Roughness effect on squeeze film pressure

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Abstract. The Stokes equations for the axial symmetric slow motion generated by a plane, approaching towards a solid surface allowing slippage, have been solved in this paper by using finite Hankel transform. The squeeze film pressure between the two rigid faces is then obtained. It is found that the roughness parameter $\beta \sim d/4$, where d is the separation between the two surfaces, causes an extremely high pressure on the surface.

Keywords. Axial symmetry; roughness parameter; squeeze film; Hankel transform.

1. Introduction

The flow behaviour of liquid films squeezed between two rigid surfaces has been of considerable interest to many investigators. There is extensive literature available on this subject. Jackson (1962) gave a theoretical analysis of this problem and Kuzma *et al* (1964) advanced the case of squeeze film with an applied magnetic film. In both these papers the fluid inertia effect has been neglected. This theory was extended by Kuzma (1967), including the effects of fluid inertia. More work in this direction has been carried out by Wolfe (1964–65), Grimm (1976), Wang & Watson (1979), Oliver *et al* (1978), Oliver (1979) and Halpern (1992).

Wolfe (1964–65) solved the Navier–Stokes equations for squeeze flow by transform methods for both rectangular and cylindrical geometries. In the present paper, we investigate the same problem but in the presence of surface roughness. Roughness has been taken into account through the condition, derived by Miksis & Davis (1994), whereby the surface velocity is proportional to the tangential stress present. This condition is analogous to Basset's (1961) slip condition but with a negative coefficient of proportionality. The squeeze film pressure is then calculated after solving the Stokes equations by the Hankel transform method (Sneddon 1951). Approximate calculation of pressure ratio for the rough and smooth surfaces shows that there exists extremely high pressure for the roughness parameter $\beta \sim d/4$, where d is the initial separation between the rigid surfaces.

Figure 1. Fluid squeezed between two solid faces.

2. Formulation of the problem

We make use of the cylindrical polar co-ordinate system (r, z) in the axial symmetric case with z as the axis of symmetry. Here v_r and v_z represent the velocity components along r and z directions respectively. As the cylindrical body of radius a descends with a velocity w in the negative z direction (figure 1), the fluid is squeezed between the two solid faces and is forced to flow along the r direction. The initial distance between two rigid faces is assumed to be d . It is further supposed that the fluid is incompressible, has a constant density ρ , viscosity μ and that the Reynolds number is taken to be sufficiently small so as to neglect the inertia terms.

The motion may be considered as quasi-steady (O'Brien & Brule 1991) and, being slow, governed by the Stokes equations (Wolfe 1964–65) which for the axial symmetric case are,

$$\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{\partial^2 v_r}{\partial z^2} = \frac{1}{\mu} \frac{\partial p}{\partial r}, \quad (1)$$

$$\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2} = \frac{1}{\mu} \frac{\partial p}{\partial z}, \quad (2)$$

where p is the pressure. Also the equation of continuity is

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0. \quad (3)$$

Except for the no-slip boundary condition $v_r = 0$, at $z = 0$, the following conditions as presented by Wolfe (1964–65) are applicable here also

$$v_r = 0, \quad \text{at } r = 0, \quad (4)$$

$$r(\partial v_r / \partial r) + v_r = 0, \quad \text{at } r = a, \quad (5)$$

$$p = 0, \quad \text{at } r = a. \quad (6)$$

The conditions (5) and (6) follow from the continuity equation (3) under the physical assumption that edge effects are negligible so that when the cylinder descends the streamlines become

radial at $r = a$ and hence $(\partial v_z / \partial z) = 0$; further, we have

$$v_z = 0, \quad \text{at } z = 0, \quad 0 \leq r \leq a, \quad (7)$$

$$v_z = -w, \quad \text{at } z = d, \quad 0 \leq r \leq a, \quad (8)$$

$$\partial v_z / \partial z = 0, \quad \text{at } z = d. \quad (9)$$

The roughness condition for the two-dimensional flow bounded by the rough plane $y = 0$ as obtained by Miksis & Davis (1994) is

$$u(x, 0, t) + \langle h \rangle \frac{\partial u(x, 0, t)}{\partial y} = 0, \quad (10)$$

where $\langle h \rangle$ is the average amplitude of the surface roughness and u is the velocity component along the x direction.

Using the same method as Miksis & Davis (1994), we have derived the roughness condition at $z = 0$ for an axial symmetric case (r, z) as

$$v_r(r, 0, t) + \beta \frac{\partial v_r(r, 0, t)}{\partial z} = 0, \quad (11)$$

where v_r is the velocity along the r direction, and $\langle h \rangle$ has been replaced by the more convenient notation β . This boundary condition replaces Wolfe's (1964–65) no-slip condition.

Applying the Hankel transform to (1), (2) and (3) as by Wolfe (1964–65) we obtain the following transformed equations

$$\frac{d^2 V_r}{dz^2} - \varepsilon_n^2 V_r = -\frac{\varepsilon_n P_n}{\mu}, \quad (12)$$

$$\frac{d^2 V_z}{dz^2} - \varepsilon_n^2 V_z = \frac{1}{\mu} \frac{dP_n}{dz}, \quad (13)$$

$$\frac{dV_z}{dz} + \varepsilon_n V_r = 0, \quad (14)$$

where V_r , V_z and P_n are the same as obtained by Wolfe (1964–65)

$$V_r = \int_0^a r v_r J_1(\varepsilon_n r) dr, \quad (15)$$

$$V_z = \int_0^a r v_z J_0(\varepsilon_n r) dr, \quad (16)$$

$$P_n = \int_0^a r p J_0(\varepsilon_n r) dr. \quad (17)$$

where the values of ε_n are obtained by using the condition

$$J_0(\varepsilon_n a) = 0. \quad (18)$$

The above condition ensures the vanishing of the derived value of p ((27) below) at $r = a$ as stipulated in the condition (6). Eliminating V_r and P_n in (12) and (14), we get the following fourth order equation for determining V_z

$$\left(\frac{d^2}{dz^2} - \varepsilon_n \right)^2 V_z = 0. \quad (19)$$

Further the transformed boundary conditions at $z = 0$ and $z = d$ provide

$$V_z = 0, \quad \text{at } z = 0, \quad (20)$$

$$V_z = -waJ_1(\varepsilon_n a)/\varepsilon_n, \quad \text{at } z = d, \quad (21)$$

$$\frac{\partial V_z}{\partial z} = 0, \quad \text{at } z = d, \quad (22)$$

$$\frac{\partial V_z}{\partial z} = -\beta \frac{\partial^2 V_z}{\partial z^2}, \quad \text{at } z = 0. \quad (23)$$

Now the differential equation (19) yields the following general solution

$$V_z = (A + Bz) \cosh(\varepsilon_n z) + (C + Dz) \sinh(\varepsilon_n z). \quad (24)$$

Next, from the boundary conditions (20)–(23) we find

$$\begin{aligned} A &= 0, \\ B &= \frac{waJ_1(\varepsilon_n a) [\sinh(\varepsilon_n d) + \varepsilon_n d \cosh(\varepsilon_n d) - 2\beta\varepsilon_n \cosh(\varepsilon_n d)]}{[\sinh^2(\varepsilon_n d) - (\varepsilon_n d)^2 + 2\beta\varepsilon_n^2 d - 2\beta\varepsilon_n \sinh(\varepsilon_n d) \cosh(\varepsilon_n d)]}, \\ C &= -\frac{B [2\beta\varepsilon_n^2 d \sinh(\varepsilon_n d) + 2\beta\varepsilon_n \cosh(\varepsilon_n d) - \varepsilon_n d \cosh(\varepsilon_n d) - \sinh(\varepsilon_n d)]}{\varepsilon_n [2\beta\varepsilon_n \cosh(\varepsilon_n d) - \varepsilon_n d \cosh(\varepsilon_n d) - \sinh(\varepsilon_n d)]}, \\ D &= \frac{B\varepsilon_n d \sinh(\varepsilon_n d)}{[2\beta\varepsilon_n \cosh(\varepsilon_n d) - \varepsilon_n d \cosh(\varepsilon_n d) - \sinh(\varepsilon_n d)]}. \end{aligned}$$

Eliminating V_r from (12) and (14), we get the value of P_n

$$P_n = \frac{\mu}{\varepsilon_n^2} \left(\frac{d^3 V_z}{dz^3} - \varepsilon_n^2 \frac{dV_z}{dz} \right). \quad (25)$$

On substituting the value of V_z from (24) in (25), we get

$$P_n = \frac{2\mu waJ_1(\varepsilon_n a) [\sinh(\varepsilon_n d) + \varepsilon_n d \cosh(\varepsilon_n d) - 2\beta\varepsilon_n \cosh(\varepsilon_n d)]}{[\sinh^2(\varepsilon_n d) - (\varepsilon_n d)^2 + 2\beta\varepsilon_n^2 d - 2\beta\varepsilon_n \sinh(\varepsilon_n d) \cosh(\varepsilon_n d)]} \quad (26)$$

Using inversion formula (Sneddon 1951) we get the value of p from P_n as

$$p = \frac{2}{a^2} \sum_n \frac{P_n(\varepsilon_n) J_0(\varepsilon_n r)}{[J_1(\varepsilon_n a)]^2}, \quad (27)$$

hence,

$$p = \frac{4\mu w}{a} \sum_n \frac{J_0(\varepsilon_n r)}{J_1(\varepsilon_n a)} \frac{[\sinh(\varepsilon_n d) d \cosh(\varepsilon_n d) - 2\beta\varepsilon_n \cosh(\varepsilon_n d)]}{[\sinh^2(\varepsilon_n d) - (\varepsilon_n d)^2 + 2\beta\varepsilon_n^2 d - 2\beta\varepsilon_n \sinh(\varepsilon_n d) \cosh(\varepsilon_n d)]} \quad (28)$$

where ε_n is obtained from the eigenvalue equation (18).

Similarly, we can easily evaluate the squeeze film pressure with the help of roughness condition for the rectangular geometry by the method outlined by Wolfe (1964-65). Thus

$$p = \frac{4\mu w}{a} \sum_{n=1}^{\infty} \frac{\sinh(\varepsilon_n a) \cos(\varepsilon_n x) [\sinh(\varepsilon_n d) + \varepsilon_n d \cosh(\varepsilon_n d) - 2\beta \varepsilon_n \cosh(\varepsilon_n d)]}{\sinh^2(\varepsilon_n d) - (\varepsilon_n d)^2 + 2\beta \varepsilon_n^2 d - 2\beta \varepsilon_n \sinh(\varepsilon_n d) \cosh(\varepsilon_n d)} \quad (29)$$

where $\varepsilon_n = (n - \frac{1}{2}) (\pi/a)$.

In the limit of zero roughness (i.e. $\beta = 0$), roughness condition (11) reduces to no-slip condition and (28) and (29) exactly match the solutions obtained by Wolfe (1964-65) for symmetrical and rectangular geometries.

3. Approximate calculations

Equation (28) gives the value of squeeze film pressure for symmetrical geometry. The expression, however, is cumbersome and to avoid this we approximate p for $d/a \ll 1$.

Putting $u = \varepsilon_n d$

$$p = \frac{4\mu w}{a} \sum_n \frac{J_0(\varepsilon_n r)}{J_1(\varepsilon_n a)} A,$$

where

$$A = \frac{[\sinh u + u \cosh u - 2\beta \varepsilon_n \cosh u]}{[\sinh^2 u - u^2 + 2\beta \varepsilon_n u - 2\varepsilon_n \beta \sinh u \cosh u]},$$

when $u \ll 1$, A can be written as

$$A = \frac{2u(1 - \beta/d) + (2/3)u^3(1 - 3\beta/d) + \dots}{(u^4/3)(1 - 4\beta/d) + (2u^6/45)(1 - 6\beta/d) + \dots}, \quad (30)$$

retaining the first term only of the above expression we obtain

$$A = \frac{6(1 - \beta/d)}{u^3(1 - 4\beta/d)}. \quad (31)$$

Thus the squeeze film pressure p can be written as

$$p \simeq \frac{24\mu w}{au^3} \frac{(1 - \beta/d)}{(1 - 4\beta/d)} \sum_n \frac{J_0(\varepsilon_n r)}{J_1(\varepsilon_n a)}. \quad (32)$$

Taking $\beta = 0$ in (32) we obtain the approximate squeeze film pressure p_0 for smooth surface viz.

$$p_0 \simeq \frac{24\mu w}{au^3} \sum_n \frac{J_0(\varepsilon_n r)}{J_1(\varepsilon_n a)}. \quad (33)$$

Figure 2. Variation of pressure ratio (P/P_0) with respect to roughness parameter (β/d).

4. Conclusion

The pressure ratio (p/p_0) between rough and smooth surface for both the geometries is given by

$$\frac{p}{p_0} = \frac{(1 - \beta/d)}{(1 - 4\beta/d)}. \quad (34)$$

The variation in the pressure ratio (p/p_0) can be explained by using the positive and negative values of the parameter (β/d) (figure 2). The negative value of (β/d) corresponds to the slip present at the rigid plane and may be achieved by chemically treating (Scheneel 1956) the surface in contact with the fluid. Further, it is seen that as $|\beta|/d$ increases, p/p_0 decreases to its asymptotic value $1/4$ starting from its maximum value 1 for the no-slip case $\beta = 0$.

On the other hand, slight positive increase in the values of β/d shows that pressure ratio (p/p_0) increases enormously and it is also seen that there is an infinite jump discontinuity at $\beta/d = 1/4$. This indicates that for rough surfaces the analysis is tenable only for small values of β ($< d/4$). There is nothing surprising in this as the underlying analysis of Miksis & Davis (1994) has been developed only for microscopic roughness. When the roughness is on macroscopic scale the problem may be investigated numerically using the no-slip condition but that entails enormous work requiring a very fine grid in the microscopic case, and to overcome this difficulty Miksis & Davis (1994) introduced the present effective boundary condition. Moreover it may be seen that in the case of classical slip ($\beta < 0$) as physically expected, no such singularity occurs.

It may be remarked that similar conclusion may be drawn for the two-dimensional situation described by the squeeze film pressure given by (29).

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