Effects of fluid layer at micropolar orthotropic boundary surface

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Abstract. Effects of a fluid layer at a micropolar orthotropic elastic solid interface to a moving point load have been studied. After using the Fourier transform an eigen value approach has been employed to solve the problem. The displacement, microrotation and stress components for a micropolar orthotropic elastic solid so obtained in the physical domain are computed numerically by applying the numerical inversion technique. Micropolarity and anisotropy effects along with that of the depth of the fluid layer on various expressions have been depicted graphically for a particular model. Some special cases of interest have been presented.

Keywords. Fluid layer; micropolar; orthotropic; eigen value approach; Fourier transform.

1. Introduction

The classical theory of elasticity does not explain certain discrepancies that occur in the case of problems involving elastic vibrations of high frequency and short wavelength, that is, vibrations due to generation of ultrasonic waves. The reason lies in the microstructure of the material which exerts a special influence at high frequencies and short wavelengths. In many engineering phenomenon, including the response of solids, geological materials and composites, the assumptions of isotropic behaviour may not capture some significant features of the continuum response. An attempt was made to eliminate these discrepancies by suggesting that the transmission of interaction between two particles of a body through an elementary area lying within the material was affected not only by the action of a force vector but also by a moment (couple) vector. This led to the existence of a couple stress in elasticity. Polycrystalline materials, materials with fibrous or coarse grain structure fall under this category. The analysis of such materials requires incorporation of theories of oriented media. For this reason, micropolar theories were developed by Eringen (1966) for elastic solids and fluids. Micropolar theory is believed to be particularly useful in investigating materials consisting of bar-like molecules that exhibit microrotation effects and can support body and surface couples.

Formulation and solution of an anisotropic problem is far more difficult and cumbersome than that of its isotropic counterpart. In recent years, the elastodynamic response of anisotropic continuum has received the attention of several researchers. In particular, transversely isotropic and orthotropic materials, which may not be distinguishable from one another in plane strain and plane stress, have been more regularly studied. A review of literature on micropolar orthotropic continua shows that Iesan (1973,1974) analysed the static problem of plane micropolar strain of a homogeneous and orthotropic elastic solid, torsion problem of homogeneous and orthotropic cylinders in the linear theory of micropolar elasticity and bending of orthotropic micropolar elastic beams by terminal couples. Nakamura *et al* (1984) applied finite element method for orthotropic micropolar elasticity. Recently, Kumar & Choudhary (2002–2004) have discussed various problems in orthotropic micropolar continua.

Dynamical response to moving loads is an interesting subject in various technological and geophysical circumstances and some recent investigations are concerned with this problem. For instance, it is of great interest in solid dynamics where ground motions and stresses can be produced by blast waves (surface pressure waves due to explosions), or by supersonic aircraft. This type of situation occurs in many branches of engineering, for e.g in bridges and railways, beams subjected to pressure waves and piping systems subjected to two-phase flow. Other applications are encountered within the context of contact mechanics like the problem of high velocity rocket sleds sliding over steel guide rails. Kumar and coworkers (Kumar & Gogna 1992; Kumar & Deswal 2000, 2002; Kumar & Choudhary 2002, 2003; Kumar & Ailawalia 2003) studied the steady state response to moving loads using the micropolar theory of elasticity. Most of the moving load problems solved so far involve the use of potential functions. However, the use of an eigen value approach has the advantage of finding solutions to equations in coupled form in matrix notation directly, whereas the potential function approach requires the decoupling of equations.

In the present investigation we apply the eigen value approach, following Fourier transformation, to the moving load problem at the fluid layer/micropolar orthotropic boundary surface. Solutions are obtained in the transformed domain and are inverted by using a numerical technique. Applications of the paper may be found in mechanics, viz. in ships and motor boats moving over the surface of rivers and lakes.

2. Formulation and solution of the problem

We consider a normal point load moving along the fluid layer (depth h) and lying over the surface of a micropolar orthotropic elastic solid. We consider a rectangular coordinate system (x, y, z) with y -axis pointing vertically downward as shown in figure 1. The region $y > 0$ is occupied by micropolar orthotropic elastic half space (medium I) and the region $-h < y < 0$ represents the non viscous fluid layer (medium II). The plane $y = -h$ is the surface on which the load is acting and $y = 0$ represents the interface of medium I and medium II. The pressure pulse $P(x + Ut)$ moves with a constant velocity U in the negative x direction. After the load has been moving for some time and the transient effects have died away, the displacements appear stationary in a coordinate system moving with the load.

If we restrict our attention to the plane deformation parallel to the xy plane with displacement vector $\mathbf{u} = (u_1, u_2, 0)$ and microrotation vector $\boldsymbol{\phi} = (0, 0, \phi_3)$, the basic equations in the dynamic theory of the plane strain of homogeneous, micropolar orthotropic elastic solid

Figure 1. Geometry of the problem.

in the absence of body forces and body couples, given by Eringen (1968), can be recalled as:

$$t_{kl,l} = \rho(\partial^2 u_k / \partial t^2), \quad (1)$$

$$m_{k3,k} + \varepsilon_{kl3} t_{kl} = \rho j (\partial^2 \phi_3 / \partial t^2), \quad k = l = 1, 2. \quad (2)$$

The constitutive relations, given by Iesan (1973), can be written as:

$$\begin{aligned} t_{11} &= A_{11}\varepsilon_{11} + A_{12}\varepsilon_{22}, & t_{12} &= A_{77}\varepsilon_{12} + A_{78}\varepsilon_{21}, & t_{21} &= A_{78}\varepsilon_{12} + A_{88}\varepsilon_{21}, \\ t_{22} &= A_{12}\varepsilon_{11} + A_{22}\varepsilon_{22}, & m_{13} &= B_{66}\phi_{3,1}, & m_{23} &= B_{44}\phi_{3,2}, \end{aligned} \quad (3)$$

where

$$\varepsilon_{kl} = u_{l,k} + \varepsilon_{lk3}\phi_3. \quad (4)$$

In these relations, we have used the following notations: t_{kl} – components of the force stress tensor, m_{k3} – components of the couple stress tensor, ε_{kl} – components of micropolar strain tensor, u_k – components of displacement vector, ϕ_3 – component of microrotation vector, $\varepsilon_{k/3}$ – permutation symbol, A_{11} , A_{12} , A_{77} , A_{78} , A_{88} , A_{22} , B_{44} , B_{66} – characteristic constants of the material, ρ – the density and j – the microinertia.

From (1)–(4), we obtain the field equations of the plane strain for micropolar orthotropic elastic solid in the form:

$$\left(A_{11} \frac{\partial^2}{\partial x^2} + A_{88} \frac{\partial^2}{\partial y^2} \right) u_1 + (A_{12} + A_{78}) \frac{\partial^2 u_2}{\partial x \partial y} - K_1 \frac{\partial \phi_3}{\partial y} = \rho \frac{\partial^2 u_1}{\partial t^2}, \quad (5)$$

$$(A_{12} + A_{78}) \frac{\partial^2 u_1}{\partial x \partial y} + \left(A_{77} \frac{\partial^2}{\partial x^2} + A_{22} \frac{\partial^2}{\partial y^2} \right) u_2 - K_2 \frac{\partial \phi_3}{\partial x} = \rho \frac{\partial^2 u_2}{\partial t^2}, \quad (6)$$

$$\left(B_{66} \frac{\partial^2}{\partial x^2} + B_{44} \frac{\partial^2}{\partial y^2} - \aleph \right) \phi_3 + K_1 \frac{\partial u_1}{\partial y} + K_2 \frac{\partial u_2}{\partial x} = \rho j \frac{\partial^2 \phi_3}{\partial t^2}, \quad (7)$$

where

$$K_1 = A_{78} - A_{88}, \quad K_2 = A_{77} - A_{78}, \quad \aleph = K_2 - K_1. \quad (8)$$

The equation which governs the motion of the fluid is given by Ewing *et al* (1957) as

$$\lambda_0 \nabla (\nabla \cdot \mathbf{u}^0) = \rho_0 \frac{\partial^2 \mathbf{u}^0}{\partial t^2}, \quad (9)$$

The stress and displacement relations are given by

$$\tau_{pn}^0 = \lambda_0 u_{r,r}^0 \delta_{pn} \cdot p = n = 1, 2, 3, \quad (10)$$

where ρ^0 is the density of fluid and λ^0 is Lamé's constant.

For a two-dimensional problem we assume,

$$\mathbf{u}^0 = (u_1^0, u_2^0, 0). \quad (11)$$

Following Fung (1968), a Galilean transformation

$$x^* = x + Ut, \quad y^* = y, \quad t^* = t, \quad (12)$$

is introduced then the boundary conditions would be independent of t^* and assuming the dimensionless variables defined by the expressions

$$\begin{aligned} x' &= \frac{\omega^*}{c_1} x^*, \quad y' = \frac{\omega^*}{c_1} y^*, \quad u_1' = \frac{\omega^*}{c_1} u_1, \quad u_2' = \frac{\omega^*}{c_1} u_2, \quad \phi_3' = \frac{A_{11}}{K_1} \phi_3, \\ t_{kl}' &= \frac{t_{kl}}{A_{11}}, \quad m_{k3}' = \frac{c_1}{B_{44}\omega^*} m_{k3}, \quad u_1^{0'} = \frac{\omega^*}{c_1} u_1^0, \quad u_2^{0'} = \frac{\omega^*}{c_1} u_2^0, \quad \tau_{pn}' = \frac{\tau_{pn}}{\lambda_0}. \end{aligned} \quad (13)$$

where

$$\omega^{*2} = \frac{\aleph}{\rho j}, \quad c_1^2 = \frac{A_{11}}{\rho},$$

in (5)–(7) and using (12) and (13) and applying Fourier transform defined by

$$\tilde{f}(\xi, y) = \int_{-\infty}^{\infty} f(x, y) e^{i\xi x} dx. \quad (14)$$

on the resulting equations (after suppressing the primes), we obtain

$$\tilde{u}_1'' = b_{11}\tilde{u}_1 + a_{12}\tilde{u}_2' + a_{13}\tilde{\phi}_3', \quad (15)$$

$$\tilde{u}_2'' = b_{22}\tilde{u}_2 + a_{21}\tilde{u}_1' + b_{23}\tilde{\phi}_3, \quad (16)$$

$$\tilde{\phi}_3'' = b_{33}\tilde{\phi}_3 + a_{31}\tilde{u}_1' + b_{32}\tilde{u}_2, \quad (17)$$

Similarly using (11)–(13) in (9) and applying transform defined by (13), we obtain,

$$\tilde{u}_2^{0''} = \alpha_5^2 \tilde{u}_2^0. \quad (18)$$

where

$$\begin{aligned}
 b_{11} &= \frac{\xi^2 (A_{11} - \rho U^2)}{A_{88}}, & b_{22} &= \frac{\xi^2 (A_{77} - \rho U^2)}{A_{22}}, & b_{23} &= -\frac{i\xi K_1 K_2}{A_{11} A_{22}}, \\
 b_{32} &= \frac{i\xi c_1^2 K_2 A_{11}}{\omega^{*2} K_1 B_{44}}, & b_{33} &= \frac{1}{B_{44}} \left[\xi^2 (B_{66} - \rho U^2 j) + \frac{8c_1^2}{\omega^{*2}} \right], \\
 a_{12} &= \frac{i\xi (A_{12} + A_{78})}{A_{88}}, & a_{13} &= \frac{K_1^2}{A_{11} A_{88}}, & a_{21} &= \frac{i\xi (A_{12} + A_{78})}{A_{22}}, \\
 a_{31} &= -\frac{A_{11} c_1^2}{B_{44} \omega^{*2}}, & \alpha_5^2 &= \frac{\xi^2 (\lambda_0 - \rho_0 U^2)}{\lambda_0}.
 \end{aligned} \tag{19}$$

Equations (15)–(17) may be written as

$$\frac{d}{dy} W(\xi, y) = A(\xi) W(\xi, y). \tag{20}$$

where

$$\begin{aligned}
 W &= \begin{pmatrix} V \\ V' \end{pmatrix}, & A &= \begin{pmatrix} O & I \\ A_1 & A_2 \end{pmatrix}, & V &= \begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{\phi}_3 \end{pmatrix}, \\
 A_1 &= \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & b_{23} \\ 0 & b_{32} & b_{33} \end{pmatrix}, & A_2 &= \begin{pmatrix} 0 & a_{12} & a_{13} \\ a_{21} & 0 & 0 \\ a_{31} & 0 & 0 \end{pmatrix},
 \end{aligned} \tag{21}$$

O and I are respectively zero and an identity matrix of order 3.

To solve (20), we assume

$$W(\xi, y) = X(\xi) e^{qy}, \tag{22}$$

which leads to eigen value problem. The characteristic equation corresponding to matrix A is given by,

$$|A - qI| = 0, \tag{23}$$

which on expansion provides us,

$$q^6 + \lambda_1 q^4 + \lambda_2 q^2 + \lambda_3 = 0, \tag{24}$$

where

$$\begin{aligned}
 \lambda_1 &= -(a_{12} a_{21} + a_{13} a_{31} + b_{11} + b_{22} + b_{33}), \\
 \lambda_2 &= a_{12} (a_{21} b_{33} - b_{23} a_{31}) + a_{13} (b_{22} a_{31} - a_{21} b_{32}) + b_{22} b_{33} - b_{23} b_{32} \\
 &\quad + b_{11} (b_{22} + b_{33}), \\
 \lambda_3 &= b_{11} (b_{23} b_{32} - b_{22} b_{33}).
 \end{aligned} \tag{25}$$

Values of matrix A are the characteristic roots of (24). The vectors $X(\xi)$ corresponding to the eigen values q_s can be determined by solving the homogeneous equation

$$[A - qI]X(\xi) = 0. \quad (26)$$

The set of eigen vectors $X_s(\xi)$, $s = 1, 2, \dots, 6$ may be obtained as

$$X_s(\xi) = \begin{pmatrix} X_{s1}(\xi) \\ X_{s2}(\xi) \end{pmatrix}, \quad (27)$$

where

$$X_{s1}(\xi) = \begin{pmatrix} q_s \\ a_s \end{pmatrix}, \quad X_{s2}(\xi) = \begin{pmatrix} q_s^2 \\ a_s q_s \\ b_s q_s \end{pmatrix}, \quad q = q_s; s = 1, 2, 3, \quad (28)$$

$$X_{j1}(\xi) = \begin{pmatrix} -q_j \\ a_j \\ b_j \end{pmatrix}, \quad X_{j2}(\xi) = \begin{pmatrix} q_j^2 \\ -a_j q_j \\ -b_j q_j \end{pmatrix}, \quad j = s + 3; q = -q_s; s = 1, 2, 3, \quad (29)$$

and

$$\begin{aligned} a_s &= [b_{11}b_{23} - q_s^2(b_{23} + a_{21}a_{13})]/\nabla_s, \\ b_s &= (q_s^2a_{31} + a_sb_{32})/(q_s^2 - b_{33}), \\ \nabla_s &= q_s^2a_{13} + a_{12}b_{23} - b_{22}a_{13}. \end{aligned} \quad (30)$$

The solution of (22) is given by

$$W(\xi, y) = \sum_{s=1}^3 [D_s X_s(\xi) \exp(q_s y) + D_{s+3} X_{s+3}(\xi) \exp(-q_s y)]. \quad (31)$$

The transformed displacements and microrotation satisfying the radiation conditions for micropolar orthotropic solid (i.e. for $y \geq 0$) are given by,

$$\begin{aligned} \tilde{u}_1 &= -q_1 D_4 e^{-q_1 y} - q_2 D_5 e^{-q_2 y} - q_3 D_6 e^{-q_3 y}, \\ \tilde{u}_2 &= a_1 D_4 e^{-q_1 y} + a_2 D_5 e^{-q_2 y} + a_3 D_6 e^{-q_3 y}, \\ \tilde{\phi}_3 &= b_1 D_4 e^{-q_1 y} + b_2 D_5 e^{-q_2 y} + b_3 D_6 e^{-q_3 y}. \end{aligned} \quad (32)$$

The transformed displacement component $\tilde{u}_2^0$ in the fluid layer (i.e. for $-h \leq y \leq 0$) is given by,

$$\tilde{u}_2^0 = B_1 e^{\alpha_5 y} + B_2 e^{-\alpha_5 y}. \quad (33)$$

3. Boundary conditions

For concentrated normal pressure, we take $P(x+Ut) = F\delta(x^*)$, where $\delta(x^*)$ is the Dirac delta function and F is the applied pressure pulse, therefore in moving coordinates the boundary conditions are,

- (i) $-\tau_{22}^0 = F\delta(x^*)$, at $y = -h$,
- (ii) Normal component of velocity of fluid = normal velocity of solid, i.e. $\partial u_2^0/\partial t = \partial u_2/\partial t$, at $y = 0$,
- (iii) $t_{21} = 0$ at $y = 0$,
- (iv) $m_{23} = 0$ at $y = 0$,
- (v) $t_{22} = \tau_{22}^0$, at $y = 0$.

(34)

Applying the Fourier transform defined by (14) on the boundary conditions (34) and using (3), (10)–(13) and (32)–(33), we obtain the expressions for displacement components, micro-rotation, force stress and couple stress for an orthotropic solid as,

$$\tilde{u}_1 = (F/\Delta) \left[q_1 \Delta'_1 e^{-q_1 y} - q_2 \Delta'_2 e^{-q_2 y} + q_3 \Delta'_3 e^{-q_3 y} \right], \quad (35)$$

$$\tilde{u}_2 = (F/\Delta) \left[a_1 \Delta'_1 e^{-q_1 y} - a_2 \Delta'_2 e^{-q_2 y} + a_3 \Delta'_3 e^{-q_3 y} \right], \quad (36)$$

$$\tilde{\phi}_3 = (F/\Delta) \left[b_1 \Delta'_1 e^{-q_1 y} - b_2 \Delta'_2 e^{-q_2 y} + b_3 \Delta'_3 e^{-q_3 y} \right], \quad (37)$$

$$\tilde{t}_{22} = (F/\Delta) \left[s_1 \Delta'_1 e^{-q_1 y} - s_2 \Delta'_2 e^{-q_2 y} + s_3 \Delta'_3 e^{-q_3 y} \right], \quad (38)$$

$$\tilde{m}_{23} = -(FK_1/\Delta A_{11}) \left[b_1 q_1 \Delta'_1 e^{-q_1 y} - b_2 q_2 \Delta'_2 e^{-q_2 y} + b_3 q_3 \Delta'_3 e^{-q_3 y} \right], \quad (39)$$

where

$$\begin{aligned} \Delta &= -2M \sum_{i=1}^3 (-1)^{i+1} f_i g_i, \quad f_\Theta = r_\sigma b_\varsigma q_\varsigma - r_\varsigma b_\sigma q_\sigma, \\ g_\Theta &= s_\Theta \cosh(\alpha_5 h) + a_\Theta M \sinh(\alpha_5 h), \quad M = [\rho_0 U^2 / (\lambda_0 - \rho_0 U^2)] \alpha_5, \\ \Delta'_\Theta &= 2M (r_\sigma b_\varsigma q_\varsigma - r_\varsigma b_\sigma q_\sigma), \quad r_\Theta = -i\xi a_\Theta A_{78} + q_\Theta^2 A_{88} \\ &\quad + (K_1/A_{11}) (A_{88} - A_{78}) b_\Theta, \\ s_\Theta &= (q_\Theta/A_{11}) (i\xi A_{12} - a_\Theta A_{22}), \quad \Theta = 1, 2, 3; \quad \sigma = 2, 1, 1; \quad \varsigma = 3, 3, 2. \end{aligned} \quad (40)$$

4. Particular cases

4.1 Micropolar transversely isotropic materials

This type of medium has only one axis of symmetry and, in this case, the elastic constants are given by

$$A_{11} = A_{22}, A_{12}, A_{77} = A_{88}, A_{78}, B_{44} = B_{66}$$

Using these values in (35)–(39), we obtain the corresponding expressions in case of micropolar transversely isotropic material.

4.2 Micropolar cubic crystal

Elastic constants in case of a micropolar cubic crystal are given by

$$A_{11} = A_{22}, A_{12}, A_{77} = A_{78} = A_{88}, B_{44} = B_{66}.$$

Use of these elastic constants in (35)–(39) yields the corresponding expressions in case of micropolar cubic crystals.

4.3 Micropolar isotropic material

Taking

$$\begin{aligned} A_{11} = A_{22} = \lambda + 2\mu + K, \quad A_{77} = A_{88} = \mu + K, \quad A_{12} = \lambda, \\ A_{78} = \mu, \quad B_{44} = B_{66} = \gamma, \end{aligned}$$

with

$$-K_1 = K_2 = \aleph/2 = K,$$

we obtain the corresponding expressions for the micropolar isotropic medium. These results tally with those obtained for a micropolar isotropic medium.

4.4 Elastic isotropic solid

If we take $\gamma = j = K = 0$ in case (c), we obtain the components for normal displacement and normal stress for an elastic solid as,

$$\tilde{u}_2 = (F/\Delta_0) \left[d_1 \Delta'_{10} e^{-p_1 y} - d_2 \Delta'_{20} e^{-p_2 y} \right], \quad (41)$$

$$\tilde{t}_{22} = (F/\Delta_0) \left[s'_1 \Delta'_{10} e^{-p_1 y} - s'_2 \Delta'_{20} e^{-p_2 y} \right], \quad (42)$$

where

$$\begin{aligned} \Delta_0 &= 2M \left[\left(r'_2 s'_1 - r'_1 s'_2 \right) \text{Cosh}(\alpha_5 h) + \left(d'_1 r'_2 - d_2 r'_1 \right) \text{Sinh}(\alpha_5 h) \right], \\ \Delta'_{10,20} &= -2Mr'_{2,1}, \quad r'_m = -i\xi d_m + p_m^2, \quad s'_m = i\xi p_m \\ &\quad - ((\lambda + 2\mu)/\lambda) p_m d_m, \\ d_m &= (p_m^2 - c_{11})/c_{12}, \quad p_{1,2}^2 = \left[-\lambda_4 \pm (\lambda_4^2 - 4\lambda_5)^{1/2} \right] / 2, \\ \lambda_4 &= -(c_{12}c_{22} + c_{11} + c_{21}), \\ \lambda_5 &= c_{11}c_{21}, \quad c_{11} = \xi^2(\lambda + 2\mu - \rho U^2)/\mu, \quad c_{12} = i\xi(\lambda + \mu)/\mu, \\ c_{21} &= \xi^2(\mu - \rho U^2)/(\lambda + 2\mu), \quad c_{22} = i\xi(\lambda + \mu)/(\lambda + 2\mu), \quad m = 1, 2. \end{aligned} \quad (43)$$

5. Inversion of the transform

To obtain the solution of the problem in the physical domain, we must invert the transform in (35)–(39) and (41)–(42). These expressions are functions of y and ξ , the parameter of the Fourier transform, and hence are of the form $\tilde{f}(\xi, y)$. To get the function $f(x, y)$ in the physical domain we invert the Fourier transform using,

$$f(x, y) = (1/2\pi) \int_{-\infty}^{\infty} \tilde{f}(\xi, y) e^{-i\xi x} d\xi \quad (44)$$

$$= (1/2\pi) \int_{-\infty}^{\infty} [\cos(\xi x) f_e - i \sin(\xi x) f_o] d\xi. \quad (45)$$

where f_e and f_o are respectively even and odd parts of the function $\tilde{f}(\xi, y)$. The method for evaluating this integral is described by Press *et al* (1986) which involves the use of Rhomberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

6. Numerical results and discussions

For numerical computations, we take the following values of relevant parameters for micropolar orthotropic solid as,

$$\begin{aligned} A_{11} &= 13.97 \times 10^9 \text{ N/m}^2, & A_{77} &= 3.0 \times 10^9 \text{ N/m}^2, \\ A_{88} &= 3.2 \times 10^9 \text{ N/m}^2, & A_{22} &= 13.75 \times 10^9 \text{ N/m}^2, \\ A_{12} &= 8.13 \times 10^9 \text{ N/m}^2, & A_{78} &= 2.2 \times 10^9 \text{ N/m}^2, \\ B_{44} &= 0.056 \times 10^5 \text{ N}, & B_{66} &= 0.057 \times 10^5 \text{ N}. \end{aligned}$$

For comparison with a micropolar isotropic solid, following Gauthier (1982), we take the following values of the relevant parameters for the case of aluminium epoxy composite as,

$$\begin{aligned} \rho &= 2.19 \times 10^3 \text{ kg/m}^3, & \lambda &= 7.59 \times 10^9 \text{ N/m}^2, & \mu &= 1.89 \times 10^9 \text{ N/m}^2, \\ K &= 0.0149 \times 10^9 \text{ N/m}^2, & \gamma &= 0.0268 \times 10^5 \text{ N}, & j &= 0.00196 \times 10^{-4} \text{ m}^2. \end{aligned}$$

The physical constants for water are given by Ewing *et al* (1957),

$$\rho_0 = 1.0 \text{ gm/cm}^3, \quad \lambda_0 = 2.14 \times 10^{10} \text{ dyne/cm}^2.$$

The values of normal displacement $U_2 = (u_2/F)$, normal force stress $T_{22} = (t_{22}/F)$ and tangential couple stress $M_{23} = (m_{23}/F)$ for a micropolar orthotropic solid (MOS), micropolar isotropic solid (MIS) and elastic isotropic solid (EIS) have been studied for different fluid layers and the variations of these components with distance x have been shown by (a) (o—o—o) for MOS, long dashes line with centered symbol (o— —o— —o) for MIS and short dashes

Figure 2. Variation of normal displacement $U_2 (= u_2/F)$ with distance x .

with centered symbols (o- - o- - o) for EIS ($h = 0.1$). (b) solid line with centered symbols (x—x—x) for MOS, long dashes line with centered symbols (x- -x- -x) for MIS and short dashes with centered symbols (x- - -x- - -x) for EIS ($h = 0.5$). These variations are shown in figures 2–4. The computations are carried out for $y = 1.0$ and $U < c_1$ in the range $0 \leq x \leq 10.0$.

7. Discussions for various cases

The values of normal displacement for EIS are large as compared to the values for MOS and MIS. The variations which are oscillatory in nature, decreases with increase in depth of fluid layer over the surface of solid. These variations of normal displacement for MOS, MIS and EIS are shown in figure 2.

Contrary to the variations of normal displacement, the variations of normal force stress for all the solids are comparable to each other. Although the variations of normal force stress for all the solids among themselves i.e. MOS, MIS and EIS for $h = 0.1$ and $h = 0.5$ are similar in nature but the difference in magnitude for different depths of fluid layer is minimum for MIS. The values of normal force stress for MOS and MIS, very close to the point of application of source, decreases with increase in depth of fluid layer. These variations of normal force stress are shown in figure 3.

It is observed from figure 4 that the variations of tangential couple stress for MOS and MIS are similar, but these variations, which are oscillatory in nature, increase with increase in depth of fluid layer over the surface of the solid. Moreover, the values of tangential couple stress are more for MOS in comparison to the values for MIS, but with increase in depth of fluid layer these variations are the opposite in nature, in the initial range. However, these values of tangential couple stress for both MOS and MIS are quite close to one another for both MOS and MIS in the range $2.0 \leq x \leq 10.0$.

Figure 3. Variation of normal force stress $T_{22}(= t_{22}/F)$ with distance x .

Figure 4. Variation of tangential couple stress $M_{23}(= m_{23}/F)$ with distance x .

8. Conclusion

Analysis of the displacement, microrotation and stresses developed in a body due to a moving source causing fracture is an interesting problem of mechanics having its applications in determining the stability of a medium. The anisotropy effect is quite significant in the study of deformation of a body. The value of the quantities are appreciably affected by the depth of fluid layer. While the magnitude of oscillations of normal displacement and normal force stress decreases with increase in depth of fluid layer over the surface of solid, the variation is opposite in nature in case of tangential couple stress. The values of normal displacement for micropolar solid (MOS and MIS) lie over a short range as compared to the values for EIS. Also, variations of normal displacement and normal stress, in the initial range, for EIS and micropolar solid (MOS and MIS) are opposite in nature.

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