Study of rectangular waveguides with grooves of irregular shape using the finite element method

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Abstract. Electromagnetic wave propagation through grooved waveguides is studied using the finite element method (FEM). The effect of grooves of irregular shape on TE₁₀, TE₂₀ mode frequencies and passband is studied. The variation in cutoff frequencies for TE₁₀, TE₂₀ mode and passband is observed.

Keywords. Rectangular waveguides; grooves of irregular shape; finite element method; cutoff frequencies.

1. Introduction

A survey of the work on grooved waveguides indicates that a grooved waveguide has the advantages of low loss transmission, low dimensional tolerance and high power handling capacity at higher frequencies. It is also noted that these waveguides can effectively reject higher order modes.

The analysis of V-grooved waveguides of open and closed type has been done by various workers (Harris & Lee 1981; Ho & Harris 1984; Oliner & Lampariello 1985; Choi *et al* 1988; Li *et al* 1991; Schutt Aine 1992) using different methods. Rectangular-grooved guides of various dimensions have also been studied (Sachidananda 1992) for cutoff wavelengths and various higher modes. The single V-grooved guide was studied by Li *et al* (1991) and Oliner & Lampariello (1985) and propagation characteristics worked out for various dimensions, using the transverse resonance technique. Choi *et al* (1988) analysed single and double V-grooved waveguides by a conformal technique. Sachidananda (1992) used a mode matching technique to study the rectangular-grooved waveguide and reported the variation of cut-off wavelength by varying plate distance for constant groove width.

The work done on grooved guide as mentioned above is restricted to two definite shapes of grooves, V-shaped and rectangular. In this paper, rectangular waveguides with irregular

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Figure 1. Cross-section of waveguide with symmetric groove of irregular shape.

grooves are studied by using the finite element method (Webb 1988; Dillon & Webb 1994; Popalghat 1995). The effect of the irregularity of grooves on the cut-off frequency for the dominant mode TE_{10} and the mode next to it, i.e. TE_{20} is studied.

2. Statement of the problem

A homogeneous rectangular waveguide, with grooves of irregular shape at the centre of both its wide walls (as shown in figure 1), is considered. The walls of the waveguide are perfectly conducting, enclosing air as the dielectric medium.

The cross-section of the grooved waveguide under study is considered in the X - Y plane of the Cartesian coordinate system as a problem domain Ω with conducting boundaries. The domain Ω under study has as its boundaries, electric walls (Webb 1988) on which normal magnetic components and tangential electric components vanish at all the boundary segments.

The electric and magnetic fields inside the waveguide satisfy Maxwell's equations. With time dependence $= \exp(j\omega t)$, Maxwell's curl equations can be combined to give

$$\nabla \times (\epsilon_r^{-1} \nabla \times \mathbf{H}) = K_0^2 \mathbf{H}, \quad (1)$$

where $K_0^2 = \omega^2 \mu_0 \epsilon_0$.

Here ω is the angular frequency, ϵ_0 the permittivity and μ_0 the permeability of free space.

Table 1. Frequencies (f) and bandwidths (BW) for grooved waveguides of irregular shape. $W = 1.6$ cm, $h = 0.15$ cm.

L/W	f (GHz)		BW (GHz)
	TE_{10}	TE_{20}	
0.1250	5.9511	12.9977	7.0467
0.2500	5.9123	12.9288	7.0165
0.3750	5.8779	12.8346	6.9567
0.5000	5.8487	12.7165	6.8678
0.6250	5.8257	12.5868	6.7611
0.7500	5.8086	12.4433	6.6347
0.8750	5.7958	12.3097	6.5139
1.1250	5.7754	12.0626	6.2872
1.2500	5.7595	11.9360	6.1764
1.3750	5.7484	11.7947	6.0463

Table 2. Frequencies (f) and bandwidths (BW) for grooved waveguides of irregular shape. $W = 1.6$ cm, $h = 0.3$ cm.

L/W	f (GHz)		BW (GHz)
	TE ₁₀	TE ₂₀	
0.1250	5.5951	12.8999	7.3048
0.2500	5.5314	12.8508	7.3194
0.3750	5.4759	12.8075	7.3317
0.5000	5.4274	12.6283	7.2009
0.6250	5.3846	12.4461	7.0615
0.7500	5.3463	12.2244	6.8781
0.8750	5.3108	11.9753	6.6645
1.1250	5.2460	11.4321	6.1860
1.2500	5.2147	11.1428	5.9282
1.3750	5.1829	10.8367	5.6538

The magnetic field $\mathbf{H}$ must satisfy the appropriate boundary condition at the conducting boundaries, the condition being that the tangential component of the electric field be zero at the conductor boundaries,

$$\mathbf{n} \times \nabla \times \mathbf{H}|_{\text{boundary}} = 0. \quad (2)$$

3. FEM formulation

The expression for functional Π is

$$\Pi = \frac{1}{2} \int_{\Omega} [(\nabla \times \mathbf{H}^*) \cdot (\epsilon_r^{-1} \nabla \times \mathbf{H}) - K_0^2 \mathbf{H}^* \cdot \mathbf{H} + (\nabla \cdot \mathbf{H}^*)(\nabla \cdot \mathbf{H})] d\Omega. \quad (3)$$

The stationary character of Π requires $\delta\Pi = 0$.

The first variation in Π is given by,

$$\delta \left[\int_{\Omega} (\nabla \times \mathbf{H}^*) \cdot (\epsilon_r^{-1} \nabla \times \mathbf{H}) d\Omega - \int_{\Omega} K_0^2 \mathbf{H}^* \cdot \mathbf{H} d\Omega + \int_{\Omega} (\nabla \cdot \mathbf{H}^*)(\nabla \cdot \mathbf{H}) d\Omega \right]. \quad (4)$$

Table 3. Frequencies (f) and bandwidths (BW) for grooved waveguides of irregular shape. $W = 1.2$ cm, $h = 0.15$ cm.

L/W	f (GHz)		BW (GHz)
	TE ₁₀	TE ₂₀	
0.1667	5.9438	12.9299	6.9861
0.3334	5.8990	12.9259	7.0269
0.5000	5.8582	12.8965	7.0383
0.6667	5.8225	12.8352	7.0157
0.8334	5.7911	12.7518	6.9607
1.1667	5.7336	12.5195	6.7860
1.3334	5.7053	12.3811	6.6758
1.5000	5.6774	12.2191	6.5417
1.6667	5.6496	12.0197	6.3701
1.8334	5.6215	11.7684	6.1469

Table 4. Frequencies (f) and bandwidths (BW) for grooved waveguides of irregular shape. $W = 1.2$ cm, $h = 0.3$ cm.

L/W	$f(\text{GHz})$		B W (GHz)
	TE ₁₀	TE ₂₀	
0.1667	5.6185	13.0254	7.4069
0.3334	5.5394	13.0286	7.4892
0.5000	5.4675	13.0037	7.5362
0.6667	5.4013	12.9384	7.5372
0.8334	5.3395	12.8214	7.4819
1.1667	5.2258	12.4002	7.1744
1.3334	5.1716	12.0962	6.9245
1.5000	5.1180	11.7362	6.6182
1.6667	5.0643	11.3244	6.2601
1.8334	5.0104	10.8673	5.8569

The domain (i.e. cross-section of the waveguide) is divided into rectangular elements with four nodes. The functional over each element is,

$$\Pi^e = \sum \frac{1}{2} \left[\int_{\Omega^e} (\nabla \times H^{e*}) \cdot (\epsilon_r^{-1} \nabla \times H^e) d\Omega^e - \int_{\Omega^e} K_0^2 H^{e*} \cdot H^e d\Omega^e + \int_{\Omega^e} (\nabla \cdot H^{e*}) (\nabla \cdot H^e) d\Omega^e \right], \quad (5)$$

where the unknown vector H^e has three components H_x, H_y, H_z . Each unknown component H_x, H_y, H_z is defined in terms of the nodal values H_x^e, H_y^e, H_z^e respectively.

Linear mapping functions are used for 4 nodes per rectangular element.

Functional Π^e can be written as,

$$\Pi^e = \sum \frac{1}{2} \{H^e\}^T ([S^e] - K_0^2 [T^e]) \{H^e\}. \quad (6)$$

The functional for the whole region Ω is given by,

$$\Pi = \frac{1}{2} \{H\}^T [S] \{H\} - \frac{1}{2} K_0^2 \{H\}^T [T] \{H\}. \quad (7)$$

The condition $\delta\pi/\delta\{H\} = 0$ leads to the following matrix equation

$$[S] \{H\} - K_0^2 [T] \{H\} = 0. \quad (8)$$

Table 5. Frequencies (f) and bandwidths (BW) for grooved waveguides of irregular shape. $W = 0.8$ cm, $h = 0.15$ cm.

L/W	$f(\text{GHz})$		B W (GHz)
	TE ₁₀	TE ₂₀	
0.2500	5.9889	12.9377	6.9488
0.5000	5.9367	12.9673	7.0306
0.7500	5.8879	12.9756	7.0877
1.2500	5.7972	12.9182	7.1210
1.5000	5.7530	12.8577	7.1047
1.7500	5.7089	12.7733	7.0645
2.0000	5.6644	12.6498	6.9855
2.2500	5.6195	12.4600	6.8405
2.5000	5.5737	12.1686	6.5949
2.7500	5.5271	11.7499	6.2228

Table 6. Frequencies (f) and bandwidths (BW) for grooved waveguides of irregular shape. $W = 0.8$ cm, $h = 0.3$ cm.

L/W	f (GHz)		B W (GHz)
	TE ₁₀	TE ₂₀	
0.2500	5.7160	13.0005	7.2844
0.5000	5.6175	13.0184	7.4010
0.7500	5.5252	13.0323	7.5071
1.2500	5.3544	13.0181	7.6637
1.5000	5.2740	12.9618	7.6878
1.7500	5.1954	12.8393	7.6439
2.0000	5.1182	12.6015	7.4833
2.2500	5.0417	12.1876	7.1459
2.5000	4.9659	11.5952	6.6294
2.7500	4.8900	10.9101	6.0202

4. Numerical calculations

Consider an irregular groove on the wide wall of an X -band rectangular waveguide of cross-sectional length $a = 2.4$ cm and breadth $b = 1.2$ cm. The groove is symmetric. By keeping base width W and height h of the groove fixed, the inclination of the grooved side is varied by varying the groove top length L , from 0.2 cm to 2.2 cm, in steps of 0.2 cm.

The analysis is repeated for three different base widths $W = 1.6$, 1.2, and 0.8 cm, and for two different heights $h = 0.15$ and 0.3 cm. The results are presented in tables 1–6.

For each W and for two different h , L is varied and the effect of this on TE₁₀ and TE₂₀ mode frequencies and bandwidth is studied. The variation of TE₁₀ and TE₂₀ are shown

Figure 2. Variation of TE₁₀ and TE₂₀ mode frequencies for waveguides with grooves of irregular shape with $W = 1.6$ cm and $h = 0.15$ cm, for different L/W ratios.

Figure 3. Variation of TE_{10} and TE_{20} mode frequencies for waveguides as in figure 2, with $W = 1.6$ cm and $h = 0.3$ cm, for different L/W ratios.

Figure 4. Variation of TE_{10} and TE_{20} mode frequencies for waveguides as in figure 2, with $W = 1.2$ cm and $h = 0.15$ cm for different L/W ratios.

Figure 5. Variation of TE₁₀ and TE₂₀ mode frequencies waveguides as in figure 2, with $W = 1.2$ cm and $h = 0.3$ cm, for different L/W ratios.

Figure 6. Variation of TE₁₀ and TE₂₀ mode frequencies for waveguides as in figure 2, with $W = 0.8$ cm and $h = 0.15$ cm, for different L/W ratios.

Figure 7. Variation of TE₁₀ and TE₂₀ mode frequencies for waveguides as in figure 2, with $W = 0.8$ cm and $h = 0.3$ cm for different L/W ratios.

Figure 8. Variation of bandwidth for waveguides as in figure 2, with $W = 1.6$ cm and $h = 0.15$ cm, for different ratios of L/W .

Figure 9. Variation of bandwidth for waveguides as in figure 2, $W = 1.6$ cm and $h = 0.3$ cm, for different ratios of L/W .

Figure 10. Variation of bandwidth for waveguides as in figure 2, with $W = 1.2$ cm and $h = 0.15$ cm, for different ratios of L/W .

Figure 11. Variation of bandwidth for waveguide as in figure 2, with $W = 1.2$ cm and $h = 0.3$ cm, for different ratios of L/W .

Figure 12. Variation of bandwidth for waveguides as in figure 2, with $W = 0.8$ cm and $h = 0.15$ cm, for different ratios of L/W .

Figure 13. Variation of bandwidth for waveguides as in figure 2, with $W = 0.8$ cm and $h = 0.3$ cm, for different ratios of L/W .

in figures 2 to 7. The variations of bandwidth with ratio L/W for each case are shown in figures 8 to 13.

5. Conclusions

Bandwidth is higher than that of a normal waveguide when $L/W < 1$ and it decreases as L/W increases. When $L/W > 1$, the bandwidth is less than that of the normal waveguide and decreases further with increase in L/W . For a specific value of L/W , the bandwidth is higher for greater groove height.

Bandwidth variations can be helpful for using waveguides as pass-band components whose pass-band range can be widened or suppressed by proper choice of groove height and suitable L/W ratios.

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