Analysis of deformed microstrip resonator using the finite element method

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Abstract. Microstrip resonator with shape deformation has been analysed using the finite element method (FEM). Keeping the front surface of the microstrip resonator fixed, the back surface is deformed. Five cases of the deformation of the back surface are considered: (i) the left vertical side of the back surface is shifted inward; (ii) both vertical sides of the back surface are shifted inward; (iii) the right vertical side of the back surface is shifted outward; (iv) both vertical sides of the back surface are shifted outward; (v) left vertical side of back surface is shifted inward and the right vertical side of it is shifted outward. Variation in cutoff frequency for the TM_{011} , TM_{110} , TM_{111} , TM_{012} , TM_{112} and TM_{210} is observed.

Keywords. Microstrip resonator; finite element method.

1. Introduction

Finite element method (FEM) is a powerful tool for the approximate solutions of differential equations governing diverse physical problems. It can tackle problems of different branches in physics and it can be used for any complicated geometry or domain of regular or irregular shape. The method can deal with the problems described by coupled equation. The method is useful for inhomogeneous and anisotropic media also.

The resonant frequencies of open microstrip ring resonators are determined by Wolff & Menzel (1975) and Pintzos & Pregla (1978). The microstrip resonator with equilateral triangular patch is studied by Wolf & Knoppic (1974), Helszajn *et al* (1979), Lyon & Helszajn (1982). The triangular and rectangular patch microstrip resonator has been analysed by Kalamse & Patil (1994) using the finite element method.

In this paper we have analysed the microstrip resonator with shape deformation for five cases:

- (i) the left vertical side of the back surface is shifted inward;
- (ii) both vertical sides of the back surface are shifted inward;

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- (iii) the right vertical side of the back surface is shifted outward;
- (iv) both vertical sides of the back surface are shifted outward;
- (v) left vertical side of back surface is shifted inward and the right vertical side of it is shifted outward.

2. Statement of the problem

Consider the rectangular microstrip resonator bounded by six faces B1, B2, B3, B4, B5, B6, as shown in figure 1a.

The two side surfaces B₁, B₂ and front and back surfaces B3, B4 are magnetic walls. The top and the bottom surfaces are electric walls.

The electric field **E** within the resonator will satisfy Maxwell's equations,

$$\text{Curl Curl } \mathbf{E} - K^2 \mathbf{E} = 0, \quad (1)$$

where $K^2 = \omega^2 \mu_0 \epsilon_0$

$$\text{grad div } \mathbf{E} - \nabla^2 \mathbf{E} - K^2 \mathbf{E} = 0. \quad (2)$$

Since the medium is charge free, $\text{div } \mathbf{E} = 0$.

$$\nabla^2 \mathbf{E} + K^2 \mathbf{E} = 0. \quad (3)$$

The electric field within the substrate has only the Z component and the magnetic field has the X and Y components. The tangential component of the magnetic field at the edge is negligible,

Figure 1. (a) Microstrip resonator without deformation. (b) Left vertical side of the back surface is shifted inward. (c) Both vertical sides of the back surface are shifted inward. (d) Right vertical side of the back surface is shifted outward. (e) Both vertical sides of the back surface are shifted outward. (f) Left vertical side of the back surface is shifted inward and right vertical side of the back surface is shifted outward.

$$\operatorname{div} \operatorname{grad} E_z + K^2 E_z = 0. \quad (4)$$

The fields within the resonator corresponding to TM modes will be generated by the equation,

$$\nabla^2 E_z + K^2 E_z = 0, \quad (5)$$

subjected to the boundary condition

$$\left. \frac{\partial E_z}{\partial n} \right|_{B_1, B_2, B_3, B_4} = 0, \quad (6)$$

$$E_z|_{B_5, B_6} = 0, \quad (7)$$

where E_z is the z component of E , and $\partial/\partial n$ represents normal derivative.

3. Variational formulation

To get the expression for the functional Π in variational formulation, multiply (4) by some weight function V and integrate it over the domain of the resonator,

$$\Pi = \int \int_{\Omega} \int V \operatorname{div} \operatorname{grad} E_z \, d\Omega + K^2 \int \int_{\Omega} \int V E_z \, d\Omega. \quad (8)$$

Using the vector identity $S \operatorname{div} \mathbf{A} = \operatorname{div}(S\mathbf{A}) - (\operatorname{grad} S) \cdot \mathbf{A}$ for the first term, (8) becomes

$$\begin{aligned} \Pi &= \int \int_{\Omega} \int \operatorname{div}(V \operatorname{grad} E_z) \, d\Omega - \int \int_{\Omega} \int (\operatorname{grad} V) \cdot (\operatorname{grad} E_z) \, d\Omega \\ &\quad + K^2 \int \int_{\Omega} \int V E_z \, d\Omega. \end{aligned} \quad (9)$$

Applying Gauss Divergence theorem to the first term in (9), we get

$$\begin{aligned} \Pi &= - \int \int_{\Omega} \int (\operatorname{grad} V) \cdot (\operatorname{grad} E_z) \, d\Omega + K^2 \int \int_{\Omega} \int V E_z \, d\Omega \\ &\quad + \int_S \int V \operatorname{grad} E_z \cdot \bar{n} \, ds. \end{aligned} \quad (10)$$

Using (6), the last term in (10) will vanish.

Table 1. Variation of cutoff frequency for different modes for the inner deformation of one vertical side.

Shift	Frequency (GHz)					
L (cm)	TM ₀₁₁	TM ₁₁₀	TM ₁₁₁	TM ₀₁₂	TM ₁₁₂	TM ₂₁₀
0.05	13.607	14.016	15.005	16.470	17.641	17.906
0.10	13.606	14.046	15.035	16.470	17.663	17.992
0.15	13.605	14.077	15.066	16.470	17.685	18.073
0.20	13.604	14.107	15.100	16.470	17.707	18.148
0.25	13.603	14.139	15.136	16.470	17.728	18.219
0.30	13.601	14.170	15.175	16.470	17.749	18.287
0.35	13.599	14.202	15.217	16.471	17.770	18.351
0.40	13.597	14.233	15.261	16.473	17.792	18.413

Table 2. Variation of cutoff frequency for different modes for the inner deformation of both vertical sides.

Shift	Frequency (GHz)					
L (cm)	TM ₀₁₁	TM ₁₁₀	TM ₁₁₁	TM ₀₁₂	TM ₁₁₂	TM ₂₁₀
0.05	13.607	14.045	15.035	16.469	17.667	17.990
0.10	13.607	14.103	15.104	16.468	17.719	18.130
0.15	13.607	14.161	15.184	16.466	17.778	18.269
0.20	13.608	14.218	15.277	16.464	17.842	18.379
0.25	13.608	14.272	15.386	16.461	17.915	18.477
0.30	13.609	14.324	15.513	16.458	17.998	18.567
0.35	13.611	14.372	15.659	16.455	18.093	18.650
0.40	13.612	14.417	15.825	16.451	18.203	18.728

By substituting $V = E_z^*$ and changing the sign, (10) becomes

$$\Pi = \frac{1}{2} \int \int_{\Omega} \int (\text{grad } E_z) \cdot (\text{grad } E_z^*) d\Omega - K^2 \frac{1}{2} \int \int_{\Omega} \int E_z E_z^* d\Omega. \quad (11)$$

$\frac{1}{2}$ is introduced since Π is bilinear functional.

The first variation $\delta\Pi$ is given by,

$$\delta\Pi = \delta \frac{1}{2} \left[\int \int_{\Omega} \int (\nabla E_z) \cdot (\nabla E_z^*) d\Omega - K^2 \int \int_{\Omega} \int E_z E_z^* d\Omega \right]. \quad (12)$$

For Π to be stationary, $\delta\Pi$ should be minimum.

4. Discretization

According to the finite element method (Reddy 1986; Akin 1988), the volume of the resonator is divided into hexahedral elements with 20 nodes. The mapping functions assumed for these elements are quadratic in nature. The functional over an element is given by

$$\Pi^e = \sum_{\text{ele}} \frac{1}{2} \{E_z^e\}^T [S^e] \{E_z^e\} - \frac{1}{2} K^2 \sum_{\text{ele}} \{E_z^e\}^T [T^e] \{E_z^e\}, \quad (13)$$

Table 3. Variation of cutoff frequency for different modes for the outer deformation of one vertical side.

Shift	Frequency (GHz)					
L (cm)	TM ₀₁₁	TM ₁₁₀	TM ₁₁₁	TM ₀₁₂	TM ₁₁₂	TM ₂₁₀
0.05	13.607	13.957	14.950	16.470	17.595	17.719
0.10	13.606	13.930	14.925	16.469	17.570	17.622
0.15	13.604	13.903	14.902	16.469	17.551	17.515
0.20	13.602	13.877	14.879	16.469	17.522	17.416
0.25	13.599	13.853	14.857	16.468	17.495	17.315
0.30	13.595	13.831	14.836	16.467	17.468	17.214
0.35	13.590	13.810	14.816	16.465	17.438	17.116
0.40	13.583	13.792	14.797	16.463	17.410	17.019

Table 4. Variation of cutoff frequency for different modes for the outer deformation of both vertical sides.

Shift	Frequency (GHz)					
L (cm)	TM ₀₁₁	TM ₁₁₀	TM ₁₁₁	TM ₀₁₂	TM ₁₁₂	TM ₂₁₀
0.05	13.607	13.928	14.926	16.469	17.574	17.620
0.10	13.607	13.872	14.882	16.468	17.533	17.416
0.15	13.607	13.817	14.843	16.465	17.494	17.211
0.20	13.607	13.765	14.808	16.461	17.457	17.014
0.25	13.607	13.714	14.777	16.453	17.422	16.826
0.30	13.606	13.666	14.749	16.437	17.388	16.656
0.35	13.606	13.621	14.723	16.388	17.355	16.528
0.40	13.606	13.577	14.700	16.269	17.323	16.479

where

$$S_{ij} = \int \left[\frac{\partial F_i}{\partial x} \frac{\partial F_j}{\partial x} + \frac{\partial F_i}{\partial y} \frac{\partial F_j}{\partial y} + \frac{\partial F_i}{\partial z} \frac{\partial F_j}{\partial z} \right] dx dy dz \quad (14)$$

and

$$T_{ij} = \int F_i F_j dx dy dz. \quad (15)$$

Here F_i is the mapping function due to the i th node and integrations are over the mesh element surface.

The functional for the whole region Ω is given by

$$\Pi = \frac{1}{2} \{E_z\}^T [S] \{E_z\} - \frac{1}{2} K^2 \{E_z\}^T [T] \{E_z\}. \quad (16)$$

The condition that variation of Π must be minimum i.e. zero, gives

$$[S] \{E_z\} - K^2 [T] \{E_z\} = 0. \quad (17)$$

Equation (17) is the matrix equation to be solved to get eigenvalues and eigenvectors.

5. Numerical calculations for microstrip resonator

A microstrip resonator of sides $2.4 \text{ cm} \times 1.2 \text{ cm} \times 2.8 \text{ cm}$ is considered. The deformation for each case is shown in figure 1(b–f).

Table 5. Variation of cutoff frequency for different modes for the inner deformation of one vertical side and outer deformation of other vertical side.

Shift	Frequency (GHz)					
L (cm)	TM ₀₁₁	TM ₁₁₀	TM ₁₁₁	TM ₀₁₂	TM ₁₁₂	TM ₂₁₀
0.05	13.606	13.987	14.976	16.470	17.615	17.816
0.10	13.603	13.991	14.973	16.471	17.605	17.820
0.15	13.599	13.996	14.969	16.472	17.588	17.827
0.20	13.593	14.004	14.964	16.474	17.565	17.836
0.25	13.586	14.013	14.956	16.476	17.537	17.848
0.30	13.577	14.025	14.948	16.479	17.505	17.862
0.35	13.568	14.037	14.938	16.482	17.469	17.879
0.40	13.557	14.051	14.926	16.485	17.430	17.898

Figure 2. Variation of cutoff frequencies for different modes for the inner deformation of one vertical side of back surface.

Figure 3. Variation of cutoff frequencies for different modes for the inner deformation of both vertical sides of back surface.

Figure 4. Variation of cutoff frequencies for different modes for the outer deformation of one vertical side of back surface.

Figure 5. Variation of cutoff frequencies for different modes for the outer deformation of both vertical sides of back surface.

Figure 6. Variation of cutoff frequencies for different modes for the inner deformation of one vertical side and outer deformation of other vertical side of back surface.

- (i) First case: Left vertical side of the back surface of the microstrip resonator is shifted inward by 0.05 cm.
- (ii) Second case: Both vertical sides of the back surface of the microstrip resonator are shifted inward by 0.05 cm.
- (iii) Third case: Right vertical side of the back surface of the microstrip resonator is shifted outward by 0.05 cm.
- (iv) Fourth case: Both vertical sides of the back surface of the microstrip resonator are shifted outward by 0.05 cm.
- (v) Fifth case: Left vertical side of the back surface is shifted inward by 0.05 cm and the right vertical side is shifted outward by 0.05 cm.

In each case, for every shift, eigenvalues and eigenvectors are obtained. TM_{011} , TM_{110} , TM_{111} , TM_{012} , TM_{112} and TM_{210} modes are identified using field plots. These eigenvalues are the values of $\omega^2 \mu_0 \epsilon_0$. From this, the corresponding cutoff frequencies are calculated in gigahertz.

For every shift, in each case, variation of cutoff frequency with the shift for the above modes is given in tables 1–5 and shown graphically in figures 2–6.

6. Conclusions

For the inner deformation of one or both vertical sides of the back surface, the cutoff frequencies increase with the shift for all the modes, except for TM_{011} and TM_{012} where they are almost constant.

For the outer deformation of one or both vertical sides of the back surface, the cutoff frequencies decrease with the shift for all the modes.

For the inward deformation of one vertical side and outward deformation of the other vertical side, values of cutoff frequency for the TM_{011} , TM_{111} and TM_{112} modes decrease with the shift, while for the TM_{110} , TM_{012} and TM_{210} modes they increase.

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