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Gas dynamic analysis of the performance of diffuser augmented wind turbine

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Abstract. A diffuser augmented wind turbine (DAWT) is considered an important application of the advanced concepts to improve the attractiveness of wind energy. The present paper aims to find a theoretical demonstration of DAWT by using theoretical analysis, mathematical models, assumptions, estimations and maximization of power coefficients and augmentation ratios, in addition to computer programs for calculations and drawings. The final results show that the maximum power coefficient (Cpd_0) and augmentation ratio (R_b) – relative to Betz – are directly proportional to pressure recovery factor (Cr), turbine factor (Ct), and maximum velocity ratio (No), but inversely proportional to overall recovery factor (C_0) of diffuser. The power coefficient (Cpd_0) of DAWT reaches 1.5 at $C_0 \cong -0.5$, $No \cong 1.0$ and $Cr \cong 0.5$, but the augmentation ratio (R_b) reaches 6.0 at $C_0 \cong -0.5$, $Cr \cong 0.9$, and reaches 7.0 at $No \cong 1.0$ and $Ct \cong 1.0$, which gives a good application for DAWT systems.

Keywords. Diffuser flow; diffuser augmented wind turbine; theoretical demonstration; design and performance.

1. Introduction

Enclosing a wind turbine by a diffuser reduces the pressure behind the turbine, thus enabling the drawing of more air for flow through the rotor, due to entrance velocity increase. Maximum power rating – for a given blade technology – can be extended substantially by installing a wind rotor at the entrance of a diffuser. The optimal performance of a wind turbine system has been studied (Badawy & Abd Raboo 1995; Mohamed & Badawy 1997), whereof the optimal power coefficients can be realized by the use of suitable pitch and twist angles in addition to design parameters such as tip speed ratio with minimum losses ($Cp \cong 0.435$). Gilbert *et al* (1977) made an experimental program guided by analytical models of various aspects of DAWT flow, where the power coefficient is 2.67 for a local turbine disk coefficient of 0.257. Forman *et al* (1976) carried out a general study on the

A list of symbols is given at the end of the paper performance of DAWT system, whereby a diffuser area ratio of 3 gives maximum augmentation. Forman & Gilbert (1979) and Gilbert & Forman (1978) carried out further investigations of DAWT and experimental demonstration for increasing augmentation. Turbomachinery diffuser design technology and review of diffuser analysis was studied by Japikse (1984), Mayer & Kneeling (1992) and Johnston (1998). Finally the unified integral method has been demonstrated to produce results which are equal to or better than the performance parameters in terms of accuracy. It is obvious from the previous work, that the optimum configuration regarding numbers, positions, pressure recovery and overall factors of diffuser, suitable area ratio and the diffuser efficiency in addition to velocity ratio must be considered to get the optimum performance of a DAWT system.

In the present paper, a theoretical demonstration of DAWT performance has been realized. Modelling and analysis of DAWT system were estimated and improved for determining the power coefficient and augmentation ratio by changing area ratio, diffuser efficiency, velocity ratio, turbine factor, pressure recovery and overall factor in the computer program.

2. Modelling and analysis

Analysis of DAWT performance depends on some assumption, such as one-dimensional potential, incompressible and inviscid flow, where the power coefficient (see figure 1) is given by

$$Cpd = \Delta P_{1-2}(\tfrac{3}{2})V_2A_2/[(1/2)\rho A_2V_0^3]. \quad (1)$$

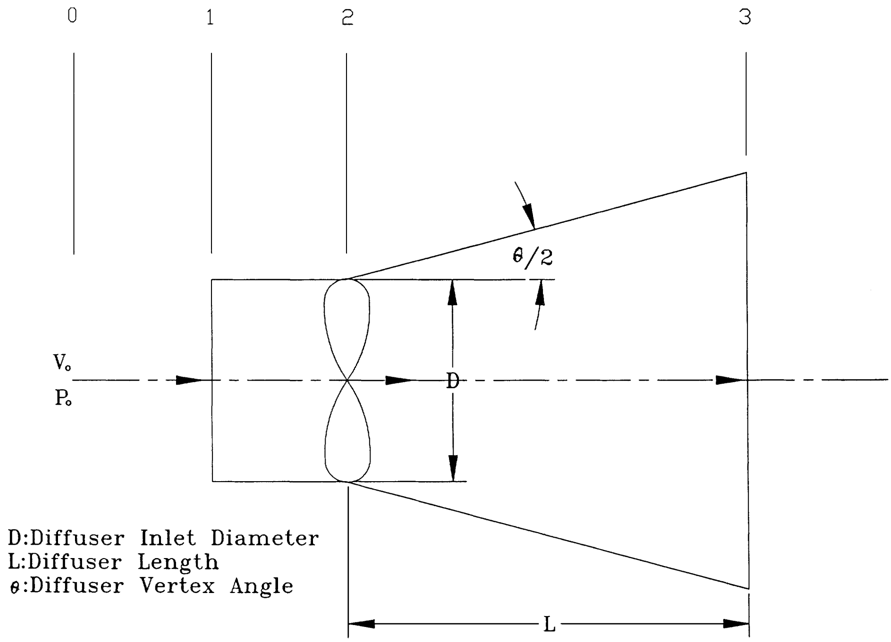

Figure 1. Layout of diffuser augmented wind turbine (DAWT).

By using of some mathematical operations, we get

$$\Delta P_{1-2}/\tfrac{1}{2}\rho V_0^2 = (\Delta P_{0-3} - \Delta P_{2-3})/\tfrac{1}{2}\rho V_0^2 = (1 - Co) - N^2(1 - Cr). \qquad (2)$$

Substituting (2) into (1), the power coefficient of DAWT is

$$Cpd = (3/2)N[(1 - Co) - N^2(1 - Cr)] = (3/2)N[Ct + (1 - N^2)], \qquad (3)$$

where

$$Cr = (P_3 - P_2)/\tfrac{1}{2}\rho V_2^2 = \eta(1 - m^2), \quad N = V_2/V_0,$$
$$\eta = (P_3 - P_2)/\tfrac{1}{2}\rho(V_2^2 - V_3^2), \quad m = A_2/A_3 = (1 + (2L/D_2)\tan(\theta/2))^{-2},$$
$$Co = (P_3 - P_0)/\tfrac{1}{2}\rho V_0^2, \quad Ct = (P_0 - P_2)/\tfrac{1}{2}\rho V_0^2.$$

The relation between Cr, Co and Ct is given by

$$Ct + Co = N^2 \times Cr. \qquad (4)$$

The maximum power coefficient of DAWT (at $\partial Cpd/\partial N \approx 0$) is given by

$$Cpd_0 = N_0(1 - Co) = N_0(1 + 3Ct),$$

where

$$No = [(1 + Ct)/3]^{1/2} = [(1 - Co)/3(1 - Cr)]^{1/2}.$$

The augmentation ratio relative to the Betz limit (R_b) is given by

$$R_b = Cpd_0/Cp_{\mathrm{Betz}} = (27/16)Cpd_0. \qquad (6)$$

3. Computations and algorithm

From the above modelling and analysis of DAWT performance, it is clear that the power output can be increased significantly by satisfying the following parameters.

(a) Maximum area ratio (m), due to optimum L/D and suitable vertex angle (θ) of diffuser to get optimum Cr;
(b) optimum diffuser efficiency (η) to find optimum Cr;
(c) relation of Cr for optimum Ct and Co;
(d) effect of area ratio (m) on Cpd at N, Co and η;
(e) optimum speed ratio (N) for optimum Cpd;
(f) maximum and suitable pressure recovery factor (Cr);
(g) maximum overall factor (Co) for optimum Cpd;
(h) optimum and suitable turbine load factor (Ct).

The algorithm of the determination and computer program is as follows.

(i) Calculating the area ratio (m) from L, D, θ;
(ii) calculating the pressure recovery factor (Cr) from m, η;
(iii) calculating Cr from Ct, Co and N;
(iv) calculating Cpd from m, Co, N and η;

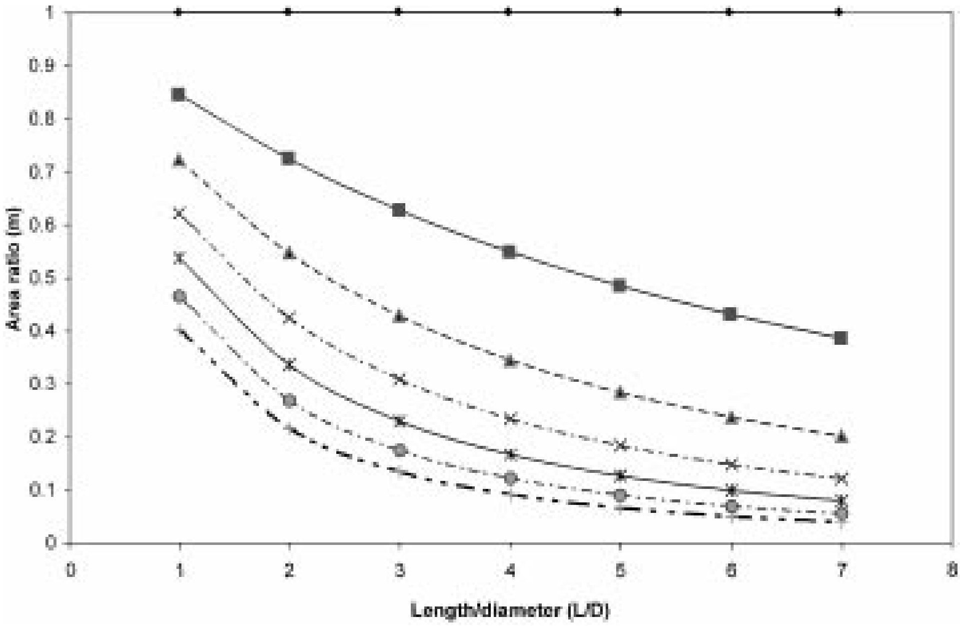

Figure 2. Effect of L/D and θ on the area ratio (m) of diffuser. $\theta = 0$ (◆), 5 (■), 10 (▲), 15 (×), 20 (∗), 25 (●), 30 (+).

(v) calculating the power coefficient (*Cpd*) from *Cr*, *Co*, *Ct*, and *N* (at different values);
(vi) calculating the maximum velocity ratio (*No*) from *Co*, *Cr*, and *Ct*;
(vii) calculating the maximum power coefficient (Cpd_0) from *No*, *Cr*, *Co* and *Ct*;

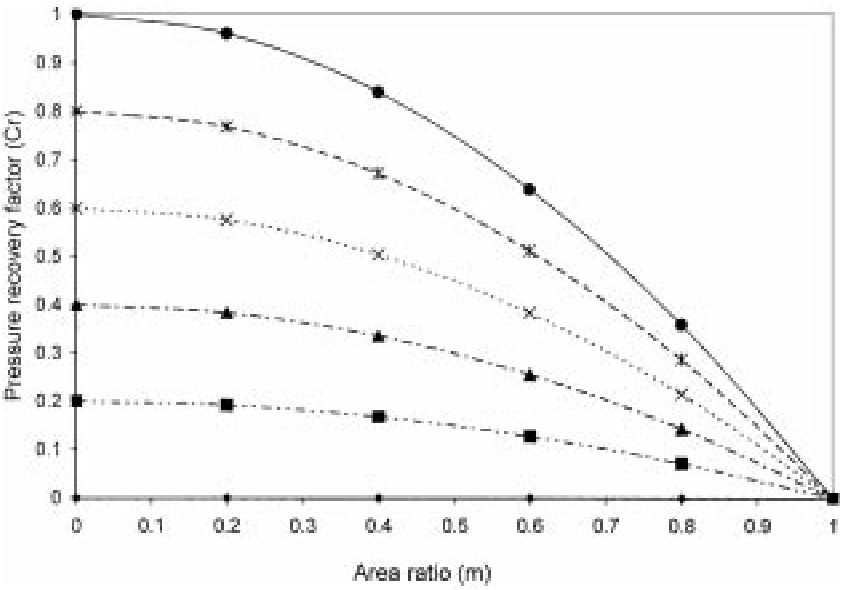

Figure 3. Effect of m and η on the pressure recovery factor (*Cr*) of DAWT. $\eta = 0$ (◆), 0.2 (□), 0.4 (▲), 0.6 (×), 0.8 (∗), 1.0 (●).

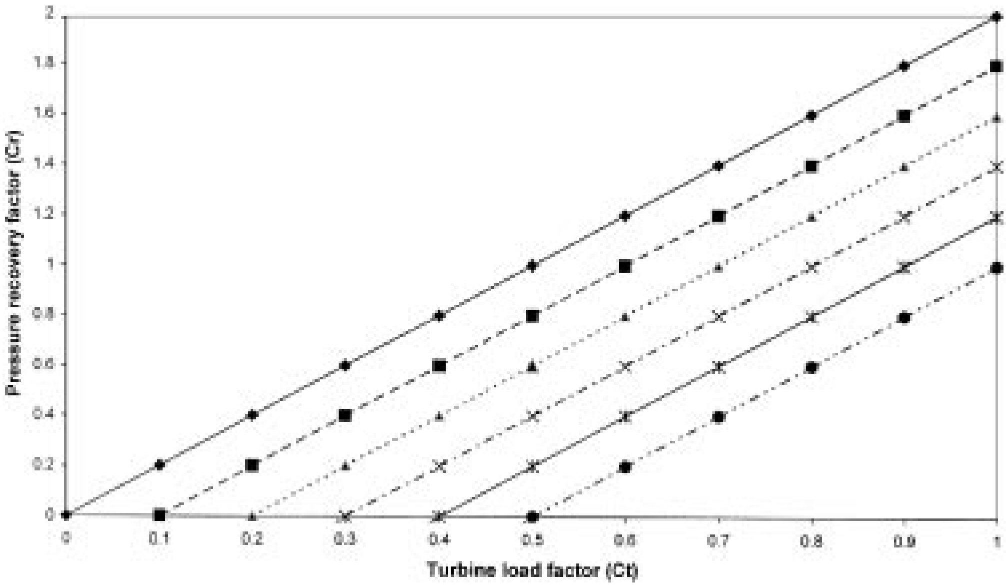

Figure 4. The effect of turbine load factor (Ct) and overall recovery factor (Co) on the pressure recovery factor (Cr) of DAWT at $N = 0.5$. $Co = 0$ (◆), -0.1 (■), -0.2 (▲), -0.3 (×), -0.4 (∗), -0.5 (●).

(viii) calculating the augmentation ratio (R_b) from Cpd_0, $Cp_{(\mathrm{Betz})}$, Co, Cr, Ct, and No;

(ix) plotting these relations to find the optimal power coefficient (Cpd_0) and the augmentation ratio (R_b) for DAWT system with Cr, Co, Ct, and No and so on.

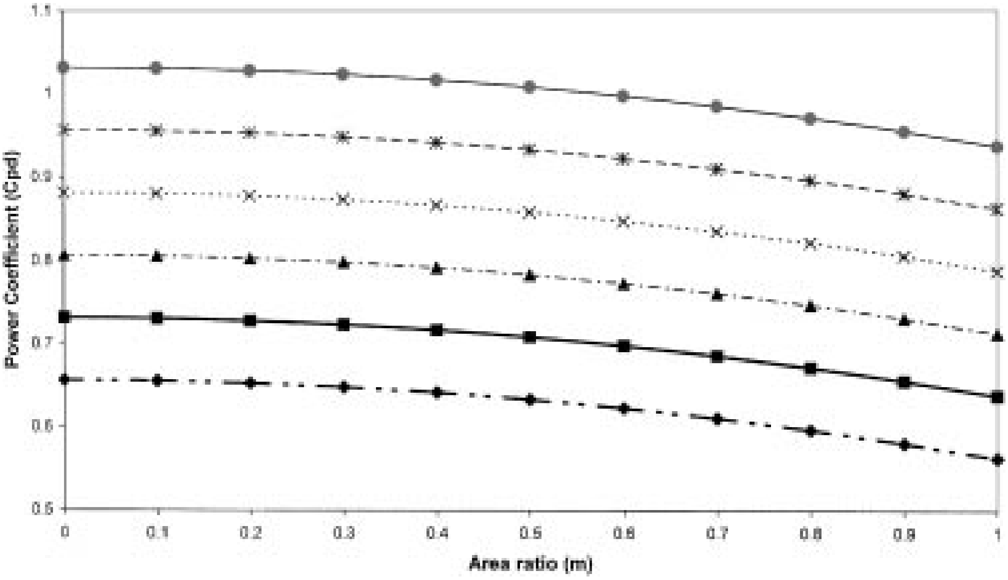

Figure 5. Effect of area ratio (m) on the power coefficient (Cpd) of DAWT at $N = 0.5$ and $\eta = 0.5$. $Co = 0$ (◆), -0.1 (■), -0.2 (▲), -0.3 (×), -0.4 (∗), -0.5 (●).

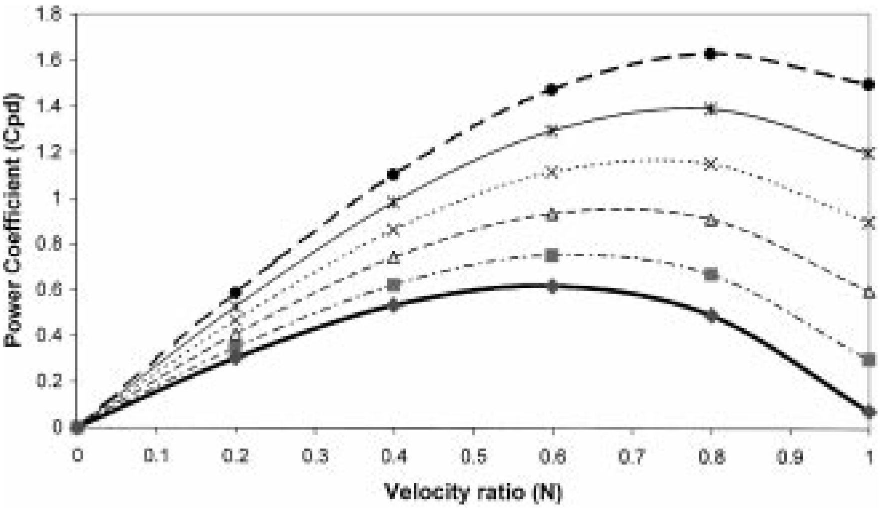

Figure 6. Effect of N and Ct on the power coefficient (Cpd) of DAWT. $Ct = 0.05$ (◆), 0.2 (■), 0.4 (△), 0.6 (×), 0.8 (∗), 1.0 (●).

4. Results and discussions

The relation between area ratio (m), L/D, and θ of diffuser is shown in figure 2, where the area ratio is inversely proportional to L/D and θ. The effect of area ratio (m) and diffuser efficiency (η) on the pressure recovery factor is shown in figure 3, where Cr is directly proportional to η, but inversely to (m), for good choice of suitable Cr for optimum power coefficient of DAWT system.

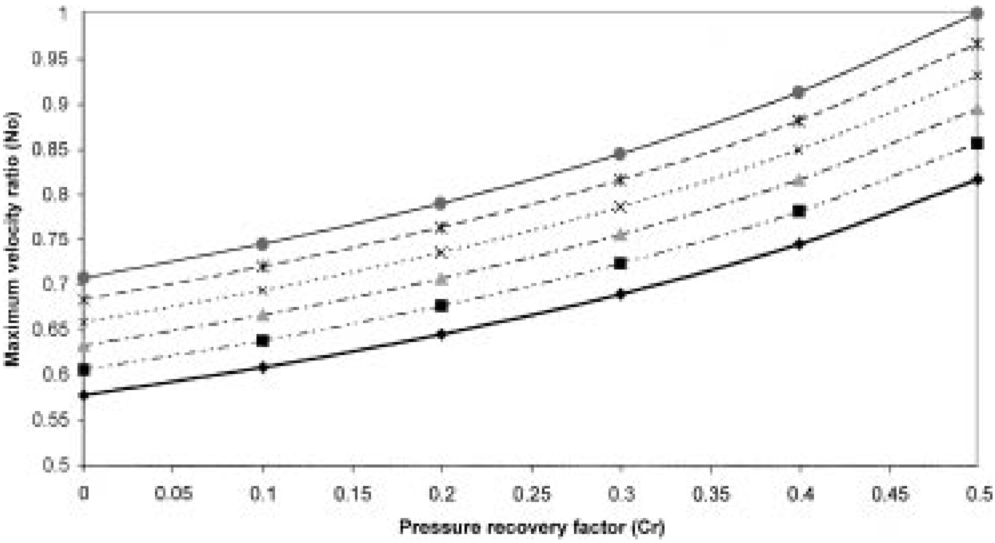

Figure 7. Effect of Cr and Co on the maximum velocity ratio (No) of DAWT. $Co = 0$ (◆), −0.1 (■), 0.2 (▲), 0.3 (×), 0.4 (∗), 0.5 (●).

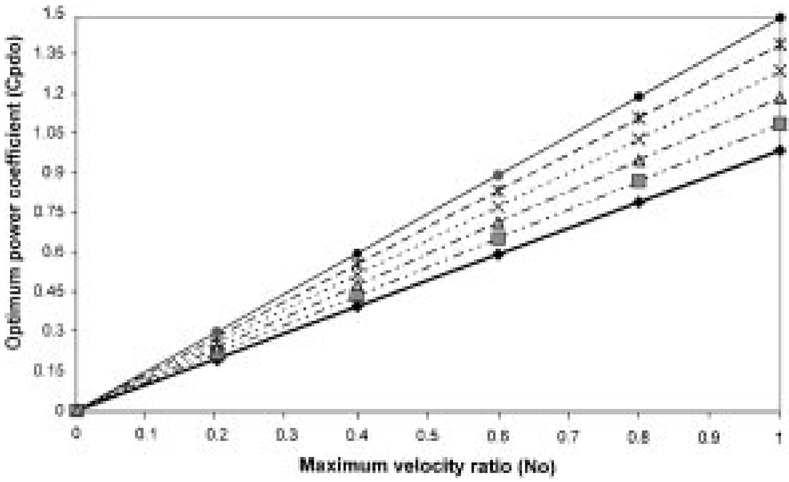

Figure 8. Effect of *No* and *Co* on the optimum power coefficient (*Cpdo*) of DAWT. $Co = 0$ (◆), -0.1 (■), -0.2 (△), 0.3 (×), 0.4 (∗), 0.5 (●).

The relation between turbine load factor (*Ct*), overall recovery factor (*Co*) and the pressure recovery factor (*Cr*) at $N = 0.5$ is directly proportional and is shown in figure 4. The power coefficient (*Cpd*) decreases with the increase of diffuser area ratio (*m*) at different values of *Co*, $N = 0.5$, $\eta = 0.5$ as shown in figure 5.

The relation between power coefficient (*Cpd*) and N, Ct is shown in figure 6 which shows it is directly proportional to *N*, and *Ct* to a maximum value and then decreases, the optimum value increasing with *N* and *Ct*. The effects of Cr, Co and *No* on the optimum

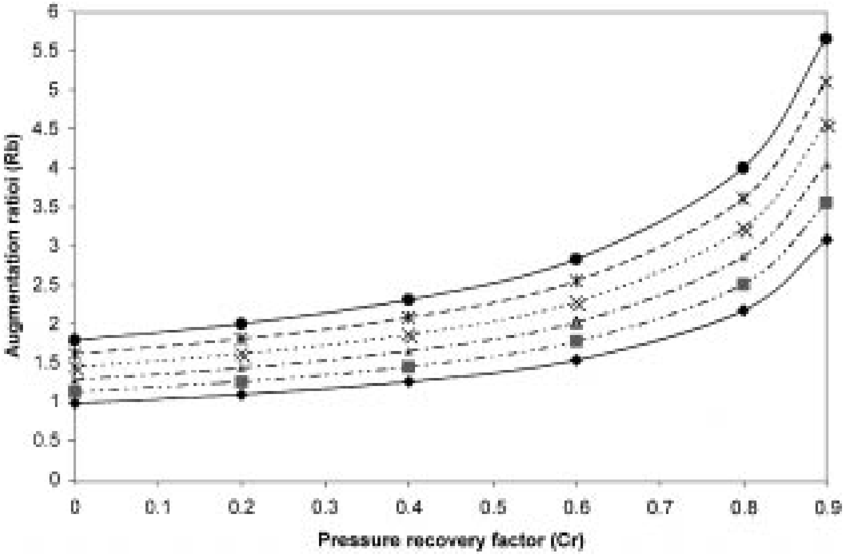

Figure 9. Effect of *Cr* and *Co* on the augmentation ratio (*Rb*) of DAWT. $Co = 0$ (◆), 0.1 (■), 0.2 (▲), 0.3 (×), 0.4 (∗), 0.5 (●).

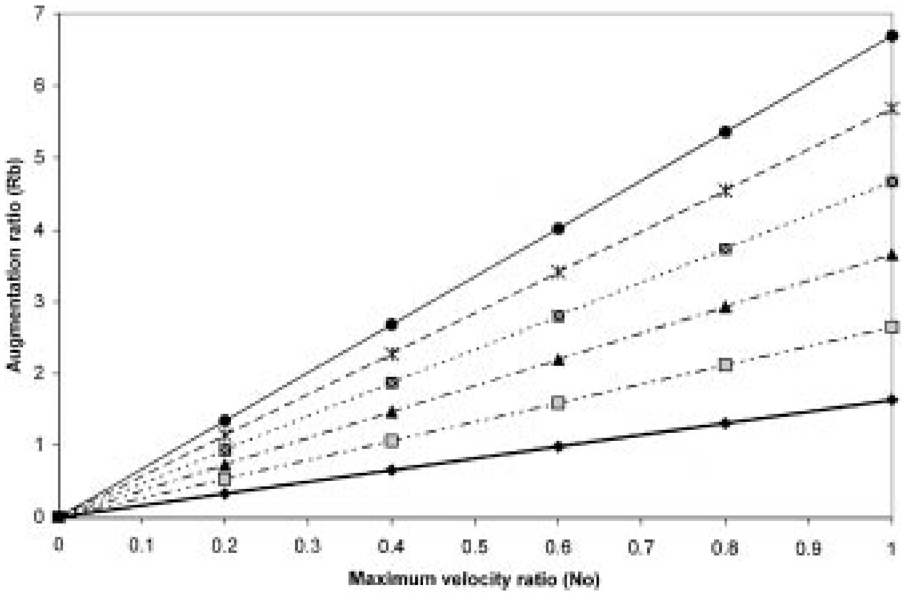

Figure 10. Effect of *No* and *Ct* on the augmentation ratio (*Rb*) of DAWT. $Ct = 0$ (◆), 0.2 (■), 0.4 (△), 0.6 (×), 0.8 (∗), 0.1 (●).

power coefficient (Cpd_0) are shown in figures 7 and 8. It is clear that Cpd_0 is directly proportional to *No* and *Co*, it has a value of 1.5 at $No \approx 1$, $Co \approx -0.5$ and $Cr \approx 0.5$.

The effects of Cr, Co, Ct and *No* on the augmentation ratio (R_b) are shown in figures 9 and 10. One can see that R_b is directly proportional to Cr, Co and *No*, where the optimum value of ≈ 6 for R_b at $Cr \approx 0.9$, $Co \approx -0.5$, $No \approx 1.0$ and $Ct \approx 1.0$.

5. Conclusions

The conclusions of this paper are as follows.

(1) The power coefficient of DAWT (*Cpd*) is directly proportional to Cr, Co, Ct, N and η, but inversely proportional to *m*, where suitable choice must be made to get optimal power.
(2) Maximum power coefficient (Cpd_0) of DAWT is directly proportional to Cr, Co, *Ct* and *No*, and reaches a value of 1.5 at $Co \approx -0.5$, $Cr \approx 0.5$, $Ct \approx 1.0$ and $No \approx 1.0$.
(3) Augmentation ratio relative to Betz (R_b) is directly proportional to Cr, Co, Ct and *No*, and reaches a value of 6 at $Ct \approx 1.0$, $No \approx 1.0$, $Cr \approx 0.9$ and $Co \approx -0.5$.
(4) By these analyses and demonstrations, the DAWT system is considered to be a good application to get more power with suitable choice of m, N, Cr, Ct, Co and *No*.

List of symbols

A	cross-sectional area at any section (m^2);
Co	overall recovery factor;
Cpd	power coefficient of DAWT;
Cpd_0	optimum power coefficient of DAWT;

Cr pressure recovery factor;
Ct turbine load factor;
M area ratio of diffuser;
N, No velocity ratio, maximum velocity ratio respectively;
P pressure at any section (N/m^2);
ΔP total pressure difference (N/m^2);
V flow velocity (m/s);
V_0 free stream velocity (m/s);
ρ air density (kg/m^3);
η diffuser efficiency.

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