Surface characterization and parameter estimation for industrial vision system

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Abstract. In this paper we have presented some geometric techniques to characterize and parametrize surfaces of industrial parts in range images. The surfaces are characterized to one of plane, sphere, cylinder and cone, because they form the majority of object surfaces in man-made industrial parts. The problem has been studied for two different situations. In the first case, *a priori* knowledge about the surface shape is assumed. In such a situation the problem of surface characterization reduces to that of surface parameter estimation. The standard deviations of the estimated parameters give a measure of uncertainty of characterizing a surface patch to one of the four surface types. In the second case, no *a priori* information regarding the shape of a surface is available. This includes partially visible surfaces also. To deal with such a situation, a fuzzy classifier is designed using the uncertainty values. The fuzzy classifier classifies the unknown surface patch (including partially visible surfaces) to one of the four surface types. Experimental results with synthetic range images are presented to highlight the distinctive features of our technique.

Keywords. Range image; geometric surface fitting; parameter estimation; fuzzy classification; occlusion.

1. Introduction

The major aim of any computer vision system is to recover useful information about the three dimensional world from huge image arrays of sensed values. In the past these image arrays were digital intensity images. At each point in an intensity image the brightness value encodes information about surface geometry, surface reflectance characteristics, surface texture, scene illumination, the distance from the camera to an object surface, the characteristics of the intervening medium and the camera characteristics. However, such images do not contain explicit information about depth.

In recent years digital range images (also known as $2\frac{1}{2}$ -D images) have become available owing to the development of various active and passive range-finding

techniques. Range data are produced in the form of an array of numbers, where the numbers quantify the distance from the sensor focal plane to the object surface within the field of view along rays emanating from points on a regularly spaced grid. Thus the range images provide direct geometrical information about the shape of visible surfaces. Therefore, the process of recognising objects by their shape is less difficult in range images than in intensity images. Such images find wide application in industrial automation such as assembly automation, automated inspection of industrial parts etc.

Surface characterization and parameter estimation are two of the major problems of computer vision utilising range images in industrial environment. The estimated surface parameters can be used efficiently by the matching process for object recognition and pose determination even in presence of many undefined or occluded surface regions. A number of approaches for surface characterization and parameter extraction have been reported in the literature. The optimization and eigenvalue analysis (Faugeras 1984) approach employs global analysis of 3-D or $2\frac{1}{2}$ -D (range) data. This approach is more general in the sense that it is applicable to all sorts of quadrics and planar surfaces. Characterization by quadric invariants (e.g. sign of curvatures) (Besl & Jain 1984, 1988; Brady *et al* 1985) employs computation of differential properties of the surface at each surface point. This technique requires spatial ordering of surface points. Surfaces are characterized according to the signs of mean and Gaussian curvatures.

Though eigenvalues or quadric invariants are good discriminators in a mathematical sense, they are often very sensitive to noise and quantization errors (Han *et al* 1987). Moreover in man-made industrial parts, the majority of object surfaces belong to planar, spherical, cylindrical and conical surfaces (Bolle & Cooper 1984). Hence, for such applications it is not necessary to work with general quadric surfaces but rather to deal with a narrow domain of surface types, such as plane, sphere, cylinder and cone. Hough transform (Muller & Mohr 1984) or modified Hough transform (Lin & Wee 1985) techniques have been used in such cases for surface characterization purposes.

Parametric estimation technique by Han *et al* (1987) calculates the geometric properties of a sufficient set of shapes, namely plane, sphere and cylinder, based upon an analysis of the surface normals. This approach uses a mixture of histogramming and parameter-fitting techniques to estimate surface parameters.

We have presented in this paper some new geometric techniques which use transformation invariant properties of surface normals and surface tangents to obtain the description of an unknown surface in terms of plane, sphere, cylinder or cone. This work presumes surfaces in range images are already segmented. Range image segmentation is not addressed here. A number of techniques for range image segmentation have been discussed by Biswas *et al* (1990). Unlike Han *et al* (1987) who use histogram analysis of surface normals to extract surface regions of a particular type, our technique uses mean and standard deviation of parameters characterizing different surfaces. The standard deviation of a parameter represents the measure of uncertainty of fitting a particular type of surface with respect to that parameter. The problem has been attempted for two different situations. In the first case, it has been assumed that *a priori* knowledge about the shape of the surface is available. In such situations, a surface of proper type is fitted to the set of points representing the surface patch and the parameters of the fitted surface are extracted. This strategy of selective surface fitting reduces the amount of computation. In the second case, when no *a priori* information regarding the shape of the unknown surface is available, the characterization of the surface is obtained in the

form of a fuzzy membership vector. A fuzzy classifier is designed using the uncertainty measures which classifies an unknown surface to one of the surface types.

Geometric surface fitting and parameter estimation is discussed in §2. Section 3 describes the fuzzy classifier to classify an unknown surface. Occluded surface characterization is discussed in §4.

2. Surface characterization through geometric method

The geometric method described in this section is guided by *a priori* information regarding the shape of a surface patch to fit an appropriate surface and extract the parameters of the fitted surface. The parameters extracted for each of planar, spherical, cylindrical and conical surfaces are the surface normal and the perpendicular distance from the origin for a plane, the centre coordinates and the radius for a sphere, the axis and the radius for a cylinder and the apex, the apex angle and the axis for a cone. The geometric method of surface fitting for these surfaces is facilitated from the existence of certain unique and transformation invariant properties of their surface normals and surface tangents as listed below:

- Plane:* All surface normals are parallel (to the normal to the plane).
- Sphere:* All surface normals are concurrent (at the centre of the sphere).
- Cylinder:* All surface normals are perpendicular to the axis of the cylinder.
- Cone:* All tangential planes to the surface intersect at a point (at the apex of the cone).

In the following discussion the input is taken to be a set of M points in three dimensional space. The i th point is referred to as $X_i \equiv (x_i, y_i, z_i)$ or equivalently by $\mathbf{X}_i$, the vector joining the origin $(0, 0, 0)$ to (x_i, y_i, z_i) . The set of corresponding normals is taken to be $\{\mathbf{N}_i | i = 1, 2, \dots, M\}$. The normal $\mathbf{N}_i$ at point $\mathbf{X}_i$ is computed by fitting a plane to the 5×5 neighbourhood of X_i using least square error method (Taylor *et al* 1989).

2.1 Characterization of a plane

A plane is characterized by the direction of the normal to the plane and the perpendicular distance of the plane from the origin (refer figure 1). The estimate of the normal to the plane approximating the set of input points is given by

$$\mathbf{N}_{av} = \sum_{i=1}^M \mathbf{N}_i \left/ \left| \sum_{i=1}^M \mathbf{N}_i \right| \right., \quad (1)$$

where $\mathbf{N}_i$ is the unit normal vector at point X_i and 'Σ' in (1) represents vector addition. The uncertainty in the estimation of the surface normal is given by the standard deviation σ_N of $\mathbf{N}_{av}$ where

$$\sigma_N^2 = \frac{1}{M} \sum_{i=1}^M |\mathbf{N}_i - \mathbf{N}_{av}|^2, \quad (2)$$

where $|\cdot|$ represents the magnitude of a vector. The lesser the value of σ_N , the more reliable is the normal estimate. The perpendicular distance d of the plane from the

Figure 1. Direction of normal to a plane and the perpendicular distance of the plane from the origin.

origin and the standard deviation σ_d of d are given by

$$d = \frac{1}{M} \sum_{i=1}^M (\mathbf{N}_{av} \cdot \mathbf{X}_i), \quad (3)$$

$$\sigma_d^2 = \frac{1}{M} \sum_{i=1}^M (\mathbf{N}_{av} \cdot \mathbf{X}_i - d)^2. \quad (4)$$

The values of σ_N and σ_d together reflect the uncertainty of fitting a plane to the set of points.

2.2 Characterization of a sphere

Ideally, all the surface normals pass through the centre of a sphere. But in practice, owing to the presence of noise in the range data and to computational round-off errors, such a unique point cannot be determined. Rather, the majority of surface normals pass through a small neighbourhood of the centre of the sphere. So, for every pair of nonparallel normals $\mathbf{N}_i$ and $\mathbf{N}_j$ (refer figure 2), the position of the line perpendicular to both of them is determined (Shanti Narayan 1985). The length of the line segment intercepted between the two normal vectors also gives the shortest distance between them. The points of intersection p_i and p_j of this common perpendicular with the two normals are taken as two estimates of the centre of the sphere. The mean position of the centre $\mathbf{X}_c$ and the standard deviation σ_x are computed from these estimates.

Once the position of the centre is found, the radius R can be estimated as

$$R = \frac{1}{M} \sum_{i=1}^M |\mathbf{X}_i - \mathbf{X}_c|. \quad (5)$$

Figure 2. Estimation of the centre of a sphere.

The standard deviations of the centre σ_x and the radius σ_R together give a measure of uncertainty in fitting the sphere to the set of points.

2.3 Characterization of a cylinder

The parameters of interest in this case are the orientation and position of the axis and the radius of the cylinder. Every normal on the surface of a cylinder is perpendicular to the direction of the axis. Thus the cross-product of any two nonparallel surface normals gives an estimate of the orientation of the cylinder axis. The cross-products are calculated for all possible pairs of nonparallel surface normals. The average orientation of all these cross-products determines the candidate axis direction. Thus, $\forall i, j$ where $\mathbf{N}_i$ and $\mathbf{N}_j$ are nonparallel, the k th estimate of axis orientation is given by the vector τ_k where

$$\tau_k = (\mathbf{N}_i \times \mathbf{N}_j) / |\mathbf{N}_i \times \mathbf{N}_j|, \quad (6)$$

From these estimates the average orientation τ_{av} and the standard deviation σ_τ are computed. The position of the axis is determined by finding a point on the axis which is done in the following manner.

At each point $\mathbf{X}_i$, the plane on which $\mathbf{X}_i$, $\mathbf{N}_i$ and τ_{av} lie is determined. The intersection of two such nonparallel planes (Shanti Narayan 1985) gives an estimate of the axis position (refer figure 3). From these estimates average estimate $\mathbf{X}_{av}$ and standard deviation σ_x are computed.

A point to be noted is that to get an estimate of the same point on the axis in all cases, the estimates are considered for the point at which the cylinder axis intersects the $x = 0$ plane if axis is not parallel to that plane, else the $y = 0$ plane if axis is not parallel to that plane, else the $z = 0$ plane.

Once the orientation and the position of the axis are fixed, at each point $\mathbf{X}_i$, the radius R_i of the cylinder (refer figure 4) can be estimated as follows.

$$R_i = |\mathbf{X}_i - \mathbf{X}_{av}| \sin \theta = |(\mathbf{X}_i - \mathbf{X}_{av}) \times \tau_{av}|, \quad (7)$$

From different estimates of R_i we compute R_{av} and σ_R .

The standard deviations of the axis orientation σ_τ , axis position σ_x and the radius σ_R together give a measure of the uncertainty of fit.

Figure 3. Estimation of the axis position of a cylinder.

2.4 Characterization of a cone

A cone is described by the position of its apex, orientation of its axis and the apex angle. Ideally the point of intersection of all tangential planes specifies the apex point.

Figure 4. Estimation of the radius of a cylinder.

However in practice, such a unique point cannot be determined due to the same reasons as stated earlier. Hence, the apex is estimated in the following manner. For all points X_i with normals N_i , the equation of the plane, tangent to the surface at X_i is obtained. Taking the intersection of three such pairwise nonparallel planes (Shanti Narayan 1985) an estimate V_k of the apex is obtained. Average of all such estimates gives the mean apex position V_{av} . The uncertainty in the estimation of the position of apex is given by the standard deviation σ_v of apex.

Once the position of the apex (V_{av}) is fixed, the axis of the conical surface is estimated in the following manner. $\forall i, 1 \leq i \leq M$, the normal h_i to the plane containing the points V_{av} , X_i and N_i (normal to the surface at X_i) is determined as follows (refer figure 5).

$$h_i = (X_i - V_{av}) \times N_i. \quad (8)$$

The orientation of the axis of the cone is given by the cross product of two such nonparallel vectors h_i and h_j , where $1 \leq i, j \leq M$ and $i \neq j$. Thus $\forall i, j$, where h_i and h_j are not parallel, the k th estimate of the axis orientation is given by the vector τ_k where

$$\tau_k = (h_i \times h_j) / |h_i \times h_j|, \quad (9)$$

From these estimates the expected orientation τ_{av} and the standard deviation σ_τ are computed.

Once the apex V_{av} and the axis orientation are available, for each point X_i an estimate of the semi apex angle γ_i is obtained

$$\sin \gamma_i = |(X_i - V_{av}) \times \tau_{av}| / |X_i - V_{av}|, \quad (10)$$

From different estimates of γ we can compute average semi apex angle γ_{av} and standard deviation σ_γ . The standard deviations of the apex, the axis and the semi apex angle together give a measure of uncertainty in fitting the conical surface to the set of points.

Figure 5. Estimation of the axis of a cone.

Table 1. Geometric surface fitting.

a) Plane samples.
 N normal to the plane; l, m, n direction cosines of the normal; d perpendicular distance from the origin; % error (N) = $\frac{1}{3}$ (% error(l) + % error(m) + % error(n)).

Sample	Original parameters				Estimated parameters				%Error in estimation		Standard deviation of estimates	
	l	m	n	d	l	m	n	d	N	d	N	d
PL-1	0.24	0.24	0.94	25.0	0.24	0.24	0.94	24.45	0.00	2.20	0.03	0.27
PL-2	0.45	0.45	0.77	25.0	0.45	0.45	0.77	24.66	0.00	1.36	0.02	0.22
PL-3	0.61	0.61	0.50	25.0	0.61	0.61	0.50	24.72	0.00	1.12	0.00	0.14
PL-4	0.23	0.59	0.78	16.67	0.23	0.59	0.78	16.40	0.00	1.62	0.01	0.21
PL-5	0.00	0.00	1.00	25.00	0.00	0.00	1.00	25.00	0.00	0.00	0.00	0.00

b) Sphere samples
 Centre centre of the sphere; x_c, y_c, z_c coordinates of the centre; R radius of the sphere; %error(centre) = $\frac{1}{3}$ (% error(x_c) + %error(y_c) + % error(z_c)).

Sample	Original parameters				Estimated parameters				%Error in estimation		Standard deviation of estimates	
	x_c	y_c	z_c	R	x_c	y_c	z_c	R	Centre	R	Centre	R
SP-1	25.00	25.00	25.00	25.0	25.02	25.02	25.19	24.45	0.31	2.2	3.11	0.28
SP-2	25.00	25.00	25.00	30.00	24.96	24.96	26.24	30.97	1.76	3.23	7.21	0.35
SP-3	25.00	25.00	25.00	9.0	25.00	25.00	24.99	9.1	0.01	1.11	1.76	0.17
SP-4	25.00	25.00	25.00	22.0	25.04	25.04	24.22	21.82	1.14	0.82	3.02	0.24
SP-5	25.00	25.00	25.00	41.00	24.92	24.92	25.50	41.03	0.88	0.07	2.95	0.25

c) Cylinder samples.

R radius of the cylinder; l, m, n direction cosines of the axis of the cylinder; x, y, z a point on the axis; % error (axis) = $\frac{1}{3}$ (% error(l) + % error(m) + % error(n)); % error(point) = $\frac{1}{3}$ (% error(x) + % error(y) + % error(z)).

Sample	Original parameters						Estimated parameters						%Error in estimation			Standard deviation of estimates				
	Original parameters			Estimated parameters			%Error in estimation			Standard deviation of estimates										
	l	m	n	x	y	z	R	l	m	n	x	y	z	R	Axis	Point	R	Axis	Point	R
CL-1	0.71	0.71	0	0	6	13	15	0.71	0.71	0	0	6.49	17.67	14.63	0	14.69	2.45	0.00	10.57	0.72
CL-2	0.66	0.66	0.34	0	6	10.94	30	0.69	0.64	0.33	0	6.94	10.63	29.02	2.5	9.3	3.3	0.30	45.98	0.39
CL-3	0.54	0.54	0.64	0	6	8.25	9	0.57	0.52	0.63	0	7.68	9.66	9.30	3.6	22.5	3.3	0.33	62.01	0.39
CL-4	0.35	0.35	0.87	0	6	3.20	22	0.37	0.34	0.86	0	6.54	2.81	21.47	3.2	10.6	2.4	0.25	48.59	0.42
CL-5	0.12	0.12	0.98	0	6	-19.08	41	0.13	0.12	0.98	0	5.42	-26.40	40.12	2.8	24.0	2.1	0.18	96.45	0.19

d) Cone samples.

θ semi apex angle of cone; l, m, n direction cosines of the axis of the cone; x_v, y_v, z_v apex of the cone; % error(axis) = $\frac{1}{3}$ (% error(l) + % error(m) + % error(n)); % error(apex) = $\frac{1}{3}$ (% error(x_v) + % error(y_v) + % error(z_v)).

Sample	Original parameters						Estimated parameters						%Error in estimation			Standard deviation of estimates				
	l	m	n	x_v	y_v	z_v	θ	l	m	n	x_v	y_v	z_v	θ	Axis	Apex	θ	Axis	Apex	θ
CN-1	0	1	0	4	10	13	30°	-0.09	0.97	0.20	5.74	9.94	13.74	28.71°	11.5	16.60	4.28	0.42	66.67	3.56
CN-2	0	0.34	0.94	4	10	13	20°	0.00	0.35	0.94	4.08	10.34	13.43	20.3°	1.5	2.90	1.5	0.12	20.41	0.36
CN-3	0	0.94	0.34	4	10	13	30°	-0.04	0.93	0.35	3.96	9.02	13.28	22.39°	2.0	3.8	11.9	0.31	24.37	0.95
CN-4	0	0.77	0.64	4	10	13	30°	0.01	0.71	0.7	4.06	13.31	13.16	34°	8.6	11.9	13.3	0.36	20.46	1.74
CN-5	0	0	1	4	10	13	40°	0.09	0.06	0.95	6.4	11.43	10.11	46.45°	3.5	32.18	16.1	0.24	43.88	5.33

2.5 Experimental results

Experimental results on synthetic range images of size 50×50 are presented. To reduce the amount of computation, surface normals are computed at some sample points, distributed uniformly over the surface patch. At each sample point, surface normal is estimated by fitting a plane to a 5×5 neighbourhood of the point, using least square error method. A set of five images from each of the classes plane, sphere, cylinder and cone are considered. For each sample the corresponding surface is fitted geometrically. The results obtained are tabulated in table 1.

It is observed from table 1 that the percentage of errors in the estimated parameter values are quite low for plane and sphere fitting ($< 3.5\%$). For the cylinder samples the percentage errors in the estimations of axis orientation are low ($< 3.6\%$) and also in the estimations of radius ($< 3.3\%$). Estimation of a point on the axis has given a large error in most cases (10.6–24%). This is due to the fact that even a slight error in estimating the axis orientation can significantly change the estimated point position. In the case of cone samples, the error in estimation of axis orientation is quite low (1.5–11.5%). Angle estimation also has low percentage error (1.5–16.1%). The error in estimation of apex position has shown wide variation (2.9% to 32.2%). From the results on synthetic range images it can be said that errors in an orientation estimation (e.g., axis, normal) or a distance estimation (e.g., radius) have been less as compared to estimation of a point position (e.g., apex, centre).

3. Fuzzy characterization of surfaces

In the previous section on geometric surface fitting it has been shown how an appropriate surface may be fitted when the input image is of planar, spherical, cylindrical or conical types. The implicit assumption has been that *a priori* information regarding the surface nature is available. But even in the absence of *a priori* information the geometric surface fitting proves to be useful. In such cases all four surfaces are fitted to the set of points and in each case the uncertainties, as given by the standard deviations of the parameters, are noted. These measures of uncertainty in parameter estimation are used to classify the surface as a planar or spherical or cylindrical or conical surface. The details of the method involved are discussed in this section.

The simplest strategy to design the classifier is to assign the input image to that class whose parameters have been estimated with the least amount of uncertainty. However, the solution is not so straightforward because parameters for different types of surfaces are obtained with different methods of estimation. Also the numbers of parameters for different types of surfaces are different. This has led to the development of a fuzzy classifier, where two thresholds are set for the standard deviation of each parameter of a particular surface. The lower threshold denotes the maximum uncertainty expected when the surface patch in the input image is of the same type as the surface fitted. The upper threshold denotes the least amount of uncertainty expected when the input is from any class other than that of the surface fitted. The input is assigned a membership to the class by comparing the standard deviation values obtained for a parameter of the fitted surface with the corresponding thresholds. Fuzziness in the classification is introduced by grading the membership if a standard deviation value falls between the two thresholds. Finally, the membership values obtained with each parameter of a fitted surface type are

combined to get an overall membership. This procedure is repeated for each of the four surface types namely, planar, spherical, cylindrical and conical surfaces. The result is a membership vector giving the membership of the input surface with respect to the four different classes. The vector itself can be taken as the characterization of the surface. Alternatively, the sample is classified into the class with the maximum membership value (the dominant class) and characterized by the corresponding parameters. Designing the classifier involves finding the thresholds, the details of which are discussed below.

3.1 Classifier design

A set of images, henceforth referred to as training images or samples, are taken from each of the four classes. All four surfaces are fitted geometrically to each of the samples. In each case the parameters and their standard deviations are noted. The fact that for any particular type of fitted surface, samples from the same class should yield lower standard deviation values as compared to samples from other classes, is used to obtain the lower thresholds. If θ is a parameter of a class denoted by class(θ), then the lower threshold for θ is defined as

$$lo_{\theta} = \text{mean}(\sigma_{\theta_i}) + \text{standard deviation}(\sigma_{\theta_i}), \quad (11)$$

where

$$\begin{aligned} i &= 1 \dots M \text{ is the set of sample images from class}(\theta), \\ \theta_i &= \text{value of parameter } \theta \text{ for the } i\text{th sample,} \\ \sigma_{\theta_i} &= \text{standard deviation in estimating } \theta_i. \end{aligned}$$

The upper threshold for parameter θ can be set by considering the standard deviations obtained with sample images from all classes other than class(θ). Thus the upper threshold is defined as

$$hi_{\theta} = \min(\sigma_{\theta_k}), \quad (12)$$

for all k , where $k = 1 \dots N$ is the set of samples not belonging to class(θ) and $\sigma_{\theta_k} > lo_{\theta}$.

The value of lo_{θ} has been selected considering the distribution of standard deviation values of the samples from the same class as that of the surface fitted. The value of hi_{θ} has been selected in order to reduce the misclassification of samples from other classes into the class of the surface fitted.

The membership function for class(θ) with respect to the parameter θ can be defined as μ_{θ} where

$$\begin{aligned} \mu_{\theta} &= 1, & \text{if } \sigma_{\theta} \leq lo_{\theta}, \\ \mu_{\theta} &= (hi_{\theta} - \sigma_{\theta}) / (hi_{\theta} - lo_{\theta}), & \text{if } lo_{\theta} \leq \sigma_{\theta} \leq hi_{\theta}, \\ \mu_{\theta} &= 0, & \text{if } hi_{\theta} \leq \sigma_{\theta}. \end{aligned} \quad (13)$$

The membership function μ_{θ} is graphically shown in figure 6.

Overall membership of a sample in a class can now be defined in terms of the membership with respect to individual parameters of the class. For a class ω if $\theta^d, d = 1 \dots D_{\omega}$ are the parameters which characterize the class, the overall class membership μ_{ω} for a sample is given by

$$\mu_{\omega} = (1/D_{\omega}) \sum_{d=1}^{D_{\omega}} \mu_{\theta^d}, \quad (14)$$

Figure 6. Fuzzy membership function (μ_θ).

where the class ω represents one of {plane, sphere, cylinder, cone} and μ_{θ^d} is the membership obtained for parameter θ^d as per (13).

3.2 Experimental results and discussions

To design the classifier five training samples from each of the four classes are taken. The standard deviations of different parameters obtained when all the four surfaces are fitted to each training sample are listed in table 2. The lower and upper thresholds for each parameter are tabulated in table 3.

Once the classifier is designed, classification results for other images from the four classes (henceforth referred to as test samples) are presented. For each test sample all the four surfaces were fitted and the standard deviations of the various parameters were noted. From the standard deviations, the membership values with respect to the different classes were derived using the corresponding membership functions ((30) and (31)). The results are tabulated in table 4. The table also shows the dominant class displayed by each of these test samples. It may be seen that only one sample out of 20 has been misclassified. Another point of interest is the nature of the membership vectors for test samples from various classes. Some planar test samples have shown conical behaviour as well (although not dominant). When a cone is to be fitted to a surface, each apex estimate is formed from three tangential planes where the line of intersection of two planes is not parallel to the third. Ideally, for a planar surface, all surface normals are parallel. Hence, fitting a cone fails at the stage of finding the apex itself, since no suitable combinations of three pairwise nonparallel tangential planes can be found. This conical behaviour is seen only in cases where the effects of quantization noise in the input image are so severe that it has been possible to find combinations of three normals to find apex estimates. In all these cases the semi apex angle is close to 90° and axis of the cone is normal to the plane. Thus conical behaviour exhibited by the planar samples is acceptable since any plane can be considered as a cone with semi apex angle of 90° . Sphere samples also have exhibited conical behaviour. Sphere samples actually represent the visible portion of a sphere which may be a hemisphere or a part of it. In all

Table 2. Standard deviations of the parameters obtained when all the four surface types are fitted to the training samples.**a) Planar training samples**

Unable to fit surface; plane: σ_N s.d. of the normal direction, σ_d s.d. of perpendicular distance from origin; sphere: σ_X s.d. of the centre, σ_R s.d. of the radius; cylinder: σ_τ s.d. of axis orientation, σ_X s.d. of a point on the axis, σ_R s.d. of the radius; cone: σ_V s.d. of the apex, σ_τ s.d. of the axis orientation, σ_θ s.d. of the semi apex angle.

Sample	Type of surface fitted									
	Plane		Sphere		Cylinder			Cone		
	σ_N	σ_d	σ_X	σ_R	σ_τ	σ_X	σ_R	σ_V	σ_τ	σ_θ
PL-1	0.00	0.00	—	—	—	—	—	—	—	—
PL-2	0.02	0.27	406.42	6.98	0.54	1248.51	8.32	—	—	—
PL-3	0.01	0.22	—	—	0.40	642.61	3.74	—	—	—
PL-4	0.00	0.14	—	—	0.44	826.92	4.62	—	—	—
PL-5	0.01	0.22	—	—	0.81	653.74	3.89	81.42	0.98	0.92

b) Spherical training samples.

Unable to fit surface; plane: σ_N s.d. of the normal direction, σ_d s.d. of perpendicular distance from origin; sphere: σ_X s.d. of the centre, σ_R s.d. of the radius; cylinder: σ_τ s.d. of axis orientation, σ_X s.d. of a point on the axis, σ_R s.d. of the radius; cone: σ_V s.d. of the apex, σ_τ s.d. of the axis orientation, σ_θ s.d. of the semi apex angle.

Sample	Type of surface fitted									
	Plane		Sphere		Cylinder			Cone		
	σ_N	σ_d	σ_X	σ_R	σ_τ	σ_X	σ_R	σ_V	σ_τ	σ_θ
SP-1	0.74	3.78	3.11	0.28	0.84	494.2	2.27	64.87	0.72	0.41
SP-2	0.67	5.77	7.21	0.35	0.81	227.1	3.38	126.77	0.78	0.73
SP-3	0.78	2.89	1.76	0.17	0.85	197.8	1.90	98.39	0.76	0.68
SP-4	0.73	4.77	3.02	0.20	0.83	522.2	3.08	95.25	0.74	0.4
SP-5	0.49	3.42	2.95	0.25	0.79	301.0	2.40	86.12	0.97	0.44

c) Cylinder training samples.

Unable to fit surface; plane: σ_N s.d. of the normal direction, σ_d s.d. of perpendicular distance from origin; sphere: σ_X s.d. of the centre, σ_R s.d. of the radius; cylinder: σ_τ s.d. of axis orientation, σ_X s.d. of a point on the axis, σ_R s.d. of the radius; cone: σ_V s.d. of the apex, σ_τ s.d. of the axis orientation, σ_θ s.d. of the semi apex angle.

Sample	Type of surface fitted									
	Plane		Sphere		Cylinder			Cone		
	σ_N	σ_d	σ_X	σ_R	σ_τ	σ_X	σ_R	σ_V	σ_τ	σ_θ
CL-1	0.57	3.27	19.20	5.14	0.00	10.57	0.72	—	—	—
CL-2	0.44	3.97	39.51	3.56	0.30	45.98	0.30	295.45	0.46	0.98
CL-3	0.59	2.16	56.66	9.02	0.33	62.00	0.58	129.59	0.80	0.37
CL-4	0.57	5.01	55.09	12.37	0.25	48.59	0.41	463.24	0.77	0.30
CL-5	0.36	3.57	85.91	39.63	0.18	96.45	0.19	629.80	0.82	0.38

(Continued)

Table 2. (Continued)**d)** Conical training samples.

Unable to fit surface; plane: σ_N s.d. of the normal direction, σ_d s.d. of perpendicular distance from origin; sphere: σ_X s.d. of the centre, σ_R s.d. of the radius; cylinder: σ_τ s.d. of axis orientation, σ_X s.d. of a point on the axis, σ_R s.d. of the radius; cone: σ_V s.d. of the apex, σ_τ s.d. of the axis orientation, σ_θ s.d. of the semi apex angle.

Sample	Type of surface fitted									
	Plane		Sphere		Cylinder			Cone		
	σ_N	σ_d	σ_X	σ_R	σ_τ	σ_X	σ_R	σ_V	σ_τ	σ_θ
CN-1	0.30	9.5	66.42	3.15	0.64	158.34	1.24	66.67	0.42	0.06
CN-2	0.74	3.1	45.31	94.11	0.44	164.12	21.73	20.41	0.12	0.01
CN-3	0.43	2.15	32.8	18.84	0.73	330.0	4.21	24.37	0.31	0.02
CN-4	0.50	3.29	35.30	2.86	0.63	784.8	7.93	20.46	0.36	0.03
CN-5	0.48	3.64	47.79	4.57	0.74	105.29	2.47	43.88	0.24	0.09

these cases, the axis of the fitted cone is perpendicular to the plane of the image, i.e., along the z -axis. The standard deviations of the apex and the axis are low enough to fall below the corresponding lower threshold. These two parameters thus contribute a membership value of 1 each. The third parameter, the apex angle, has given a high standard deviation above the higher threshold. Thus contribution of this parameter to overall membership function is 0. Thus an overall membership of 0.67 has resulted for these samples when a cone is fitted. The 0.33 membership with respect to cylindrical classes shown by sphere samples can be explained similarly. The point estimate has given a low standard deviation below the lower threshold in all the cases while the other two parameters have crossed the higher threshold. Cylinder samples have shown no other behaviour significantly apart from cylindrical behaviour. Two cone samples have shown some amount of planar behaviour out of which one cone sample has been misclassified as planar because in that case value of the semiapex angle was very high.

Table 3. Lower and upper thresholds of different parameters for different surface classes

PL = planar surface; SP = spherical surface; CL = cylindrical surface; CN = conical surface.

Surface type	Parameters	Lower threshold	Upper threshold
PL	Normal N	0.015	0.30
	Distance d	0.28	2.15
SP	Centre X_c	5.48	19.2
	Radius R_c	0.31	2.86
	Axis τ	0.33	0.40
CL	Point X	80.42	105.29
	Radius R	0.63	1.24
	Apex V	53.15	64.87
CN	Axis τ	0.39	0.46
	Angle θ	0.07	0.30

Table 4. Fuzzy classification of plane, sphere, cylinder and cone samples.

Sample	Membership vector				Dominant class	Correctly classified (Y/N)
	PL	SP	CL	CN		
PL-6	0.93	0.06	0.00	0.67	PL	Y
PL-7	1.00	0.00	0.00	0.67	PL	Y
PL-8	0.98	0.38	0.00	0.88	PL	Y
PL-9	0.94	0.00	0.00	0.67	PL	Y
PL-10	1.00	0.00	0.00	0.00	PL	Y
SP-6	0.00	1.00	0.33	0.67	SP	Y
SP-7	0.00	1.00	0.33	0.67	SP	Y
SP-8	0.00	1.00	0.33	0.67	SP	Y
SP-9	0.00	1.00	0.33	0.67	SP	Y
SP-10	0.10	1.00	0.33	0.67	SP	Y
CL-6	0.00	0.00	1.00	0.00	CL	Y
CL-7	0.00	0.00	1.00	0.00	CL	Y
CL-8	0.07	0.00	1.00	0.30	CL	Y
CL-9	0.00	0.00	1.00	0.00	CL	Y
CL-10	0.00	0.00	0.54	0.22	CL	Y
CN-6	0.00	0.00	0.03	0.33	CN	Y
CN-7	0.00	0.00	0.00	0.52	CN	Y
CN-8	0.00	0.09	0.00	0.41	CN	Y
CN-9	0.53	0.00	0.00	1.00	CN	Y
CN-10	0.59	0.00	0.00	0.22	PL	N

Abbreviations as in table 3

Classification results for general quadratic surfaces not belonging to any of the four classes, plane, sphere, cylinder and cone, are also presented. The aim is to see if a general pattern of membership vectors is derivable for a set of samples from the same quadratic class. This may be useful in characterising that quadratic class. The results of this experimentation are given in table 5. The classification based on the dominant class behaviour is also indicated in the table. It is interesting to note that samples from the same quadratic class show the same dominant class behaviour. For example cylindrical behaviour is predominantly shown by samples from elliptical and hyperbolic cylinder classes, while ellipsoids are predominantly spherical, hyperboloids are predominantly conical.

4. Occluded surface characterization

In a range image with more than one object, usually some objects may be partially occluded by the others. This makes the problem of understanding range images more difficult since certain regions in the image representing surfaces of objects may be missing or partially occluded. Since due to occlusion the object surfaces are partially visible, local surface properties should be used to characterize them. The geometric method for surface characterization, discussed in §2, obtains surface descriptions through surface normal analysis. Since the normal to a surface at a point is the local property of the surface at the point with respect to the point's neighbourhood,

Table 5. Fuzzy classification of general quadric surface samples.

Surface type	Sample	Membership vector				Dominant class
		PL	SP	CL	CN	
Elliptic cylinder	1	0.00	0.19	0.67	0.00	CL
	2	0.40	0.00	1.00	0.33	CL
	3	0.00	0.19	0.67	0.00	CL
	4	0.43	0.15	1.00	0.00	CL
	5	0.00	0.00	0.33	0.00	CL
Hyperbolic cylinder	1	0.00	0.00	0.33	0.00	CL
	2	0.07	0.28	1.00	0.00	CL
	3	0.00	0.00	0.67	0.00	CL
	4	0.00	0.09	0.78	0.00	CL
	5	0.00	0.00	0.33	0.00	CL
Ellipsoid	1	0.00	0.40	0.33	0.33	SP
	2	0.00	0.53	0.33	0.25	SP
	3	0.11	0.93	0.33	0.33	SP
	4	0.16	0.93	0.33	0.36	SP
	5	0.22	0.99	0.46	0.67	SP
Elliptic cone	1	0.00	0.00	0.00	1.00	CN
	2	0.27	0.55	0.00	1.00	CN
	3	0.00	0.00	0.11	1.00	CN
	4	0.00	0.00	0.00	0.67	CN
	5	0.00	0.00	0.00	0.33	CN
Hyperboloid	1	0.00	0.00	0.00	0.33	CN
	2	0.00	0.00	0.00	0.33	CN
	3	0.00	0.00	0.00	0.33	CN
	4	0.00	0.00	0.31	0.33	CN
	5	0.00	0.00	0.00	0.33	CN
Hyperbolic paraboloid	1	0.00	0.00	0.00	0.33	CN
	2	0.00	0.00	0.00	0.27	CN
	3	0.00	0.00	0.00	0.33	CN
	4	0.00	0.00	0.00	0.33	CN
	5	0.00	0.00	0.00	0.33	CN

Abbreviations as in table 3

the geometric method also forms a reliable tool for characterizing partially visible surfaces.

The shape of a surface (i.e., planar, spherical, cylindrical or conical) is uniquely determined by the global parameters of the surface. For example, a spherical surface is uniquely characterized by its centre and radius. The geometric method estimates the global surface parameters from the locally calculated surface normals. Since the estimation of global parameters at different regions of a surface should be very close to each other, it is also possible to determine if two or more partially visible surface patches are parts of the same surface. This is achieved in the following manner.

Let a surface class ω , $\omega \in \{\text{plane, sphere, cylindrical, cone}\}$, be characterized by a set of parameters $\{\theta^d | 1 \leq d \leq D_\omega\}$, where D_ω is constant whose value is different for different surface classes. This set of parameters defines a D_ω -dimensional space, henceforth referred to as parameter space, such that the mean values of the parameters θ_i^d describing a surface S_i of class ω represent a point p_i in the parameter space. Because

of the uncertainty in the measurement of θ_i^d , different estimates of θ_i^d may occur anywhere in the region $(\theta_i^d - hi_{\theta_i^d})$ to $(\theta_i^d + hi_{\theta_i^d})$ where $hi_{\theta_i^d}$ is the upper threshold used in the fuzzy classifier and is given in (29). Thus different estimates of the point p_i may occur anywhere within a hyper rectangular parallelepiped bounded within the region $[\theta_i^d - hi_{\theta_i^d}, \theta_i^d + hi_{\theta_i^d}]$ for $1 \leq d \leq D_\omega$. This region is referred to as region of occurrence of S_i . Let this region for surface S_i be denoted by R_i . Now two partially visible surface patches S_i and S_j , both of same surface type ω , are assumed to be possible parts of the same surface if $R_i \cap R_j \neq \phi$. If this condition is satisfied then surfaces are fitted over the region $S_i \cup S_j$. If the fitted surface also shows a dominant class ω then S_i and S_j are identified as parts of the same surface and the parameters of the fitted surface are extracted. If the fitted surface shows a dominant class other than ω then S_i and S_j are identified as different surfaces.

4.1 Experimental results

Experimental results on six synthetic range images, each containing occluded surfaces, are presented. All the synthetic images are of size 128×128 and are shown in figures 7a–7f. These images are referred to as image-1, image-2, image-3, image-4, image-5 and image-6 respectively. Boundaries of the surface patches in these images are shown in figures 8a–f respectively, where each surface region is marked with an integer. Each of these images contain one or more occluded surface patches. The fuzzy classification results are shown in table 6. The table shows the original class and the original parameters of the surfaces, the dominant class of the fitted surfaces and the estimated parameters when surfaces of respective dominant classes are fitted.

From table 6 it is seen that the occluded surfaces, i.e., the cone in image-1 (figure 7a), the cylinder in image-2 (figure 7b) and the sphere in image-3 (figure 7c), are correctly classified and the estimated parameters of the fitted surfaces closely approximate the original parameters. Image-4 (figure 7d) contains two partially visible hemispherical surfaces (surface patches 2 and 4 in figure 8d), one with radius 55 and the other with radius 30. Both of them have the same centre (63, 63, 150). Both these hemispherical surfaces are correctly classified and parametrized individually. But since the region of occurrence of one hemisphere does not intersect that of the other, they are not considered as part of a single surface. Whereas in image-5 (figure 7e), though the regions of occurrence of two partially occluded hemispheres (surface 2 with radius 50 and surface 4 with radius 55 in figure 8e) intersect, the combined surface shows conical behaviour though individually they are dominantly spherical. Hence they also cannot comprise a single surface. Two partially visible conical surface patches (2 and 4 in figure 8f) in image-6 (figure 7f) together construct a single conical surface whose estimated parameters closely approximate the estimated parameters of the individual surfaces and also those of the original surface. The cylindrical surfaces in all these three images are correctly classified and parametrized.

5. Conclusion

In this paper various geometric methods for surface fitting and surface characterization have been considered. Geometric surface fitting techniques give best results when the input is from the restricted domain of plane, sphere, cylinder and cone. The problem

Figure 7. Synthetic range images containing occluded surfaces: (a) Image-1, (b) image-2, (c) image-3, (d) image-4, (e) image-5, and (f) image-6.

has been attempted for two different situations. In the first case, it has been assumed that *a priori* knowledge about the shape of the surface is available. In such situations, a surface of the proper type is fitted to the set of points representing the surface patch and the parameters of the fitted surface are extracted. This strategy of selective surface fitting reduces the amount of computation.

In the second case, when no *a priori* information regarding the shape of the unknown surface is available, the characterization of the surface is obtained in the form of a fuzzy membership vector. The unknown surface is taken as belonging to the class with maximum membership value. In this case the burden of fitting all four classes of surfaces cannot be avoided. The design of the fuzzy classifier plays a crucial role in determining the performance of the system. The performance of the classification scheme depends upon how correctly it can classify unknown surfaces. Thus the choice

Figure 8. Boundaries of surface patches in synthetic range images: (a) Image-1, (b) image-2, (c) image-3, (d) image-4, (e) image-5, and (f) image-6.

of training samples to design the classifier is also very important. Experimental results on partially visible surfaces indicate that the geometric method for surface fitting along with the fuzzy classification scheme forms a reliable tool for handling occlusion.

Table 6. Fuzzy classification of synthetic range images containing occluded surface patches.

Image no.	Surface no. and type	Original parameters	Membership vector				Dominant Class	Estimated parameters
			PL	SP	CL	CN		
1	2 CN	Apex 60, 10, 120 Axis $l = 0, m = 1, n = 0$ Semi apex angle 30°	0.32	0	0	0.67	CN	Apex 46.8, 49.6, 107.1 Axis $l = 0.3, m = 0.8, n = 0.4$ Semi apex angle 35.2°
	3 CL	Axis $l = 0, m = 1, n = 0$ Axis point 90, 0, 60 Radius 30	0	0	1	0	CL	Axis $l = 0, m = 1, n = 0$ Axis point 90.7, 0, 58.9 Radius 29.4
2	2 CL	Axis $l = 0, m = 1, n = 0$ Axis point 80, 0, 150 Radius 40	0	0	1	0.33	CL	Axis $l = 0, m = 1, n = 0$ Axis point 79, 0, 146 Radius 37.9
	3 SP	Centre 45, 80, 110 Radius 40	0.3	0.9	0.3	0	SP	Centre 45.25, 80.26, 107.27 Radius 38.43
3	2 CN	Apex 70, 20, 100 Axis $l = 0, m = 1, n = 0$ Semi apex angle 25°	0	0	0	0.52	CN	Apex 64.8, 38.9, 95.4 Axis $l = 0, m = 0.9, n = 0.1$ Semi apex angle 38.6°
	3 SP	Centre 45, 80, 150 Radius 40	0	1	0.33	0.67	SP	Centre 45.2, 80, 149.7 Radius 40.26
4	2 SP	Centre 63, 63, 150 Radius 30	0	1	0.3	0.7	SP	Centre 62.8, 63.7, 149.4 Radius 30.4
	3 CL	Axis $l = 1, m = 0, n = 0$ Axis point 0, 63, 90 Radius 10	0	0	1	0	CL	Axis $l = 1, m = 0, n = 0$ Axis point 0, 62.5, 91.6 Radius 10.5
4	4 SP	Centre 63, 63, 150 Radius 55	0	1	0.3	0.3	SP	Centre 63.2, 62.9, 149.5 Radius 55

5	2 SP	Centre 63, 63, 150 Radius 50	0	1	0.3	0.3	SP	Centre 63.2, 62.8, 150 Radius 50
	4 SP	Centre 63, 63, 150 Radius 55	0	1	0.3	0.3	SP	Centre 63.2, 62.9, 149.5 Radius 55
	24 SP		0	0.5	0.3	0.7	CN	
6	2 CN	Apex 80, 20, 150 Axis $l = 0, m = 1, n = 0$ Semi apex angle 30°	0	0	0	0.33	CN	Apex 78, 27.7, 146.6 Axis $l = 0.1, m = 0.9, n = 0.2$ Semi apex angle 34.6°
	4 CN	Apex 80, 20, 150 Axis $l = 0, m = 1, n = 0$ Semi apex angle 30°	0	0.2	0.3	0.7	CN	Apex 81.9, 39.2, 141.9 Axis $l = 0, m = 0.9, n = 0.1$ Semi apex angle 37.4°
	24 CN	Apex 80, 20, 150 Axis $l = 0, m = 1, n = 0$ Semi apex angle 30°	0	0	0	0.62	CN	Apex 80.75, 34.7, 143.6 Axis $l = 0, m = 0.9, n = 0.1$ Semi apex angle 35.8°
	3 CL	Axis $l = -0.8, m = -0.5, n = 0$ Axis point 0, 120, 100 Radius 10	0	0	1	0.3	CL	Axis $l = -0.8, m = -0.4, n = 0$ Axis point 0, 118.2, 100.1 Radius 10.5

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