net firing rule. Therefore, the majority of the well-developed petri net methods for analysing properties, such as liveliness, boundedness, safety, deadlock detection, reversibility and coverability are applicable.

The focus of new developments will be on the following.

- Creating automated agents that are capable of finding exactly right information in imprecise and uncertain environment, which are inherent in supply chain networks.
- The future work will emphasize on merging an execution of petri net with a graphical simulation and the realization that simulation action takes place as a result of the firing a transition.

The authors are indebted to Prof Y Narahari and Dr N R S Raghavan and the two anonymous referees for constructive suggestions that led to considerable improvement of an earlier version of the manuscript.

References

Cachon G P, Fisher M 2000 Supply chain inventory management and the value of shared information. *Manage. Sci.* 46: 1032–1048

Cardoso J, Valette R, Dubois D 1999 Possibilistic petri nets. *IEEE Trans. Syst. Man Cybern.* B29: 573–582

Chen S M 1994 A knowledge acquisition scheme for rule-based expert systems based on fuzzy petri nets. *Int. J. Inf. Manage.* 1(3): 45–54

Desrochers A A, Aljaar R Y 1994 *Applications of petri nets in manufacturing systems, modelling, control, and performance analysis* (New York: IEEE Press)

Gavirneni S, Kapuscinski R, Tayur S 1999 Value of information in capacitated supply chains. *Manage. Sci.* 45(1): 16–24

Generich H L, Lautenbach K 1979 *The analysis of distributed system by means of predicate/transition nets* Lecture notes on computer Science, 70, Semantics and Concurrent Computation (New York: Springer-Verlag) pp 123–146

Goyal S K, Gopalkrishnan M 1996 Production lot-sizing model with insufficient production capacity. *Product. Planning Control* 7: 222–224

Jackson S, Browne J 1992 AI-based decision support tool for strategic decision making in factory for the future. *Comput. Integrated Manuf. Syst.* 5(2): 83–90

Jain V, Swarnkar R, Tiwari M K 2003 Modelling and analysis of wafer fabrication scheduling via generalized stochastic petri net and simulated annealing. *Int. J. Product. Res.* 41: 3501–3527

Jeng M D, Xie X, Chou S W 1998 Modelling, qualitative analysis, and performance evaluation of the etching area in an IC wafer fabrication system using petri nets. *IEEE Trans. Semicond. Manuf.* 11: 358–373

Jeng M, Xie X, Hung W Y 2000 Markovian timed petri nets for performance analysis of semiconductor manufacturing systems. *IEEE Trans. Syst. Man Cybern.* 30: 757–771

Kulp S C 2002 The effect of information precision and information reliability on manufacturer-retailer relationships. *Acc. Rev.* 77: 653–677

Lee H L, Padmanabhan V, Whang S 1997 The bullwhip effect in supply chains. *Sloan Manage. Rev.* 38(3): 97–102

Lin L R, Huang H P 1996 Real time networking for the implementation of CIM. *Fourth Int. Conf. Automation Technology* (Hsinchu, Taiwan: Mech. Ind. Res. Labs/ITRI)

Looney C G 1994 *Fuzzy petri nets and applications, fuzzy reasoning in information, decision and control systems* (Dordrecht: Kluwer Academic) : 511–527

Mahadevan B 2003 Making sense of emerging market structures in Business to Business E Commerce. *California Manage. Rev.* 46(1): 86–100

Mohanty R, Deshmukh S G 2000 Reengineering of a supply chain management system: a case study. *Prod. Planning Control* 11(1): 90–104

Murata T 1989 Petri nets properties, analysis and applications, *Proc. IEEE* 77: 541–580

Peterson J L 1981 *Petri net theory and the modelling of systems)* (Englewood Cliffs, NJ: Prentice Hall)

Petrovic D, Roy R, Petrovic R 1999 Supply chain modelling using fuzzy sets. *Int. J. Product. Econ.* 59: 443–453

Scarpelli H, Gomide F 1993 Fuzzy reasoning and fuzzy petri nets in manufacturing systems modelling. *J. Intell. Fuzzy Syst.* 1: 225–242

Scarpelli H, Gomide F, Yager R 1996 A reasoning algorithm for high level fuzzy petri nets. *IEEE Trans. Fuzzy Syst.* vol. 4: 282–294

Sugeno M, Yasukawa T 1993 A fuzzy logic based approach to qualitative modelling. *IEEE Trans. Fuzzy Syst.* 1: 7–31

Sun R 1999 *Sensitivity analysis of constraint-based factory scheduling*. Ph D thesis, University of North Carolina at Charlotte, Charlotte, NC

Turksen I B 1992 Fuzzy expert systems for IE/OR/MS. *Fuzzy Sets Syst.* 51(1): 1–27

Turksen I B, Zarandi M H 1999 *Production planning and scheduling: Fuzzy and crisp approaches*. (Boston: Kluwer Academic) pp 479–529

Vishwanadham N, Narahari Y 1994 *Performance modelling of automated manufacturing systems* (New Delhi: Prentice Hall of India)

Vishwanadham N, Raghavan N R S 2000 Performance analysis and design of supply chains: A petri net approach. *J. Oper. Res. Soc.* 51: 1158–1169

Wadhwa S, Browne J 1989 Modelling flexible manufacturing systems with decision petri nets. *Int. J. Flexible Manuf. Syst.* 1: 255–280

Wilding R 1998 The supply chain complexity triangle: Uncertainty generation in the supply chain. *Int. J. Phys. Distrib. Logis. Manage.* 28: 599–616

Yeung D S, Tsang E C C 1994 Fuzzy knowledge representation and reasoning using petri nets. *Expert Syst. Appl.* 7: 281–289

Zadeh L A 1973 Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Trans. Syst. Man Cybern.* 3: 28–44

Zenie A 1985 Coloured stochastic petri nets. In *Int. Workshop on Timed Petri Nets*, Turin, Italy, pp 262–271

Zhou M C, Jeng M D 1998 Modelling, analysis, simulation, scheduling, and control of semiconductor manufacturing systems: a petri net approach. *IEEE Trans. Semicond. Manuf.* 11: 333–357

Zimmerman H J 1996 *Fuzzy set theory and its applications*. (Boston, MA: Kluwer Academic)