Effects of inhomogeneity on surface waves in anisotropic media

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Abstract. This paper investigates the effects of anisotropy and inhomogeneity on surface waves in elastic media. Exponential variation in properties are assumed for the elastic parameters and material density. The classical equations of motion for propagation of waves in an inhomogeneous transversely isotropic elastic solid are deduced. The equations of motion for surface waves are derived and general surface waves are investigated. This general theory is then utilized to investigate Rayleigh, Love and Stoneley waves. Results obtained in the above cases reduce to the corresponding well-known classical results when inhomogeneity and anisotropy are not present. It is seen that inhomogeneity has significant effects on dispersion characteristics. Numerical calculations are included for Love waves and some conclusions have been drawn from the above calculations.

Keywords. Exponential inhomogeneity; transversely isotropic; surface waves; Rayleigh waves; Love waves; Stoneley waves; wave velocity equation.

1. Introduction

It is known that surface waves in a homogeneous isotropic elastic solid medium play an important role in the study of earthquakes, seismology, geophysics and geodynamics. The theory of surface waves has been widely investigated by Rayleigh (1885), Stoneley (1924), Ewing *et al* (1957), Hunter (1960), Bland (1960), Biot (1965), Flugge (1967), and Jeffreys (1970). The effects of the anisotropic character of the medium on surface waves have been studied to a limited extent. The problem of wave-propagation in an inhomogeneous medium also plays a role, because any realistic model of the earth must take into account continuous changes in the vertical direction of the elastic properties of the material. Some problems of wave propagation, in which viscosity, rigidity and density are functions of depth, have been studied by several authors (Dutta 1963; Bhattacharya 1969; Chattopadhyay 1975). In this paper, we consider the problem of surface waves

when inhomogeneity and anisotropy occur simultaneously. We assume the medium to be transversely isotropic which is a particular case of anisotropy. Following the papers of Dey & Sengupta (1978) and Das *et al* (1992), we assume the inhomogeneities of the lower medium (M_1) and the upper medium (M_2) to be respectively of the form:

$$\begin{aligned} A &= A_1 e^{\nu z}, & N &= N_1 e^{\nu z}, & F &= F_1 e^{\nu z}, \\ C &= C_1 e^{\nu z}, & L &= L_1 e^{\nu z}, & \rho &= \rho_1 e^{\nu z}, \end{aligned} \quad (1)$$

$$\begin{aligned} A' &= A'_1 e^{\nu' z}, & N' &= N'_1 e^{\nu' z}, & F' &= F'_1 e^{\nu' z}, \\ C' &= C'_1 e^{\nu' z}, & L' &= L'_1 e^{\nu' z}, & \rho' &= \rho'_1 e^{\nu' z}, \end{aligned} \quad (2)$$

where $A_1, N_1, F_1, C_1, L_1, \rho_1$ and their primes are constants for an anisotropic medium. In the case of Love waves, the inhomogeneities in two media are assumed to be of the form:

$$N = N_1 e^{\nu z}, \quad L = L_1 e^{\nu z}, \quad \rho = \rho_1 e^{\nu z}, \quad \text{for } 0 \leq Z \leq \infty, \quad (3)$$

$$N' = N'_1 e^{\nu' z}, \quad L' = L'_1 e^{\nu' z}, \quad \rho' = \rho'_1 e^{\nu' z}, \quad \text{for } -H \leq Z \leq 0. \quad (4)$$

We first deduce a set of dynamical equations of motion for a three-dimensional problem with inhomogeneity and transverse isotropy. The general theory of surface waves is then deduced to highlight the effects of anisotropy and inhomogeneity. It is believed that the above type of problem has not so far been considered.

2. Formulation of the problem and its solution

Let M_1 and M_2 be two nonhomogeneous transversely isotropic semi-infinite elastic solid media, which are in perfectly welded contact along $Z = 0$, M_2 being above M_1 (figure 1). We consider a system of orthogonal Cartesian axes $Oxyz$, the origin O being on the interface $Z = 0$ and OZ is normal to the interface vertically downwards.

We consider the possibility of a type of wave along the positive direction of the x -axis and largely confined to the neighbourhood of $Z = 0$; and the particles in YZ plane with equal phase. The first assumption asserts that the wave is a surface wave and the second

Figure 1. Schematic of the problem.

assumption leads to zero partial derivatives with respect to Y . The equations of motion are

$$\begin{aligned}\frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} + \frac{\partial \sigma_{13}}{\partial z} &= \rho \frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} + \frac{\partial \sigma_{23}}{\partial z} &= \rho \frac{\partial^2 v}{\partial t^2}, \\ \frac{\partial \sigma_{31}}{\partial x} + \frac{\partial \sigma_{32}}{\partial y} + \frac{\partial \sigma_{33}}{\partial z} &= \rho \frac{\partial^2 w}{\partial t^2},\end{aligned}\quad (5)$$

where u, v, w are the components of displacement and the stress tensors σ_{ij} ($i, j = 1, 2, 3$) are given in the following forms (Ewing *et al* 1957; Love 1967)

$$\begin{aligned}\sigma_{11} &= A \frac{\partial u}{\partial x} + (A - 2N) \frac{\partial v}{\partial y} + F \frac{\partial w}{\partial z}, \\ \sigma_{22} &= (A - 2N) \frac{\partial u}{\partial x} + A \frac{\partial v}{\partial y} + F \frac{\partial w}{\partial z}, \\ \sigma_{33} &= F \frac{\partial u}{\partial x} + F \frac{\partial v}{\partial y} + C \frac{\partial w}{\partial z}, \\ \sigma_{12} &= N \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \\ \sigma_{23} &= L \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right), \\ \sigma_{31} &= L \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right),\end{aligned}\quad (6)$$

where A, N, F, C , and L are of the form given by (1). In view of the variation of the elastic parameters we have

$$\begin{aligned}\sigma_{11} &= e^{\nu z} \left[A_1 \frac{\partial u}{\partial x} + (A_1 - 2N_1) \frac{\partial v}{\partial y} + F_1 \frac{\partial w}{\partial z} \right], \\ \sigma_{22} &= e^{\nu z} \left[(A_1 - 2N_1) \frac{\partial u}{\partial x} + A_1 \frac{\partial v}{\partial y} + F_1 \frac{\partial w}{\partial z} \right], \\ \sigma_{33} &= e^{\nu z} \left[F_1 \frac{\partial u}{\partial x} + F_1 \frac{\partial v}{\partial y} + C_1 \frac{\partial w}{\partial z} \right], \\ \sigma_{12} &= e^{\nu z} N_1 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \\ \sigma_{23} &= e^{\nu z} L_1 \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right), \\ \sigma_{31} &= e^{\nu z} L_1 \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right).\end{aligned}\quad (7)$$

Introducing (7) in (5) we get

$$A_1 \frac{\partial^2 u}{\partial x^2} + (F_1 + L_1) \frac{\partial^2 w}{\partial x \partial z} + L_1 \frac{\partial^2 u}{\partial z^2} + \nu L_1 \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) = \rho_1 \frac{\partial^2 u}{\partial t^2},$$

$$\begin{aligned}
N_1 \frac{\partial^2 v}{\partial x^2} + L_1 \frac{\partial^2 v}{\partial z^2} + v L_1 \frac{\partial v}{\partial z} &= \rho_1 \frac{\partial^2 v}{\partial t^2}, \\
L_1 \frac{\partial^2 w}{\partial x^2} + (F_1 + L_1) \frac{\partial^2 u}{\partial x \partial z} + C_1 \frac{\partial^2 w}{\partial z^2} + v \left(F_1 \frac{\partial u}{\partial z} + C_1 \frac{\partial w}{\partial z} \right) &= \rho_1 \frac{\partial^2 w}{\partial t^2}. \quad (8)
\end{aligned}$$

For the solution of the above equations in the case of medium M_1 we take

$$\begin{aligned}
u(x, z, t) &= P(z)e^{i\eta(x-ct)}, \\
v(x, z, t) &= Q(z)e^{i\eta(x-ct)}, \\
w(x, z, t) &= R(z)e^{i\eta(x-ct)}, \quad (9)
\end{aligned}$$

and similar set of displacements for the solution of equations for the medium M_2 , the functions P, Q, R being replaced by P', Q', R' . This helps us to have a particular type of solutions corresponding to a group of simple harmonic waves of wave length $2\pi/\eta$, travelling with velocity c . Inserting (9) in (8) we get a set of differential equations for the medium M_1 as follows:

$$\begin{aligned}
[L_1 D^2 + v L_1 D + \eta^2(\rho_1 c^2 - A_1)]P + i\eta[(F_1 + L_1)D + v L_1]R &= 0, \\
i\eta[(F_1 + L_1)D + v F_1]P + [C_1 D^2 + v C_1 D + \eta^2(\rho_1 c^2 - L_1)]R &= 0, \\
[L_1 D^2 + v L_1 D + \eta^2(\rho_1 c^2 - N_1)]Q &= 0, \quad (10)
\end{aligned}$$

where $D = d/dz$, $D^2 = d^2/dz^2$.

For medium M_2 we get similar set of differential equations as follows:

$$\begin{aligned}
[L'_1 D^2 + v' L'_1 D + \eta^2(\rho'_1 c^2 - A'_1)]P' + i\eta[(F'_1 + L'_1)D + v' L'_1]R' &= 0, \\
i\eta[(F'_1 + L'_1)D + v' F'_1]P' + [C'_1 D^2 + v' C'_1 D + \eta^2(\rho'_1 c^2 - L'_1)]R' &= 0, \\
[L'_1 D^2 + v' L'_1 D + \eta^2(\rho'_1 c^2 - N'_1)]Q' &= 0. \quad (11)
\end{aligned}$$

Equations (10) and (11) must have exponential solutions for surface waves with u, v , and w and hence, P, Q , and R tend to zero as $Z \rightarrow \infty$ in medium M_1 . Similarly P', Q', R' tend to zero as $Z \rightarrow -\infty$ in medium M_2 . Hence for medium M_1

$$\begin{aligned}
u(x, z, t) &= \{K_1 e^{-im_1 \eta z} + K_2 e^{-im_2 \eta z}\} e^{i\eta(x-ct)} \\
w(x, z, t) &= \{\bar{K}_1 e^{-im_1 \eta z} + \bar{K}_2 e^{-im_2 \eta z}\} e^{i\eta(x-ct)}, \\
v(x, z, t) &= K_3 e^{-im_3 \eta z} e^{i\eta(x-ct)}. \quad (12)
\end{aligned}$$

For medium M_2 ,

$$\begin{aligned}
u'(x, z, t) &= \{K_4 e^{im'_1 \eta z} + K_5 e^{im'_2 \eta z}\} e^{i\eta(x-ct)}, \\
w'(x, z, t) &= \{\bar{K}_4 e^{im'_1 \eta z} + \bar{K}_5 e^{im'_2 \eta z}\} e^{i\eta(x-ct)}, \\
v'(x, z, t) &= K_6 e^{im'_3 \eta z} e^{i\eta(x-ct)}, \quad (13)
\end{aligned}$$

where for $M_1, m_j (j = 1, 2)$ are the roots of

$$m^4 + 2i \frac{v}{\eta} m^3 - \left\{ L_1(\rho_1 c^2 - L_1) + \frac{v^2}{\eta^2} L_1 C_1 + C_1(\rho_1 c^2 - A_1) + (F_1 + L_1)^2 \right\},$$

$$\begin{aligned} & \frac{1}{L_1 C_1} m^2 - i \frac{\nu}{\eta} \{L_1(\rho_1 c^2 - L_1) + C_1(\rho_1 c^2 - A_1) + (F_1 + L_1)^2\} \frac{1}{L_1 C_1} m \\ & + \left\{ (\rho_1 c^2 - L_1)(\rho_1 c^2 - A_1) + \frac{\nu^2}{\eta^2} L_1 F_1 \right\} \frac{1}{L_1 C_1} = 0. \end{aligned} \quad (14)$$

For M_2 , m'_j ($j = 1, 2$) are the roots of

$$\begin{aligned} & m'^4 + 2i \frac{\nu'}{\eta} m'^3 - \left\{ L'_1(\rho'_1 c^2 - L'_1) + \frac{\nu'^2}{\eta^2} L'_1 C'_1 + C'_1(\rho'_1 c^2 - A'_1) + (F'_1 + L'_1)^2 \right\} \\ & \frac{1}{L'_1 C'_1} m'^2 - i \frac{\nu'}{\eta} \{L'_1(\rho'_1 c^2 - L'_1) + C'_1(\rho'_1 c^2 - A'_1) + (F'_1 + L'_1)^2\} \frac{1}{L'_1 C'_1} m' \\ & + \left\{ (\rho'_1 c^2 - L'_1)(\rho'_1 c^2 - A'_1) + \frac{\nu'^2}{\eta^2} L'_1 F'_1 \right\} \frac{1}{L'_1 C'_1} = 0, \end{aligned} \quad (15)$$

with positive real part and

$$\begin{aligned} \bar{K}_1 &= \alpha_1 K_1, & \bar{K}_2 &= \alpha_2 K_2, \\ \bar{K}_4 &= \alpha'_1 K_4, & \bar{K}_5 &= \alpha'_2 K_5, \end{aligned} \quad (16)$$

where

$$\alpha_j = \frac{L_1 m_j^2 + i \frac{\nu}{\eta} L_1 m_j - (\rho_1 c^2 - A_1)}{(F_1 + L_1) m_j + i \frac{\nu}{\eta} L_1},$$

and

$$\alpha'_j = \frac{L'_1 m_j'^2 + i \frac{\nu'}{\eta} L'_1 m'_j - (\rho'_1 c^2 - A'_1)}{i \frac{\nu'}{\eta} L'_1 - (F'_1 + L'_1) m'_j}, \quad j = 1, 2. \quad (17)$$

Also m_3 is a root of

$$L_1 m^2 + i \frac{\nu}{\eta} L_1 m - (\rho_1 c^2 - N_1) = 0, \quad (18)$$

and m'_3 is a root of

$$L'_1 m'^2 + i \frac{\nu'}{\eta} L'_1 m' - (\rho'_1 c^2 - N'_1) = 0. \quad (19)$$

3. Boundary conditions

- (i) The displacement components at the interface between M_1 and M_2 must be continuous.
- (ii) The stresses σ_{31} , σ_{32} , σ_{33} are also continuous at the interface, where

$$\begin{aligned} \sigma_{31} &= e^{\nu z} L_1 \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right), \\ \sigma_{32} &= e^{\nu z} L_1 \frac{\partial v}{\partial z}, \\ \sigma_{33} &= e^{\nu z} \left(F_1 \frac{\partial u}{\partial x} + C_1 \frac{\partial w}{\partial z} \right). \end{aligned}$$

4. Solutions of the problem

Applying the above boundary conditions we have

$$\begin{aligned}
 K_1 + K_2 - K_4 - K_5 &= 0, \\
 \alpha_1 K_1 + \alpha_2 K_2 - \alpha'_1 K_4 - \alpha'_2 K_5 &= 0, \\
 K_3 &= K_6, \\
 L_1(m_1 - \alpha_1)K_1 + L_1(m_2 - \alpha_2)K_2 + L'_1(m'_1 + \alpha'_1)K_4 + L'_1(m'_2 + \alpha'_2)K_5 &= 0, \\
 (C_1 m_1 \alpha_1 - F_1)K_1 + (C_1 m_2 \alpha_2 - F_1)K_2 + (C'_1 m'_1 \alpha'_1 + F'_1)K_4 \\
 + (C'_1 m'_2 \alpha'_2 + F'_1)K_5 &= 0, \\
 L_1 m_3 K_3 + L'_1 m'_3 K_6 &= 0.
 \end{aligned} \tag{20}$$

The third and the sixth conditions in (20) imply that there is no transverse component of displacement.

Eliminating constants K_1, K_2, K_4, K_5 from (20) we get

$$\det(a_{ij}) = 0, \quad i, j = 1, 2, 3, 4, \tag{21}$$

where

$$\begin{aligned}
 a_{11} &= 1, \quad a_{12} = 1, \quad a_{13} = -1, \quad a_{14} = -1 \\
 a_{21} &= \alpha_1, \quad a_{22} = \alpha_2, \quad a_{23} = -\alpha'_1, \quad a_{24} = -\alpha'_2 \\
 a_{31} &= L_1(m_1 - \alpha_1), \quad a_{32} = L_1(m_2 - \alpha_2), \\
 a_{33} &= L'_1(m'_1 + \alpha'_1), \quad a_{34} = L'_1(m'_2 + \alpha'_2) \\
 a_{41} &= c_1 m_1 \alpha_1 - F_1, \quad a_{42} = c_1 m_2 \alpha_2 - F_1, \\
 a_{43} &= C'_1 m'_1 \alpha'_1 + F'_1, \quad a_{44} = C'_1 m'_2 \alpha'_2 + F'_1.
 \end{aligned}$$

The roots of (21) determine the wave velocity of the generalized interface wave propagating along the common boundary of two inhomogeneous transversely isotropic elastic solid media. If we take $v = v' = 0$ i.e., a homogeneous medium, we obtain the equation obtained by Dey & Sengupta (1978), where in M_1

$$\begin{aligned}
 m_1^2 + m_2^2 &= \{L_1(\rho_1 c^2 - L_1) + C_1(\rho_1 c^2 - A_1) + (F_1 + L_1)^2\} \frac{1}{L_1 C_1}, \\
 m_1^2 \cdot m_2^2 &= (\rho_1 c^2 - A_1)(\rho_1 c^2 - L_1) \frac{1}{L_1 C_1}, \\
 \alpha_1 &= \frac{L_1 m_1^2 - (\rho_1 c^2 - A_1)}{(F_1 + L_1) m_1}, \\
 \alpha_2 &= \frac{L_1 m_2^2 - (\rho_1 c^2 - A_1)}{(F_1 + L_1) m_2},
 \end{aligned}$$

and similar expressions in M_2 .

5. Rayleigh waves

For Rayleigh waves, M_2 is replaced by vacuum. Hence from (20)

$$\begin{aligned}(m_1 - \alpha_1)K_1 + (m_2 - \alpha_2)K_2 &= 0, \\ (C_1 m_1 \alpha_1 - F_1)K_1 + (C_1 m_2 \alpha_2 - F_1)K_2 &= 0.\end{aligned}\quad (22)$$

Eliminating K_1, K_2 from (22) we get

$$(\alpha_2 - \alpha_1)(C_1 m_1 m_2 - F_1) + (m_1 - m_2)(C_1 \alpha_1 \alpha_2 - F_1) = 0. \quad (23)$$

Equation (23) gives the wave velocity for Rayleigh waves in a transversely isotropic nonhomogeneous medium, the nonhomogeneity being of the assumed forms (1), (2). It is evident from (23) that the values of m_1, m_2 depend on the particular value of η and hence the wave velocity c depends on η creating a dispersion of the general wave form.

Equation (23) is of the same form as obtained by Dey & Sengupta (1978) except that m_1, m_2 are different. Now m_1, m_2 depend upon ν as evident from (14) whereas for homogeneous media they are obtained from

$$\begin{aligned}m^4 - \{L_1(\rho_1 c^2 - L_1) + C_1(\rho_1 c^2 - A_1) + (F_1 + L_1)^2\} \frac{1}{L_1 C_1} m^2 \\ + \{(\rho_1 c^2 - A_1)(\rho_1 c^2 - L_1)\} \frac{1}{L_1 C_1} = 0.\end{aligned}\quad (24)$$

6. Love waves

We take the origin of the coordinate system at the interface between the upper stratum of thickness H ($-H \leq Z \leq 0$) and the lower semi-infinite medium ($0 \leq Z < \infty$) with x -axis and z -axis taken along the interface and vertically downwards respectively (figure 2).

For Love wave propagation along the x -axis we have $u = 0, w = 0, v = v(x, z, t)$. Therefore, the only equation of motion for medium M_1 is

$$N_1 \frac{\partial^2 v_1}{\partial x^2} + \nu L_1 \frac{\partial v_1}{\partial z} + L_1 \frac{\partial^2 v_1}{\partial z^2} = \rho_1 \frac{\partial^2 v_1}{\partial t^2}, \quad (25)$$

Figure 2. Love waves.

and for medium M_2

$$N_1' \frac{\partial^2 v_2}{\partial x^2} + v' L_1' \frac{\partial v_2}{\partial z} + L_1' \frac{\partial^2 v_2}{\partial z^2} = \rho_1' \frac{\partial^2 v_2}{\partial t^2}. \quad (26)$$

Putting $v_j = V_j(x, z)e^{i\lambda t}$ ($j = 1, 2$ and λ is the wave frequency) (25) and (26) reduce to

$$N_1 \frac{\partial^2 V_1}{\partial x^2} + v L_1 \frac{\partial V_1}{\partial z} + L_1 \frac{\partial^2 V_1}{\partial z^2} + \rho_1 \lambda^2 V_1 = 0, \quad (27)$$

$$N_1' \frac{\partial^2 V_2}{\partial x^2} + v' L_1' \frac{\partial V_2}{\partial z} + L_1' \frac{\partial^2 V_2}{\partial z^2} + \rho_1' \lambda^2 V_2 = 0. \quad (28)$$

Let $\bar{f}(K, Z)$ be the Fourier transform of $f(x, z)$ defined by the relation

$$\bar{f}(K, Z) = \int_{-\infty}^{\infty} f(x, z) e^{ikx} dx,$$

so that

$$f(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{f}(K, Z) e^{-ikz} dk. \quad (29)$$

Applying Fourier transformation defined by (29) we obtain equations of motion (27) and (28) in the form

$$\frac{d^2 \bar{V}_1}{dz^2} + v \frac{d \bar{V}_1}{dz} + P_1^2 \bar{V}_1 = 0, \quad (30)$$

$$\frac{d^2 \bar{V}_2}{dz^2} + v' \frac{d \bar{V}_2}{dz} + P_2^2 \bar{V}_2 = 0, \quad (31)$$

where

$$\begin{aligned} P_1^2 &= \frac{\rho_1}{L_1} \lambda^2 - \frac{N_1}{L_1} K^2, \\ P_2^2 &= \frac{\rho_1'}{L_1'} \lambda^2 - \frac{N_1'}{L_1'} K^2. \end{aligned} \quad (32)$$

We take solutions of (30) and (31) as follows.

$$\begin{aligned} \bar{V}_1 &= S e^{-\gamma_1 z}, \\ \bar{V}_2 &= \gamma e^{-\gamma_2 z} + T e^{\gamma_3 z}, \end{aligned} \quad (33)$$

where

$$\begin{aligned} \gamma_1 &= \frac{v}{2} - i \left(\frac{\lambda^2}{\beta_1^2} - \frac{N_1}{L_1} K^2 - \frac{v^2}{4} \right)^{1/2}, \\ \gamma_2 &= \frac{v'}{2} - i \left(\frac{\lambda^2}{\beta_2^2} - \frac{N_1'}{L_1'} K^2 - \frac{v'^2}{4} \right)^{1/2}, \\ \gamma_3 &= -\frac{v'}{2} - i \left(\frac{\lambda^2}{\beta_2^2} - \frac{N_1'}{L_1'} K^2 - \frac{v'^2}{4} \right)^{1/2}, \end{aligned} \quad (34)$$

$$\beta_1^2 = \frac{L_1}{\rho_1}, \quad \beta_2^2 = \frac{L_1'}{\rho_1'}. \quad (35)$$

Thus the displacements in these two media are

$$V_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} S e^{-\gamma_1 z} e^{-ikx} dk,$$

and

$$V_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} (Y e^{-\gamma_2 z} + T e^{\gamma_3 z}) e^{-ikx} dk. \quad (36)$$

Applying the boundary conditions

$$\left. \begin{aligned} (i) \quad & V_1 = V_2 \\ (ii) \quad & L_1 \frac{\partial V_1}{\partial z} = L'_1 \frac{\partial V_2}{\partial z} \end{aligned} \right\} \text{ on } z = 0, \quad (37)$$

and

$$(iii) \quad L'_1 \frac{\partial V_2}{\partial z} = 0, \text{ on } Z = -H,$$

we get

$$\begin{aligned} S - Y - T &= 0, \\ L_1 \gamma_1 S - L'_1 \gamma_2 Y + L'_1 \gamma_3 T &= 0, \\ \gamma_2 e^{\gamma_2 H} Y - \gamma_3 e^{-\gamma_3 H} T &= 0. \end{aligned} \quad (38)$$

Eliminating S, Y, T from the set of equations (38) and taking $\lambda = cK$, where c is the common wave velocity propagating along the x -axis we obtain the equation of dispersion as

$$\begin{aligned} & \tan \left\{ KH \left(\frac{c^2}{\beta_2^2} - \frac{N'_1}{L'_1} - \frac{v'^2}{4K^2} \right)^{1/2} \right\} \\ &= \left\{ \frac{L_1}{L'_1} \left[\frac{c^2}{\beta_2^2} - \frac{N'_1}{L'_1} - \frac{v'^2}{4K^2} \right]^{1/2} \left(\frac{v}{2K} + \left[\frac{N_1}{L_1} + \frac{v^2}{4K^2} - \frac{c^2}{\beta_1^2} \right]^{1/2} \right) \right\} \div \\ & \left\{ \frac{c^2}{\beta_2^2} - \frac{N'_1}{L'_1} - \frac{L_1}{L'_1} \cdot \frac{v'}{2K} \left(\frac{v}{2K} + \left[\frac{N_1}{L_1} + \frac{v^2}{4K^2} - \frac{c^2}{\beta_1^2} \right]^{1/2} \right) \right\}. \end{aligned} \quad (39)$$

For the real solution of (39) we note that

$$\left(\frac{N'_1}{L'_1} + \frac{v'^2}{4K^2} \right)^{1/2} < \frac{c}{\beta_2} < \frac{\beta_1}{\beta_2} \left(\frac{N_1}{L_1} + \frac{v^2}{4K^2} \right)^{1/2}. \quad (40)$$

If the nonhomogeneity of the material media be neglected, putting $v = v' = 0$ in the above dispersion equation, we obtain the wave velocity equation for Love waves in a transversely isotropic medium as

$$\begin{aligned} & \tan KH \left[\frac{c^2}{\beta_2^2} - \frac{N'_1}{L'_1} \right]^{1/2} \\ &= \frac{L_1}{L'_1} \left\{ \left[\frac{c^2}{\beta_2^2} - \frac{N'_1}{L'_1} \right]^{1/2} \left[\frac{N_1}{L_1} - \frac{c^2}{\beta_1^2} \right]^{1/2} \right\} \Bigg/ \left\{ \frac{N'_1}{L'_1} - \frac{c^2}{\beta_2^2} \right\}, \end{aligned} \quad (41)$$

Table 1. Numerical calculations for Love wave.
 $L_1 = 6.53 \times 10^6 \text{ N/cm}^2$, $L_1' = 4.0 \times 10^6 \text{ N/cm}^2$, $N_1 = 8.665 \times 10^6 \text{ N/cm}^2$, $N_1' = 6.30 \times 10^6 \text{ N/cm}^2$, $\rho_1 = 2.7 \text{ g/cm}^3$, $\rho_1' = 7.1 \text{ g/cm}^3$.

	Values of KH									
C/β_2	1.36817	1.48135	1.59453	1.70771	1.82090	1.93408	2.04726	2.16044	2.27362	2.27362
$\nu'/2k = \nu/2k = 0$	2.25904	1.37291	0.97403	0.73245	0.56448	0.43673	0.33302	0.24257	0.15426	0.15426
$\nu'/2k = 0.00999$ $\nu/2k = 0$	2.28940	1.38560	0.98100	0.73667	0.56698	0.43840	0.33404	0.24312	0.15449	0.15449
$\nu'/2k = 0$ $\nu/2k = 0.00999$	2.26512	1.37873	0.97959	0.73777	0.56940	0.44153	0.33764	0.24697	0.15877	0.15877
$\nu'/2k = \nu/2k = 0.00999$	2.29556	1.39147	0.98661	0.74203	0.57209	0.44323	0.33867	0.24754	0.15901	0.15901

where

$$\left(\frac{N'_1}{L'_1}\right)^{1/2} < \frac{c}{\beta_2} < \frac{\beta_1}{\beta_2} \left(\frac{N_1}{L_1}\right)^{1/2}. \quad (42)$$

From the above it is observed that the inhomogeneity of the material media has a distinct effect on the range of admissible values of the phase velocity of Love waves.

7. Stoneley waves

The generalized form of Rayleigh waves which propagate along the interface of M_1 and M_2 are known as Stoneley waves according to the classical theory. Hence, in the present case of anisotropic inhomogeneous elastic media, Stoneley waves propagating along the common boundary of M_1 and M_2 are determined by the roots of the wave velocity equation (21). It is seen from this equation that Stoneley wave velocity equation is influenced by the nonhomogeneous and anisotropic nature of the material media. Putting $\nu = \nu' = 0$ in (21) one obtains the wave velocity equation for Stoneley waves in transversely isotropic homogeneous elastic media.

8. Discussion

Table 1 shows that the inhomogeneity has an important role to play on Love wave propagation in transversely isotropic elastic media. The table also indicates that the phase velocity decreases with increase of KH , i.e. the thickness of the layer. Moreover, for a particular value of wave velocity, the values of KH increase with increase of the inhomogeneous character of material media.

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