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Entropy generation in non-Newtonian fluid flow in a slider bearing

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Abstract. In the present study, entropy production in flow fields due to slider bearings is formulated. The rate of entropy generation is computed for different fluid properties and geometric configurations of the slider bearing. In order to account for the non-Newtonian effect, a special type of third-grade fluid is considered. It is found that the rate of entropy generation is influenced significantly by the height of the bearing clearance and the non-Newtonian parameter of the fluid.

Keywords. Slider bearing; non-Newtonian fluid; entropy generation.

1. Introduction

Bearings find application in many mechanical components in order to reduce the frictional losses between two rotating or sliding mechanical parts. The lubricant used in the bearing systems is usually non-Newtonian, such as powdered graphite, and the carrier fluid is ethylene glycol. In this case, the hydrodynamic solution of the flow system in the bearing requires extensive computational effort, since the governing equations of flow are coupled due to the different species that exist in the system. However, the assumption of uniform fluid with non-Newtonian behaviour enables the solution of flow equations analytically. Various models are developed to account for the non-Newtonian behaviour of the fluid flow. One group of models includes differential type fluids. Third-grade fluid is a special model of differential type of fluid which has received attention in recent years.

Considerable research studies have been carried out to explain the flow field in bearing systems. The approximate analytical solution for Reynolds equation of a slider bearing with a smooth surface was presented by Wang (1991). He showed that lubricant rheological behaviour and surface roughness had an important influence on the load capacity and

*For communication
A list of symbols is given at the end of the paper

friction drag at the surface of the bearing. Non-Newtonian effects on the static characteristics of one-dimensional slider bearings in the internal flow regime were investigated by Hashimoto (1994). He applied the modified Reynolds equation to one-dimensional slider bearings and solved the resulting equation analytically using the perturbation technique. The non-Newtonian effects of powder-lubricant slurries in hydrostatic and squeeze film bearings were studied by Wu (1994). They showed that the damping factor was increased with the addition of powdered graphite into the carrier fluid. Non-Newtonian temperature and pressure effects of graphite powder lubricant when added to a Newtonian carrier fluid and applied in a rotating hydrostatic step bearing was studied by Peterson *et al* (1994). They showed that temperature increased with bearing rotational speed and compared favorably with the mathematical predictions. The transient response of a two-lobe journal bearing with non-Newtonian lubricant was studied by Sinhasan & Goyal (1995). They showed from the non-linear trajectories that the two-lobe journal bearing system became unstable when the critical journal mass was less than that obtained from Routh's criterion. The model study of double-layered porous Rayleigh-step bearings with second-order fluid as lubricant was presented by Naduvinamani (1997). He indicated that maximum dimensionless load-carrying capacity occurred at a slightly larger step ratio as compared with the conventional porous Rayleigh-step bearings. The lubrication of slider bearings with a special third-grade fluid was considered by Yurusoy & Pakdemirli (1999). They used a perturbation method to obtain approximately velocity and pressure fields in the bearings. The theoretical study concerning the effect of nonlinear behaviour of the lubricant on the performance of a slot-entry journal bearing was carried out by Sharma *et al* (2000). He showed that the combined effect of nonlinearity factor and beating flexibility affected the performance characteristics of slot-entry journal bearing significantly. A slider bearing with second- and third-grade fluids as lubricant was analyzed by Yurusoy (2002). He presented the pressure distribution in the bearing analytically. The thermodynamic analysis of the misaligned conical-cylinderical bearing with non-Newtonian lubricants was carried out by Yang & Jeng (2003). He indicated that the normal load carrying capacity was enhanced by higher values of flow behaviour index, higher eccentricity ratios, and larger misalignment factors.

Thermodynamic irreversibility occurring in the flow system lowers the availability and enhances the thermodynamic losses within the system. Entropy analysis enables us to quantify thermodynamic irreversibility associated with the system when it operates at different conditions. Consequently, optimal operating conditions minimizing entropy generation rate enables the setting to optimum operating parameters for efficient processing. Considerable research studies were carried out to examine entropy generation in thermal systems. Entropy minimization for efficient operation of thermal systems was presented by Bejan (1995). He demonstrated that the minimum entropy concept can be used for optimal design of engineering systems. A method of finding an approximate solution for the maximum entropy method in bearing estimations was presented by Tsao (1990). He indicated that the maximum entropy was an all-pole spectrum and the coefficients of its polynomial could be found from the estimation error for the signals. The maximum entropy method for fault diagnosis of rolling bearings was considered by Xu *et al* (2001). They indicated that the spectrum produced by maximum entropy had many advantages including the high frequency resolution ratio. The non-isothermal viscoelastic flows and entropy generation were investigated by Peters & Baaijens (1997). They presented the partitioning between dissipated and elastically stored energy as well as discussed the difference between entropy and energy elasticity. Irreversibility analysis of concentrically rotating annuli was carried out by Mahmud & Fraser (2002). The distributions of volumetric average entropy generation rate were presented for both isothermal and isoflux boundary conditions. Yilbas (2001) studied entropy generation in bearings when subjected to external heating. He indicated that entropy generation rate became minimum at certain heating conditions.

In the open literature, neither experimental nor theoretical work addressing entropy generation in slider bearings due to non-Newtonian fluid is found. Therefore, the present work is carried out to formulate and examine entropy generation due to lubrication of a slider bearing with a special third-grade fluid. Entropy generation rate in the bearing system was formulated analytically. Isothermal flow situation is assumed and a third-grade fluid model is accommodated to account for the non-Newtonian effects. Entropy generation numbers are determined for different non-Newtonian parameters and bearing clearance ratios.

2. Entropy generation number

The geometry of the slider bearing is shown in figure 1. The dimensionless quantities are defined from the dimensional parameters (denoted by asterisk) as follows:

$$x = \frac{x^*}{L}, \quad y = \frac{y^*}{b}, \quad m = \frac{b_2}{b_1}, \quad u = \frac{u^*}{U}, \quad v = \frac{Lv^*}{bU}, \quad p = \frac{p^*}{(\mu U L/b_1^2)}. \tag{1}$$

The dimensional viscous dissipation for a special third-grade fluid, in which the second-grade effects are negligible, is (Massoudi & Christie 1995):

$$\phi^* = \mu\left(\partial u^*/\partial y^*\right)^2 + 2\beta\left(\partial u^*/\partial y^*\right)^4. \tag{2}$$

Following Yurusoy & Pakdemirli (1999), one assumes

$$\beta = \varepsilon\mu. \tag{3}$$

The clearance is a linear function of x (figure 1):

$$b = b_1[1 - (1 - m)x]. \tag{4}$$

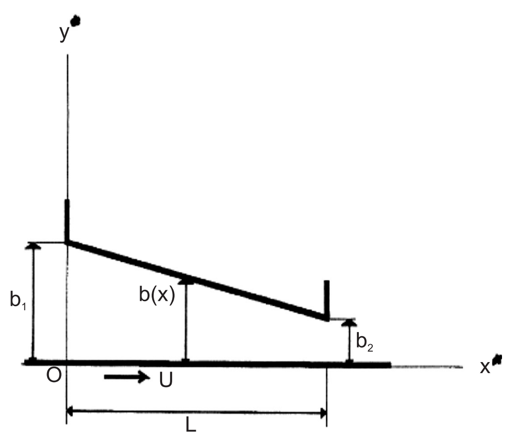


Figure 1. Schematic view of slider bearing.

Knowing that the dimensional entropy generation rate is $S'''_{\text{gen}} = \phi^*/T_0$ for isothermal flow (T_0 is a reference temperature) and substituting (4), (3) and (1) into (2), the dimensional entropy generation rate becomes

$$S'''_{gen} = \frac{\mu U^2}{T_0 b_1^2}\frac{1}{[1-(1-m)x]^2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + 2\frac{\varepsilon U^2}{b_1^2}\frac{1}{[1-(1-m)x]^2}\left(\frac{\partial u}{\partial y}\right)^4\right]. \tag{5}$$

A reference entropy generation rate and a non-Newtonian parameter are defined as follows

$$S'''_G = \mu U^2/(T_0 b_1^2), \qquad k = \varepsilon U^2/b_1^2. \tag{6}$$

The dimensionless entropy generation number is obtained by dividing by the reference entropy generation number and using the non-Newtonian parameter definition

$$Ns = \frac{1}{[1-(1-m)x]^2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \frac{2k}{[1-(1-m)x]^2}\left(\frac{\partial u}{\partial y}\right)^4\right]. \tag{7}$$

Taking the approximate velocity and pressure profiles from Yurusoy & Pakdemirli (1999) and expressing them in terms of dimensionless quantities yield

$$\begin{aligned} u &= [1-(1-m)x]^2\left(\frac{y^2}{2}-\frac{y}{2}\right)\frac{\mathrm{d}p}{\mathrm{d}x} + 1 - y \\ &\quad - k\left\{[1-(1-m)x]^4\left(\frac{y^4}{2} - y^3 + \frac{3y^2}{4} - \frac{y}{4}\right)\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)^3\right. \\ &\quad \left. + [1-(1-m)x]^2(-2y^3+3y^2-y)\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)^2 + 3(y^2-y)\frac{\mathrm{d}p}{\mathrm{d}x}\right\} + \dots, \end{aligned} \tag{8}$$

$$\begin{aligned} p &= p_\infty + \frac{6x(1-x)(1-m)}{(1+m)[1-(1-m)x]^2} + k\left\{\frac{6}{25(1-m)[1-(1-m)x]^6(1+m)^3}\right. \\ &\quad \big(648m^2\,[1-(1-m)x](1+m) + 140(1+m)^3[1-(1-m)x]^3 - 360m^3 \\ &\quad - 480m[1-(1-m)x]^2(1+m)^2\big) + \frac{24(1-m)(13+13m^2-m)}{25m(1+m)^3[1-(1-m)x]^2} \\ &\quad \left. - \frac{24(13+13m^2+8m)}{25m(1-m)(1+m)^3}\right\} + \dots . \end{aligned} \tag{9}$$

The velocity and pressure gradients are obtained by differentiating the above expressions

$$\frac{\partial u}{\partial y} = [1-(1-m)x]^2\left(y-\frac{1}{2}\right)\frac{\mathrm{d}p}{\mathrm{d}x} - 1$$
$$- k\left\{[1-(1-m)x]^4\left(2y^3-3y^2+\frac{3y}{2}-\frac{1}{4}\right)\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)^3\right.$$
$$\left.+[1-(1-m)x]^2(-6y^2+6y-1)\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)^2 + 3(2y-1)\frac{\mathrm{d}p}{\mathrm{d}x}\right\} + \dots, \tag{10}$$

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{6(1-x)(1-m)}{(1+m)[1-(1-m)x]^2} - \frac{6x(1-m)}{(1+m)[1-(1-m)x]^2}$$
$$+ \frac{12x(1-x)(1-m)^2}{(1+m)[1-(1-m)x]^3} + k\left\{\frac{36}{25[1-(1-m)x]^7(1+m)^3}\right.$$
$$\big(648m^2[1-(1-m)x](1+m) + 140(1+m)^3[1-(1-m)x]^3 - 360m^3$$
$$-480m[1-(1-m)x]^2(1+m)^2\big) + \frac{6}{25(1-m)[1-(1-m)x]^6(1+m)^3}$$
$$\big(-648m^2\,(1-m)^2 - 420(1-m)(1+m)^3[1-(1-m)x]^2$$
$$+960m(1-m)(1+m)^2[1-(1-m)x]\big)$$
$$\left.+\frac{48(1-m)^2(13+13m^2-m)}{25m(1+m)^3[1-(1-m)x]^3}\right\} + \dots. \tag{11}$$

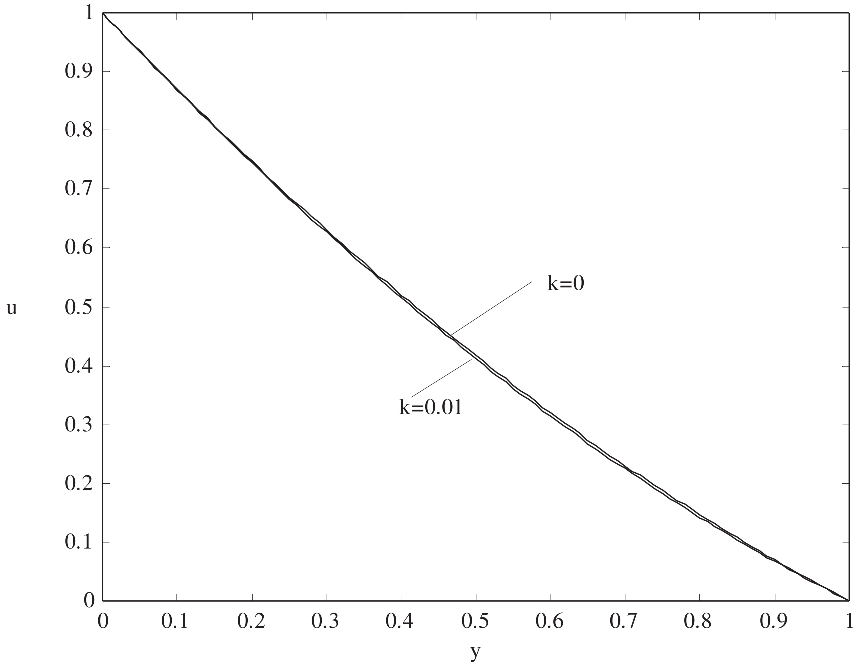
Substituting (11) into (10) and then the result into (7) yields the entropy generation number. Numerical plots for different parameters are presented in the next section.

3. Results and discussions

Non-Newtonian flow due to lubrication of slider bearing is considered. A special third-grade fluid is accommodated to account for the non-Newtonian effect. The bearing clearance is assumed to vary linearly as shown in figure 1. The flow situation is considered as isothermal due to low fluid velocity. Since the velocity and the velocity gradients all depend on pressure gradients, a care was taken in numerical calculations of the pressure gradients. According to the perturbation theory, correction terms (terms multiplied by non-Newtonian coefficient (k) in (11) should be small compared to the leading terms. In the present case, this can be ensured by selecting appropriate values of both k and m. Therefore, to satisfy the requirements for the application of the perturbation method, m values should be greater than 0 and less than 1. In addition, k values should not be greater than 0·01.

Entropy generation in the flow field due to fluid friction is formulated for the slider bearing. Entropy generation rate is computed for different non-Newtonian parameters k and bearing clearance ratios m.

Figure 2 shows velocity profiles along the y-axis at mid point of the bearing length for different non-Newtonian parameters and two clearance ratios. In the case of Newtonian fluid



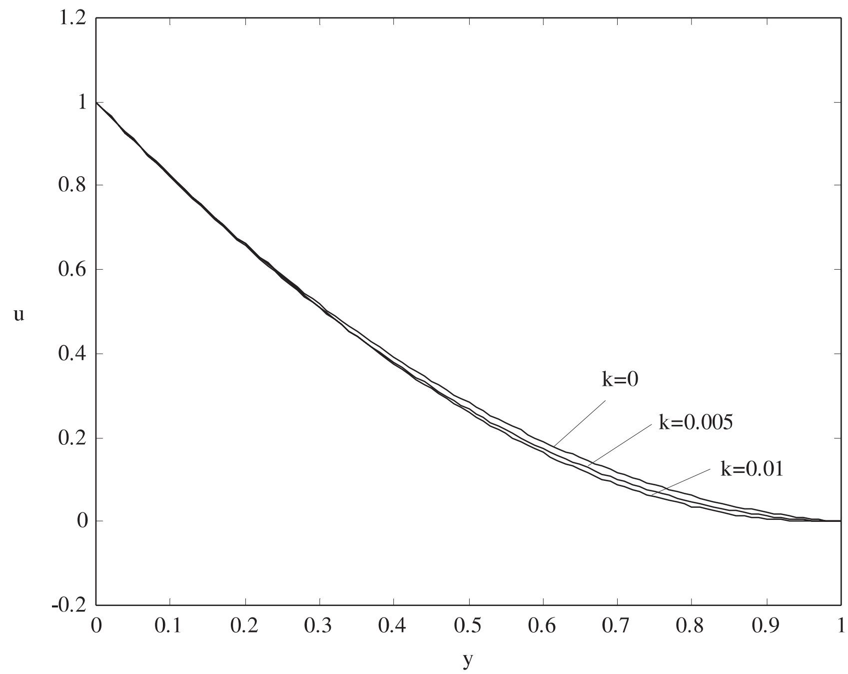


Figure 2. Velocity distribution along the y-axis for different non-Newtonian parameters: **(a)** $(x = 1/2, m = 0{\cdot}5)$, $k = 0, 0{\cdot}01$; **(b)** $(x = 1/2, m = 0{\cdot}3)$, $k = 0, 0{\cdot}005, 0{\cdot}01$.

($k = 0$) velocity profile decays gradually along the y-axis similar to that which occurs in Couette flow. This is more pronounced when clearance ratio increases to $m = 0{\cdot}5$ (figure 2a). However, velocity profile along the y-axis changes significantly as the non-Newtonian parameter increases, particularly for the axial location $x = 1/2$ (figure 2b), reducing its minimum gradually to a point $y = 1$. The location of minimum velocity gradient along the y-axis varies slightly with changing non-Newtonian parameter, i.e. it moves closer to the flat surface towards the edge of the wedge along the y-axis with increasing non-Newtonian parameter. This suggests that the rate of fluid strain across the bearing clearance varies with increasing non-Newtonian parameter; in which case, it reduces to the minimum at some point in the fluid. Consequently, shear rate close to the bearing wall is modified with changing the non-Newtonian parameter. This is more pronounced at high wedge angle where the clearance ratio is low. Therefore, the influence of geometric feature of the bearing and non-Newtonian parameter alter the rate of shear strain.

Figure 3 shows entropy generation number along the y-axis for three x-axis locations and two different clearance ratios, while k is kept constant. Entropy generation number is high in the region close to the horizontal surface of the bearing (figure 3a). This is due to sharp increase in the velocity gradient in this region, which enhances the entropy generation, which is more pronounced for the clearance ratio of 0·3. Moreover, the behaviour of the entropy number changes significantly with varying clearance ratio. In this case, entropy number reduces gradually to reach its minimum at some y-axis location for a clearance ratio of 0·5 while it decays sharply for a clearance ratio of 0·3. This is because of the large change in the rate of fluid strain in the flow field with varying the clearance ratio, i.e. it modifies the entropy generation rate. However, the existence of minimum entropy generation number at a y-axis location of 0·85 for clearance ratio 0·3 suggests that the frictional loss is minimum in this region. Moreover, the frictional losses reduce at the x-axis location of $x = 1/3$ and increasing the x-axis location results in enhancement of frictional losses. Consequently, increasing bearing length enhances entropy generation rate and frictional losses in the bearing.

Figure 4 shows entropy generation number along the x-axis for different y-axis locations in the bearing for two clearance ratios while k is kept constant. Entropy generation number remains almost steady along the x-axis towards the bearing edge for all y-axis locations when clearance ratio is 0·3, provided that in the vicinity of the bearing edge, entropy generation number increases substantially, particularly for $y = 1$ (figure 4b). This occurs because of the velocity gradient, which increases significantly in this region. In the case of clearance ratio of 0·5 (figure 4a), entropy generation increases considerably along the x-axis. Moreover, in the lower surface vicinity of the bearing, where $y = 0$, entropy generation rate attains high values for $x \leq 0{\cdot}5$ and as the distance along the x-axis increases, entropy generation rate decreases sharply towards the edge of the bearing. However, the opposite is true at the y-axis location at $y = 1$. Consequently, entropy generation rate in the bearing shows that complex fluid strain developed in the bearings, which is more pronounced for increased clearance ratios.

Figure 5 shows entropy generation number with clearance ratio at the mid points of the x and y-axes. Increasing clearance ratio beyond 0·45 lowers the entropy generation rate gradually. This is particularly true for large non-Newtonian parameters. In this case, entropy generation rate reduces as bearing wedge angle reduces, i.e. it becomes geometrically almost that of parallel plates. Consequently, complex flow field reduces to Couette-like flow, which in turn lowers the rate of fluid strain in the flow field. Increasing entropy generation number at high non-Newtonian parameter suggests that the flow field is influenced considerably by the non-Newtonian effect despite the fact that bearing geometrically approaches parallel plate like behaviour.

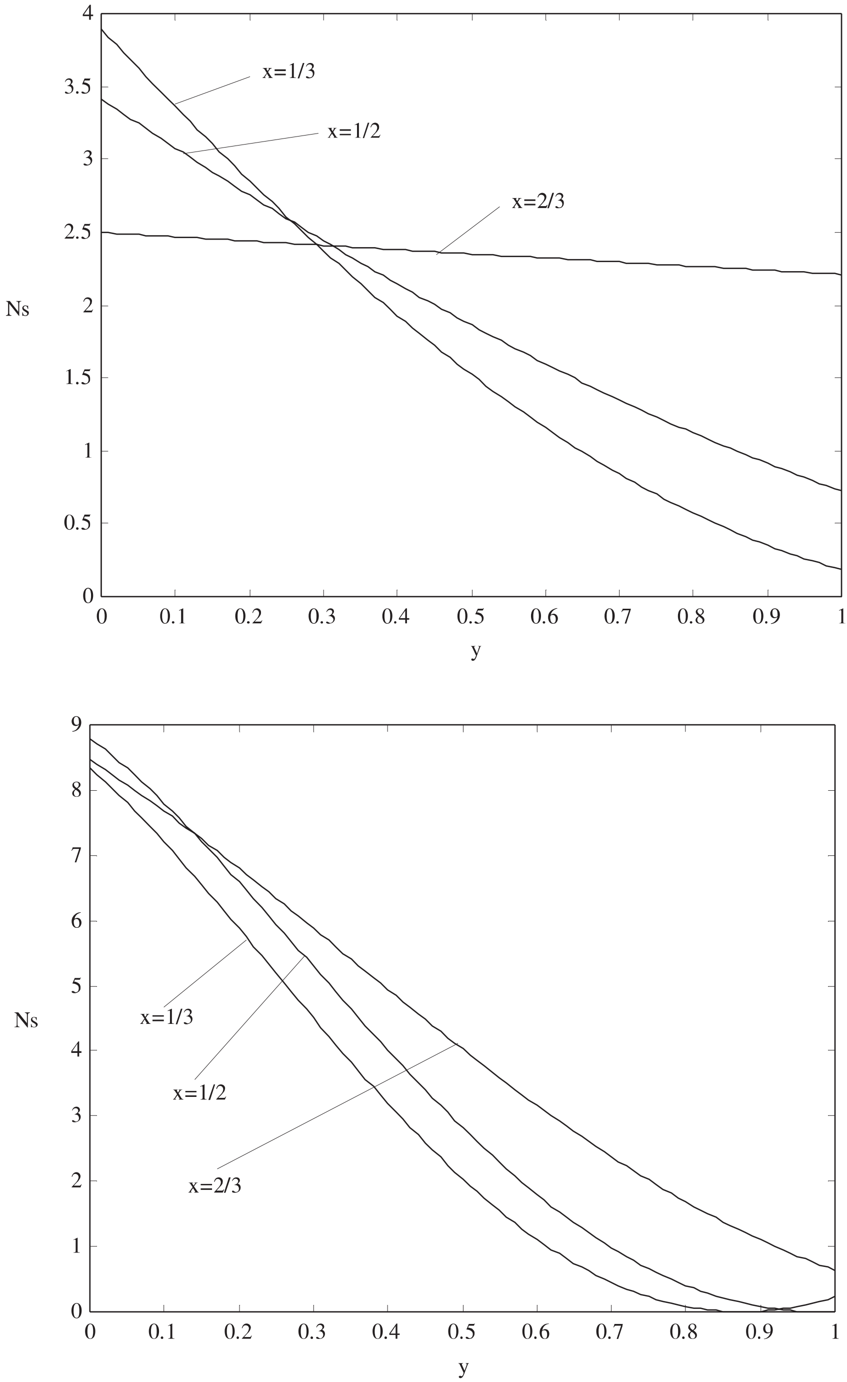


Figure 3. Entropy generation number for three x-axis locations: **(a)** ($k = 0{\cdot}01, m = 0{\cdot}5$), $x = 1/3, 1/2, 2/3$; **(b)** ($k = 0{\cdot}01, m = 0{\cdot}3$), $x = 1/3, 1/2, 2/3$.

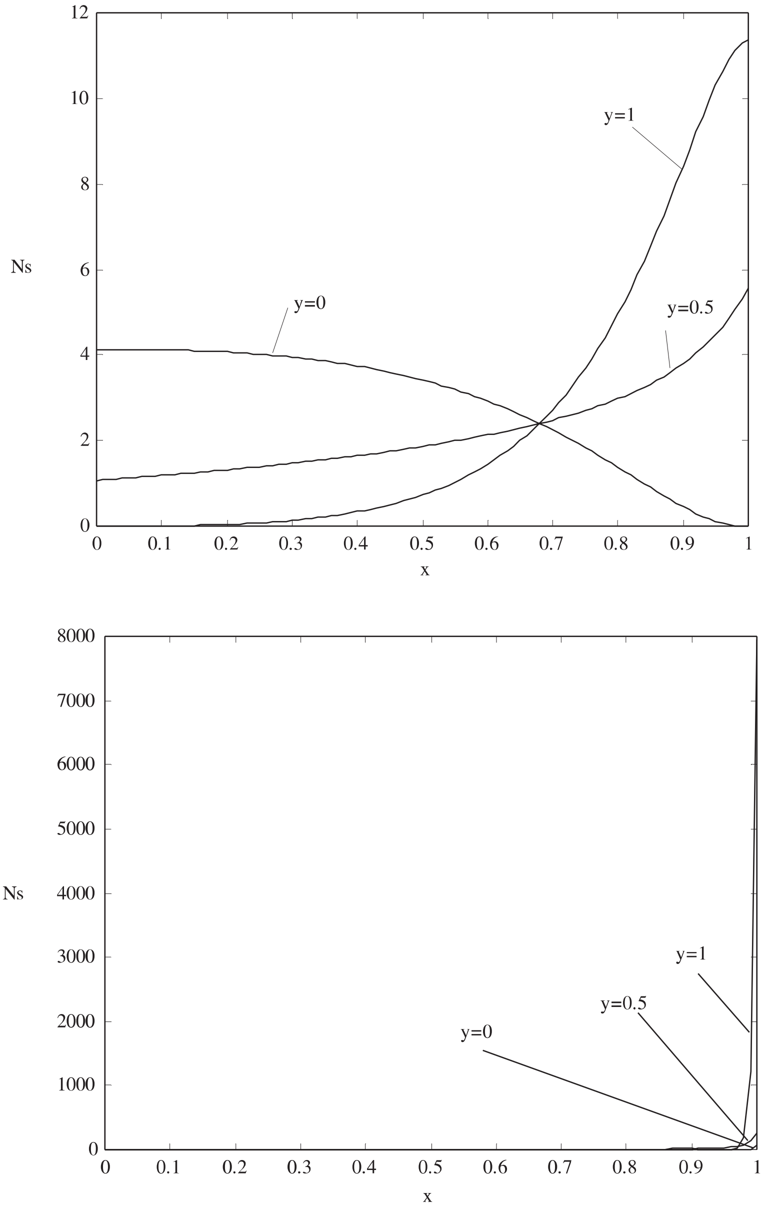


Figure 4. Entropy generation number along the x-axis for different y-axis locations: **(a)** ($m = 0{\cdot}5$ and $k = 0{\cdot}01$); **(b)** ($m = 0{\cdot}3$ and $k = 0{\cdot}01$).

Figure 6 shows entropy generation number with non-Newtonian parameter at a location midway in the bearing and for the clearance ratio $m = 0{\cdot}5$. Entropy generation number increases almost linearly with increasing non-Newtonian parameter. It should be noted that $k = 0$ corresponds to Newtonian fluid. Consequently, entropy generation number attains minimum for $k = 0$ and deviates considerably from Newtonian fluid case as non-Newtonian parameter increases. Entropy generation rate enhances as the non-Newtonian parameter

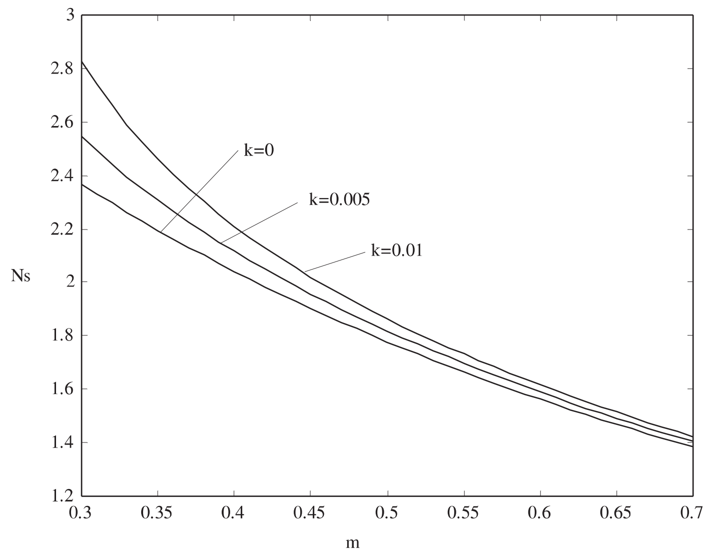


Figure 5. Entropy generation number with clearance ratio for different non-Newtonian parameters ($x = 1/2$, $y = 1/2$).

increases, i.e. Newtonian fluid results in less entropy generation rather than that corresponding to non-Newtonian fluid.

4. Conclusions

Entropy generation in a slider bearing due to non-Newtonian fluid flow is considered. Entropy generation rate is formulated in terms of velocity field, geometric configuration of slider

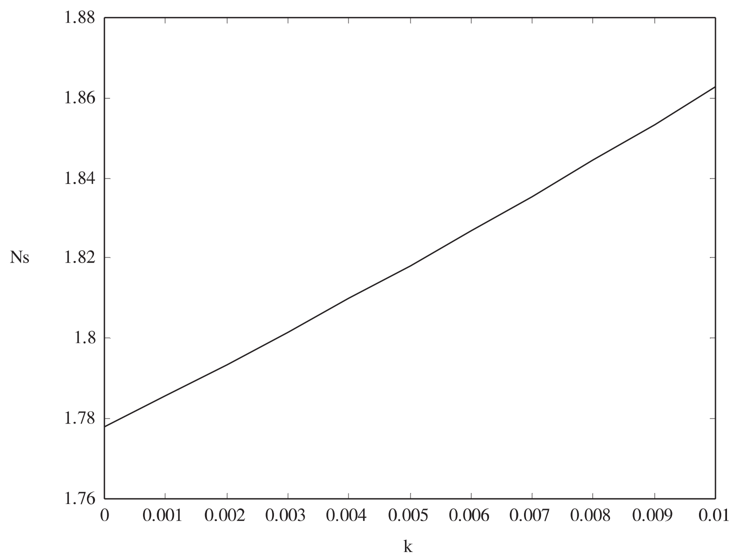


Figure 6. Entropy generation number with non-Newtonian parameters ($x = 1/2$, $y = 1/2$, $m = 0{\cdot}5$).

bearing, and non-Newtonian parameter. To account for the non-Newtonian effect, a third-grade fluid is considered. Perturbation method is accommodated to obtain the analytical solution for the entropy generation rate in the slider bearing. Isothermal flow situation is assumed in the analysis, and it is found that geometric configuration of the bearing influences the entropy generation rate considerably. In this case, low clearance ratio of the bearing results in high entropy generation rate. Moreover, increasing clearance ratio ($m \to 0{\cdot}45$) results in Couette-like flow in the slider bearing, which in turn lowers fluid strain and frictional losses in the bearing. Moreover, increasing the non-Newtonian parameter together with lowering the clearance ratio enhances entropy generation rate. Moreover, as the fluid deviates from Newtonian behaviour, entropy generation increases, regardless of the clearance ratio of the bearing. This suggests that the rates of fluid strain and friction losses in the flow system are enhanced considerably as the fluid deviates from Newtonian behaviour.

List of symbols

b	bearing clearance distance (m);
b_1	bearing clearance distance at the left side (m);
b_2	bearing clearance distance at the right side (m);
k	non-Newtonian parameter (s^{-4});
L	axial length of the bearing (m);
m	bearing clearance ratio;
Ns	entropy generation number;
p^*	dimensional pressure in the bearing (Pa);
p	dimensionless pressure in the bearing;
p_∞	pressure outside the bearing (Pa);
S'''_{gen}	entropy generation rate (W/m^{3}K);
S'''_G	reference entropy generation rate (W/m^{3}K);
T_0	reference temperature (K);
u^*	dimensional velocity in x^* direction (m);
u	dimensionless velocity in x direction;
U	bearing surface axial velocity (m/s);
v^*	dimensional velocity in y^* direction (m/s);
v	dimensionless velocity in y direction;
x^*	dimensional axial location (m);
x	dimensionless axial location;
y^*	dimensional clearance coordinate;
y	dimensionless clearance coordinate (m);
β	material constant for third-grade fluid (N.s/m^2);
ε	ratio of β to μ;
ϕ^*	dimensional viscous dissipation (N/s.m^2);
μ	viscosity (N.s/m^2).

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