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Incipient motion of gravel and coal beds

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Abstract. An experimental study on incipient motion of gravel and coal beds under unidirectional steady-uniform flow is presented. Experiments were carried out in a flume with various sizes of gravel and coal samples. The critical bed shear stresses for the experimental runs determined using side-wall correction show considerable disagreement with the standard curves. The characteristic parameters affecting the incipient motion of particles in rough-turbulent regime, identified based on physical reasoning and dimensional analysis, are the Shields parameter, particle Froude number, non-dimensional particle diameter and non-dimensional flow depth. Equations of critical bed shear stress for the initial movement of gravel and coal beds were obtained using experimental data. The method of application of critical bed shear stress equations is also mentioned.

Keywords. Granular material; sedimentary bed; incipient motion; sediment transport; sediment threshold; fluvial hydraulics.

1. Introduction

The erosion and sedimentation of fluvial beds can be predicted if the critical shear stress for the initiation of bed particle motion is accurately determined. Shields (1936) has been the pioneer in describing the initiation of motion of uniform sediment particles. Although his diagram is widely used, Mantz (1977), Miller *et al* (1977), Yalin & Karahan (1979) and Dey (1999) expressed reservations about it. Incipient motion of uniform sediments was also studied by Iwagaki (1956), Yang (1973), Yalin & Karahan (1979), Ling (1995), and others. Several investigations on incipient motion of non-uniform sediment mixtures were also reported (Egiazaroff 1965; Nakagawa *et al* 1982; Parker *et al* 1982; White & Day 1982; Wiberg & Smith 1987; Wilcock & Southard 1988; Bridge & Bennett 1992; Wilcock 1992, 1993; Kuhnle 1993; Patel & Ranga Raju 1999; Dey 1999; Dey *et al* 1999; Dey & Debnath 2000). Furthermore, experimental studies with gravel beds were put forward by Bathust *et al* (1987) and field studies with gravel and boulder beds river data were reported by Andrews (1983, 1994), Andrews & Erman (1986), Andrews & Parker (1987), Ashworth & Ferguson (1989) and Bathurst (1987).

The aim of the present investigation is to report the results of experiments on incipient motion of gravel and coal beds under a unidirectional steady-uniform flow. The experimental results are used to put forward simple equations for critical bed shear stress.

2. Experimentation

2.1 *Set-up and procedure*

The experiments were performed in a rectangular flume 14 m long, 0.51 m wide and 0.25 m deep. The sides of the flume, in the test section, were made of glass enabling observation of the movement of bed particles. Before starting an experimental run, the gravel or coal bed of the test section was leveled. The downstream control valve of the flume was closed initially. Water was introduced to the set-up by gradually opening an upstream valve. After the bed was completely submerged, the downstream valve was opened gradually. At the same time, the upstream discharge was adjusted so that the incipient condition was reached when all fractions of bed particles (on the surface) had movement over a period of time. The incipient condition was revealed by collecting the removed particles in a wire-net downstream. It is always difficult to observe the real beginning of bed particle motion (Neill & Yalin 1969). However, a certain degree of established movement of bed particles can be considered as the condition of incipient motion. Once the incipient condition was reached, discharge Q and corresponding flow depth h were registered. The flow depth h is a distance from the free surface of flow to the virtual bed level. The virtual bed level was considered to be at $0.25d_{50}$ below the top level of the bed particles, as was done by van Rijn (1984), Dey (1999) and Dey *et al* (1999). Here, d_{50} is the 50 percent finer particle diameter. Table 1 summarizes the characteristics of gravel and coal samples used along with the flow conditions and energy slope S_e. The effort was given to have a uniform flow in the test section, which had a slope almost equal to S_e. Detailed data are given by Raju (1989). The choice of these samples was dictated by a desire to vary the geometric standard deviation σ_g of the particle size distribution given by $(d_{84}/d_{16})^{1/2}$.

2.2 *Estimation of effective diameter*

The effective diameter d_e of the sample, which is the diameter of a uniform spherical particle that behaves in the same way as the sample, was determined analytically from the particle size distribution curve following the relationship by Christensen (1969), given below.

$$d_e = 1/\int_0^1 \frac{\mathrm{d}F}{d} = 1/\sum_{i=1}^{i=n}\left[\frac{F_i - F_{i-1}}{d_i d_{i-1}}\right]\left[\frac{d_i - d_{i-1}}{\ln d_i - \ln d_{i-1}}\right], \tag{1}$$

where $\mathrm{d}F$ = increment in fraction finer of particles, and d = particle diameter.

2.3 *Estimation of bed shear stress*

The equation of bed shear stress τ_b as a function of dynamic pressure is used here, which is

$$\tau_b = (f_b/8)\rho V_b^2, \tag{2}$$

Table 1. Experimental data.

d_e (mm)	d_{65} (mm)	d_{50} (mm)	$\hat{d}$	σ_g	h (cm)	$\hat{h}$	V (m/s)	F_d	$\hat{\tau}$	R$*$	S_e
Gravel ($s = 2.56$)											
7.11	7.51	7.18	1.056	1.11	3.54	0.201	0.561	2.124	0.027	387	0.0112
7.16	7.54	7.23	1.054	1.12	3.90	0.183	0.528	1.993	0.023	359	0.0116
6.87	7.45	7.05	1.085	1.20	3.78	0.182	0.523	2.015	0.024	343	0.0114
6.87	7.47	7.11	1.087	1.29	3.17	0.217	0.566	2.180	0.030	386	0.0107
6.63	7.61	7.06	1.149	1.27	2.86	0.232	0.590	2.314	0.036	400	0.0129
6.45	7.70	7.07	1.194	1.36	3.84	0.168	0.548	2.179	0.028	339	0.0104
6.27	7.82	7.10	1.247	1.44	3.11	0.202	0.634	2.556	0.043	401	0.0119
6.12	7.89	7.14	1.289	1.47	3.66	0.167	0.491	2.003	0.024	293	0.0107
5.95	7.93	7.19	1.333	1.58	3.81	0.156	0.532	2.202	0.029	306	0.0109
5.83	8.10	7.25	1.390	1.64	3.84	0.152	0.500	2.091	0.026	283	0.0129
5.66	8.14	7.28	1.437	1.72	3.26	0.174	0.667	2.830	0.052	381	0.0133
5.55	8.34	7.34	1.503	1.79	3.41	0.163	0.698	2.992	0.058	388	0.0117
5.53	8.45	7.52	1.529	1.80	3.84	0.144	0.610	2.620	0.042	330	0.0126
5.55	5.81	5.65	1.046	1.12	2.62	0.212	0.567	2.430	0.036	308	0.0120
4.54	5.44	4.93	1.199	1.32	2.71	0.167	0.430	2.038	0.024	187	0.0105
4.37	5.59	4.89	1.280	1.40	2.86	0.153	0.490	2.367	0.032	204	0.0117
4.22	5.48	4.97	1.299	1.58	2.01	0.210	0.536	2.635	0.047	233	0.0110
4.11	5.87	5.12	1.427	1.68	2.59	0.159	0.467	2.325	0.034	189	0.0108
4.11	5.96	5.35	1.452	1.79	2.22	0.185	0.557	2.775	0.052	234	0.0118
4.16	6.59	5.64	1.583	1.93	2.80	0.149	0.437	2.162	0.030	181	0.0104
4.26	6.62	5.96	1.553	1.86	2.83	0.151	0.521	2.548	0.041	220	0.0101
4.38	7.05	6.31	1.608	2.09	2.56	0.171	0.478	2.305	0.036	216	0.0110
4.51	7.56	6.60	1.678	2.11	2.68	0.168	0.561	2.668	0.049	262	0.0107
8.72	8.91	8.77	1.022	1.06	3.20	0.273	0.664	2.270	0.035	599	0.0108
8.94	8.94	9.07	1.000	1.10	3.90	0.229	0.550	1.857	0.022	487	0.0126
8.93	9.19	9.21	1.029	1.21	3.63	0.246	0.619	2.091	0.029	560	0.0146
8.74	9.53	9.18	1.091	1.25	3.51	0.249	0.664	2.268	0.035	597	0.0146
8.43	9.66	9.17	1.146	1.33	3.44	0.245	0.666	2.316	0.037	582	0.0154
8.05	10.02	9.20	1.245	1.42	3.99	0.202	0.605	2.153	0.030	492	0.0132
7.65	10.14	9.25	1.325	1.53	3.35	0.228	0.769	2.807	0.056	623	0.0149
7.32	10.33	9.32	1.411	1.68	3.60	0.203	0.611	2.280	0.036	467	0.0114
7.07	10.28	9.39	1.454	1.82	4.02	0.176	0.613	2.328	0.036	440	0.0118
6.89	10.82	9.47	1.571	1.63	3.78	0.182	0.734	2.824	0.056	527	0.0169
Coal ($s = 1.39$)											
30.90	33.62	31.75	1.088	1.68	4.36	0.709	0.427	0.776	0.030	1837	0.0104
26.57	32.60	28.60	1.227	1.18	5.06	0.525	0.546	1.069	0.050	1907	0.0080
21.69	30.05	25.18	1.385	1.42	3.44	0.631	0.629	1.364	0.100	1977	0.0185
20.19	29.53	23.98	1.463	1.79	4.57	0.442	0.564	1.267	0.071	1496	0.0098
19.63	28.75	23.51	1.465	1.16	5.46	0.359	0.417	0.950	0.035	1013	0.0134
22.07	23.05	22.44	1.044	1.29	4.54	0.486	0.483	1.038	0.041	1302	0.0109
21.88	25.88	24.04	1.183	1.52	4.30	0.509	0.474	1.023	0.044	1333	0.0130
20.06	26.05	23.41	1.298	1.60	4.21	0.477	0.570	1.285	0.071	1482	0.0075
19.45	26.53	23.13	1.364	1.66	4.57	0.426	0.505	1.156	0.055	1249	0.0128
19.21	26.40	23.01	1.374	1.64	4.63	0.415	0.547	1.260	0.065	1327	0.0137
15.45	16.90	15.88	1.094	1.18	3.14	0.492	0.452	1.161	0.053	865	0.0106
15.26	17.10	16.22	1.121	1.32	3.44	0.444	0.510	1.318	0.065	942	0.0102
15.52	18.62	17.53	1.200	1.50	4.39	0.354	0.479	1.228	0.052	862	0.0073
16.45	21.65	19.32	1.316	1.65	4.14	0.397	0.451	1.123	0.049	913	0.0112
17.20	23.48	20.54	1.365	1.74	4.36	0.394	0.533	1.298	0.066	1139	0.0103

where f = friction factor, ρ = mass density of fluid, and V = mean velocity of flow. Subscript b refers to the quantities associated with the bed. The Colebrook–White equation, used to evaluate f_b, is,

$$\frac{1}{\sqrt{f_b}} = -0.86 \ln\left[\frac{k_s P_b}{14.8 A_b} + \frac{2.51}{\mathrm{R}_b\sqrt{f_b}}\right], \tag{3}$$

where k_s = equivalent roughness height, A = flow area, P = wetted perimeter, and R = Reynolds number of flow. In the present study, the bed, is rough, consisting of gravel or coal bed and the side-walls are smooth. As a result of this, f_w is considerably different from f_b, where subscript w refers to the quantities associated with the side-walls. Consequently, τ_w is significantly different from τ_b. Therefore, Vanoni's (1975) method of *side-wall correction* for a rectangular flume is used here. Thus, the discharge Q is expressed as

$$Q = AV = A_w V_w + A_b V_b. \tag{4}$$

The mean velocity V, considered same as V_w and V_b, can be computed once Q is known. The equation of force along the stream-wise direction is

$$-A\frac{\mathrm{d}p}{\mathrm{d}x} = \rho\frac{f}{8}V^2 P = \rho\frac{f_w}{8}V_w^2 P_w + \rho\frac{f_b}{8}V_b^2 P_b, \tag{5}$$

where $\mathrm{d}p/\mathrm{d}x$ = stream-wise pressure gradient. Using $V = V_w = V_b$ (Vanoni 1975) in (5), we get

$$Pf = P_w f_w + P_b f_b. \tag{6}$$

As the hydraulic grade line is the same for the smooth side-wall and rough bed regions, equating forces to the wall and bed regions yields

$$\frac{Pf}{A} = \frac{P_w f_w}{A_w} = \frac{P_b f_b}{A_b}. \tag{7}$$

Reynolds numbers of flow for the different regions are

$$\mathrm{R} = \frac{4VA}{\upsilon P},\ \mathrm{R}_w = \frac{4VA_w}{\upsilon P_w},\ \mathrm{R}_b = \frac{4VA_b}{\upsilon P_b}, \tag{8}$$

where υ = kinematic viscosity of fluid. Inserting (7) into (8), one gets

$$\frac{\mathrm{R}}{f} = \frac{\mathrm{R}_w}{f_w} = \frac{\mathrm{R}_b}{f_b}. \tag{9}$$

As the wall is smooth, the Blasius equation can be used to evaluate f_w,

$$f_w = 0.316/R_w^{0.25}. \tag{10}$$

Using (4)–(10), the following is obtained

$$f_b = 0.316 R_b\left[\frac{4VA}{\upsilon P_w} - \frac{R_b P_b}{P_w}\right]^{-1.25}. \tag{11}$$

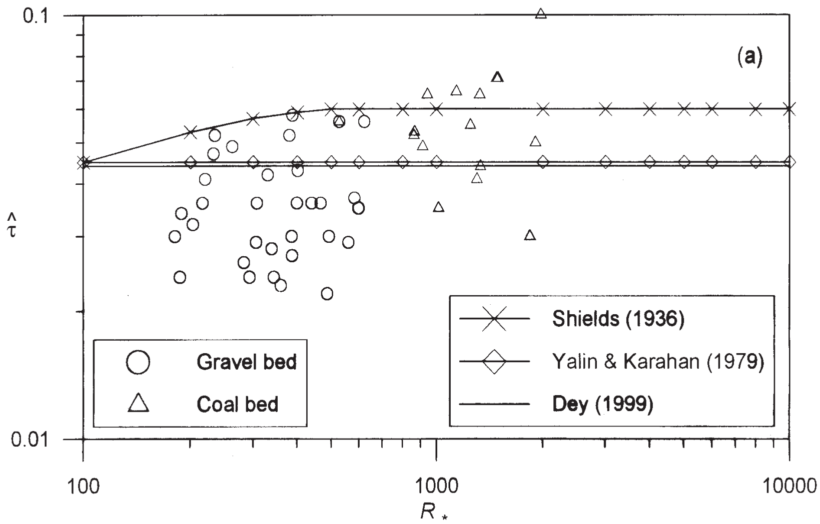
Again, using (8) in (3), the Colebrook–White equation becomes

$$\frac{1}{\sqrt{f_b}} = -0.86 \ln\left[\frac{k_s V}{3.7\upsilon R_b} + \frac{2.51}{\mathrm{R}_b\sqrt{f_b}}\right]. \tag{12}$$

Here, k_s is assumed to be d_{65}, as was done by Wiberg & Smith (1987) and Patel & Ranga Raju (1999). For given values of A, V, P, P_w, P_b, υ, ρ and d_{65}, the unknowns R_b and f_b can be determined numerically solving (11) and (12). Then, (2) is used to estimate τ_b.

3. Critical bed shear stress equation

The experimental critical bed shear stresses τ_b were estimated using the methodology described in the preceding section for various runs. Table 1 presents the details of experimental data collected for the incipient motion of bed particles. The non-dimensional critical bed shear stress or Shields parameter $\hat{\tau}(= \tau_b/\Delta g \rho d_e$; where $\Delta = s - 1$, s = relative density of bed particles, and g = gravitational constant) and corresponding particle Reynolds number $R* (= d_e(\tau_b/\rho)^{1/2}/v)$ are plotted in figure 1a. It is observed that the experimental data are



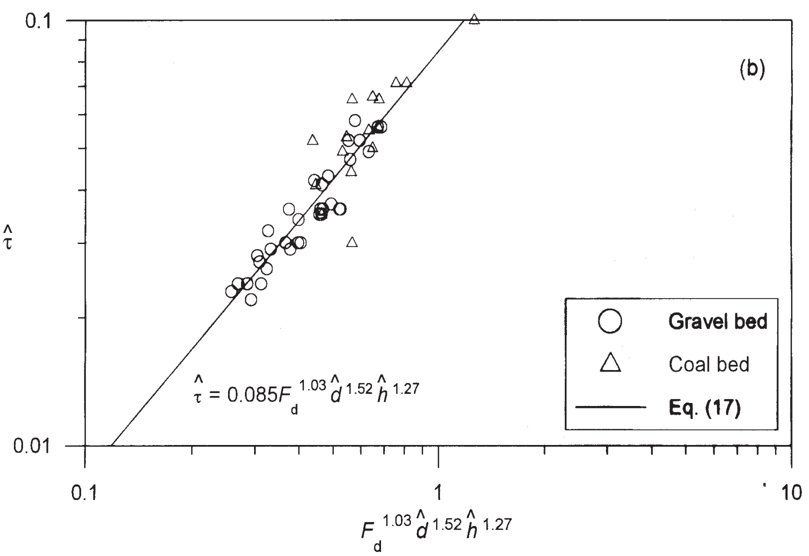


Figure 1. **(a)** Comparison of the experimental data with the curves ($\hat{\tau}$ versus R$*$) proposed by Shields (1936), Yalin & Karahan (1979) and Dey (1999) in rough-turbulent regime; and **(b)** comparison of the results of $\hat{\tau}$ obtained using (17) with the present experimental data for gravel and coal beds.

in complete disagreement with the Shields (1936), Yalin & Karahan (1979) and Dey (1999) curves in rough-turbulent regime. This disagreement is not at all uncommon for gravel beds, as it was reported in the literature (Andrews 1983; Kuhnle 1993).

The set of characteristic parameters appropriate for the beginning of bed particle motion phenomenon can be given in functional form as,

$$\tau_b = f_1(V, d_e, h, \rho, \rho_s, k_s, g, v, \sigma_g), \tag{13}$$

where ρ_s = mass density of bed particles. For two-phase flow phenomena involving sediment–water mixture, the terms ρ and ρ_s should not appear as independent parameters in (13). However, a better representation is $\Delta\rho$. Moreover, as fully developed rough-turbulent flow regime ($R* \gg 70$) occurred in each run, the inclusion of v becomes insignificant in the present analysis. Therefore, (13) becomes

$$\tau_b = f_2(V, d_e, h, \Delta\rho, g, k_s, \sigma_g). \tag{14}$$

Using the Buckingham π-theorem and selecting the parameters τ_b, g and d_e as repeating variables, yields

$$\hat{\tau} = f_3(F_d, \hat{d}, \hat{h}, \sigma_g), \tag{15}$$

where F_d = particle Froude number, that is $V/(gd_e)^{0.5}$, $\hat{d} = k_s/d_e$, and $\hat{h} = d_e/h$. The term σ_g represents the sediment gradation on the initiation of bed particle motion. The values of σ_g in table 1 being below two (except in two samples) indicate that most of the samples are uniform. Therefore, the parameter σ_g is also insignificant. Thus, (15) reduces to

$$\hat{\tau} = f_3(F_d, \hat{h}, \hat{d}). \tag{16}$$

The above non-dimensional parametric representation appropriate to the present study may be justified as follows.

- The term F_d indicates the mobility of the bed particles under stream velocity.
- The term $\hat{d}$ refers to the role of particle size on the initiation of particle motion. It was reported that $\hat{d}$ is a function of angle of repose ϕ (Dey 1999; Dey *et al* 1999). Thus, it also includes the effect of ϕ on the incipient motion.
- The term $\hat{h}$ represents the effect of relative submergence of the particle on the incipient motion.

Altogether forty-eight runs were made with various gravel and coal beds (table 1). A multiple-linear regression analysis of the experimental data yields the following equation of non-dimensional critical bed shear stress or Shields parameter,

$$\hat{\tau} = 0.085 F_d^{1.03} \hat{d}^{1.52} \hat{h}^{1.27}. \tag{17}$$

Comparisons of $\hat{\tau}$ obtained from (17) with the experimental data (table 1) are shown in figure 1b. The value of the correlation coefficient between the experimentally obtained and computed $\hat{\tau}$ is 0.924, which indicates that the above equation can adequately be used for the estimation of critical bed shear stress for gravel and coal beds in rough-turbulent regime.

Equation (17) is applicable for the range of $d_{50} = 4.89 - 31.75$ mm. Also, the two data sets (gravel and coal) were analysed separately to have the following two equations:

$$\hat{\tau} = 0.013 F_d^2 \hat{d}^{0.48} \hat{h}^{0.49}, \quad \text{for gravel bed,} \tag{18}$$

$$\hat{\tau} = 0.058 F_d^2 \hat{d}^{0.62} \hat{h}^{0.63}, \quad \text{for coal bed.} \tag{19}$$

The comparisons of $\hat{\tau}$ obtained from (18) and (19) with the experimental data (table 1) are shown in figures 2a and 2b, respectively. The correlation coefficient between the experimentally obtained and computed values of $\hat{\tau}$ is 0.998 for both the cases. This indicates that (18)

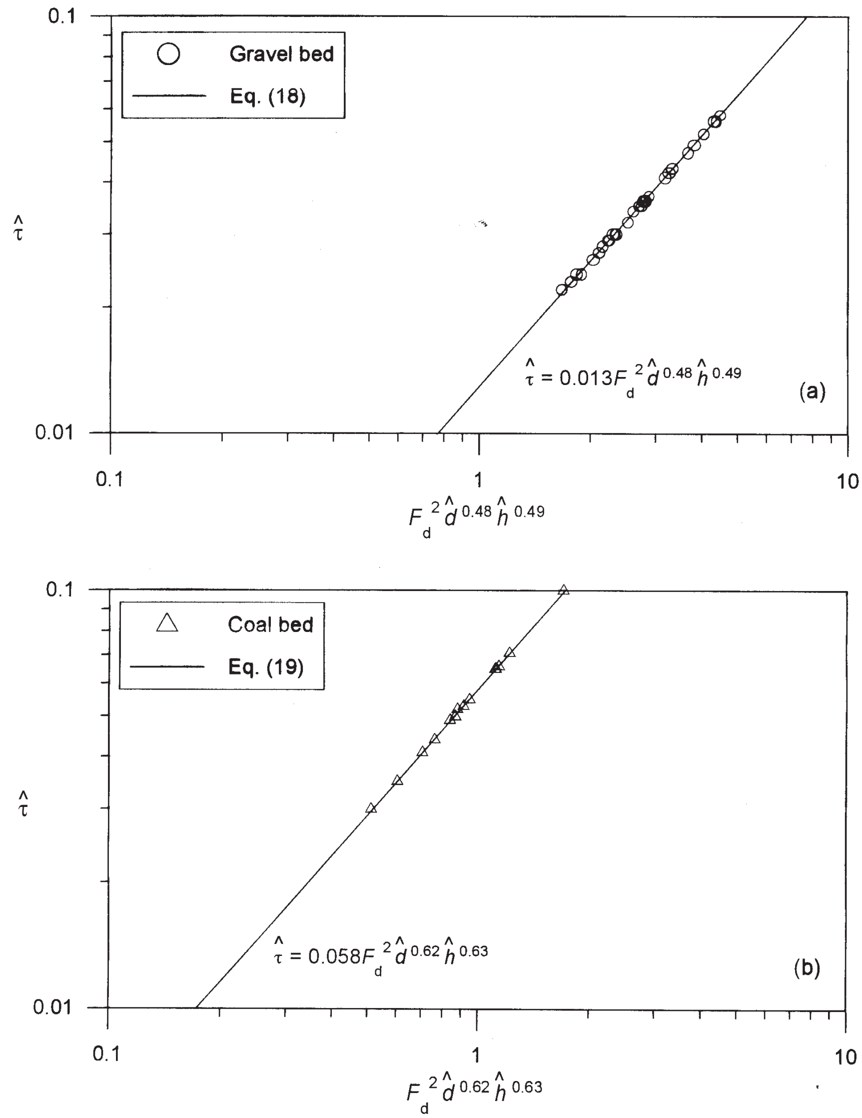


Figure 2. **(a)** Comparison of the results of $\hat{\tau}$ obtained using (18) with the present experimental data for gravel bed; and **(b)** comparison of the results of $\hat{\tau}$ obtained using (19) with the present experimental data for coal bed.

and (19) do an excellent job in predicting critical bed shear stress for gravel and coal beds respectively, in the rough–turbulent regime.

4. Applications

In the field, the following data are required as the input data to calculate the critical bed shear stress $\hat{\tau}$ of the bed material.

- The longitudinal bed slope S of the site is to be determined, as the gravel-bed rivers usually have considerable longitudinal bed slopes.
- The relative density s, effective diameter d_e, and equivalent roughness height $k_s (= d_{65})$ are to be determined through appropriate sampling of the bed material from the river site.

In (17)–(19), the values of F_d, $\hat{h}$ and $\hat{\tau}$ are not known. Hence, the resistance equation for gravel-bed rivers given by Aberle *et al* (1999) can be used as an auxiliary equation. It can be written, on modification for the incipient condition, as

$$\left[2.96 - \frac{1}{\sqrt{8S}}\right]\frac{F_d}{\sqrt{\hat{h}}} = 2.03 \ln \hat{h} + 1.01. \tag{20}$$

Also, $\hat{\tau}$ can be expressed as

$$\hat{\tau} = \frac{\tau_b}{\Delta \rho g d_e} = \frac{\rho g h S}{\Delta \rho g d_e} = \frac{S}{\Delta \hat{h}}. \tag{21}$$

If the river is gravel-bed, (18) can be solved numerically for $\hat{\tau}$ with the help of (20) and (21). However, it is needless to mention that (18) is always a better predictor than (17). Similarly, for coal beds, (19) is to be used.

5. Conclusions

Experiments were conducted to determine the critical bed shear stress for the initiation of motion of gravel and coal beds under a unidirectional steady-uniform flow. The experimental data have a considerable disagreement with the standard curves proposed by Shields (1936), Yalin & Karahan (1979) and Dey (1999) in rough-turbulent regime. Critical bed shear stresses for the incipient motion of gravel and coal beds have been represented by simple empirical equations.

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