The coordinating model of the Ganga Basin

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Abstract. A scheme of model studies required to analyse a complex large scale system through decomposition approach is outlined. First, the total system is proposed to be modelled to identify interlinkages of the subsystems. Then the subsystems are analysed in detail to develop response functions. Finally the optimal system functioning is obtained by modelling the entire system in terms of response functions of the components.

A linear programming model is developed for modelling the total system to identify the interlinkages. Its application is demonstrated in the Greater Ganga Basin using published data. Trade-off is developed between power generation and irrigated area in various seasons.

Keywords. Large scale systems; multilevel hierarchical approach; co-ordinating screening model; linear programming model

1. Introduction

Systems analysis of large systems is an integrated-hierarchical multilevel and multistage activity because of the complexity, scale and dynamic nature of the problem. Multilevel hierarchical analysis may be carried out in several ways (Lasdon 1970; Haimes 1977; Chaturvedi 1974). In the first instance, the total system may be modelled to identify the order of interlinkage of the sub-systems. In the second study, the sub-systems may be examined in detail and their response expressed as a function of the interlinkage over the likely range as anticipated from the first study. Finally, the optimal system functioning may be obtained by modelling the entire system in terms of the response functions of each of the sub-systems as obtained from the second study.

The type of model depends upon the characteristics, scale and complexity of the system, as well as on the objectives of the analysis. The first study carried out through a model called the coordinating-screening model is for coarse screening and deterministic modelling dividing the year into two or three time periods. A linear-programming approach may be used. Further, by adapting the inflows over a range to cover the likely variability of inflows, the implications of the stochastic nature of inputs may be studied (Jacoby & Loucks 1972; Chaturvedi & Srivastava 1981). For the second series of models for sub-systems analysis, any appropriate systems analysis approach using deterministic and stochastic models may be used (Hall & Dracup 1970; Biswas 1976; Chaturvedi 1984). And finally for the third-stage analysis, mathematical programming models and simulation models, singly or hierarchially, may be used. For studying the Ganga basin, the hierarchical modelling scheme shown in figure 1 was

Figure 1 Water resources systems analysis modelling morphology

developed. The scheme with slight modifications has been applied to study the Punjab water resources (Chaturvedi 1979).

It must be emphasised at the outset that the study brings out only an approach to analysis and the quantitative figures given are meant only to illustrate the approach for the decomposition of the system and have no real life implication.

2. The model

The first stage preliminary screening of the Ganga basin is discussed in this section.

For the first coordinating-screening model, a simple linear model was constructed to coordinate all the demands placed upon the system (Chaturvedi 1974; Chaturvedi and Rogers 1975). This model is then used as a tool to explore the various goals to be attained and the constraints on development. Once the system has been fully explored by this inexpensive model, it can be broken up into smaller, more manageable pieces for more detailed analysis. However, the overall model can still be used to coordinate the results of the detailed studies as they become available, or the second coordination can be in terms of the response functions of each of the sub-systems.

Figure 2 shows the schematic layout of the Ganga basin used in the coordinating model. There are 49 river-schematic nodes of interest at which junctions, diversions, groundwater pumping and return flows occur. The model considers 5 storage reservoirs, 6 hydropower plants, and 25 major irrigation works as already developed and looks for the optimal allocation of the water resources for agriculture and power subject to meeting downstream flows for navigation, water supply and salinity control in the delta.

Table 1 summarises the system sketch and the water continuity relationships. The number in each row shows the connection and the flow direction for each of the parameters or division variables— X , river flow; Y , diversion; Z , diversion for irrigation; P , flow for hydroelectric generation; V , storage locations; A , area for irrigation; T ,

Figure 2. Schematic diagram of the Ganga basin

groundwater development; D , groundwater return flow; H , the hydrological input. For example, a negative number ($-k$) from a row of node i indicates that water flows from node k towards node i . On the other hand, a positive number (k) implies that water flows away from node i towards node k .

2.1 Objective function

The major uses for water in the basin are to produce energy and for agriculture; therefore, the coordinating-screening model is formulated on the basis of these two uses. Other uses of water (such as water quality control, navigation, urban water supply, flood control, etc.) are not fully studied and are constrained by minimum or maximum river flows.

An objective function is thus constructed to include the major water uses for energy production and irrigation. This objective function will enable the model to select the best mix of these uses of water. The objective is to maximise agriculture and energy in the following form.

$$\max Z = f_a \cdot \sum_i \sum_j (A_{ij}) + f_p \cdot \sum_j \sum_i (P_{ij} \cdot K_i), \quad (1)$$

Table 1. System identification table.

Node no.	X	Y	Z	P	V	A	T	D	H
1	1-2							-2	1
2	2-3	2	2			2	2	-3	
3	3-4-19		3			3	3	-4-19	
4	4-5-15		4			4	4	-5-15	
5	5-6-19		5			5	5	-6-19	5
6	6-7-16		6			6	6	-7-16	
7	7-8-17		7			7	7	-17	7
8	8-9-21								8
9	9-10-28							-28	9
10	10-11-32							-32	
11	11-12-29							-12-29	11
12	12-13	-29	12			12	12	-13	
13	13-14		13			13	13		
14	14								14
15	15		15			15	15		15
16	16		16			16	16		16
17	17-18		17			17	17		
18	18-20								18
19	19		19			19	19		19
20	20	20		20	20				20
21	21-22-24							-24	
22	22-23	22							
23	23								23
24	24-25		24			24	24		
25	25-26	-22						-26	
26	26-27		26			26	26		
27	27								27
28	28		28			28	28		28
29	29-30	29	29			29	29		
30	30	30		30	30				30
31	31								31
32	32-33-35		32			32	32	-33-35	
33	33-34		33			33	33		
34	34	34							34
35	35-36-38		35			35	35	-36-38	35
36	36-37		36			36	36		
37	37	37		37	37				37
38	38-31-39		38			38	38		38
39	39-40-44							-40-44	39
40	40-41		40			40	40		
41	41-42	41		41					
42	42-43	42		42	42				
43	43	43		43	43				43
44	44-45		44			44	44	-45	44
45	45-46		45			45	45		45
46	46-47-48								
47	47								47
48	48								48
49	49		49			49	49		49

where f_a is the net benefit of agriculture due to irrigation and f_p is the net benefit of energy production.

2.2 The constraints

The constraints of the coordinating model are formulated according to system identification in table 1 and the schematic layout in figure 2. There are six major types of constraints, (i) hydrological constraints, (ii) agricultural constraints, (iii) capacity constraints, (iv) municipal water supply, (v) economic constraints, (vi) environmental constraints, and others. A complete description of the constraint set for each node is given in appendix A.

2.2a Hydrological constraints (i) Continuity constraints—a typical constraint of this kind will be

$$X_{ij} + Z_{ij} = H_{ij} + X_{i+1,j} + D_{i+1,j}. \quad (2)$$

The amount of water that flows away from node i (river flow X_{ij} , irrigation diversion Z_{ij}) equals the amount of water that flows towards node i (river flow $X_{i+1,j}$, groundwater return $D_{i+1,j}$ and hydrological input H_{ij}).

(ii) Groundwater returns—a certain fraction (ξ_j) of the unpumped groundwater is returned to the river at a downstream node,

$$D_{ij} = \xi_j \left[\theta_i AG_i - \sum_j T_{ij} \right]. \quad (3)$$

For example, at node i , the groundwater pumped in a year is $\sum_j T_{ij}$ and the amount of groundwater left is $\theta_i AG_i - \sum_j T_{ij}$, of which ξ_j portion is returned to the river at a downstream node during the j th period.

2.2b Agricultural constraints (i) Field irrigation requirements for each crop are provided by surface water and groundwater irrigation,

$$T_{ij} + Z_{ij} = \alpha_{ij} A_{ij}. \quad (4)$$

It is assumed for simplicity that the irrigation water is lost to the river system. This water is either evapotranspired or percolated into the aquifer.

(ii) Culturable land and current irrigation level: irrigation cannot exceed the culturable land at each district and it should reach at least the current irrigation level,

$$AL_i \leq A_{ij} \leq AH_i. \quad (5)$$

(iii) An artificial constraint is imposed in the model to make the summer season a land preparation season for the following kharif season (i.e., irrigated acreage at any district during the summer season equals that of the kharif season):

$$A_{i2} = A_{i3}, \quad (6)$$

where A_{i1} , A_{i2} , A_{i3} are areas irrigated in rabi, summer and kharif seasons respectively.

2.2c Capacity constraints (i) Irrigation channel: new capacity for irrigation channels at each agricultural district is provided in order to divert more surface water for irrigation. The increased channel capacity at node i becomes $ZH_i + Z_{\text{NEW}_i}$,

$$Z_{ij} \leq m_j \cdot (ZH_i + Z_{\text{NEW}_i}), \quad (7)$$

where m_j is the number of months during the j th period.

(ii) Groundwater storage: annual groundwater irrigation at any agricultural district cannot exceed the available groundwater storage at that district,

$$\sum_j T_{ij} \leq \theta_i AG_i. \quad (8)$$

(iii) Diversion channel: the amount of water diverted cannot exceed the capacity of the channel,

$$Y_{ij} \leq m_j \cdot YH_i. \quad (9)$$

(iv) Reservoir live storage: The reservoir cannot store more water than its live storage capacity during any period of time,

$$V_{ij} \leq VH_i. \quad (10)$$

(v) Hydropower plant capacity: The water that drives the turbine cannot exceed the generating capacity. New turbines can be installed for necessary expansion of the power plant,

$$P_{ij} \leq m_j (PH_i + P_{NEW_i}). \quad (11)$$

2.2d Municipal water supply Water is diverted from the Ganga to meet the municipal water demand at Calcutta,

$$Y_{ij} \geq m_j \cdot YL_i. \quad (12)$$

2.2e Economic constraint Power demand distribution: the energy generated during each period must satisfy the distributional demand (η_j),

$$\sum_i (P_{ij} \cdot K_i) \geq \eta_j \sum_j \sum_i (P_{ij} \cdot K_i). \quad (13)$$

2.2f Environmental and other constraints In order to maintain river flows for navigation, water quality control, irrigation requirements downstream and other purposes, several minimum flows are provided,

$$X_{ij} \geq m_j \cdot XL_i. \quad (14)$$

3. Data used

Input data of the coordinating-screening model contains hydrological inputs (H_{ij}), field irrigation requirement given in appendix B (α_{ij}), culturable land (AH_i), low flow requirement (XL_i), groundwater availability ($\theta_i AG_i$), fractions of groundwater return (ξ_j), reservoir live storage (VH_i), hydropower plant capacity (PH_i), power conversion factor (K_i), and power demand distribution η_j .

3.1 Hydrological input

Since the model is deterministic, the hydrology of a typical single year is required. For flood protection and other purposes, a hydrology of extreme events which would have a chance of realization of one year out of every 50 or 100 years may be more appropriate. The hydrological inputs (H_{ij}) at various nodes are given in table 2. This table is constructed according to the estimated average monthly flows in the sub-basins of the Ganga by Briscoe (1974).

Table 2. Hydrological inputs (m cu m/season)

Node no	Rabi	Summer	Kharif
1	80561	43891	346812
5		680	12925
7		773	14694
8		1814	34474
9		421	7995
11		636	12092
35		188	3580
38		188	3580
39		188	3580
44		471	8949
45		471	8949
14	3823	3330	22806
15	6501	8030	37461
16	1052	350	14838
18	8338	7229	53410
19	259	57	931
20	688	229	9697
23	5551	4785	32928
27	5551	4785	32928
28	370	123	5217
30	764	654	4489
31	575	194	8236
34	575	194	8236
37	575	194	8236
43	2506	777	32984
47	936	809	5569
48	936	809	5569
49	259	57	931

Total:

(1) Include Brahmaputra (node)

Rabi	Summer	Kharif
119620	182327	742096
1044043 m cu m/year.		

(2) Exclude Brahmaputra (node)

Rabi	Summer	Kharif
39059	38436	395284
472779 m cu m/year.		

3.2 Field irrigation requirements

Monthly field irrigation requirements (α_{ij}) are estimated as per crop, rainfall, temperature, wind, humidity, sunshine, solar radiation and other factors at each agricultural district. These monthly requirements are then aggregated into three seasons for the simplified three-season model: (a) rabi season (November, December, January, February and March); (b) summer season (April, May and June); and (c) kharif season (July, August, September and October). Field irrigation requirements for different seasons at various agricultural districts are presented in table 3.

Table 3. Field irrigation requirements at various districts
(Seasonal)–(meter/unit area)

Node no	$A_{\max}$	$A_{\min}$	Rabi	Summer	Kharif
2	6600	660	0-5860	0-0510	0-3300
3	6600	660	0-6200	0-1310	0-3800
4	1900	190	0-6200	0-1310	0-3800
5	1900	190	0-6200	0-1310	0-3800
6	2600	260	0-6200	0-1310	0-3800
7	3000	300	0-6200	0-1310	0-3800
12	24000	2400	0-5490	0-2730	0-5720
13	20000	2000	0-5210	0-2150	0-5340
15	19000	1900	0-6200	0-1310	0-3800
16	19000	1900	0-6200	0-1310	0-3800
17	33000	3300	0-6340	0-1430	0-3700
19	26100	2610	0-6610	0-1390	0-3950
24	40000	4000	0-6140	0-1990	0-5020
26	36800	3680	0-6140	0-1990	0-5020
28	19000	1900	0-6490	0-1400	0-3750
29	20800	2080	0-4920	0-2250	0-4820
32	20000	2000	0-6220	0-1960	0-5140
33	3100	310	0-6450	0-1610	0-4560
35	20000	2000	0-6220	0-1960	0-5140
36	4700	470	0-6450	0-1610	0-4560
38	20000	2000	0-6220	0-1960	0-5140
40	100000	5000	0-6360	0-2280	0-7240
44	50000	2000	0-5320	0-2690	1-039
45	50000	2000	0-5100	0-2440	0-7800
49	26100	2610	0-6610	0-1390	0-3950

Total $A_{\max} = 57.42$ m ha
 $A_{\min} = 4.64$ m ha

3.3 Culturable land and current irrigation level

Information about culturable land at each agriculture district is reported by the Irrigation Commission (1972). Table 3 also gives the culturable land districtwise. The current irrigation level has been assumed at 10 % of the culturable land in most districts.

3.4 Current irrigation capacity

Table 4 lists some capacities of irrigation channels as recorded in the Irrigation Commission (1972). New capacity ($Z_{\text{NEW},i}$) is constructed in the coordinating model to allow more surface water diversion for irrigation.

3.5 Groundwater storage

Groundwater storage at an agriculture district is the product of annual groundwater availability (θ_i) and area (AG_i) over which groundwater is available. Aquifer is assumed to be fully recharged annually. These groundwater storages are given in table 5

3.6 Reservoir live storage, hydro-power plant capacity, and power conversion factor

Figures of reservoir live storages, current power plant capacities, and power conversion factors are presented in table 6. Although preliminary study indicates that there is a

Table 4. Current channel capacities

Node no	m cu m/mon	Node no	m cu m/mon
2-7	—	29	5.7
12	155.7	32	—
13	297.5	33	56.6
15	283.2	35	—
16	311.5	36	85.0
17	283.2	38	—
19	28.3	40	260.5
24	283.2	44	90.6
26	396.4	45	424.8
28	28.3	49	28.3

Table 5. Groundwater availability

Node no	m cu m	Node no	m cu m	Node no	m cu m
2, 3	2255	15	6462	32, 35, 38	2987
4, 5	677	16	4454	33	579
6	902	17	11659	36	877
7	621	19	11277	40	23816
12	7467	24	8960	44, 45	2987
13	2987	26	4480	49	10600
		28	2838		
		29	3734		

Total groundwater capacity: 122512 m cu m.

Table 6. Power conversion factors, power plant capacities and reservoir live storages.

Node no.	$K = \frac{\text{MW}}{\text{m cu m/month}}$	PH (MW)	VH (m cu m)
20	0.1816	195	10608
30	0.2270	130	2220
37	0.0454	13	780
41	0.0272	64	
42	0.0908	111	1567
43	0.1090	75	6920

reservoir and a power plant at node 34 (KEN), they are removed from the model because they are not reported in the Irrigation Commission (1972). Appropriate new power plant capacity (P_{NEW_i}) may be installed in order to attain higher energy production. Power production factor (K_i) is calculated by using a head of one half dam height.

3.7 Power demand distribution

Distributional constraints may be imposed on the energy production so that the energy output meets the monthly or seasonal demand (η_j). Initial values of the demand

distribution are taken at 0.0955, 0.0797, 0.0762, 0.0515, 0.0630, 0.0784, 0.0797, 0.0797, 0.0938, 0.1233, 0.1039, and 0.0753 for the respective twelve months. These distributions should be reevaluated according to the power requirements for groundwater pumping and other irrigation facilities.

3.8 Other water demands

Water uses such as water quality control, navigation, and other purposes are not well developed. These water uses are considered in the model as minimum river flows and are presented in table 7.

4. Matrix Generation

The coordinating-screening model described above is a form of linear program and can be solved by IBM/MPSX. The matrix preparation for this IBM/MPSX used in Italy on the Tiber River Basin by Rogers & Chi (1973) is employed to generate the matrix of the Ganga Basin in the MPSX input format. The Ganga Basin linear program for a simplified three-season model consists of 490 constraint rows of which most are continuity equations, about 1050 decision variables and 3500 non-zero matrix elements.

5. Solutions

In this simplified three-season model solutions using various assumptions have been obtained. In each solution the summer season (June irrigation) is assumed to be land preparation and the model is constrained so that the area under irrigation during the summer season at each district must also be irrigated in the kharif season. In most solutions energy is maximized subject to different constraint sets.

5.1 Solution 1

Maximizing energy under current power plant capacity,

$$\max Z = f_P \cdot \sum_j \sum_i (P_{ij} \cdot K_i). \quad (15)$$

In this solution, energy production is unrestricted by the distributional constraints. The maximum energy under the current generating capacity is 55,863 GWh

Table 7. Low flow requirements.

Node no.	<i>XL</i> (m cu m/month)
2	100
5	50
7	240
9	285
10	255
44	38
45	180

(19,876 GWh in rabi season, 15,427 GWh in summer season, and 20,560 GWh for kharif season; which are 35 %, 28 % and 37 % respectively). Compared with the installed capacity (95,093 GWh/year),* hydropower facilities are about 60 % (55·863/95·093) utilized.

The energy output of the system is limited by water availability at power plant nodes 20 and 30 (Ramganga). On the other hand, the installed turbine capacities and reservoir storage capacities have restricted the energy production at power plant nodes 37 (Betwa), 41, 42 and 42 (Chambal) where large quantity of water is bypassed from reservoirs through diversion channels.

5.2 Solution 2

Maximizing energy with new power plant capacities,

$$\max Z = f_p \cdot \sum_j \sum_i (P_{ij} \cdot K_i). \quad (16)$$

If new turbines are installed when necessary, the energy production reaches 103, 932 GWh ignoring the distributional constraints**. All hydrological inflow upstream of power plants are utilized for energy.

New capacity is not needed at nodes 20 (Rihand) and 30 (Ramganga) since they are restricted by the availability of hydrological inputs. The system requires new turbines at nodes 37 (Betwa), 41, 42 and 43 (Chambal). These additional capacities*** are 72 MW at node 37, 103 MW at node 41, 445 MW at node 42 and 635 MW at node 43. When new turbines are installed, the energy production will only be limited by the hydrological inputs. However, excessive new capacities are required at these power plants (nodes 37, 41, 42 and 43) due to the extremely high kharif season inflows, which the reservoirs cannot regulate.

The annual irrigated acreage under this scheme reaches 56·8 m ha (17·2 m ha in rabi season and 39·6 m ha in the kharif season). This irrigation level is also the maximum area that can be fully irrigated under the current constraints.

5.3 Solution 3

Maximizing energy subject to various levels of irrigation, while energy output also meets the distributional constraints,

$$\max Z = f_p \cdot \sum_j \sum_i (P_{ij} \cdot K_i). \quad (17)$$

Since energy and irrigation are the major water uses in the model, it is useful to understand how these two interact with each other. For this purpose, energy is maximized according to demand distribution subject to different levels of irrigation usage.

* This is the installed hydropower capacity at 65 % load factor.

** If energy production must satisfy demand distribution, then the energy output will only reach 74,172 GWh even when new turbines are installed.

***The new installed turbines have capacities at 110, 158, 685 and 977 MW for power plants at nodes 37, 41, 42 and 43 respectively at a 65 % load factor.

To facilitate the solution, the power distributional constraints are formulated in the following form:

$$\sum_i (P_{ij} K_i) \geq \eta_j \cdot \sum_j \sum_i (P_{ij} K_i), \quad (18)$$

where distribution values of η_j are subject to adjustment to simulate real demands.

With an irrigation level lower than 55 m ha (15.9 m ha in the rabi season, 39.1 m ha in the kharif season) the energy output reaches a maximum level at 54,254 GWh (23,362 GWh in rabi, 10,465 GWh in summer and 20,427 GWh in kharif). On the other hand, when energy production is less than 36,461 GWh (15,700 GWh in rabi, 7,033 GWh in summer and 13,728 GWh in kharif), the irrigation level is maximized at 56.8 m ha (17.2 m ha in rabi and 39.6 m ha in kharif). It is not possible to generate energy higher than 54,254 GWh by sacrificing irrigation and it is not feasible to insist on an irrigation level higher than 56.8 m ha by giving up energy production. For any level of irrigation between these two extremes (55 and 56.8 m ha), there is a corresponding energy production (between 36,461 and 54,254 GWh). No further parametric study on energy and irrigation in these ranges is made since the difference between the two irrigation levels is relatively small. Solutions of these two extremes are shown in figure 3.

Any point inside the boundary *ABCD* is feasible but not efficient. Furthermore, points between *A* and *B* are not efficient because the irrigation level can be increased from zero to 55 m ha without any loss of energy. Similarly, points between *C* and *D* are not efficient. Points that fall on the line *BC* denote the production frontier. Point *C* represents operating the system for agricultural maximization and treating energy as a residual. The marginal rate of transformation of energy for agriculture at any point on the line *BC* is the gradient of the line segment *BC*. This rate is estimated at

Figure 3. Trade-off between energy and total irrigation

– 10,070 GWh/m ha (i.e., to gain 1 m ha of irrigation between points *B* and *C* 10,070 GWh energy is sacrificed).

The range of irrigated acreage is only 1.8 m ha (56.8 – 55) while energy production has a relatively wide range (from 36,461 to 54,254 GWh) of variation. No substantial irrigation can be gained by giving up energy production (at a rate of 10,070 GWh for only 1 m ha).

Irrigated acreage is at capacity level at most downstream districts where water is available through accumulation of inflows. Reservoirs are full in the kharif season to regulate the excessive monsoon water for irrigation in the following rabi season.

5.4 Solution 4

Trade-off between rabi irrigation and kharif irrigation,

$$\max Z = \sum_i A_{i3}, \text{ subject to various levels of } \sum_i A_{i1}. \quad (19)$$

Since the reservoirs and groundwater storage can regulate the water for irrigation from one season to the next, there is a definite trade-off between rabi irrigation and kharif irrigation. Solutions (points *B*, *C*, *D*, *E*, *F*, *G* and *H*) shown in figure 4 illustrate the

Figure 4. Trade-off between kharif and rabi irrigations

corresponding irrigation level in the kharif season for each irrigation level in the rabi season. It is not feasible to irrigate in the rabi season at a level higher than 27.4 m ha. The maximum level of irrigation in kharif can be achieved at a higher level (43.6 m ha). Point *D* (at which rabi irrigation is 17.2 m ha and kharif irrigation is 39.6 m ha) represents operating the system for maximum annual total irrigation. Steeper gradients (-2.96 , -3.73) at points *E*, *G* in figure 4 indicate that 2.96 and 3.73 units of irrigation acreage in kharif season would be given up for every unit increment of irrigation in the rabi season. This is because the June irrigation (summer season) binds the irrigation level of the kharif season ($A_{i2} = A_{i3}$); therefore, a large quantity of river flow in the kharif season is wasted.

5.5 Solution 5

Maximizing energy subject to different levels of rabi and kharif irrigations,

$$\max Z = f_P \cdot \sum_j \sum_i (P_{ij} K_i), \quad (20)$$

this solution is obtained assuming that no new power capacity is provided and energy production must satisfy the demand distributional constraints. For every rabi irrigation level (according to §5.4) the energy is maximized while increasing kharif irrigation level until infeasibility. To obtain solutions among rabi irrigation, kharif irrigation and the corresponding energy production, the parametric procedure of linear programming is adopted.

6. Results

Solutions (rabi irrigation, kharif irrigation and energy production) are presented in table 8 and figure 5. The convex plane *DGHIJKLN'E* represents the operating production frontier. Any point that falls on or inside the feasible region but not on this plane (*DGHIJKLN'E*)* is not efficient and is dominated by that plane. Point *I* in figure 5 represents operating the system for agricultural maximization and treating energy production as a residual. Points that fall on the line *DE* represent operating the system for energy maximization. The rates of marginal transformation are calculated between energy and kharif irrigation for each given rabi irrigation as shown in figure 6.

The range of variation for energy production as defined in figure 6 is 17,793 GWh (54,254 – 36,461 GWh) and irrigation acreage has a variation of 15.3 m ha (56.8 – 41.5 GWh). The range of these variations is relatively narrow and no substantial amount of energy will be sacrificed for further agricultural development and vice versa. Basrur (1975) discussed the difficulties of identifying a trade-off between energy and irrigation under the system described in this paper.

At some agricultural districts, June irrigation has bound the following kharif irrigation because less water is available during the summer season.

Reservoirs are used to store the excessive monsoon water to their storage capacities for the irrigation of the following rabi season.

* Point *N'* in figure 5 is a linear extrapolation point on the line *EN* to prevent the production frontier from going to points *M* and *N* where the rates of marginal transformation are negligible.

Table 8. Solutions of maximizing energy subject to different levels of A_1 and A_3 .

Point in figure 5	A_1 m ha	A_3 m ha	$A_1 + A_3$ m ha	Energy GWh
	7.5	18.12	25.62	54254
		40.0	47.5	54254
		43.58	51.08	45443
	10	15.61	25.61	54254
		40.0	50.0	54254
		43.58	53.58	45443
G				
	15	11.49	26.49	54254
D		40.0	55.0	54254
H		41.24	56.24	36576
	17.15	9.89	27.04	54254
		35.0	52.152	54254
I		39.61	56.76	36461
	20	7.8	27.8	54254
		30.0	50.0	54254
P		35.0	55.0	50190
J		36.04	56.04	36461
	22.5	6.92	29.42	54254
		25.0	47.5	54254
Q		30.0	52.5	43592
K		30.74	53.24	36461
	25	6.20	31.20	54254
E		20.0	45.0	54254
L		23.35	48.35	36461
N	26.09	15.44	41.53	36461
	27.40	6.27	33.67	15200
H		10.0	37.40	15200
K		14.38	41.78	15152

Power plant capacity and reservoir storage capacity have severely limited the energy production at power plant nodes 37 (Betwa), 41, 42 and 43 (Chambal). At power plant nodes 20 (Rihand) and 30 (Ramganga) the annual hydrological inflows have restricted the energy production.

6.1 Irrigation level

(a) The maximum total irrigated acreage (rabi and kharif combined) occurs at point *I* as shown in figure 5. This irrigation level is 56.8 m ha (17.2 m ha in rabi, and 39.6 m ha in kharif) and its corresponding energy output is 36,461 GWh. Breakdown of annual irrigation level (rabi and kharif, surface water irrigation and groundwater irrigation by states inside India) are presented in tables 9, 10 and 11.

At point *I* in figure 5, surface water irrigation is 257.2 billion m^3 annually (38.3, 29.8 and 189.1 for rabi, summer and kharif), groundwater irrigation is 122.5 billion m^3 (65.1, 43.6 and 13.7 for the respective seasons). Groundwater irrigation in the kharif season is only 11% of total annual groundwater irrigation while surface water

Figure 5. Trade-off among energy, kharif irrigation and rabi irrigation. At point I, $A_1 = 17.152$, $A_3 = 39.615$ and energy = 36,461, where $A_1 + A_3 = 56,167$ is the maximum irrigated acreage that can be achieved

irrigation is nearly 74% of the total surface water irrigation. This is because of the monsoon water availability in the kharif season. Surface water and groundwater irrigation relationship is presented in table 12.

(b) Maximum energy production that satisfies the distributional constraints occurs on the line DE as shown in figure 5. This energy output is 54,254 GWh and the corresponding irrigation levels are between 45 m ha and 55 m ha.

6.2 Cropping intensity

The Irrigation Commission (1972) reported that the agricultural level in the Ganges Basin had a net sown area of 43.824 m ha, a gross sown area of 54.757 m ha, a net irrigated area of 10.817 m ha and a gross irrigated area of 12.544 m ha. Thus, the cropping intensity in 1972 was 125% and the irrigation intensity was 23%.

The cropping and irrigation intensities in the future according to the above solutions are as follows* (a) The cropping intensity is improved to 148% when rabi irrigation is

* In order to calculate these intensities, it is assumed that all culturable land is cultivated in the kharif season with or without irrigation and only the area that is irrigated in the rabi season is cultivated in that season.

Figure 6. Trade-off among energy, kharif irrigation and rabi irrigation

maximized (point *M* in figure 5). (b) The cropping intensity is 130% when the total irrigation is maximized (point *I* in figure 5).

All the interesting cropping intensities and irrigation intensities from the solutions obtained in this paper can be found in table 13.

6.3 Degree of complementarity

It is possible to assess the degree of complementarity of the three-season model by assuming net benefits of energy and irrigation. When these net benefits are absent, ranges of degree of complementarity are obtained.

When the system is operated as a single-purpose one for energy the annual energy production is 54,254 GWh; and when the system is operated for agriculture maximization, the annual irrigation level is 56.8 m ha. Therefore, if the full potential of the

Table 9. Breakdown of irrigation level between states and seasons

Sub-basin	H.P.	Haryana	Delhi	Rajasthan	U.P.	M.P.	Bihar	W.B.	Total by basin
Chambal	A_1			1.428	0.017	1.087			2.532
	A_3			1.908	0.022	1.453			3.383
Yamuna	0.018	0.296	0.010	0.243	0.558	0.526			1.651
	0.106	1.780	0.061	1.463	3.359	3.163			9.932
Ramganga					1.283				1.283
					0.471				0.471
Tons & others					0.071	0.081	0.038		0.190
					0.529	0.607	0.284		1.420
Gomati & Ghaghara					2.260		0.078		2.338
					6.777		0.233		7.010
Son					0.115	0.918	0.417		1.450
					0.296	2.279	1.035		3.600
Gandak & others					0.083		3.471	0.696	4.250
					0.083		3.471	0.696	4.250
Right bank tributaries							1.195	0.837	2.032
							3.191	2.237	5.428
Main Ganga					1.427				1.427
					4.120				4.120
Total by State	0.018	0.296	0.010	1.671	5.814	2.612	5.199	1.533	17.153
	0.106	1.780	0.061	3.71	15.647	7.502	8.214	2.933	39.614

Table 10. Annual surface irrigation ($10^9 \text{ m}^3/\text{year}$)

Sub-basin	H.P.	Haryana	Delhi	Rajasthan	U.P.	M.P.	Bihar	W.B.	Total by basin
Chambal				13.814	0.162	10.521			24.497
Yamuna	0.823	13.781	0.469	11.328	26.008	24.493			76.902
Ramganga					5.907				5.907
Tons and others					2.128	2.441	1.141		5.710
Gomati and Ghaghara					48.393		1.667		50.060
Son					1.219	9.707	4.406		15.332
Gandak and others					0.697	29.049	5.827		35.573
Right bank tributaries							7.821	5.482	13.303
Main Ganga					29.959				29.959
Total by State	0.823	13.781	0.469	25.142	114.473	76.211	20.862	5.482	257.243

Table 11. Annual groundwater irrigation ($10^9 \text{ m}^3/\text{year}$)

Sub-basin	H.P.	Haryana	Delhi	Rajasthan	U.P.	M.P.	Bihar	W.B.	Total by basin
Chambal				13-430	0-157	10-229			23-816
Yamuna	0-175	2-937	0-100	2-414	5-543	5-221			16-390
Ramganga					3-734				3-734
Tons & others					1-058	1-213	0-567		2-838
Gomati & Ghaghara					12-992		0-448		13-440
Son					0-976	7-774	3-529		12-280
Gandak & others					0-245		10-203	2-047	12-495
Right bank tributaries							15-911	11-153	27-064
Main Ganga					10-454				10-454
Total by State	0-175	2-937	0-100	15-844	35-159	24-437	30-658	13-200	122-510

Table 12. Water utilisation of various solutions

Point	Surface water 10^9 m^3	% by season	Ground-water 10^9 m^3	% by season	Water total 10^9 m^3	Irrigated acreage m ha	Energy GWh
I							
rabi	38-6	15-0	64-9	53-0	103-5	17-2	15-70
summer	29-5	11-5	43-9	35-8	73-4	—	7-03
kharif	189-1	73-5	13-7	11-2	202-8	39-6	13-73
total	257-2	100-0	122-5	100-0	379-6	56-8	36-46
D							
rabi	37-5	24-0	114-5	95-3	152-0	25-0	23-36
summer	28-0	17-9	5-4	4-5	33-4	—	10-47
kharif	90-8	58-1	0-2	0-2	91-0	20-0	20-43
total	156-3	100-0	120-1	100-0	276-4	45-0	54-26
E							
rabi	42-7	16-1	48-4	45-1	91-0	15-0	23-36
summer	33-1	12-5	41-2	38-4	74-3	—	10-47
kharif	188-9	71-4	17-7	16-5	206-6	40-0	20-43
total	264-7	100-0	107-3	100-0	371-9	55-0	54-26
N							
rabi	45-3	42-4	121-1	98-8	166-4	27-4	6-55
summer	15-3	14-3	1-2	1-0	16-5	—	2-93
kharif	46-4	43-3	0-2	0-2	46-6	6-3	5-72
total	107-0	100-0	122-5	100-0	229-5	33-7	15-20
N'							
rabi	44-0	23-7	84-7	73-8	128-7	26-1	15-70
summer	24-2	13-0	21-2	18-6	45-4	—	7-03
kharif	117-6	63-3	9-0	7-6	126-6	15-4	13-73
total	185-8	100-0	114-9	100-0	300-7	41-5	36-46

Table 13. Cropping intensities and irrigation intensities in future development

Point	A ₁ m ha	A ₃ m ha	Gross irrigated area A ₁ + A ₃ m ha	Gross sown area A ₁ + 57.4*	Cropping intensity	Irrigation intensity
E	25	20	45	82.4	144	55
D	15	40	55	72.4	126	76
G	10	43.6	53.6	67.4	117	80
H	15.4	41.2	56.6	72.8	127	78
I	17.2	39.6	56.8	74.6	130	76
J	20	36	56	77.4	135	72
K	22.5	30.7	53.2	79.9	139	67
L	25	23.4	48.4	82.4	144	59
M	27.4	14.4	41.8	84.8	148	49
N'	26.1	15.4	41.5	83.5	145	50
P	20	35	55	77.4	135	71
Q	22.5	30	52.5	79.9	139	66

* Assuming all the culturable land is sown during kharif season in future development.

Ganges basin for both energy and agriculture is realized the total benefit would reach $f_a \cdot 56.8 + f_p \cdot 54254$. The degree of complementarity of any solution is:

$$\frac{f_a \sum_i (A_{i1} + A_{i3}) + f_p \sum_j \sum_i (P_{ij} K_i)}{f_a \cdot 56.8 + f_p \cdot 54254}, \quad (21)$$

and the values for various solutions are given in table 14.

7. Summary

A summary of solutions obtained in this study is presented in table 15. This table lists the objective function, constraint, objective value and remarks for each solution. In each solution, the same hydrological data as shown in tables 2 to 6 are used. This table also indicates the maximum energy production, maximum rabi irrigation and maximum kharif irrigation that can be achieved.

Several solutions are compared with the 1972 agriculture status in table 16.

(i) If the objective is to maximize total irrigation (rabi and kharif) the solution is

Table 14. Degree of complementarity for solutions

Point	A ₁ (n ha)	A ₃ (m ha)	Energy GWH	Range of degree of complementarity
I	17.2	39.6	36461	0.67–0.99
D	15	40	54254	0.97–0.99
E	25	20	54254	0.79–0.99
Points on DE	15–25	20–40	54254	0.79–0.99
P	20	35	50190	0.93–0.97
Q	22.5	30	43592	0.80–0.92
H	15	41.241	36576	0.69–0.99

Table 15. Summary of solutions

Objective function	Constraints	Objective value	Remarks
1. Maximizing energy $\sum_j \sum_i (P_{ij}, K_i)$	1. Basic constraints (hydrological inputs. . . .) 2. Minimum water supply, navigation etc. 3. Minimum flow at Farakka 4. No new power capacity 5. No power distributional constraint.	55,863 GWh	B NB B
2. Maximizing energy $\sum_j \sum_i (P_{ij}, K_i)$	1. Basic constraints 2. Minimum water supply, navigation etc. 3. Minimum flow at Farakka 4. New power capacity 5. No power distribution.	103,932 GWh	NB NB
3. Maximizing energy $\sum_j \sum_i (P_{ij}, K_i)$	1. Basic constraints 2. Minimum water supply, navigation, etc. 3. Minimum flow at Farakka 4. No new power capacity 5. Power distribution 6. Various levels of irrigation (41.8 to 56.8 m ha)	from 36,461 to 54,254 GWh	B B in rabi season B B B
4. Maximizing kharif irrigation $\sum_i A_{i3}$	1. Basic constraints 2. Minimum water supply, navigation, etc. 3. Minimum flow at Farakka 4. Various levels of rabi irrigation (10 to 27.4 m ha)	from 14.4 to 43.6 m ha	B B in rabi season B

Table 16. Present (1972) Irrigation and future irrigation (m ha)

	(1)	(2)	(3)	(4)	(5)	(6)
Rabi			57.42	17.15	40.27	57.42
Summer			57.42	39.62	17.80	57.42
Kharif			57.42	39.62	17.80	57.42
Total*	12.54	102.30	114.84	56.77	58.07	114.84

* Total equals rabi plus kharif. (1) Present (1972) gross irrigated area; (2) Present (1972) gross non-irrigated area; (3) Total = (1) + (2); (4) Future area under irrigation; (5) Future area not under irrigation; (6) Total = (4) + (5)

point *I* in figure 5. The total irrigation level is 56.8 m ha which is an improvement of 44.2 m ha over the 1972 gross irrigation level (12.5 m ha) or 2.5 times better.

(ii) If the objective is to maximize rabi irrigation, then point *M* in figure 5 dominates the other solutions. The total irrigation level is 41.8 m ha (27.4 in rabi, 14.4 in kharif) which is a 29.3 m ha improvement over 1972 level.

(iii) In order to avoid substantial energy sacrifice when rabi irrigation is maximized, a compromise solution point *N'* in figure 5 is obtained. The total irrigation level is 41.5 m ha (26.1 in rabi, 15.4 in kharif). This is a 29 m ha improvement over the 1972 level. In this case, the reduction of rabi irrigation is only 1.3 m ha from that of case (ii) while the energy output is increased from 15,200 GWh to 36,461 GWh.

Since this model of the Ganga basin considers only the benefit to the Indian states it is necessary to understand the water availability when the Ganga enters Bangladesh downstream; River flows at Farakka (which enter Bangladesh) are given in table 17. Values shown in this table are under the maximum irrigation level (56.8 m ha) in Indian states. Table 18 summarizes the results for each of the 9 sub-basins and compares them with the Irrigation Commission (1972) estimates for resource use in 1968–69.

8. Conclusion

The objective of this model is to maximize the net benefit of energy and irrigation:

$$f_a \cdot \sum_i (A_{i1} + A_{i3}) + f_p \cdot \sum_j \sum_i (P_{ij} K_i).$$

This objective function has been replaced by a single-purpose objective (either agriculture or energy) in every solution. It either maximizes energy subject to various irrigation levels or vice versa. However, if the net benefits of energy and irrigation are estimated and the objective function

$$f_a \sum_i (A_{i1} + A_{i3}) + f_p \sum_j \sum_i (P_{ij} K_i)$$

is adopted, the solution should fall upon the plane *DGHIJKLN'E* as shown in figure 5. Solutions obtained by a comprehensive parametric study on values f_a and f_p will cover points on the same plane *DGHIJKLN'E* in figure 5.

The model presented in this paper aims to achieve ultimate development for Indian states within the Ganga basin. It is equally important to provide necessary water downstream of the Ganga for optimal development in Bangladesh. This is expressed in the model in the form of minimum irrigation requirements in Bangladesh (XL_2). It is found that the rabi season river flow at Farakka is critical. River flow in this season

Table 17. River flows at Farakka, in Bangladesh, and diversion to Calcutta (million cubic metres)

	River flows at Farakka	Water available after Brahmaputra joins Ganga	Diversion to Calcutta
Rabi	500	81,061	14,680
Summer	6,580	150,471	8,808
Kharif	173,226	520,038	11,744
Total	180,306	751,570	11,744

Table 18. Ganga basin water use and irrigation

Basin	Average annual runoff (m cu m)	Surface water (m cu m)	Ground water (m cu m)	Irrigated areas (ha × 10 ⁶)	
I	24,312	24,497 (8,305)	23,816 (5,562)	K 3-383 R 2-532	(0-886)
II	65,621	76,902 (15,523)	16,390 (9,513)	9-932 1-651	(3-054)
III	18,626	5,907 (7,144)	3,734 (1,446)	0-471 1-283	(0-382)
IV	14,123	5,710 (3,819)	2,838 (1,126)	1-420 0-190	(0-394)
V	122,793	50,060 (21,280)	13,440 (13,186)	7-101 2-338	(2-143)
VI	42,308	15,332 (10,335)	12,280 (838)	3-600 1-450	(0-336)
VII	134,277	35,573 (26,006)	12,495 (1,068)	4-250 4-250	(0-374)
VIII	45,145	13,303 (28,366)	27,064 (1,552)	5-428 2-032	(1-872)
IX	42,533	29,959 (11,643)	10,454 (6,604)	4-120 1-427	(1-914)
Total	509,738	257,243 (132,421)	122,510 (40,895)	39-614 17-153	(11-36)

Figures in parentheses represent Irrigation Commission estimates of 1968-69 resource use.

(X_{21}) is binding at the minimum level. It is important to study the relation between the minimum flow requirement at Farakka and the irrigation level in Indian states.

There are few trade-off points found between energy and irrigation in this three-season model. This is because under the assumed system no water is diverted for irrigation upstream of a hydropower plant. The same water that drives the turbine can be used to irrigate an agricultural district downstream as discussed by Basrur (1975). The model in its present form considers the Ganga basin within Indian states as a unit despite possible different local interests. If various targets are required by different states in the basin, then more trade-off points can be expected.

Finally, the power conversion factor is estimated according to one half of the dam height. The real energy production will be less than the solutions obtained in this paper. Although seasonal average height of water surface in the reservoir is more appropriate, it is not used in this present study.

It must be clarified as emphasized at the outset that the study brings out only an approach to analysis and the quantitative figures given above are meant only to illustrate the approach for the decomposition of the system and have no real-life implication. However, it must also be added that the systems approach is a powerful tool for planning and if the currently available data with the various governmental agencies were made available, significant improvements in real-life water resources development could be made. This model is only the starting point for a detailed study.

Appendix A: Mathematics of the Ganga basin model

The complete Ganga basin model is described in this appendix. In the following algebraic equations, j is an index for month or season ($j=1, \dots, 12$ for a 12-month model and $j=1, \dots, 3$ for a 3 season model); i is an index for node number ($i=1, \dots, 49$). I is a set that contains the nodes of 25 agricultural districts 2, 3, 4, 5, 6, 7, 12, 13, 15, 16, 17, 19, 24, 26, 28, 29, 32, 33, 35, 36, 38, 40, 44, 45 and 49.

The constraints

(i) Hydrological constraints

(a) Continuity constraints

- (1) $X_{1j} = H_{1j} + X_{2j} + D_{2j}$,
- (2) $X_{2j} + Y_{2j} = X_{3j} + D_{3j} - Z_{2j}$,
- (3) $X_{3j} = X_{4j} + X_{19,j} + D_{4j} - Z_{3j}$,
- (4) $X_{4j} = X_{5j} + X_{15,j} + D_{15,j} + D_{5j} - Z_{4j}$,
- (5) $X_{5j} = X_{6j} + X_{49,j} + D_{6j} + D_{49,j} - Z_{5j} + H_{5j}$,
- (6) $X_{6j} = X_{7j} + X_{16,j} + D_{7j} + D_{16,j} - Z_{6j}$,
- (7) $X_{7j} = X_{8j} + X_{17,j} + D_{17,j} - Z_{7j} + H_{7j}$,
- (8) $X_{8j} = X_{9j} + X_{21,j} + H_{8j}$,
- (9) $X_{9j} = X_{10,j} + X_{28,j} + D_{28,j} + H_{9j}$,
- (10) $X_{10,j} = X_{11,j} + X_{32,j} + D_{32,j}$,
- (11) $X_{11,j} = X_{12,j} + X_{29,j} + D_{12,j} + D_{29,j} + H_{11,j}$,
- (12) $X_{12,j} = X_{13,j} + Y_{29,j} + D_{13,j} - Z_{12,j}$,
- (13) $X_{13,j} = X_{14,j} - Z_{13,j}$,
- (14) $X_{14,j} = H_{14,j}$,
- (15) $X_{15,j} = H_{15,j} - Z_{15,j}$,
- (16) $X_{16,j} = H_{16,j} - Z_{16,j}$,
- (17) $X_{17,j} = X_{18,j} - Z_{17,j}$,
- (18) $X_{18,j} = X_{20,j} + H_{18,j}$,
- (19) $X_{19,j} = H_{19,j} - Z_{19,j}$,
- (20) $X_{20,j} = P_{20,j} + Y_{20,j}$,
 $X_{20,j} = H_{20,j} - (V_{20,j} - V_{20,j+1})$,
- (21) $X_{21,j} = X_{22,j} + X_{24,j} + D_{24,j}$,
- (22) $X_{22,j} + Y_{22,j} = X_{23,j}$,
- (23) $X_{23,j} = H_{23,j}$,
- (24) $X_{24,j} = X_{25,j} - Z_{24,j}$,
- (25) $X_{25,j} = X_{26,j} + Y_{22,j} + D_{26,j}$,
- (26) $X_{26,j} = X_{27,j} - Z_{26,j}$,
- (27) $X_{27,j} = H_{27,j}$,
- (28) $X_{28,j} = H_{28,j} - Z_{28,j}$,
- (29) $X_{29,j} + Y_{29,j} = X_{30,j} - Z_{29,j}$,
- (30) $X_{30,j} = P_{30,j} + Y_{30,j}$,
 $X_{30,j} = H_{30,j} - (V_{30,j} - V_{30,j-1})$,
- (31) $X_{31,j} = H_{31,j}$,

- (32) $X_{32,j} = X_{35,j} + X_{33,j} + D_{33,j} + D_{35,j} - Z_{32,j}$,
 (33) $X_{33,j} = X_{34,j} - Z_{33,j}$,
 (34) $X_{34,j} = H_{34,j}$,
 (35) $X_{35,j} = X_{36,j} + X_{38,j} + D_{36,j} + D_{38,j} + H_{35,j} - Z_{35,j}$,
 (36) $X_{36,j} = X_{37,j} - Z_{36,j}$,
 (37) $X_{37,j} = P_{37,j} + Y_{37,j}$,
 $X_{37,j} = H_{37,j} - (V_{37,j} - V_{37,j-1})$,
 (38) $X_{38,j} = X_{31,j} + X_{39,j} + H_{38,j} - Z_{38,j}$,
 (39) $X_{39,j} = X_{40,j} + X_{44,j} + D_{40,j} + D_{44,j} + H_{39,j}$,
 (40) $X_{40,j} = X_{41,j} - Z_{40,j}$,
 (41) $X_{41,j} = P_{41,j} + Y_{41,j}$,
 $X_{41,j} = X_{42,j}$,
 (42) $X_{42,j} = P_{42,j} + Y_{42,j}$,
 $X_{42,j} = X_{43,j} - (V_{42,j} - V_{42,j-1})$,
 (43) $X_{43,j} = P_{43,j} + Y_{43,j}$,
 $X_{43,j} = H_{43,j} - (V_{43,j} - V_{43,j-1})$,
 (44) $X_{44,j} = X_{45,j} + D_{45,j} + H_{44,j} - Z_{44,j}$,
 (45) $X_{45,j} = X_{46,j} + H_{46,j} - Z_{45,j}$,
 (46) $X_{46,j} = X_{47,j} + X_{48,j}$,
 (47) $X_{47,j} = H_{47,j}$,
 (48) $X_{48,j} = H_{48,j}$,
 (49) $X_{49,j} = H_{49,j} - Z_{49,j}$.

(b) Groundwater returns

$$D_{ij} = \xi_j \left[\theta_i AG_i - \sum_j T_{ij} \right] i \in I, \forall_j$$

(ii) Agricultural constraints

(a) Field irrigation requirements

$$T_{ij} + Z_{ij} = \alpha_{ij} A_{ij}, i \in I, \forall_j$$

(b) Culturable land and current irrigation level

$$AL_i \leq A_{ij} \leq AH_i, i \in I, \forall_j$$

(c) Artificial constraints

$$A_{i2} = A_{i3}, i \in I$$

(iii) Capacity constraints

(a) Irrigation channels

$$Z_{ij} \leq m_j (ZH_i + Z_{NEW_i}), i \in I$$

(b) Groundwater storage

$$\sum_i T_{ij} \leq \theta_i AG_i, i \in I.$$

(c) Diversion channels

$$Y_{ij} \leq m_j YH_i, i \in I, \forall_j.$$

(d) Reservoir live storages

$$V_{ij} \leq VH_i, i = 20, 30, 37, 42, 43, \forall_j$$

(e) Hydropower plant capacity

$$P_{ij} \leq m_j (PH_i + P_{NEW_i}), i = 20, 30, 37, 41, 42, 43, \forall_j.$$

(iv) Municipal water supply

$$Y_{ij} \geq m_j YL_i, i = 2, \forall_j.$$

Diversion to Calcutta for municipal water supply.

(v) Economic constraints

$$\sum_i (P_{ij} K_i) \geq \eta_i \sum_j \sum_i (P_{ij} K_i) \\ i = 20, 30, 37, 41, 42, 43, \forall_j$$

This is the power demand distributional constraint.

(vi) Environmental and other constraints

Minimum river flow

$$X_{ij} \geq m_j XL_i, i = 2, 5, 7, 10, 44, 45 \forall_j$$

The objective function

The objective of this model is to maximize the net benefit

$$\max Z = f(f_a, A_{ij}, f_p, P_{ij}, K_i; f_a, f_p, K_i, i, j),$$

$$\text{or} = f_a \sum_i [A_{i1} + A_{i3}] + f_p \sum_j \sum_i [P_{ij} K_i].$$

Appendix B. Field irrigation requirements

In this appendix field irrigation requirements (α_{ij}) in the Ganga basin are examined. Only the Chambal sub-basin (node 40) is illustrated because the procedure for obtaining these requirements is similar. The monthly or seasonal field irrigation requirement is a function of (i) evaporation; (ii) cropping pattern; (iii) rainfall and (iv) soil type, conveyance, storage and operational losses.

Evaporation

Christiansen *et al* (1966) have developed an empirical formula for estimation of a class A pan evaporation:

$$E_p = 0.459 R C_t C_w C_h C_s C_e,$$

where E_p is class A pan evaporation, R is extraterrestrial radiation; C_t , C_w , C_h , C_s and C_e are coefficients of temperature, wind velocity, relative humidity, sunshine percentage and elevation. These coefficients for node 40 are presented in table B.1.

Cropping pattern

Evapotranspiration (consumptive use) of a crop is the product of evaporation (E_p) and the consumptive use coefficients (k) of each crop. Consumptive use coefficients of the kharif crop (mainly rice) and the rabi crop (mainly wheat) are taken from Ministry of Agriculture (1971) estimates.

The consumptive use coefficients of rice are 0.97, 1.21, 1.24 and 0.80 for the four months of the kharif season July to October respectively. The consumptive use coefficients of wheat are 0.26, 0.75, 0.90, 0.81 and 0.61 for the months November to March respectively. During the summer season the consumptive use coefficient of 0.8 is assigned to June irrigation for land preparation. There is no irrigation requirement in the months of April and May.

Rainfall

Net irrigation requirements (NIR) are the differences of evapotranspiration and effective rainfall. Effective rainfall depends on the normal rainfall and the consumptive use. These effective rainfalls (R_e) in table B.1 are estimated from Ministry of Agriculture (1971) data.

Table B.1. Field irrigation requirements in Chambal sub-basin (node 40)

Month	R	T_c	C_T	E_p	E_t	Rainfall	R_e	NIR (mm)	FIR (mm)
Jan.	304	15	0.839	119	107	10.47	7.54	99	132
Feb.	321	19	0.965	145	117	7.46	5.50	111	148
Mar.	422	24	1.135	224	137	5.83	4.60	132	176
Apr.	464	27.5	1.250	272	—	3.74	—	—	—
May	513	29	1.305	313	—	9.08	—	—	—
Jun.	507	31	1.370	325	260	91.42	88.68	171	228
Jul.	520	32.5	1.425	347	337	283.42	280.00	57	76
Aug.	496	31	1.370	318	385	252.65	252.00	133	177
Sep.	434	29	1.305	265	329	137.26	137.00	192	256
Oct.	381	27.5	1.250	223	178	19.73	17.00	161	215
Nov.	309	25	1.167	169	44	9.30	6.00	38	51
Dec.	289	20	1.000	135	101	5.42	4.00	97	129
Total	4959	25.88 mean		2855	1995	835.78	802.32	1191	1588

R radiation in mm at latitude of 26°N ; T_c monthly average temperature in $^\circ \text{C}$; C_T temperature coefficient in Christiansen formula; E_p class A pan evaporation (assuming $C_w C_h C_s = 1$) in mm; E_t evapotranspiration (consumption use) in mm, $E_t = k E_p$.

Rainfall: monthly rainfall in mm (weighted average 58% of East Rajasthan and 42% of West Madhya Pradesh); R_e effective rainfall in mm; $\alpha_{40,1} = 636 \text{ mm}$, $\alpha_{40,2} = 228 \text{ mm}$, $\alpha_{40,3} = 724 \text{ mm}$.

Node 40—Latitude 26°N , Elevation 500 m, Mean temperature 25.88°C , Annual rainfall 835.78 mm

Monthly rainfall of the Chambal sub-basin is a weighted average of East Rajasthan (about 58%) and West Madhya Pradesh (42%). Rainfall statistics of Chambal and other sub-basins are obtained from the Ministry of Irrigation and Power (1972) report.

Losses

Due to the unavoidable water losses through soil percolation, conveyance, storage and operation, more irrigation water than the estimated net irrigation requirement (NIR) is required. This field irrigation requirement (FIR) is determined by NIR and the field application efficiency (FAC). The efficiency is assumed to include all the losses for simplicity.

$$\text{FIR} = \text{NIR} / \text{FAC}.$$

An FAC of 75% is assumed for the Ganga basin model. The field irrigation requirements of Chambal sub-basin are given in the last column (FIR) in table B.1. Thus the FIRs at node 40 for a three-season model are 0.636, 0.228 and 0.724 meters per unit area for the rabi, summer and kharif seasons respectively.

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List of symbols

- AG_i area (10^6 m^2) at node i over which groundwater is available
 AH_i culturable land (10^6 m^2) at district i

AL_i	minimum irrigation level (current irrigation level) at district i
D_{ij}	quantity of water (m cu m) left from upstream river node i that returns to the river node downstream
H_{ij}	hydrological inputs (m cu m) at river node i during j th period
K_i	power conversion factor (MW/m cu m/mon) at power plant i
m_j	parameter specifying use constraint of a system capacity
PH_i	capacity of water (m cu m/mon) driving the turbine at power plant i
P_{ij}	quantity of water (m cu m) driving the turbine at power plant i
P^{NEW}_i	new capacity (m cu m/mon) to be added at power plant i
T_{ij}	quantity of groundwater (m cu m) pumped for irrigation district i during j th period
VH_i	capacity (live storage in m cu m) at reservoir i
V_{ij}	storage (m cu m) at reservoir i during j th period
X_{ij}	quantity of water in 10^6 m^3 (m cu m) flowing through a river away from node i downstream during j th period
XL_i	minimum monthly flow (m cu m) at node i
Y_{ij}	quantity of water (m cu m) diverted from node i in j th period
YH_i	capacity of diversion channel from node i
YL_i	minimum water diversion requirement from node i
Z_{ij}	quantity of water (m cu m) diverted for irrigation from node i during j th period
ZH_i	capacity of diversion channel (irrigation) from node i
Z^{NEW}_i	new capacity of diversion channel to be added to irrigation
α_{ij}	field irrigation requirement (m cu m/ $10^6 \times \text{m}^2$) at district i
η_j	power demand distribution for j th period
θ_i	quantity of annual groundwater availability (m cu m/ $10^6 \times \text{m}^2$) at district i
ξ_j	the fraction of total unpumped groundwater returns to river system in j th period