Investigation of CTARA wood-burning stove. Part II. Analytical investigation

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Centre for Technology Alternatives for Rural Areas, Indian Institute of Technology, Powai, Bombay 400 076, India

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Abstract. A theoretical framework for predicting the performance of the CTARA stove under different geometric and operating conditions has been developed by writing energy balance conditions over appropriate control volumes under a steady-state assumption. Equations are derived for predicting the fuel burning rate, air flow ingress through primary and secondary ports, wall temperatures, gas temperatures, wood-surface temperature and the vessel heat absorption rate. Complete chemical reaction is assumed to take place inside the stove and radiation heat transfer between products of combustion and the enclosing surfaces has been accounted. The analytical framework results in thirteen strongly coupled algebraic equations that are solved iteratively on a computer.

The framework predicts magnitudes of efficiency and excess-air factors for various geometric parameters (e.g. height of stove, height of the grate, port-opening diameter, stove diameter etc.) and operating parameters (power of the stove, ratio of volatiles to char in wood, vessel dimensions etc.). The predictions are found to be plausible in magnitude and in trends and compare favourably with experimental results presented in Part I with one exception. Whereas experimental results show a negligible effect of power on efficiency, the predictions indicate an increase in efficiency with power.

Keywords. Wood-burning stove; energy balance conditions; steady-state assumption; geometric parameter; radiation heat transfer.

1. Theoretical analysis

1.1 *The purpose*

Development of a theoretical model of the CTARA wood-burning cook-stove essentially involves derivation of equations which will quantify:

- (a) rate of heat release as a result of wood combustion;

A list of symbols is given at the end of the paper.

- (b) rate of heat absorption at the vessel;
- (c) rate of heat losses from the stove walls and through the exhaust gases.

The derivation of equations involves identification of the energy transfer mechanism for each of the above mentioned components. The energy transfer process is influenced by fluid flow (driven by the buoyancy forces and retarded by the internal flow resistance), by heat transfer (by convection and radiation) and by chemical reaction (between hydrocarbons in the wood and oxygen in the air). These three processes are strongly interlinked. Hence, when appropriate equations representing the effects of all the transfer mechanisms are developed, there results a solvable set of equations (with necessary empirical inputs); the set being strongly coupled and requiring iterative solution on a computer.

In deriving the necessary equations, approaches of varying degrees of complexity may be adopted. It is easy to recognise however that wood combustion and energy transfer in the stove is a three-dimensional time-dependent phenomenon. Hence, the most complex approach would necessitate solution of three-dimensional time-dependent partial differential equations for mass, momentum, energy and species concentrations.

A much simplified approach (and the one adopted here) is to treat the wood-stove as comprising only a *few control volumes* and to assume that a *steady-state* prevails. As a result of these assumptions, the equations developed are nonlinear algebraic equations rather than differential ones. The task of solving them is then a much simpler affair. The limitation of the approach is that parameters such as temperature, velocity, concentration, heat transfer coefficient etc. take their space- and time-averaged values, and 'local' variations are ignored. Further the "sensible heat" stored in the stove body is also ignored.

The solution of equations requires certain input information related to the geometric design and operating variables, besides providing empirical inputs which the reader will appreciate more fully in subsequent subsections. The output of the solution predicts:

- (a) the fuel burning rate;
- (b) the air flow rate through primary, secondary and feed ports;
- (c) the temperatures at the walls and in the hot gases;
- (d) the heat absorption rate; and
- (e) the efficiency.

The computer program developed thus enables prediction of the effects of changing the basic geometry as well as changing of operating parameters on the performance of the stove. The program developed not only offers an opportunity to verify the experimental results presented earlier but also proves to be a guide for optimising the stove design and for indicating directions for more detailed experimental investigations. In fact, the program was developed simultaneously when experiments were being conducted and was used to guide the experimental work.

1.2 The parameters of interest

Figure 1 shows the schematic of the CTARA wood-burning stove where the major parameters of interest are defined. The combustion zone above the grate is divided

Figure 1. Computational model of the CTARA stove.

into two parts: One comprising the clay enclosure and the other the metal enclosure. The average gas temperatures in these are T_{fec} and T_{fem} respectively. The temperature of the surface of the burning wood is designated as T_s . The temperature of the clay enclosure wall and the metal enclosure wall are respectively designated as T_{w_2} and T_{w_4} . The temperature of the outgoing flue gases is designated as T_{f1} . The locations of the ports for primary air flow from below the grate ($\dot{m}_{p1}$), above the grate through the wood feed port ($\dot{m}_{p2}$) and the secondary air flow rate ($\dot{m}_s$) are also shown in the figure. T_{sec} represents the temperature of the secondary air as it enters the metal enclosure.

In order to facilitate the radiation analysis (to be presented later), the various surfaces are designated with numbers 1 to 5 as shown.

In the next section we present equations governing temperatures T_{fec} , T_{fem} , T_{f1} , T_{sec} , T_{w_2} and T_{w_4} . These are derived by writing energy balances over appropriate control volumes. The temperature of the wood burning surface T_s is obtained from an Arrhenius type relationship governing the wood burning rate $\dot{m}_{fu}$ and T_s .

The temperature equations require information on radiation fluxes at surfaces 1 to 5. The necessary equations are derived from the enclosure theory in § 1.4.

Section 1.5 develops an equation for heat absorption rate at the pan bottom. Here an empirical equation for heat transfer coefficient is introduced.

By postulating flow paths, well-known flow resistance expressions can be approximated. These are used in § 1.6 to estimate the air mass flow rates $\dot{m}_{p1}$, $\dot{m}_{p2}$, and $\dot{m}_s$ driven by the appropriate buoyancy forces. Finally by postulating the energy balance mechanism at the wood burning surface, an expression for fuel burning rate $\dot{m}_{fu}$ is developed in § 1.7. The expression requires an empirical relationship between heat of gasification (or latent heat of wood) and the diameter of wood.

Section 1.8 discusses the method used for accounting for the effect of induced swirl. Section 1.9 presents the results of computations where effect of varying different input parameters is discussed.

1.3 Prediction of temperatures

With reference to figure 1 again the clay enclosure (T_{w2}) and metal enclosure (T_{w4}) wall temperatures are predicted by

$$T_{w2} = (q_{rad, 2} + \alpha_{2i} T_{fec} + Z_1 T_{sec}) / (\alpha_{2i} + Z_1), \quad (1)$$

where

$$Z_1 = 1 / [(b_2/k_2) + (1/\alpha_{20})],$$

and

$$T_{w4} = (q_{rad, 4} + \alpha_{4i} T_{fem} + Z_2 T_{sec}) / (\alpha_{4i} + Z_2), \quad (2)$$

where

$$Z_2 = 1 / [(b_4/k_4) + (1/\alpha_{40})].$$

The gas temperature (T_{fec}) in the conical clay enclosure is predicted by

$$\begin{aligned} F_1 \dot{m}_{fu} C_{a_{wo}} + (\dot{m}_{p_1} + \dot{m}_{p_2}) C_{pa} (T - 273) \\ = q_{rad, 2} + \alpha_{2i} (T_{fec} - T_{w2}) A_2 \times A_2 (\dot{m}_{fu} + \dot{m}_{p_1} + \dot{m}_{p_2}) C_{ps} (T_{fec} - 273.0), \end{aligned} \quad (3)$$

where F_1 is the fraction of the total heat released in the conical clay enclosure, and is given by

$$F_1 = \frac{2}{3} x,$$

where x is the fraction of volatiles in the wood. Typically x varies between 0.6 and 0.9.

Similarly, the gas temperature (T_{fem}) in the metal enclosure is given by

$$\begin{aligned} F_2 \dot{m}_{fu} C_{a_{wo}} + (\dot{m}_{p_1} + \dot{m}_{p_2}) C_{pa} (T_{\infty} - 273.0) + \dot{m}_s C_{pa} (T_{sec} - 273.0) - q_{rad_2} + \\ + \alpha_{2i} (T_{fec} - T_{w2}) A_2 - q_{rad_4} + \alpha_{4i} (T_{fem} - T_{w4}) A_4 \\ = (\dot{m}_{fu} + \dot{m}_{p_1} + \dot{m}_{p_2} + \dot{m}_s) C_{ps} (T_{fem} - 273.0), \end{aligned} \quad (4)$$

where $F_2 (> F_1)$ is assigned a value of 0.9.

Finally the out-going fluegas temperature T_{f1} is given by:

$$\begin{aligned} \dot{m}_{fu} C_{a_{wo}} + (\dot{m}_{p_1} + \dot{m}_{p_2}) C_{pa} (T - 273.0) + \dot{m}_s C_{pa} (T_{sec} - 273.0) - \dot{H}_a - \\ - q_{rad_2} + \alpha_{2i} (T_{fec} - T_{w2}) A_2 - q_{rad_4} + \alpha_{4i} (T_{fem} - T_w) A_4 \\ = (\dot{m}_{fu} + \dot{m}_{p_1} + \dot{m}_{p_2} + \dot{m}_s) C_{ps} (T_{f1} - 273.0) \end{aligned} \quad (5)$$

Note that complete heat release inside the stove is assumed. The values of C_{ps} (i.e. specific heat of hot gas) are calculated as given in appendix A.

The temperature of the secondary air (T_{sec}) as it enters the metal enclosure is estimated from:

$$\dot{m}_s C_{pa} (T_{sec} - T) = Z_2 (T_{w4} - T_{sec}) A_4 + Z_1 (T_{w2} - T_{sec}) A_2. \quad (6)$$

The wood surface temperature is the most difficult parameter to specify. Not only does the temperature vary along the surface, but it also varies with time. Besides wood burns in two separate phases i.e. that during which volatiles evolve and that when char is combusted at the surface. Typically wood surface temperatures vary

from 300 to 550°C (Blackshear & Murthy 1965) during the total period of combustion. Different pieces and different parts of the surface burn in different phases at a given time. This makes specification of average steady state temperature a difficult task.

However, we may postulate an Arrhenius type relationship between wood-burning rate and its surface temperature. The volatile evolution phase typically occupies 40% and the char burning 60% (Blackshear & Murthy 1965; Tinney 1965). The burning rates during the two phases are also different. Typically it is observed that 75% of weight loss is by volatile evolution and 25% by char combustion. Taking these factors into account we may postulate an expression for the average burning rate as:

$$\dot{m}_{fu}'' = (\rho_{wo} D_{wo}/8) [0.4(2-x)v_1 \exp(-E_1/RT_1) + 0.6(1-x)v_2 \exp(-E_2/RT_2)], \quad (7)$$

where T_1 and T_2 are average temperatures during volatile evolution and char burning, respectively, and x is the volatile fraction by weight; the v 's and the E 's are the rate constants in the respective phases of burning.

We now specify the average temperatures T_1 and T_2 in the two phases of burning, as:

$$T_1 = T_s - 50; \quad T_2 = T_s + 50, \quad (8)$$

and following Tinney (1965), who specifies a range of values of the rate constants, we may assume the following:

D_{wo} (cm)	v_1 (h ⁻¹)	v_2 (h ⁻¹)	E_1/R (K)	E_2/R (K)
2.54	2.16×10^{11}	1.44×10^{12}	15,000	21,500
1.26	1.26×10^{12}	4.3×10^{12}	15,000	19,900
0.63	2.7×10^{12}	7.2×10^{12}	15,000	18,300

Equation (7) thus determines T_s when $\dot{m}_{fu}''$ is obtained, or specified.

1.4 Radiation fluxes

The radiation fluxes at surfaces 1, 2, 4 and 5 (see figure 1) are derived from the enclosure theory by postulating an imaginary surface 3 that divides the combustion zone into the conical and the cylindrical enclosures. Emissivities of all surfaces are assumed to be unity. Then:

$$q_{rad,1} = \sigma [Z_{13} T_3^4 + Z_{12} T_{w_2}^4 + \epsilon_{fec} T_{fec}^4] - \sigma T_s^4, \quad (9)$$

$$q_{rad,2} = \sigma [Z_{23} T_3^4 + Z_{21} T_s^4 + Z_{22} T_{w_2}^4 + \epsilon_{fec} T_{fec}^4] - \sigma T_{w_2}^4, \quad (10)$$

$$q_{rad,4} = \sigma [Z_{43} T_3^4 + Z_{45} T_5^4 + Z_{44} T_{w_4}^4 + \epsilon_{fem} T_{fem}^4] - \sigma T_w^4, \quad (11)$$

and

$$q_{rad,5} = \sigma [Z_{53} T_3^4 + Z_{54} T_{w_4}^4 + \epsilon_{fem} T_{fem}^4] - \sigma T_5^4, \quad (12)$$

where $Z_{ij} = F_{ij} \tau_{ij}$.

In the above T_3 (imaginary surface temperature) and T_5 (the pan bottom

where

$$\Delta T = T_{ps} - T_{\infty},$$

T_{ps} was assigned a value of 343 ± 5 (K).

1.6 Air flow rates

From the first law of thermodynamics, the air flow rate expressions are given by (see figure 3a)

$$\dot{m}_{p1}^2 = \frac{2 \rho_{\infty} H_{gr} (1 - T_{\infty}/T_{fem}) A_{gr}^2}{K_i + K_{bend} + (1 - X_2)^2 X_1^2 + (T_m/T_{\infty}) + [(K_{gr} + 1) T_s/T_{\infty}]}, \quad (19)$$

where

$$T_m = 0.2 T_s + 0.8 T_{\infty},$$

$$X_1 = 2 (D_{gr}/D_{pr})^2 / N_{pr},$$

$$X_2 = (D_{pr}/D_{gr}) / N_{pr},$$

$$K_i = 0.6,$$

$$K_{bend} = 1.0,$$

and

$$K_{gr} = 0.75 [(\delta + D_{rod})/\delta - 1.0]^{1.33} \quad (\text{Kazantsev 1977}).$$

With reference to figure 3b, $\dot{m}_{p2}$ is given by the solution of the following quadratic

$$A \dot{m}_{p2}^2 + B \dot{m}_{p2} + C = 0 \quad (20)$$

Figure 3. Assumed flow patterns. a) Flow pattern of $\dot{m}_{p1}$; b) flow pattern of $(\dot{m}_{p1} + \dot{m}_{p2} + \dot{m}_{fu})$; c) flow pattern of $(\dot{m}_{p1} + \dot{m}_{p2} + \dot{m}_{fu} + \dot{m}_s)$.

where

$$\begin{aligned}
 A &= (K_{ent}/A_{ent}^2) + Z_4, \\
 B &= 2(\dot{m}_{p_1} + \dot{m}_{fu}) Z_4, \\
 C &= (\dot{m}_{p_1} + \dot{m}_{fu})^2 (Z_4 + Z_5) - 2\rho_\infty^2 g (H_{th} - H_{gr}) (1 - T_\infty/T_{fec}), \\
 Z_4 &= (T_3 + K_{cont} + T_{fec})/T_\infty/A_{th}^2, \\
 Z_5 &= (K_{bed} - 1)T_s/T_\infty/A_{gr}^2, \\
 K_{ent} &= 0.6, \\
 K_{cont} &= 0.5 \sin \phi (1 - A_{th}/A_{gr}) \quad (\text{Kazantsev 1977}), \\
 K_{bed} &= 1.12 (0.66 N_{row} + 0.5), \\
 \phi &= (D_{gr} - D_{th}) / [(D_{gr} - D_{th})^2 + (H_{th} - H_{gr})^2]^{1/2}.
 \end{aligned}$$

Similarly the secondary air flow rate ($\dot{m}_s$) is given by (see figure 3c)

$$A\dot{m}_s^2 + B\dot{m}_s + C = 0, \quad (21)$$

where

$$\begin{aligned}
 A &= K_{sec}/A_{sec}^2 + C_2, \\
 B &= 2C_2(\dot{m}_{p_1} + \dot{m}_{p_2} + \dot{m}_{fu}), \\
 C &= (C_1 + C_2)(\dot{m}_{p_1} + \dot{m}_{p_2} + \dot{m}_{fu})^2 - 2\rho_\infty^2 g (H_t - H_{th}) (1 - T_\infty/T_{fem}), \\
 C_1 &= [(K_{exp} - 1)/A_{th}^2] (T_3/T_\infty), \\
 C_2 &= (K_{st}/A_s^2) (T_{fem}/T_\infty) + (1/A_{gap}) (T_{fl}/T_\infty), \\
 K_{sec} &= 1.2, \\
 K_{exp} &= (1 - A_{th}/A_s)^2, \\
 K_{st} &= (A_s/A_{gap})^2 - 1.0 + [(A_s/A_{gap}) - 1]^2 \quad (\text{Krishna Prasad \& Buss-} \\
 &\text{man 1986}).
 \end{aligned}$$

1.7 Fuel burning rate

We have already noted that the fuel burning rate ($\dot{m}_{fu}$) is a function of time. However, we have chosen to define the average rate. This rate can be specified in two ways.

If power input to the stove pkw (kW) is specified, then

$$\dot{m}_{fu} = pkw \times 860 / Ca_{wo}. \quad (22)$$

However, by determining the energy balance at the surface of the wood, it is also possible to obtain an equation for the burning flux $\dot{m}_{fu}''$ (kg/h m²), where

$$\dot{m}_{fu}'' = \dot{m}_{fu} / A_{wo}, \quad (23)$$

and A_{wo} is the burning wood surface area.

The energy balance at the wood surface yields

$$q_{rad, 1} \frac{A_{gr}}{A_{wo}} + K \left. \frac{\partial T}{\partial n} \right|_s = \dot{m}_{fu}'' (h_s - h_T - \lambda_{wo}), \quad (24)$$

where

$$K \frac{\partial T}{\partial n} \Big|_s = \text{convective heat transfer to wood.}$$

If the average gas temperature past the surface of the wood is assumed to be $(T_\infty + T_s)/2$ and the heat transfer coefficient at the wood surface is taken as (Hollman 1975)

$$\alpha_s = 1.32 [(T_\infty - T_s)/2D_{wo}]^{0.25}, \quad (25)$$

then

$$K \frac{\partial T}{\partial n} \Big|_s = 1.32 [(T_\infty - T_s)/2D_{wo}]^{0.25} (T_\infty - T_s)/2.$$

The average wood temperature is assumed to be $T_s - 150$,

hence

$$h_T = C_{pwo}(T_s - 150)$$

where

$$C_{wo} = 0.246 \text{ kcal/kg K.} \quad (26)$$

There is no agreement on the value of λ_{wo} in the literature. Values ranging from 30 kcal/kg to 500 kcal/kg have been suggested (Blackshear & Murthy 1965; Simmons & Lee 1965; Tinney 1965). We have chosen the expression for λ_{wo} as

$$\lambda_{wo} = 2.5 (4926 D_{wo} + 38); \quad (27)$$

which yields $\lambda_{wo} = 250$ kcal/kg for $D_{wo} = 0.0126$ m. Combining the above results, it is now possible to obtain the approximate relation for $\dot{m}_{fu}$ as

$$\dot{m}_{fu}'' = \frac{q_{rad,1} (A_{gr}/A_{wo}) + 0.555 D_{wo}^{-0.25} (T_\infty - T_s)^{1.25}}{C_{ps}(T_s - 273) - C_{pwo}(T_s - 423) - 2.5(4926 D_{wo} + 38)} \quad (28)$$

The above relationship predicts that $\dot{m}_{fu}''$ decreases with increase in D_{wo} . This trend has been observed experimentally (Blackshear & Murthy 1965; Tinney 1965).

1.8 Accounting for the effect of swirl

In the CTARA stove, the swirl is induced by placing a swirling device in the throat of the combustion chamber. The device can be expected to produce the following three effects,

- (i) increased resistance to gas flow through the throat and in the metal cylinder;
- (ii) increased resistance to the lateral entry of secondary flow because of the presence of swirling gas flow in the metal cylinder;
- (iii) augmentation in the value of the heat transfer coefficient at the pan surface.

These considerations suggest that values of K_{cont} in (20), K_{exp} , K_{sec} and K_{st} in (21) and α_{pf} in (17) must be augmented. The amount of augmentation will of course differ. In the absence of any precise data on these aspects, all the above coefficients are augmented by multiplying them by a single factor K_{swirl} which is greater than 1.0.

1.9 Method of solution

We have so far provided relationships governing all parameters of interest as well as given values of other empirical inputs. The relationships are nonlinear and coupled. Hence a successive-substitution type iterative algorithm was used to solve the equations with necessary under-relaxation. We summarise in table 1 the parameters and the equations governing them.

Computer solution of the equations requires

- 1) Geometric information on the stove (i.e. all parameters mentioned in figure 1)
- 2) Empirical inputs such as shape factors, mean beam lengths, flow resistance coefficients etc.
- 3) Operating parameters like pan diameter, volatile fraction, wood moisture content, and wood-burning surface area or the power of the stove.

A computer program called STOVE has been written which has three subroutines DATA, PARAM and QRAD. The main program STOVE iteratively calculates the equations; the DATA routine provides information on stove geometry and operating parameters; the shape factors, mean beam lengths and resistance coefficients are calculated according to the specified geometry in the routine PARAM. Temperature-dependent values of transmittivities and radiation fluxes are calculated in the routine QRAD.

Solution of equations required use of an under-relaxation factor of 0.1. Typically 50 to 80 iterations were required starting with a fixed set of initial values. That the equations converged to the same result irrespective of the initial guess was verified. The convergence was checked by a fractional change criterion. The solutions were terminated when the maximum fractional change among all variables was less than 10^{-3} .

The outputs of the program are, of course, the values of temperatures, wood-burning rate, air mass flow rates and values of intermediate parameters such as the temperature-dependent heat transfer coefficients, transmittivities and radiation fluxes. From these the two following parameters are evaluated.

$$\text{Efficiency } (\eta) = \dot{H}_a / \dot{H}_r = \dot{H}_a / (\dot{m}_{fu}'' A_{wo} Ca_{wo}) \quad (29)$$

Table 1. Model equations

Symbol	Name	Equation(s)
Ca_{wo}	Lower calorific value	(8) (Part I)
T_{w2}	Wall temperature	(1)
T_{w4}	Wall temperature	(2)
T_{fec}	Combustion temperature	(3)
T_{fem}	Combustion temperature	(4)
T_{fl}	Flue gas temperature	(5)
T_s	Wood surface temperature	(7)
T_{sec}	Secondary inlet temperature	(6)
$q_{rad, 1-5}$	Surface radiation fluxes	(9) to (12)
T_3	Intermediate combustion temperature	(13)
H_a	Heat absorption rate	(15)
$\dot{m}_{p_1}$	Primary air flow rate-1	(19)
$\dot{m}_{p_2}$	Primary air flow rate-2	(20)
$\dot{m}_{sec}$	Secondary air flow rate	(21)
$\dot{m}_{fu}''$	Wood-burning flux	(28)

and

$$\text{excess air factor } (X_{air}) = \frac{\dot{m}_{p_1} + \dot{m}_{p_2} + \dot{m}_s}{[\dot{m}_{fu}(A/F)_{stoic}]}, \quad (30)$$

where the stoichiometric air-fuel ratio is 6.5 as determined in appendix A.

2. Results and discussion

2.1 Manner of presentation

The purpose of developing a theoretical model has been to verify the experimental findings presented in Part I and then to study the effect of changing various geometric dimensions and operating parameters on the performance of the stove. Hence, computer runs were made by varying the input parameters.

The reader is reminded of the fact that we have adopted a steady state model as such values of η and X_{air} etc. cannot be compared directly with the results of the wood-burning stove tests. Therefore the model can only provide trends that can be compared with experiments.

In order to facilitate understanding of the results, we have chosen a reference stove (see § 2.2 below). Individual parameters are then varied keeping all other parameters constant. Complete details of all computed parameters are given for the reference case; but for all other cases only values of η and X_{air} are plotted. This is done to preserve the clarity of the figures plotted.

2.2 The reference case

The reference case comprised the following input parameters.

Geometric inputs

$H_t = 31 \text{ cm},$	$H_{th} = 22 \text{ cm},$	$H_{gr} = 7.5 \text{ cm},$	$H_g = 2 \text{ cm},$
$D_{gr} = 22 \text{ cm},$	$D_C = 8 \text{ cm},$	$D_{th} = 8 \text{ cm},$	$D_p = 31 \text{ cm},$
$D_{pr} = 4.5 \text{ cm},$	$D_{sec} = 1 \text{ cm},$	$D_{ent} = 10 \text{ cm},$	$D_s = 3 \text{ cm},$
$N_{pr} = 5,$	$N_{sec} = 24,$	$N_s = 6,$	$H_p = 15 \text{ cm}.$

Wood parameters

$$D_{wo} = 1.26 \text{ cm}, \quad A_{wo} = 400 \text{ cm}^2,$$

$$f = 15\%, \quad x = 75\%.$$

Starting with initial guesses for temperatures, air flow rates, fuel burning rate, heat absorption rate etc., a converged solution was obtained in 54 iterations yielding the following predictions.

Output parameters

$\eta = 28.67\%,$	power = 3.38 kW,	$\dot{m}_{fu}'' = 20.93 \text{ kg/h m}^2,$
$\dot{m}_{p_1} = 5.75 \text{ kg/h},$	$\dot{m}_{p_2} = 8.011 \text{ kg/h},$	$\dot{m}_s = 13.88 \text{ kg/h},$

$$\begin{array}{lll}
X_{air} = 5.08, & T_{fec} = 671.3 \text{ K}, & T_{fem} = 675.2 \text{ K}, \\
T_{fl} = 595.5 \text{ K}, & T_{sec} = 349.7 \text{ K}, & T_{w_2} = 625.4 \text{ K}, \\
T_{w_4} = 532.9 \text{ K}, & T_s = 672.4 \text{ K}, & \alpha_s = 12.49 \text{ kcal/h m}^2 \text{ K}, \\
q_{rad,1} = -2362 \text{ kcal/h m}^2, & q_{rad,2} = 1472 \text{ kcal/h m}^2, & q_{rad,4} = 476 \text{ kcal/h m}^2, \\
q_{rad,5} = 3800 \text{ kcal/h m}^2, & \alpha_{pf} = 28.35 \text{ kcal/h m}^2 \text{ K}, & \dot{H}_a = 833.2 \text{ kcal/h}, \\
\dot{H}_r = 2906 \text{ kcal/h}, & \varepsilon_{fem} = 0.0098, & \varepsilon_{fec} = 0.01642.
\end{array}$$

The efficiency of 28.67% may be compared with the experimentally observed value 28.31% for 3.62 kW power mentioned in table 4 of Part I.

For the wood dimensions chosen, the predicted power is 3.38 kW and the excess air factor is 5.08. The total air flow rate is 27.65 kg/h. This value compares favourably with the electrical stove measurements where the measured air flow rate is of the order of 6 kg/h kW power input. The excess air factor also compares favourably with the values quoted in the literature (Krishna Prasad & Bussman 1986).

The maximum gas temperature occurs in the metal enclosure ($T_{fem} = 675.2 \text{ K}$) and the wood surface temperature is predicted at $T_s = 672.4 \text{ K}$. The radiation flux at surface 1 is negative, suggesting an outward radiation from the wood surface, whereas the radiation flux to the pan is $q_{rad,5} = 3800 \text{ kcal/h m}^2$. With $D_p = 31 \text{ cm}$, the radiation contribution to the pan is 286 kcal/h and the convective contribution is $(\dot{H}_a - q_{rad,5} A_s) = 547 \text{ kcal/h}$. Thus our stove is convectively dominated as far as heat absorption to pan is concerned. This was also observed in the electrical stove where the convective contribution dominated over the radiative one.

The outgoing flue gas temperature is $T_{fl} = 595.5 \text{ K}$ representing a loss of 2055 kcal/h.

The predicted fuel burning rate ($\dot{m}''_u = 20.93 \text{ kg/h m}^2$) also compares favourably with the values quoted in literature (Blackshear & Murthy 1965). Thus most detailed predictions yield plausible values both in direction and magnitude.

2.3 Effect of geometric parameters

We now study the effect of changing individual parameters (keeping other things constant) on the stove performance. The predicted variations are plotted in graphs (figures 4 to 18) where the reference case point is circled thus $\odot$.

Effect of D_c : The reference case corresponds to $D_c = D_{th} = 8 \text{ cm}$. As such computations were performed for $8 \text{ cm} \leq D_c \leq 15 \text{ cm}$. Results are plotted in figure 4. It is seen that the efficiency as well as the excess air factor drops with increase in D_c . This effect was also observed in wood-burning stove experiments as seen in tables 2 and 3 of Part I. The increase in D_c has a marginal effect on the predicted power for the same wood-burning surface area. The excess air factor also reduces with increase in D_c , since indicated temperatures are lower resulting in reduced buoyancy.

Effect of D_{gr} : Figure 5 shows the effect of increasing the grate diameter (or the stove diameter). Here the efficiency increases with D_{gr} suggesting that the desirable aspect ratio (H_t/D_{gr}) for our stove should be about 1.0. The X_{air} is marginally affected.

Figure 4. Effect of the diameter of the cylindrical enclosure.

Effect of H_t : A similar trend is observed in figure 6 where the total height of the stove is reduced from 31 cm to 25 cms. As H_t comes close to the reference $D_{gr} = 22$ cm, the efficiency is seen to increase substantially, with a small reduction in the excess air factor. Thus one can expect high efficiency without excessive smoke.

Effect of H_{th} : Figure 7 shows effect of varying the throat height. As the throat is brought closer to the pan, the efficiency increases marginally with similar reduction

Figure 5. Effect of the grate diameter.

Figure 6.. Effect of the height of the stove.

in excess air factor. For the same wood surface area, the power of the stove (or $\dot{m}''_{fu}$) increases with H_{th} .

Effect of H_{gr} : Effect of varying H_{gr} is seen to be marginal on efficiency and X_{air} in figure 8. The power of the stove however is affected.

Effect of H_g : The gap between the pan and the stove has a dramatic influence on the efficiency of the stove as seen from figure 9. The reduction in the gap offers greater resistance to air flow resulting in an increase in temperature which in turn increases

Figure 7. Effect of the throat height.

Figure 8. Effect of the grate height.

the buoyancy. Thus, the gap has little influence on the excess air factor but has a substantial effect on heat absorption because of the higher local temperature differences.

Effect of D_{pr} , D_{sec} , D_{ent} : Figures 10 to 12 show the effects of varying air inlet port diameters. It is seen that the effect of changes in D_{pr} and D_{ent} is less than 10% on efficiency and negligible on X_{air} factor. The effect of D_{sec} however is substantial. The wood stove experiments corroborate the effects predicted in relation to primary and

Figure 9. Effect of the height of the gap between the pan and the stove.

Figure 10. Effect of the primary port diameter.

Figure 11. Effect of the fuel feed port diameter.

entry port diameters. However the increase in efficiency observed with preventing secondary air flow was found to be minimal, whereas the predictions show substantial increase. In the model, reduction in $\dot{m}_s$ increases T_{sem} , resulting in increase in convective and radiative contributions. The electrical stove, where only one inlet port was provided, also showed substantial decrease in efficiency (see

Figure 12. Effect of the secondary port diameter.

figure 6 of Part I) with reduction in inlet port area. Thus, the predictions stress the importance of decreasing the diameter of the secondary ports. This is however likely to result in increase in smoke. Further experiments with reduced D_{sec} would be worthwhile.

2.4 Effect of operating parameters

Effect of power: These predictions were made by eliminating (28) for $\dot{m}_{fu}''$ and specifying $A_{wo} = 1 \text{ m}^2$. The power of the stove was thus an input parameter and $\dot{m}_{fu}''$ was the output. It was noticed that when this procedure was adopted, the program predicted the same efficiency for 3.35 kW as was predicted by specifying $A_{wo} = 400 \text{ cm}^2$ and including (28). This confirms the validity of (28) which has diameter-dependent empirical inputs.

Figure 13 shows the effect of varying the power input. Unlike the wood-burning stove (see tables 1 and 9 of Part I) and the electric stove (see figure 6 of Part I), the program predicts increase in efficiency and reduction in excess air with power input. This is the most significant departure noted between prediction and experiment. The deviation cannot be simply attributed to the ignorance of sensible heat (i.e. the steady state assumption) in the programmed model since the steady state was achieved to a considerable extent (over 60% of the total time of the experiment) in the electrical stove, though not in the wood-burning stove. Still it may be noted that in the experiments both the heat losses and the air flow rates are time-dependent. The model however assumes zero heat loss from the stove body.

The discrepancy thus needs further investigation and it is conceivable that splitting the control volume into greater number of sections may yield predictions in accord with the experiment besides accounting for the sensible heat stored in the stove body.

Effect of pan diameter D_p : Figure 14 shows the effect of reduction in pan

Figure 13. Effect of power.

Figure 14. Effect of the pan diameter.

diameter from the reference value. As in the wood-stove experiment, the efficiency reduces drastically with reduction in diameter of the pan. The smaller the heat absorption area, the smaller is the heat absorption rate and hence there is a reduction in efficiency. This figure confirms the desirability of shielding pans of smaller diameters, which will force the hot flue gases to travel past the vertical sides of the pan as has been shown in Krishna Prasad & Bussman (1986).

Effect of wood parameters D_{wo} , f , x : Figure 15 shows the effect of varying the diameter of the wood. As was measured in wood-stove experiments (see table 9 of Part I), the efficiency of the stove drops with increase in wood diameter. The excess air factor however increases with D_{wo} since higher temperatures are predicted with larger wood diameters. The power of the stove is also slightly affected because of reduction in $\dot{m}_{fu}''$ with increase in D_{wo} .

Figure 16 shows the expected result that the drier the wood the higher is the efficiency, although the increase is not significant.

Figure 17 shows the effect of variation in the volatile content (x) of the wood. The predictions show a negligible effect of x on efficiency in spite of the fact that F_1 (i.e. the fraction of heat released in the clay enclosure) increases with x . This is unlike the trend observed in the electrical stove (see figure 9 of Part I) where increase in flame to grate power resulted in increase in efficiency. The main reason for this is that although F_1 increases with x , the ratio of flame to grate heat release does not change significantly in the model. This suggests that the electrical stove is an oversimplification of the reality of the wood-burning phenomenon. Nonetheless, the wood-burning stove does show effects of the wood species used (see table 9 of Part I) to some extent.

2.5 Effect of swirl: In § 1.8, it was shown that the effect of swirl can be

Figure 15. Effect of the diameter of the wood pieces.

Figure 16. Effect of moisture content.

approximately accounted for by an indicative parameter K_{swirl} . In figure 18, the value of K_{swirl} is varied from 1 to 2. It is seen that efficiency increases drastically with K_{swirl} as was observed in the wood-stove experiments (see table 7 of Part I). Thus most of the effects attributed to swirl appear to be well represented. The model thus also confirms the desirability of imparting swirl. The main reason for increase in efficiency appears to be the reduction in the flue gas temperature by nearly 60°C when K_{swirl} is increased from 1 to 2, thus reducing the flue loss.

Figure 17. Effect of volatile fractions in wood.

Figure 18. Effect of swirl.

3. Conclusions

A comprehensive theoretical framework suitable for predicting the performance of a single-pot wood-burning stove has been developed in this paper. The framework enables prediction of the effects of varying different geometric and operating parameters. In fact the experimental development of the CTARA stove described in Part I was assisted by the predictions of the computer program STOVE which is based on the theoretical framework presented here.

The theoretical framework has predicted almost all trends observed in the experiment correctly except in the case of power. The theory predicts increasing efficiency with power whereas the experimental efficiencies are independent of power. It is possible that the neglect of sensible heat stored in the stove body results in this deviation from the experimental finding. Further tests with metal stoves are however necessary.

Plausible trends predicted with respect to other geometric and operating parameters however provide confidence in the several empirical inputs used in the theoretical framework. As such the framework may be extended to other stove geometries developed by several investigators.

With respect to the CTARA stove the theoretical model has shown that for good efficiencies the aspect ratio of the stove (or the ratio of height to its diameter) should be of the order 1. Also the height of the throat (H_{th}) should be approximately 3/4th the total height of the stove. The throat diameter and the metal cylinder diameter should also be equal. The dramatic effect of the gap between the stove and the vessel has been brought out by the model suggesting that the gap should be of the order of 1 cm. The grate height, the primary port diameter, and the feed port diameter have little influence on efficiency. The secondary port diameter should however be about 6 to 7.5 mm. Finally, the introduction of the swirling device is very essential to improve efficiency.

With the availability of the theoretical model, it is now possible to vary several

design parameters. Such a large number of solutions will enable optimisation of the design through a programming model. This task will be taken up in future.

Appendix A

Determination of C_{ps} and X_{air}

Complete combustion of wood in air yields

where

$$n = 5.555 [f/(1-f)],$$

and f is the percentage moisture content of wood.

From (A1) it is clear that stoichiometric air must provide

4.735 moles of O_2 per 100 kg of dry wood,

or 4.735×32 kg of O_2 per 100 kg of dry wood.

The stoichiometric air fuel ratio $(A/F)_{stoic}$ is thus given by

$$\begin{aligned} (A/F)_{stoic} &= 4.735(32 + 3.77 \times 28)/100 \\ &= 6.51 \text{ kg of air/kg of dry wood.} \end{aligned} \quad (A2)$$

Also the excess air factor X_{air} is given by:

$$X_{air} = y/4.735 = (\dot{m}_{p_1} + \dot{m}_{p_2} + \dot{m}_s)/\dot{m}_{fu}/6.51. \quad (A3)$$

The product composition can now be written as

$$\begin{aligned} &4.42 CO_2 + [3.05 + 5.555 f/(1-f)] H_2O + 3.78 \times 4.735 N_2 + \\ &+ 4.735 (X_{air} - 1) O_2. \end{aligned} \quad (A4)$$

The total weight of the products is

$$\begin{aligned} W_{pr} &= 4.42 \times 44 + [(3.05 + 5.555 f/(1-f))]18 + 3.78 \times 4.735 \times 28 + \\ &+ 4.735 (X_{air} - 1) 32 \\ &= 599 + 151.52 X_{air} + 100 f/(1-f). \end{aligned} \quad (A5)$$

The mass fraction of each component is

$$\begin{aligned} w_{CO_2} &= 194.5/W_{pr}, \\ w_{H_2O} &= (54.9 + 100 f/(1-f))/W_{pr}, \\ w_{O_2} &= 151.52 (X_{air} - 1)/W_{pr}, \\ w_{N_2} &= 501.15/W_{pr}. \end{aligned} \quad (A6)$$

The specific heat of the mixture C_{ps} is given by

$$C_{ps} = w_{N_2} C_{p, N_2} + w_{O_2} C_{p, O_2} + w_{CO_2} C_{p, CO_2} + w_{H_2O} C_{p, H_2O}, \quad (A7)$$

where in the range of temperatures encountered,

$$C_{p, N_2} = 0.244, C_{p, O_2} = 0.225, C_{p, CO_2} = 0.23, C_{p, H_2O} = 0.46.$$

List of symbols

A	surface area;
b_2, b_4	thicknesses of surfaces 2 and 4;
C_{pa}	specific heat of ambient air;
C_{ps}	specific heat of hot gases;
C_{pwo}	specific heat of wood;
Ca_{wo}	calorific value of wood;
D_c	diameter of cylindrical enclosure;
D_{ent}	fuel feed port diameter;
D_{gr}	grate diameter;
D_p	pan diameter;
D_{pr}	primary port diameter;
D_{rod}	grate-rod diameter;
D_{sec}	secondary port diameter;
D_{wo}	diameter of wood;
f	percentage moisture content of wood;
F_{ij}	shape factor;
h	enthalpy;
h_s, h_T	specific enthalpy at the surface ($C_{ps} T_s$) and average specific enthalpy of wood, respectively;
$\dot{H}_a$	heat absorption rate at the pan;
H_{gap}	height of gap between pan and stove;
H_{gr}	height of the grate;
H_p	height of the pan;
H_t	height of the stove;
H_{th}	height of the throat;
K	resistance coefficients;
K_2, K_4	conductivities of walls 2 and 4;
K_{cont}	loss coefficient for contraction;
$\dot{m}_{p_1}$	mass flow rate through primary ports;
$\dot{m}_{p_2}$	mass flow rate through fuel entry port;
$\dot{m}_{fu}$	mass flux of fuel;
$\dot{m}_{fu}$	mass flow rate of fuel;
$\dot{m}_s$	mass flow rate of secondary air;
N_{pr}	number of primary ports;
N_{row}	number of rows of wood pieces in the bed;
N_{sec}	number of secondary ports;
q_{conv}	convective heat flux;
q_{loss}	heat loss from the pan;
q_{rad}	radiation flux;
T	temperature;
T_w	wall temperature;
x	fraction of volatiles in wood;
X_{air}	excess air factor;
α	heat transfer coefficient;
σ	Stefan–Boltzman constant;
ε	emissivity;
τ	transmitivity;

ρ	density;
δ	grate rod diameter;
λ	latent heat;
λ_{wo}	heat of gasification (kcal/kg).

Suffixes

<i>fec</i>	flue gas in clay enclosure;
<i>fem</i>	flue gas in metal enclosure;
<i>fl</i>	flue gas at exit;
<i>gap</i>	between pan and stove;
<i>gr</i>	grate;
<i>i</i>	inside;
<i>o</i>	outside;
<i>pf</i>	between pan and flue gases;
<i>ps</i>	between pan and surroundings;
<i>s</i>	wood surface condition;
<i>sec</i>	secondary air;
<i>st</i>	stagnation point;
<i>th</i>	throat;
<i>wo</i>	wood;
∞	ambient condition.

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