A review of computerized tomography with application to two-phase flows

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Abstract. The technique of computerized tomography (CT) has established itself as a leading tool in diagnostic radiology over the past twenty years and is catching on fast in the non-destructive evaluation area in a variety of situations.

CT instrumentation involves a source and a detector system to scan the object of interest. The source can be acoustic, microwave, X-ray, gamma-ray, etc. depending upon the type of material being investigated. For fluid-flows, gamma-rays are quite suitable. There are basically two types of data collection geometries – fan-beam and parallel beam. Fan-beam requires less number of sources as one source feeds several detectors arranged in a fan-beam.

The use of CT in multi-phase flow studies has been limited presently to only a few laboratory experiments and the results indicate that CT does hold a lot of promise as an effective investigative methodology to understand some of the complex phenomena encountered in multi-phase flows. Some new CT algorithms developed specifically for pipe-flows have shown good results on some air–water flow data for a 15 cm dia pipe.

Keywords. Tomography; image reconstruction; two-phase flow; void-fraction.

1. Introduction

The area of medical imaging has been revolutionized with the introduction of X-ray based scanning machines involving the concept of computerized tomography (CT). The first CT scanner was developed by Hounsfield (1973) who was later on awarded the Nobel prize for medicine. These CT machines have been in the market for the past 17 years and have proved to be excellent tools in the diagnosis of cancerous tissues at a very early stage. The power of CT was quickly recognised by the researchers in the non-destructive testing (NDT) area and several machines have been developed

specifically for NDT work, e.g., Miyoshi *et al* (1987) and McClellan & Tow (1986). The engineering and physics of CT is discussed in great detail in the work of Herman (1980) while the mathematics of CT has been elaborately described by Natterer (1986). The book by Deans (1983) lists several uses of CT other than the two applications mentioned above.

Tomographic scanners incorporating ultrasonics have also been developed but are not discussed here. This paper deals specifically with radiation based CT which generally involves X-ray or gamma-ray sources and detectors.

1.1 Historical perspective

Radon (1917) proved that any arbitrary function could be recovered from its set of line integrals taken along various chords and various directions. However, the method of Radon could not be implemented due to inherent mathematical complexities. Bracewell (1956) reported an application of CT in radio astronomy and Cormack (1963) derived an inversion formula, both of which were closer to being implementable as compared to Radon's solution. Bracewell & Riddle (1967) and Ramachandran & Lakshminarayan (1970) showed how computations involved in the CT methodology could be accelerated by adopting the use of convolutions. However, it was almost 50 years after the publication of Radon's paper that a CT machine was pioneered by Hounsfield (1973).

1.2 Preliminaries

Single beam mono-energetic radiation attenuation phenomenon in a plane can be represented by

$$N = N_0 \exp \left[- \int_c \mu(r, \phi) dl \right]. \quad (1)$$

The value of μ is characteristic of a material for a given type of radiation and the energy of radiation. Equation (1) considers μ to be a two-dimensional function of position, as the path of radiation is assumed to be restricted to a plane. The energy dependence of μ is also ignored. This simplifying assumption is valid for mono-energetic radiation sources, e.g. γ -rays, and suitable corrections are incorporated to handle X-ray sources which have a distributed energy spectrum. We note that when the cross-section of interest has a uniform distribution of the absorbing material then (1) reduces to,

$$N = N_0 \exp(-\mu \Delta l), \quad (2)$$

where μ is no longer a function of position.

In general, any cross-section of interest will have non-uniform distribution. Rewriting (1) as,

$$p = \int_c \mu(r, \phi) dl, \quad (3)$$

where

$$p = \ln(N_0/N). \quad (4)$$

We observe that recovering μ from the given data p is not a straightforward

proposition. Radon (1917) showed that it is possible to recover μ from a set of several p -values measured along various chords, c . These μ -values can then be suitably calibrated to give the density values if so desired by the user.

2. Data collection modes

The CT methodology requires attenuation data [given by (3)] collected by an array of radiation detectors for the reconstruction of the function $\mu(r, \phi)$. Two popular data collection modes are termed as parallel-beam geometry (PBG) and fan-beam geometry (FBG).

2.1 Parallel-beam geometry

CT scanners based on PBG mode have several pairs of radiation source and radiation detector systems which scan the object completely. Figure 1 depicts the configuration for a generic PBG scanner. The source-detector pairs are spaced uniformly and the object to be imaged is stationed on a table which can be rotated to give different values of θ . The line SD represents the path of the data ray. The perpendicular distance of the ray from the centre of the object, which happens to be the origin of the frame of reference chosen for derivational and computational convenience, is denoted by s . Several SD pairs collect the data p for a given θ . This set of p is known as 'projection'. The object table is rotated to get several sets of p for different values of θ , i.e., different 'views'. The data thus collected is denoted by $p(s; \theta)$, the semi-colon indicating the procedure of data collection.

We note that if the object has to be kept stationary, then the SD system has to be rotated to collect the data for various 'views'.

2.2 Fan-beam geometry

The medical scanners incorporate the FBG collection mechanism which involves a rotating module including a source and an array of detectors stationed in an arc around the patient. Figure 2 shows the FBG configuration. To distinguish fan-beam

Figure 1. Parallel-beam geometry.

Figure 2. Fan-beam geometry.

from parallel-beam data, the FBG data are represented by $g(\sigma, \beta)$. Here, a single source views several detectors simultaneously and data are collected by different detectors corresponding to various angles, σ . This process is repeated for different values of β to get the CT data, $g(\sigma, \beta)$.

The FBG mode can be implemented for a rotating object system also. The data collection mechanism remains the same as that for the PBG case.

3. Tomographic inversion formula

The process of retrieving (or reconstructing) a two-dimensional function from its projection data is similar for the FBG and PBG cases. The parallel beam CT formulation involves Fourier transforms while the fan-beam formulation incorporates Hilbert transforms in the inversion process.

3.1 Parallel-beam formulation

The inversion formula for the PBG case is based on the 'central-slice' theorem for Fourier transforms of two-dimensional functions. The theorem states that the one-dimensional Fourier transform of the projection data, $p(s; \theta)$, with respect to the first variable s , is equal to the two-dimensional Fourier transform of the object function f being imaged. Mathematically,

$$\hat{f}(F; \theta) = \hat{p}(F; \theta). \quad (5)$$

Taking the inverse Fourier transform of (5) we get

$$f(r, \phi) = \int_0^\pi \int_{-\infty}^\infty \hat{p}(F; \theta) \exp[-i2\pi Fr \cos(\theta - \phi)] |F| dF d\theta. \quad (6)$$

Here, for a given θ ,

$$\hat{p}(F; \theta) = \int_{-R}^R p(s; \theta) \exp[-i2\pi Fs] ds. \quad (7)$$

Equation (6) requires projection data on a continuous basis for all values of s and θ . For computational feasibility a finite cut-off has to be incorporated for the Fourier frequency. This cut-off is usually implemented in the form of a filter function, $W(F)$, which vanishes for $|F|$ greater than A , the cut-off frequency. Thus the filtered version of (6) becomes,

$$\tilde{f}(r, \phi) = \int_0^\pi \int_{-\infty}^{\infty} \hat{p}(F; \theta) \exp[-i2\pi Fr \cos(\theta - \phi)] W(F) |F| dF d\theta. \quad (8)$$

Here, the reconstruction, $\tilde{f}$, is approximate due to the finite cut-off introduced in the computations. Also, the discrete nature of the CT data forces the practical implementation of A to be governed by the sampling theorem (Brigham 1974), i.e.,

$$A \geq [1/(2\Delta s)].$$

The band-limiting filter, introduced in electron micrography by Ramachandran & Lakshminarayanan (1970) is given by

$$W(F) = \begin{cases} 1, & |F| < A, \\ 0, & |F| \geq A, \end{cases}$$

and a popular 'sinc' filter used by Shepp & Logan (1974) is given by,

$$W(F) = \begin{cases} [\sin(\pi F/2A)]/(\pi F/2A), & |F| < A, \\ 0, & |F| \geq A. \end{cases}$$

3.2 Fan-beam formulation

The inversion formula for the FBG case was first derived by Herman & Naparstek (1978) and is given by,

$$\tilde{f}(r, \phi) = (1/4\pi^2) \int_0^{2\pi} \int_{-B}^B [1/\sin(\sigma' - \sigma)] D_\sigma g(\sigma, \beta) d\sigma d\beta, \quad (9)$$

where,

$$g(\sigma, \beta) = \text{data for the ray represented by } (\sigma, \beta),$$

$$D_\sigma g(\sigma, \beta) = (1/U) [(\partial g/\partial \sigma) - (\partial g/\partial \beta)],$$

$$\sigma' = \tan^{-1} \{ [r \cos(\beta - \phi)] / [D + r \sin(\beta - \phi)] \},$$

$$U = \{ [r \cos(\beta - \phi)]^2 + [D + r \sin(\beta - \phi)]^2 \}^{1/2},$$

and the other variables are as in the PBG case (see figure 2). We note that U is the distance of the radiation source from (r, ϕ) , the point being reconstructed, and σ' is the angular displacement of the particular data ray passing through that point (r, ϕ) . A major difference [compared to (6)] is the computation of partial derivatives of the data, g .

4. Convolution back-projection algorithm

The reconstruction algorithms based on the inversion formulae summarised in §3, are known as 'transform methods'. It was observed by several researchers that these transform methods were slow and required very accurate interpolation schemes

in computing the two-dimensional inverse Fourier transforms. The introduction of convolutions (Bracewell & Riddle 1967; Ramachandran & Lakshminarayanan 1970) eliminated the step of computing the Fourier transform of the data and the subsequent time consuming two-dimensional Fourier inversion.

4.1 Parallel-beam algorithm

Exploiting the convolution property of Fourier transforms, (8) can be rewritten as,

$$\tilde{f}(r, \phi) = \int_0^\pi \int_{-R}^R p(s; \theta) q(s' - s) ds d\theta, \quad (10)$$

where,

$$q(s) = \int_{-A}^A W(F) |F| \exp[i2\pi Fs] dF, \quad (11)$$

and,

$$s' = r \cos(\theta - \phi).$$

Here, q known as the convolving function and is the inverse Fourier transform of the filter function, $W(F)$. Equations (10)–(11) are the fundamental equations of the convolution back-projection (CBP) algorithm of CT. The inner integral of (10) is a convolution and the outer integral was termed back-projection by the early researchers. The CBP algorithm is the most widely used method of reconstruction employed by commercial CT scanners available for medical imaging purposes.

4.2 Fan-beam algorithm

The inversion formula given by (9) involves a singular integral thus forcing a regularization of the integral for computational purposes. A regularized version of (9) is given by (Herman & Naparstek 1978)

$$\tilde{f}(r, \phi) = \frac{D}{4\pi^2} \int_0^{2\pi} \frac{1}{U^2} \int_{-R}^R (F_1 + F_2) g(\sigma, \beta) d\sigma d\beta, \quad (12)$$

where,

$$F_1 = Q^1(\sigma' - \sigma) \cos \sigma,$$

$$F_2 = Q^2(\sigma' - \sigma) \cos \sigma',$$

$$Q^1(u) = [-uQ(u)]/\cos^2(u),$$

$$Q^2(u) = [Q(u) + u(dQ/du)]/\sin(u),$$

$$Q(u) = -2 \int_0^A W(F) \sin(2\pi Fu) dF.$$

A suitable choice of the filter function, $W(F)$, leads to the convolving functions Q^1 and Q^2 . An implementation of (12) for two-phase air–water bubbly flow in a pipe has been reported by Munshi *et al* (1987).

4.3 Filter selection for the convolution algorithms

A basic problem of the CBP algorithm of CT is the choice of an appropriate filter, $W(F)$, for both the parallel-beam and fan-beam cases. Chang & Herman (1980) have

conducted an exhaustive study of several types of filters; other details are given by Herman (1980). Recently, a theoretical study was completed by Munshi (1989) to study the reconstruction quality of the CBP algorithm employing the popular filters reported in literature. Certain 'higher order' filters, based on the existing Fourier filters, have also been developed and some preliminary results involving simulated data have been reported by Arora *et al* (1988).

A new filter has been proposed by Rathore *et al* (1989) for the CBP algorithm and the simulated data results appear to be quite encouraging. Unlike the Fourier filters, this new filter is induced from the configuration of the data collection geometry.

5. Algebraic reconstruction methods

The previous sections summarised the tomographic reconstruction formula for the PBG and the FBG modes of data collection which were classified as 'transform' methods. A whole group of iterative schemes have also been developed which reconstruct the function from its projection data. Such methods are known as 'algebraic' techniques.

These iterative algorithms start off with an assumed distribution, f_0 , of the object function as the first step. Next, projection data are generated on the computer for f_0 . The mismatch in the generated data and the actual data is computed and f_0 is corrected to get the next best approximation, f_1 . A sequence of such approximations is shown (Tanabe 1971) to converge to the actual solution f , the distribution of the object function. This method was originally proposed by Kaczmarz (1937) for solving a system of linear equations.

Hounsfield (1973) employed this algebraic reconstruction technique (ART) to obtain images in the first CT scanner. At the same time Gordon *et al* (1970) also proposed ART independently for reconstructions. Later on, several variations of ART were also investigated; details are available elsewhere (Censor 1983).

All the ART-type methods (Censor 1983) have been reported to consume much more computer time than the transform methods (Lewitt 1983), in particular the CBP algorithm. This is the reason for medical scanners employing CBP although the first set of successful reconstructions was obtained using ART.

6. Application in multi-phase flow studies

The previous sections briefly outlined the theory and the implementation aspects of some tomographic techniques, the CBP algorithm in particular. Current two-phase flow research, and multi-phase flow studies in general, need an accurate measurement of void-fraction distribution across the flow cross-section. Tomographic reconstruction of void-fraction in air-water flow was studied by Munshi (1979) and related findings were reported by Schlosser *et al* (1980), DeVuono *et al* (1980) and Kulacki *et al* (1980). Improved reconstruction results were reported by Seshadri *et al* (1986).

The collection of CT data can be quite easy for the flow situation where the density distributions do not change rapidly with time as was the case in a previous study (Munshi 1979). For such situations a simple single source and single detector system suffices. The source can be a γ -ray source of at least milli-curie strength but a higher source is always beneficial in reducing the data collection time and/or reducing the statistical errors encountered in the counting of photons by the detector. The single

source-detector system, mounted on a platform can be rotated to collect projection data in either the FBG mode or the PBG mode. Such a simple mechanism does not require rotation of the object and this is crucial for flow experiments as rotation of objects will generally be ruled out as is the case with the medical CT scanners where patients just cannot be rotated. However, if sufficient detectors are available then they can be arranged in a fan beam mode with a single γ -ray source to collect CT data at a much faster rate and also for flows which have void/density distributions changing with time. We note that the interval of counting (say 10 s) will be the interval during which the flow distribution will still have to be assumed to be constant. This interval of counting can be reduced by using radiation sources of higher strength. A simulation study has already been reported by DeVuono *et al* (1980).

The CT data thus obtained can be processed with any standard algorithm but the CBP method is recommended. The reconstructed function will be a distribution of the absorption coefficient, μ , hence suitable calibration is required to convert the CT numbers to the density values. For a two-phase gas-liquid flow, the mixture density, ρ_m , is related to the phase densities by the following relation,

$$\rho_m = (1 - \alpha)\rho_l + \alpha\rho_g,$$

If the gas-density is much smaller than the liquid density, then

$$\rho_m \approx (1 - \alpha)\rho_l,$$

and for steam-water flow we can write,

$$\rho_m = 1 - \alpha.$$

7. New tomographic algorithms

All the previous studies employing the CBP algorithm appear to be an overkill for the bubbly air-water flow situation where the projection data were roughly radially symmetric.

A series of tomographic methods were developed by Rathore *et al* (1986, 1987, 1988) for performing void-fraction reconstruction in radially symmetric situations generally occurring in pipe flows. A simplification of the CBP method leads to the Bessel function algorithm (Rathore *et al* 1987a, 1988a) incorporating the CBP filters. The algorithm involving radial polynomials (RAP) reported earlier (Rathore *et al* 1986, 1987b) results in a continuous approximation of the object function. The chord-segment inversion (CSI) technique (Rathore *et al* 1987c, 1988b) incorporates a discrete ring-type formulation.

All these three methods give a one-shot solution to the reconstruction process as data are required only from one 'view' for a radially symmetric situation. The CBP implementation requires convolution computations to be performed for all values of θ as per (8). Thus computation time is considerably reduced if the new methods are used instead of the CBP method. For a 'rough' density distribution the CSI formulation appears to be the most suited while for a relatively 'smooth' distribution the RAP method is likely to give better results. The Bessel algorithm is a direct consequence of the CBP inversion technique for radial symmetry but it requires computation of the Fourier transform of the CT data. A summary of these methods appears elsewhere (Munshi & Rathore 1990).

The distributions generally encountered in engineering situations are likely to be non-smooth and, hence, the CSI algorithm is recommended as a first-choice CT method. All the tomographic methods hold a lot of promise as diagnostic tools in studying the two-phase flow phenomenon. Multi-phase flow situations can also be handled with similar hardware and software facilities as the number of phases do not affect the CT method which regards density/void-fraction as a general function of position.

8. Some sample reconstruction results

This section outlines some of the results from a previous study (Rathore *et al* 1988c).

8.1 Results with simulated data

A general practice adopted in the studies was to test the algorithms by performing reconstructions with data generated by some known test functions. The test functions selected in most cases were of the following types: (i) constant; (ii) linear; (iii) exponential; (iv) step-function.

Figure 3. Reconstruction results for the (a) constant, (b) linear, (c) negative exponential and (d) positive exponential for a simulation of flow in a circular pipe of unit radius.

These functions are reasonable simulations of the likely density/void distributions encountered in bubbly and annular two-phase flow situations. Figure 3 shows the reconstruction results for the constant, linear, and positive/negative exponentials for a simulation of flow in a circular pipe of unit radius. The excellent agreement in the reconstructed values and the original values is evident. Figure 4 depicts a similar reconstruction exercise for two simulations of annular flows. Again, the reconstructed values match well with the true values.

8.2 Results for air–water flows

The bubbly air–water flow data (Munshi 1979) was processed for several cases of cross-sectionally averaged values of density/void distribution and several sets of data were taken. Reconstructions for one such set, designated Scan 1, is shown in figure 5. The density/void measurements are reported as 10 points in a pipe of 7.5 cm radius. In the experiment, CT data were collected at 10 equally-spaced angular positions (intervals of 2.5°) in a fan-beam mode, hence the reconstruction is made available for 10 radial positions. In case a finer grid is required, data have to be collected for a finer angular-spacing. For better accuracy a stronger γ -ray source (greater than the 13.6 milli-curie one used in the study) is recommended.

List of symbols

A	Fourier cut-off frequency;
B	angle of extreme data-rays;
c	chord (path) of integration;
D	distance of source from origin;
D_v	partial derivative of $g(\sigma, \beta)$;

Figure 4. Reconstruction results for simulated annular flows.

Figure 5. Reconstruction results for air-water flow for scan 1.

$f(r, \phi)$	function being reconstructed;
$\tilde{f}(r, \phi)$	reconstruction of $f(r, \phi)$;
$F, \hat{A}$	Fourier frequency and Fourier transform, respectively;
$g(\sigma, \beta)$	projection data (CT data) for fan beams;
l	variable of integration (cm);
Δl	thickness of absorbing medium;
N	radiation detector reading (counts/s);
N_0	radiation source strength (counts/s);
$p(s; \theta)$	projection data (CT data) for parallel beams;
r, ϕ	cylindrical co-ordinates;
R	radius of the object;
s	perpendicular distance of data-ray from origin;
s'	s -distance of data-ray passing through (r, ϕ) ;
Δs	spacing between data-rays;
U	distance of point (r, ϕ) from the source;
$W(F)$	window (filter) function;
α	void-fraction at point where ρ_m is measured;

β	source position (fan beam geometry);
γ	gamma (ray);
θ	source position (parallel beam geometry);
μ	absorption co-efficient (cm^{-1});
ρ	density (g/cm^3);
ρ_l, ρ_g, ρ_m	density of liquid and gaseous phases, and of the mixture, respectively;
σ	angle of data-ray from reference centre-line;
σ'	angle of data-ray passing through the point (r, ϕ);
$\langle \rangle$	cross-sectionally averaged quantity.

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