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Surface wave propagation in a double liquid layer over a liquid-saturated porous half-space

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Abstract. The frequency equation is derived for surface waves in a liquid-saturated porous half-space supporting a double layer, that of inhomogeneous and homogeneous liquids. Asymptotic approximations of Bessel functions are used for long and short wavelength cases. Certain other problems are discussed as special cases. Velocity ratio (phase and group velocity) is obtained as a function of wavenumber and the results are shown graphically.

Keywords. Surface wave; liquid-saturated porous layer; velocity ratio.

1. Introduction

Elastic wave propagation in liquid-saturated porous media has been a subject of continued interest due to its importance in various fields, such as earthquake engineering, seismology, geophysical exploration etc. For instance, Crampin (1987) has explained that the liquid present in the pores plays an important role in the preperation of earthquake. Biot (1956) established a systematic theory for the propagation of elastic waves in such solids and showed the existence of two dilatational waves along with one shear wave.

Porous solids such as sandstone or limestone saturated with oil or groundwater are often present in the Earth's crust. The liquid present in the pores of the poroelastic solids has significant effects on the surface wave characteristics, such as phase and group velocities. Therefore, many researchers have investigated the surface wave propagation at the boundaries of liquid-saturated porous solids. Deresciewicz (1961, 1962, 1964, 1974), Gazetas (1982), Yamamoto (1983), Sharma *et al* (1990, 1991), Kumar & Miglani (1996) etc. have studied surface wave propagation in liquid-saturated porous solids with different models.

Earth's structure is not homogeneous throughout. Field observations indicate the presence of inhomogeneity in the upper part of the Earth. So, some parts or the whole may be considered inhomogeneous. Propagation of plane waves in inhomogeneous media was discussed by Pekeris (1935, 1946), Scholte (1961, 1962), Eason (1967) and Scott (1970) among many others. Wave propagation in inhomogeneous liquid media was discussed by Gupta (1965), Gogna (1969), Kumari (1971), Doomra (1981) and others.

The effect of an incompressible ocean on Rayleigh waves propagating along the bottom was studied by Brownwich (1898) and Stoneley (1926). Gogna (1969) discussed Rayleigh type surface wave propagation in a model of the oceanic crust, involving a double liquid layer of inhomogeneous and homogeneous liquids lying over an anisotropic half-space. Following Gogna (1969), we consider a similar model of a double liquid layer over a liquid-saturated porous half-space, as the liquid-saturated porous materials are often present below oceans in the form of sandstone and limestone, and discuss Rayleigh type surface wave propagation. Also, this can be considered as the generalization of the problem of surface wave propagation in a liquid-saturated porous solid half-space lying under a homogeneous liquid layer as discussed by Deresiewicz (1964b).

As surface waves absorb information on the properties of the areas they traverse, which is reflected in the form of dispersion, this problem of surface wave propagation in such a realistic model is of practical interest in the field of earthquake engineering and geophysical exploration.

2. Formulation of the problem

A model consisting of a double layer of liquid, the upper layer M_1 being inhomogeneous and of thickness h_1 and the lower layer M_2 being homogeneous and of thickness h_2, lying over a liquid- saturated porous solid half-space M_3 is considered. Referring to the rectangular Cartesian co-ordinate system, the z-axis is chosen in the direction of increasing depth and $z = 0$ is taken as the free surface of the inhomogeneous liquid layer. Therefore, the media M_1, M_2 and M_3 occupy the region $0 \leq z < h_1$, $h_1 \leq z < h_1 + h_2$ and $z \geq h_1 + h_2$ respectively, as shown in figure 1.

We are discussing the two-dimensional problem with wavefronts parallel to y–z plane, so that the components of displacement along the x- and z-directions are independent of the y-coordinate and the components along the y-direction are zero.

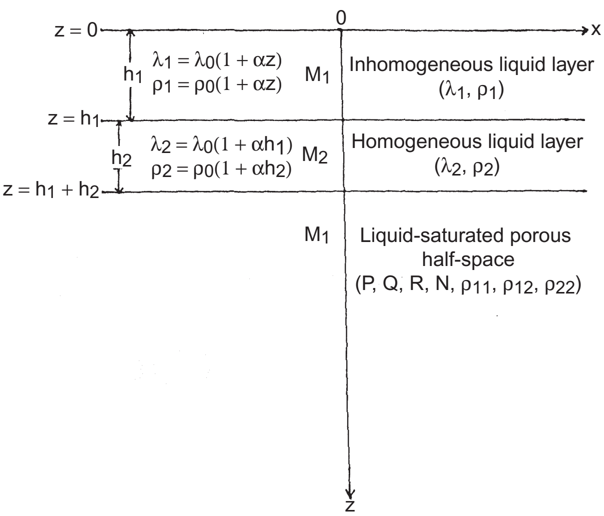

Figure 1. Geometry of the problem

3. Basic equations and their solutions

For the medium M_1, following Gogna (1969), we consider inhomogeneity varying with depth as,

$$\lambda_1 = \lambda_0(1 + \alpha z),\ \ \rho_1 = \rho_0(1 + \alpha z), \tag{1}$$

where λ_0, ρ_0 are the bulk modulus and density at the free surface, respectively, the equations of motion are

$$\frac{\partial}{\partial x}(\lambda_1 \vartheta) = \rho_1 \frac{\partial^2 u_1}{\partial t^2},\ \frac{\partial}{\partial z}(\lambda_1 \vartheta) = \rho_1 \frac{\partial^2 w_1}{\partial t^2}, \tag{2}$$

where

$$\vartheta = \frac{\partial u_1}{\partial x} + \frac{\partial w_1}{\partial z},$$

u_1, w_1 are the displacement components along x- and z- directions respectively.

For the surface waves moving along the direction of x-axis with speed c, the solution for (2) may be given as

$$\left.\begin{aligned} u_1 &= \frac{ik}{a}\left[P_1 I_0'\left(a\frac{1+\alpha z}{\alpha}\right) + P_2 K_0'\left(a\frac{1+\alpha z}{\alpha}\right)\right] e^{ik(x-ct)}, \\ w_1 &= \left[P_1 I_0\left(a\frac{1+\alpha z}{\alpha}\right) + P_2 K_0\left(a\frac{1+\alpha z}{\alpha}\right)\right] e^{ik(x-ct)}, \end{aligned}\right\} \tag{3}$$

where P_1 and P_2 are arbitrary constants,

$$a^2 = k^2\left(1 - \frac{c^2}{c_1^2}\right),\ c_1^2 = \frac{\lambda_0}{\rho_0}, \tag{4}$$

I_0, K_0 are the modified Bessel functions of the first and second kind of order zero and dash denotes the derivative with respect to z. The stress components are given by

$$(\tau_{xx})_1 = (\tau_{zz})_1 = \lambda_1 \vartheta. \tag{5}$$

For the medium M_2, the bulk modulus λ_2 and density ρ_2 are given as

$$\lambda_2 = \lambda_0(1 + \alpha h_1),\ \ \rho_2 = \rho_0(1 + \alpha h_1), \tag{6}$$

which are their respective values in the inhomogeneous layer at the interface $z = h_1$. The equations of motion in terms of displacement components u_2, w_2 are given by

$$\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial x \partial z} = \frac{1}{c_1^2}\frac{\partial^2 u_2}{\partial t^2},\ \ \frac{\partial^2 u_2}{\partial x \partial z} + \frac{\partial^2 w_2}{\partial z^2} = \frac{1}{c_1^2}\frac{\partial^2 w_2}{\partial t^2}. \tag{7}$$

For the time harmonic waves moving along x-direction, the solution for (7) may be written as

$$u_2 = \frac{ik}{a}\left[Q_1 e^{az} - Q_2 e^{-az}\right] e^{ik(x-ct)},\ w_2 = \left[Q_1 e^{az} + Q_2 e^{-az}\right] e^{ik(x-ct)}, \tag{8}$$

where Q_1, Q_2 are arbitrary constants. The stress components are given by

$$(\tau_{xx})_2 = (\tau_{zz})_2 = \lambda_2 \vartheta, \tag{9}$$

where

$$\vartheta = \frac{\partial u_2}{\partial x} + \frac{\partial w_2}{\partial z}.$$

For the medium M_3, the field equations are given by Biot (1956) as

$$N\nabla^2 \mathbf{u} + \text{grad } \{(D+N)e + Q\varepsilon\} = \frac{\partial^2}{\partial t^2}\{\rho_{11}\mathbf{u} + \rho_{12}\mathbf{U}\} + b\frac{\partial}{\partial t}(\mathbf{u} - \mathbf{U}), \tag{10}$$

$$\text{grad } \{Qe + R\varepsilon\} = \frac{\partial^2}{\partial t^2}\{\rho_{12}\mathbf{u} + \rho_{22}\mathbf{U}\} - b\frac{\partial}{\partial t}(\mathbf{u} - \mathbf{U}), \tag{11}$$

where $\mathbf{u}$ and $\mathbf{U}$ are the displacements in the solid and liquid parts of the porous aggregate respectively; $e = \text{div}\,\mathbf{u}$ and $\varepsilon = \text{div}\,\mathbf{U}$ are the corresponding dilatations. D, N, Q, and R are the elastic constants for the solid-liquid aggregate, D and N correspond to the Lame modulii of the material, Q is a measure of coupling between the volume change of solid and liquid, and R is the pressure that must be exerted on the liquid to force a given volume of it into the porous aggregate while the total volume remains same; $\rho_{11}, \rho_{12}, \rho_{22}$ are the dynamical coefficients, where ρ_{12}, represents the mass coupling parameter between fluid and solid; b is the dissipation coefficient.

Using the Helmholtz decomposition of vectors as

$$\mathbf{u} = \text{grad}\,\phi + \text{curl}\,\mathbf{H},\ \mathbf{U} = \text{grad}\,\psi + \text{curl}\,\mathbf{G}, \tag{12}$$

in (10) and (11), further, by assuming the motion to be time harmonic ($e^{i\omega t}$) and

$$\phi = \phi_1 + \phi_2, \tag{13}$$

we get

$$\left\{\nabla^2 + \frac{\omega^2}{\alpha_j^2}\right\}\phi_j = 0,\ \ (j = 1, 2),\ \ \left\{\nabla^2 + \frac{\omega^2}{\alpha_3^2}\right\}\mathbf{H} = 0, \tag{14}$$

where

$$\left.\begin{array}{l} \alpha_1^2 = \dfrac{B + (B^2 - 4AC)^{1/2}}{2C},\ \alpha_2^2 = \dfrac{B - (B^2 - 4AC)^{1/2}}{2C},\ \alpha_3^2 = \dfrac{N(\rho_{22} + i\,b/\omega)}{C}, \\ A = PR - Q^2,\ B = (\rho_{11} + i\,b/\omega)R + (\rho_{22} + i\,b/\omega)P - 2(\rho_{12} - i\,b/\omega)Q, \\ C = (\rho_{11} + i\,b/\omega)(\rho_{22} + i\,b/\omega) - (\rho_{12} - ib/\omega)^2,\ P = D + 2N. \end{array}\right\} \tag{15}$$

Also, we have

$$\left.\begin{array}{l} \psi = \mu_1\phi_1 + \mu_2\phi_2,\ \text{where} \\ \mu_j = \dfrac{(\rho_{11} + i\,b/\omega)R - (\rho_{12} - i\,b/\omega)Q - A/\omega\alpha_j^2}{(\rho_{22} + i\,b/\omega)Q - (\rho_{12} - i\,b/\omega)R},\ \ (j = 1, 2) \end{array}\right\} \tag{16}$$

and

$$\mathbf{G} = \alpha_0\, \mathbf{H}, \text{ where } \alpha_0 = -(\rho_{12} - i\, b/\omega)/(\rho_{22} + i\, b/\omega). \tag{17}$$

The solution for (14) may be given as

$$\phi_j = \left(B_j e^{-kz\xi_j}\right) e^{ik(x-ct)}, \ (j = 1, 2, 3) \tag{18}$$
$$\phi_3 = -(\mathbf{H})_y,$$

where $B_j\,(j = 1, 2, 3)$ are arbitrary constants, and

$$\xi_j = \left\{1 - \left(c^2/\alpha_j^2\right)\right\}^{1/2}, \ (j = 1, 2, 3). \tag{19}$$

The stresses in the solid $\sigma_{ij}\ (i, j = x, y, z)$ and liquid σ are given by

$$\sigma_{ij} = (De + Q\varepsilon)\, \delta_{ij} + 2N\, \varepsilon_{ij}, \ \sigma = Qe + R\varepsilon, \tag{20}$$

where δ_{ij} is the Kronecker delta and

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{21}$$

4. Boundary conditions

The appropriate boundary conditions are as follows.

(a) At the free surface $z = 0$,

(i) vanishing of the normal stress component

$$(\tau_{zz})_1 = 0. \tag{22}$$

(b) At the interface $z = h_1$,

(i) continuity of normal stress component

$$(\tau_{zz})_2 = (\tau_{zz})_1, \tag{23}$$

(ii) continuity of the normal displacement component

$$w_2 = w_1. \tag{24}$$

(c) At the interface $z = h_1 + h_2$ (following Deresiewicz & Skalak 1963),

(i) continuity of normal stress component

$$\sigma_{zz} + \sigma = (\tau_{zz})_2, \tag{25}$$

(ii) vanishing of the shear stress component

$$\sigma_{zx} = 0, \tag{26}$$

(iii) continuity of liquid pressure

$$(1/\beta)\sigma = (\tau_{zz})_2, \tag{27}$$

(iv) continuity of the normal component of velocity averaged over the bulk area

$$(1-\beta)\dot{w} + \beta\dot{W} = \dot{w}_2, \tag{28}$$

where the dot represents the time differential and β is the porosity of the porous aggregate.

Making use of (3), (5), (8), (9), (12) and (20) with the help of (18) and (21) in the boundary conditions (22)–(28), we obtain a set of seven homogeneous equations in P_1, P_2, Q_1, Q_2, B_1, B_2 and B_3. The non-trivial solution of this system of equations requires

$$|a_{ij}| = 0, \ (i, j = 1, 2, \ldots\ldots\ldots\ldots, 7), \tag{29}$$

which gives

$$(\delta_1\Omega_1 + \delta_2\Omega_2)\cosh(ah_2) + (\delta_1\Omega_2 + \delta_2\Omega_1)\sinh(ah_2) = 0, \tag{30}$$

where

$$\Omega_1 = I_0'\left(\frac{a}{\alpha}\right) K_0\left(a\frac{1+\alpha h_1}{\alpha}\right) - K_0'\left(\frac{a}{\alpha}\right) I_0\left(a\frac{1+\alpha h_1}{\alpha}\right),$$

$$\Omega_2 = I_0'\left(\frac{a}{\alpha}\right) K_0'\left(a\frac{1+\alpha h_1}{\alpha}\right) - K_0'\left(\frac{a}{\alpha}\right) I_0'\left(a\frac{1+\alpha h_1}{\alpha}\right),$$

$$\delta_1 = \Delta_3 - \Delta_5, \delta_2 = \Delta_1 + \Delta_2 - \Delta_4,$$

$$\Delta_1 = 2ZH[(L_2 - \beta H_2)\xi_1 - (L_1 - \beta H_1)\xi_2],$$

$$\Delta_2 = H[(L_1 - \beta H_1)M_2 - (L_2 - \beta H_2)M_1](1 + \xi_3^2),$$

$$\Delta_3 = (L_1H_2 - L_2H_1)\,(1 + \xi_3^2),\ \Delta_4 = 4\beta H(\xi_2 M_1 - \xi_1 M_2)\xi_3,$$

$$\Delta_5 = 4(\xi_2 L_1 - \xi_1 L_2)\xi_3,$$

$$H_j = 2 - \left\{\left(\frac{P}{N} + \frac{Q}{N}\right) + \left(\frac{Q}{N} + \frac{R}{N}\right)\mu_j\right\}\frac{c^2}{\alpha_j^2},\ L_j = -\left(\frac{Q}{N} + \frac{R}{N}\mu_j\right)\frac{c^2}{\alpha_j^2},$$

$$M_j = \{(1-\beta) + \beta\mu_j\}\xi_j,\ (j = 1, 2),$$

$$H = -\frac{c^2/c_1^2}{(1 - c^2/c_1^2)^{1/2}}\frac{\lambda_0}{N}(1 + \alpha h_1),\ Z = (1-\beta) + \beta\alpha_0. \tag{31}$$

Equation (30) is the required frequency equation relating the phase velocity c to the wave length $2\pi/k$. Wavelength is a multivalued function of phase velocity, each value corresponding to a different mode of propagation indicating the dispersive nature of the existing wave. The existence of such a surface wave is possible if, and only if, (30) has a real solution satisfying $c < \min\ (\alpha_1, \alpha_2, \alpha_3)$. Also, for the purpose of numerical calculations and to obtain the real wave velocity, we assume the liquid saturated porous media to be non-dissipative.

The group velocity U_0 can be obtained by using the formula

$$U_0 = c + k(\mathrm{d}c/\mathrm{d}k). \tag{32}$$

4.1 *Frequency equation for waves of long wavelengths*

If ah_1 and ah_2 are so small that their second and higher powers can be neglected, then we have

$$\Omega_1 \cong \alpha/a, \Omega_2 \cong h_1\alpha. \tag{33}$$

Making use of (33) in the frequency equation (30), we get

$$\delta_1 + a(h_1 + h_2)\delta_2 = 0, \tag{34}$$

Equation (34) is the frequency equation in case of long wavelengths.

4.2 *Frequency equation for waves of short wavelengths*

For waves of short wavelengths, k will be large and therefore a/α and $a[(1+\alpha h_1)/\alpha]$ can be made as large as we please provided c is not taken very close to c_1. Making use of asymptotic approximations of Bessel functions (Watson 1958), we obtain

$$\Omega_1 \cong \left[\alpha/\pi a\,(1+\alpha h_1)^{1/2}\right]\cosh(ah_1),\ \ \Omega_2 \cong \left[\alpha/\pi a\,(1+\alpha h_1)^{1/2}\right]\sinh(ah_1). \tag{35}$$

Thus, the frequency equation for waves of short wavelengths becomes

$$\delta_1 \cosh a(h_1 + h_2) + \delta_2 \sinh a(h_1 + h_2) = 0. \tag{36}$$

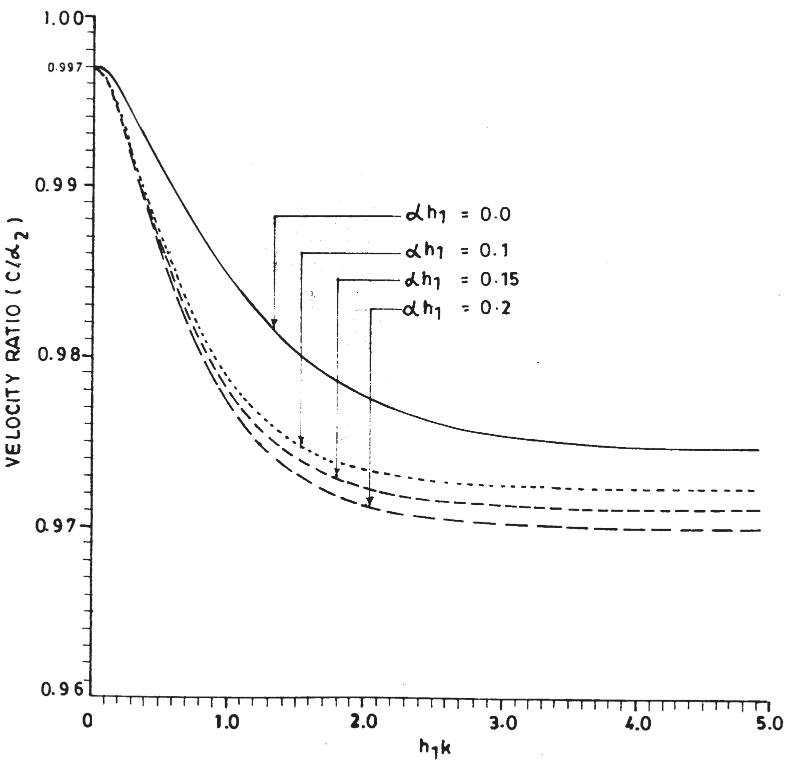

Figure 2. Variation of phase velocity with wave number for $h_2/h_1 = 0.5$.

5. Special cases

Case 1: If we remove the overlying inhomogeneous liquid layer by taking $h_1 = 0$, the frequency equation (30), after some calculations, reduces to the frequency equation for Rayleigh type surface wave propagation in a liquid layer overlying a liquid-saturated porous half-space as discussed by Deresiewicz (1964b). Further, if we take $kh_2 \to 0$, that is the wavelength is large as compared to the width of the liquid layer and hence the effect of layer becomes negligible, the frequency equation becomes

$$\Delta_3 - \Delta_5 = 0. \tag{37}$$

This equation gives the velocity of Rayleigh waves in a liquid-saturated porous half-space with free surface. If, we take $kh_2 \to \infty$, the frequency equation becomes

$$\Delta_1 + \Delta_2 - \Delta_4 = 0, \tag{38}$$

where the symbols have the same meaning as defined earlier (with $h_1 = 0$) and this equation represents the frequency equation of Stoneley waves at the interface between liquid and liquid-saturated porous half-spaces.

Case 2: If we remove the homogeneous liquid layer by taking $h_2 = 0$, the frequency equation (30) reduces to the frequency equation for Rayleigh type surface wave prop-

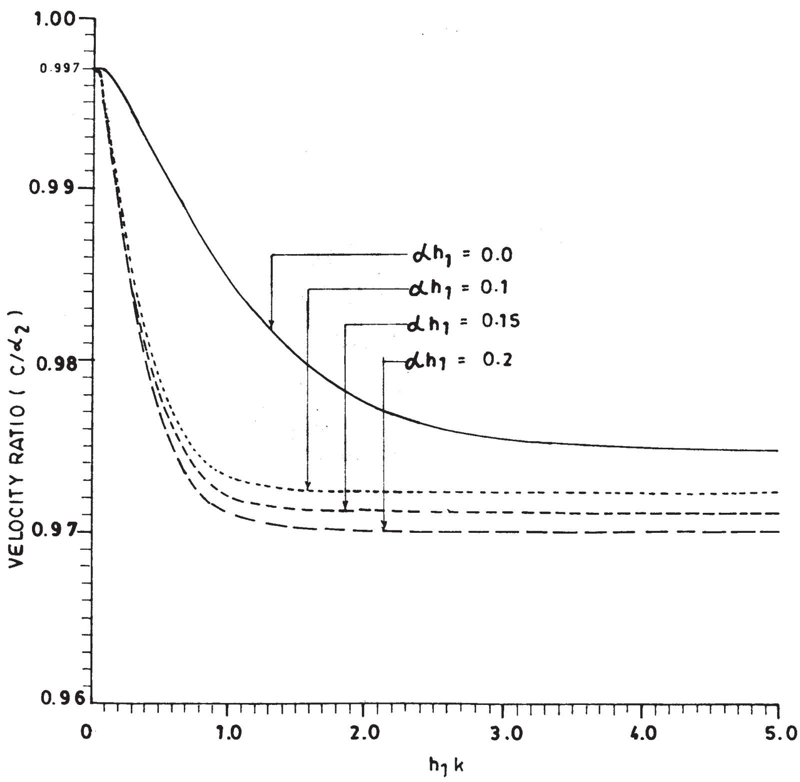

Figure 3. Variation of phase velocity with wave number for $h_2/h_1 = 2.0$.

agation in an inhomogeneous liquid layer over a liquid-saturated porous half-space i.e.

$$\Omega_1\delta_1 + \Omega_2\delta_2 = 0. \tag{39}$$

6. Numerical results and discussion

The numerical calculations of the surface wave propagation discussed above along the x-direction have been made by considering a particular model, keeping in view the availability of numerical data. The elastic parameters for the liquid-saturated porous solid are taken as those of sandstone saturated with kerosene as given by Yew & Jogi (1976),

$$\begin{aligned} &P = 0.99663 \times 10^{10}\ \text{N/m}^2, && Q = 0.07435 \times 10^{10}\ \text{N/m}^2,\\ &R = 0.03262 \times 10^{10}\ \text{N/m}^2, && N = 0.2765 \times 10^{10}\ \text{N/m}^2,\\ &\rho_{11} = 1.926137 \times 10^3\ \text{kg/m}^3, && \rho_{12} = -0.002137 \times 10^3\ \text{kg/m}^3,\\ &\rho_{22} = 0.215337 \times 10^3\ \text{kg/m}^3, && \beta = 0.26, \end{aligned}$$

and those for the water layers, following Ewing *et al* (1957), are taken as

$$\lambda_0 = .214 \times 10^{10}\ \text{N/m}^2, \quad \rho_0 = 1.0 \times 10^3\ \text{kg/m}^3.$$

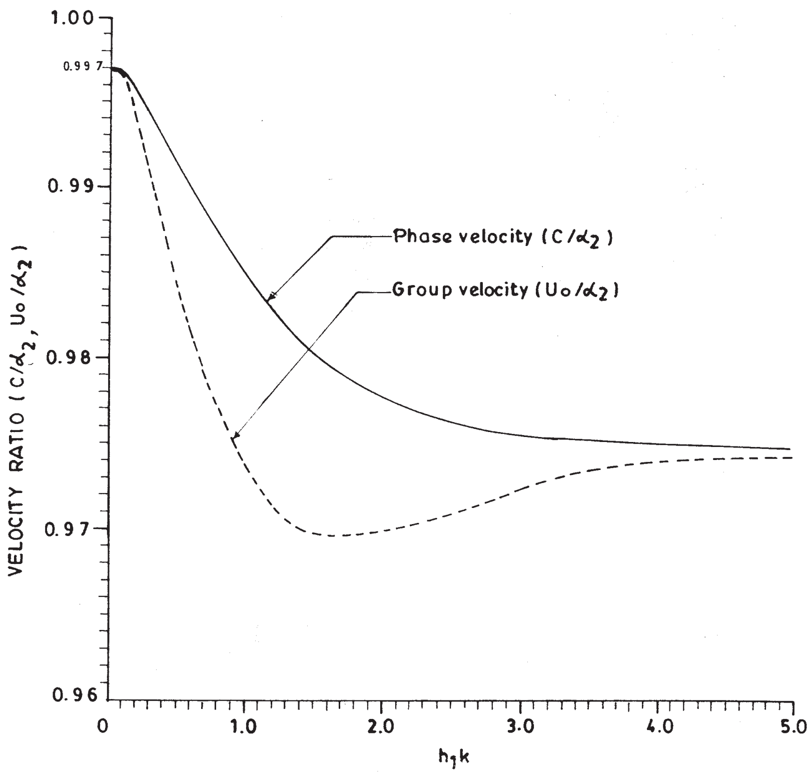

Figure 4. Variation of phase and group velocity with wave number in the absence of inhomogeneous liquid layer.

The thicknesses of the homogeneous and the inhomogeneous liquid layers are taken in the ratio form, i.e., h_2/h_1 and the numerical calculations are performed for two different values of h_2/h_1, i.e., $h_2/h_1 = 0.5$ and 2.0. This gives us the effect of the width of different liquid layers on the dispersion curves. Further, to show the effect of the inhomogeneity of the inhomogeneous liquid layer, the numerical calculations are performed for three different values of the inhomogeneity factor αh_1, i.e., $\alpha h_1 = 0.1, 0.15, 0.2$, alongwith the case where there is no inhomogeneous liquid layer (i.e., $\alpha h_1 = 0.0$) in the model considered.

The group velocity in each case is calculated by using the numerical differentiation in the formula

$$\frac{U_0}{\alpha_2} = \frac{c}{\alpha_2} + h_1 k \frac{\mathrm{d}(c/\alpha_2)}{\mathrm{d}(h_1 k)}.$$

Solving (30) for the above values of the material parameters, using a computer programme in Fortran-IV on a PC, the non-dimensional phase velocity $c/\alpha_2[\alpha_2 = \min(\alpha_1, \alpha_2, \alpha_3)]$, and the group velocity U_0/α_2 are calculated as a function of non-dimensional wavenumber $h_1 k$. The results obtained are plotted as c/α_2, U_0/α_2 against $h_1 k$ in figures 2–7.

Figures 2–3 show the effect of the inhomogeneity of the inhomogeneous liquid layer on the dispersion curves. From the figures, it is observed that for both the values of h_2/h_1, the velocity ratio (c/α_2) decreases with the increase in wavenumber and becomes constant

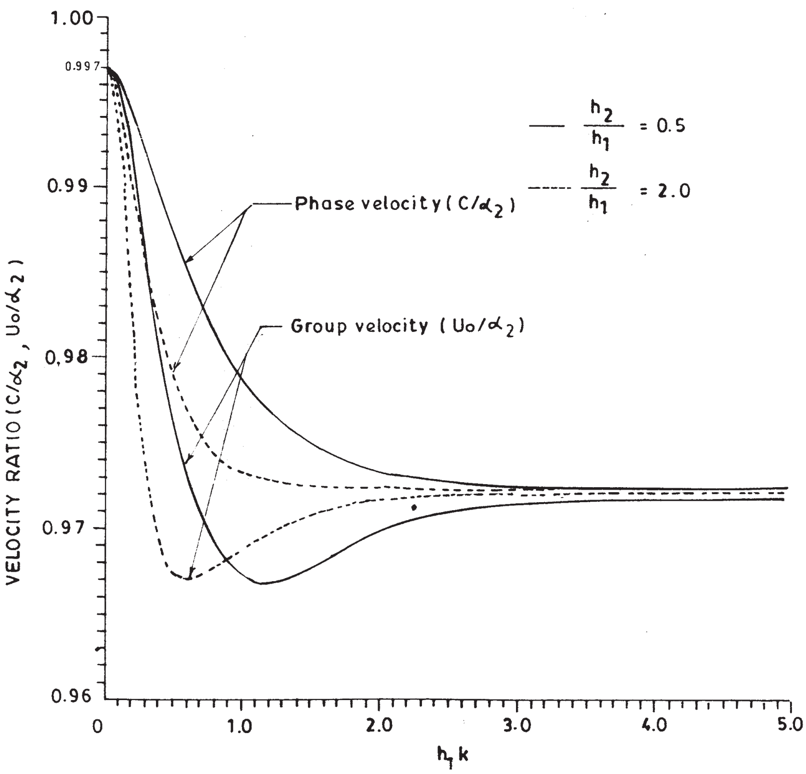

Figure 5. Variation of phase and group velocities with wave number ($\alpha h_1 = 0.1$).

ultimately. But, the rate of decrease of the velocity ratio and the ultimate constant value are different for different values of inhomogeneity factor. The rate of decrease increases and the ultimate constant value decreases with the increase in inhomogeneity factor, which shows the effect of the inhomogeneity. Also, the comparison between the figures 2 and 3 reveals that as the ratio h_2/h_1 increases from 0.5 to 2.0, the rate of decrease also increases, but the constant value for the particular case (i.e. same inhomogeneity factor) remains the same, i.e. the width of the layers affects the velocity ratio only for large wavelengths while for short wavelengths, it does not affect the ratio. Further, it is noticed that as h_1k tends to zero, i.e., the wavelength becomes infinitely large and hence the effect of the liquid layers becomes negligible, the value of the phase velocity is that of the Rayleigh wave propagation in a liquid-saturated porous half-space, i.e. the energy travels without the liquid overburden.

Deresiewicz (1964b) theoretically discussed the problem of surface wave propagation in a liquid-saturated porous half-space under a homogeneous liquid layer. Figure 4 shows the phase and the group velocity curves for the same problem, which is obtained from the model considered here, by taking the depth of the inhomogeneous liquid layer to be zero, i.e. $\alpha h_1 = 0$.

Figures 5–7 represent the velocity ratio (phase and group velocity) curves against wavenumber for the different inhomogeneous cases, i.e. $\alpha h_1 = 0.10, 0.15, 0.20$, respectively, for $h_2/h_1 = 0.5$ and 2.0. In each case both the phase and the group velocity curves approach the

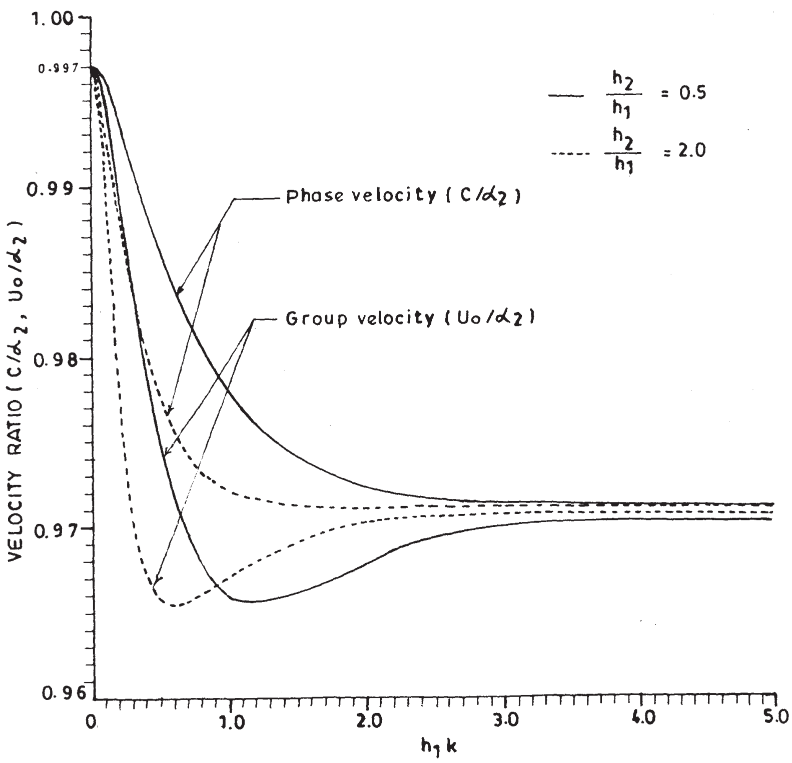

Figure 6. Variation of phase and group velocities with wave number ($\alpha h_1 = 0.15$).

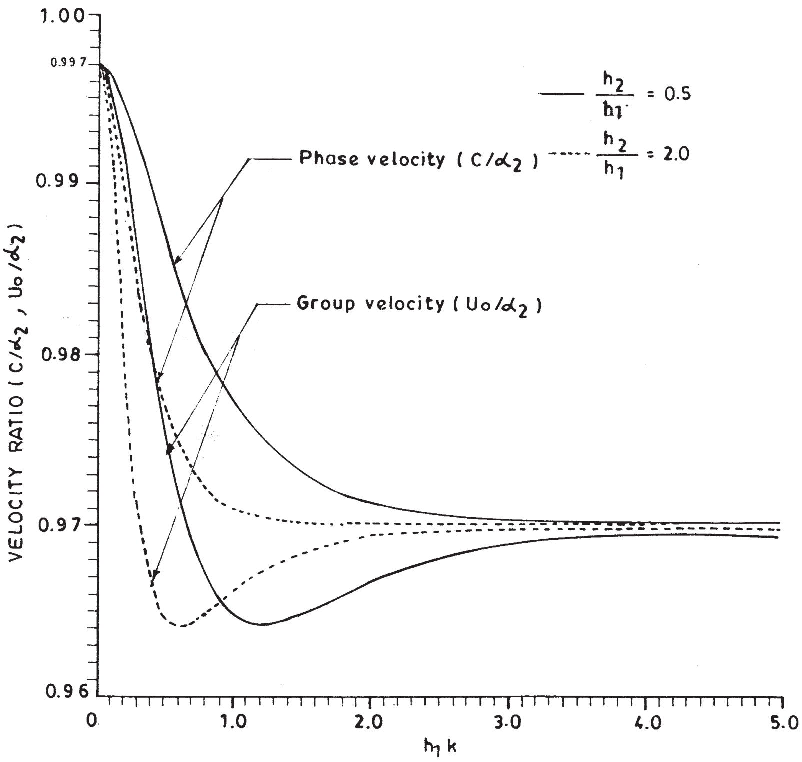

Figure 7. Variation of phase and group velocities with wave number ($\alpha h_1 = 0.2$).

same constant value, but this constant value is different for different inhomogeneity factors. Comparison between figures 5–7 and figure 4 shows the effect of the inhomogeneity of the inhomogeneous liquid layer.

The above calculations indicate that the presence of the double liquid layer, the upper part of which is inhomogeneous, has the quantitative effect on the dispersion curves of the case of only homogeneous liquid layer over a liquid-saturated porous half-space (Deresiewicz 1964b).

Thus, using such experimental values, we could calculate the dispersion curves and compare them with observed values.

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