A new theory of development of force and torque in variable reluctance stepping motors

V V ATHANI* and S K KHURANA**

*Electrical Engineering Department, Indian Institute of Technology, Bombay 400 076, India

**Khurana and Sons, I B Patel Road, Goregaon East, Bombay 400 062, India

MS received 11 June 1982; revised 14 June 1984

Abstract. This paper presents a new theory of force and torque production in doubly slotted magnetic structures encountered in variable reluctance machines. This general theory is applicable to any point on the B-H curve of iron. It is capable of explaining all known phenomena including those for which satisfactory explanations are not available in existing theories. It enables optimization of all dimensions defining the rotor geometry to be carried out. It thus lays a sound theoretical basis for designing variable reluctance machines.

Keywords. Doubly-slotted magnetic structures; variable reluctance machines; B-H curve of iron.

1. Introduction

A variable reluctance (vr) machine is characterized by the fact that its rotor does not carry a winding or permanent magnet. It is a soft iron structure having a slotting identical to that of the stator. Doubly-slotted magnetic structures are thus encountered in variable reluctance machines.

The phenomenon of force and torque production in variable reluctance machines is a challenging problem in electromagnetic field theory applied to electrical machines. It was first attempted by Mukherji & Tustin (1974) who showed that the average peripheral force $\bar{F}_m$ per unit peripheral arc is given by

$$\bar{F}_m = G(B_m^2/2\mu_0), \quad (1)$$

where G is the figure of merit defined as

$$G = \frac{a-b}{(p_s/g) \cdot (p_r/g)}. \quad (2)$$

They showed that the average force $\bar{F}_m$ could be maximized by optimizing the figure of merit G with respect to rotor parameters. The optimum values obtained were: $t/\lambda = 0.38$ and $R \approx 8$.

This was further investigated by Mukherji & Neville (1971) who standardized the terminology for the airgap as shown in figure 1. They defined permeance ratios P_1 and P_2 corresponding to two extreme positions of the rotor as shown in figure 2, viz, (i) rotor teeth in complete alignment with stator teeth corresponding to minimum

Figure 1. One phase of a multistack VR machine

Figure 2. Two extreme positions of the rotor

reluctance and (ii) rotor teeth completely out of alignment with stator teeth corresponding to maximum reluctance. They presented valuable data in graphical form on the variation of P_1 and P_2 with respect to t/g and s/g ratios. They computed the aligning force $\bar{F}$ and showed that $\bar{F}$ is maximized by optimizing the figure of merit

$$f_1 = (P_1 - P_2)/(\lambda/g)^2 \quad (3)$$

with respect to rotor parameters. They further showed that f_1 possesses a broad maximum at $t/g = 3.05$ and $s/g = 5$.

The computation of P_1 and P_2 , initiated by Mukherji & Neville, was extended by Ward & Lawrenson (1977) who presented exhaustive data, again in graphical form, on the variation of P_1 and P_2 with respect to t/λ and λ/g .

The next milestone in computing forces and torques in doubly-slotted magnetic structures was due to Bakhuizen (1975) and Bakhuizen & deBoer (1976) who showed that, by deeply saturating the magnetic circuit, forces and torques considerably higher

than in the linear case could be obtained. They showed that the average force $\bar{F}$ is given by

$$\bar{F} = \frac{B_m^2}{\mu_0} \cdot L_T \cdot \lambda \cdot Z \cdot f_2, \quad (4)$$

where f_2 is a figure of merit defined by

$$f_2 = \left(\frac{1}{P_2} - \frac{1}{P_1} \right) / (\lambda/t)^2. \quad (5)$$

They plotted f_2 against t/λ for various values of λ/g as shown in figure 3 and concluded that f_2 reaches maxima for $t/\lambda \approx 0.42$ and that these maxima keep increasing as the ratio λ/g increases. They have generated a wealth of experimental data to support their deductions.

Harris *et al* (1975, 1977) reported extensive work dealing with computation of specific torque in variable reluctance stepping motors on the basis of both linear theory and nonlinear theory. Their linear theory yields specific torque

$$\bar{T}/V_R = (2B_s^2/\mu_0) \cdot f_3, \quad (6)$$

where f_3 is the figure of merit given by

$$f_3 = (t/\lambda)^2 \frac{P_1 - P_2}{P_1^2}. \quad (7)$$

The optimum geometry that maximizes f_3 is shown to be $t/\lambda = 0.47$ and $\lambda/g = 15$. Their nonlinear theory results in the same figure of merit f_2 as defined by Bakhuizen & deBoer (1976) and given in (5).

2. The need for a new theory

The existing theories surveyed above have certain shortcomings. Since Mukherji & Neville's theory is based on linear operation of the iron parts of the magnetic circuit, it is of little use in the construction of actual variable reluctance machines.

Figure 3. Force parameter $f_2 = ((1/P_1) - (1/P_2))/(\lambda/t)^2$ vs t/λ

Bakhuizen's theory, based as it is on operation in deep saturation region, is useful in the construction of actual variable reluctance machines. There are, however, certain phenomena for which satisfactory explanations are not available. Some of these phenomena are mentioned below:

- (i) Bakhuizen's theory predicts the maxima of the figure of merit f_2 at $t/\lambda \approx 0.42$. His experimental results, shown in figure 4, however, indicate that the maxima of f_2 , actually occur at $t/\lambda \approx 0.26$. This disparity has, however, not been explained.
- (ii) Bakhuizen's theory predicts that the average torque should decrease hyperbolically with tooth length L_T as shown in figure 5. The experimental results, however, indicate that for small values of L_T , the average torque $\bar{\tau}$ increases until a maximum is reached and then decreases hyperbolically with L_T as shown in figure 5. He has attributed this vital divergence to leakage fluxes; but this explanation is hardly convincing.

Figure 4. Experimental curve of average torque vs t/λ

Figure 5. Vibration of torque with tooth length

(iii) Fair agreement is shown to exist between calculated and experimental values of torque by Bakhuizen & deBoer (1976). However, for small values of t/λ , experimental points differ widely from the calculated ones, sometimes by as much as 80% or more.

These and many other discrepancies reveal the shortcomings of the existing theories and call for development of a new theory. In fact, Harris *et al* (1975) have stated in their paper: 'Neither theory is capable of predicting the optimum geometry or the limiting torque, and the need for a more rigorous treatment of saturation is clear'.

3. The new theory

The force and torque developed in doubly-slotted magnetic structures may be obtained by computing the energy E available for performing mechanical work. As pointed out by Mukherji & Neville (1971), the rotor assumes two extreme positions as it rotates inside the stator: (i) rotor teeth are in complete alignment with stator teeth, and (ii) rotor teeth are completely out of alignment with stator teeth. These rotor positions are shown in figure 2. The B-H curves of the machine for rotor positions (i) and (ii) are shown in figure 6 as curves I and II respectively. The shaded area between curves I and II represents the energy available for mechanical work.

The computation of the shaded area in figure 7 is facilitated by splitting it up into two parts: (a) $\Phi - I$ curves for the iron parts alone, and (b) $\Phi - I$ curves for the airgap in rotor positions (i) and (ii). These are shown in figure 7(a) as curves $P(0)$ and $P(-x)$ respectively. The $P(0)$ and $P(-x)$ curves have been redrawn as shown in figure 7(b) for a particular current I . From figure 7(b) we get the energy E available for doing mechanical work as the area of ΔACD . Thus

$$\begin{aligned} \text{energy } E &= \text{area of triangle } ACD \\ &= \text{area of (trapezium } OADQ - \Delta OAC - \Delta CDQ) \end{aligned}$$

or

$$E = \frac{\Phi_m \mathcal{F}_m}{2} \left[\frac{P(0) - P(-x)}{P(0)} \right]. \quad (8)$$

Figure 6. Magnetization curves for a VR machine for two extreme alignments of stator and rotor (I and II refer to rotor positions)

Figure 7. Φ vs mmf curves for two extreme positions of the rotor

However, $P(0) = \mu_0 L_T Z P_1$ and $P(-x) = \mu_0 L_T Z P_2$

Therefore $E = \frac{1}{2} (\Phi_m \mathcal{F}_m) \cdot (P_1 - P_2) / P_1$. (9)

The mmf absorbed by the iron parts is usually very small compared to the mmf absorbed by the airgap. Consequently, we may write $\mathcal{F}_m \approx \mathcal{F}$, the applied mmf. Equation (9) then becomes

$$E \approx (\Phi \mathcal{F} / 2) \cdot [(P_1 - P_2) / P_1]. \quad (10)$$

This shows that, for the same peak flux Φ or the same peak airgap mmf $\mathcal{F}_m$, the maximum E

is obtained by maximizing the figure of merit

$$g_1 = (P_1 - P_2)/P_1 \quad (11)$$

A graph of g_1 vs t/λ is plotted in figure 8 for various values of λ/g . It is seen that maxima of g_1 occur at $t/\lambda \approx 0.27$ and that the maximum values of g_1 go on increasing as λ/g increases.

The average force developed may be obtained from (10) as

$$\bar{F} = \frac{E}{\lambda/2} = \frac{\Phi \mathcal{F}}{\lambda} \cdot \left[\frac{P_1 - P_2}{P_1} \right] = \frac{\Phi \mathcal{F}}{g} \cdot \left[\frac{P_1 - P_2}{P_1 \cdot (\lambda/g)} \right]. \quad (12)$$

This shows that, for the same peak flux Φ or same airgap mmf $\mathcal{F}_m$, the maximum $\bar{F}$ is obtained by maximizing the figure of merit

$$g_2 = (P_1 - P_2)/P_1 (\lambda/g). \quad (13)$$

Figure 9 shows a plot of g_2 against λ/g . It is clear that g_2 reaches a maximum at $t/\lambda = 0.27$ and $\lambda/g = 12$. Equation (12) also shows clearly that the smaller the airgap g , the larger the $\bar{F}$.

4. Two special cases

The above results have been obtained assuming any general operating point on the B-H curve for iron. Two special cases of this general theory are of interest: (i) deep saturation, and (ii) linear operation. These are considered below.

Figure 8. Variation of g_1 of (11) with t/λ

Figure 9. Variation of g_2 of (13) with λ/g

4.1 Deep saturation

This case occurs when the iron parts are driven deeply into saturation. In this case, the point C in figure 7 rises to coincide with point Q . The resulting Φ vs I curve is shown in figure 10. The flux Φ crossing the airgap remains constant for all positions of the rotor and is related to $\mathcal{F}$ by

$$\mathcal{F} = \Phi/P(-x). \quad (14)$$

Substituting in (10) and simplifying, we obtain

$$E = \frac{\Phi^2}{2\mu_0 L_T Z} \cdot [P_1 - P_2/P_1 P_2]. \quad (15)$$

Under deep saturation the figure of merit thus becomes

$$g_3 = (P_1 - P_2)/P_1 P_2. \quad (16)$$

The graph of g_3 vs t/λ shown in figure 11 exhibits maxima at $t/\lambda \approx 0.15$.

Figure 10. Machine operation in deep saturation

Figure 11. Variation of g_3 of (16) with t/λ . Numbers on the curves refer to λ/g

The average force under deep saturation is given by

$$\bar{F} = \frac{\Phi^2}{\mu_0 L_T Z} \cdot \frac{1}{g} \cdot \frac{P_1 - P_2}{P_1 P_2 (\lambda/g)} \quad (17)$$

$\bar{F}$ may be maximized by optimizing the figure of merit

$$g_4 = \frac{P_1 - P_2}{P_1 P_2 (\lambda/g)} \quad (18)$$

Variation of g_4 with λ/g is shown in figure 12. It is seen that the maximum of g_4 occurs at $t/\lambda \approx 0.15$ and $\lambda/g \approx 12$.

Figure 12. Variation of g_4 of (18) with λ/g

4.2 Linear operation

The linear region of operation of the B-H curve corresponds to point *D* merging with point *Q* in figure 7(b). The resulting Φ vs *I* curve is shown in figure 13. In this case, Φ is related to mmf $\mathcal{F}$ by

$$\Phi = \mathcal{F} \cdot P(0). \quad (19)$$

Substituting this relation in (10), we obtain

$$E = \frac{\Phi^2}{2\mu_0 \cdot L_T \cdot Z} \cdot [P_1 - P_2/P_1^2]. \quad (20)$$

The figure of merit for maximizing *E* now becomes

$$g_5 = (P_1 - P_2)/P_1^2. \quad (21)$$

g_5 exhibits an optimum at $t/\lambda = 0.45$ as shown by the graph of g_5 vs t/λ in figure 14.

The average force under linear operation is given by

$$\bar{F} = \frac{\Phi^2}{2\mu_0 \cdot L_T \cdot Z \cdot g} \cdot \left[\frac{P_1 - P_2}{P_1^2(\lambda/g)} \right] \quad (22)$$

Optimization of average force in this case involves optimization of the figure of merit

$$g_6 = \frac{P_1 - P_2}{P_1^2(\lambda/g)}, \quad (23)$$

and yields an optimum at $t/\lambda = 0.45$ and $\lambda/g = 35$, as seen from the graph of g_6 vs λ/g in figure 15.

4.3 Comments

The above special cases are hardly encountered in practical machines as operation in the linear region is uneconomical with respect to flux while operation in deep saturation is uneconomical with respect to mmf. Thus none of the above cases truly represents the real machine. However, our new general theory remains valid under all conditions of

Figure 13. Φ vs mmf curves for linear operation

Figure 14. Variation of g_5 of (21) with t/λ

Figure 15. Variation of g_6 of (23) with λ/g

operation of real machines. Accordingly, the optimum values of $t/\lambda = 0.27$ and $\lambda/g = 12$ remain valid in real machines.

5. Optimization of tooth length

The axial tooth length L_T was generally considered as an independent parameter. Bakhuizen & deBoer (1976) were the first to consider the variation of torque with respect to L_T . They predicted a hyperbolic fall of torque with respect to L_T as shown in figure 5. However, their experimental results yielded an increase in torque with L_T for small values of L_T upto a maximum and hyperbolic fall thereafter as shown in figure 5. This phenomenon was left unexplained by Bakhuizen & deBoer. A logical explanation for this phenomenon may be given on the basis of our general theory as follows:

When the machine is operating in saturated condition, the average force is given by

$$\bar{F} = \frac{\Phi^2}{\mu_0 L_T Z} \cdot \left[\frac{P_1 - P_2}{P_1 P_2} \right]. \quad (24)$$

Assuming that t/λ and λ/g have already been optimized, we find from (24) that $\bar{F} \propto 1/L_T$. Accordingly $\bar{F}$ and therefore $\bar{\tau}$ fall hyperbolically as L_T is increased.

The variation of $\bar{F}$ for small values of L_T may be obtained as follows: The airgap flux density B_A is given by

$$B_A = \Phi / L_T \cdot \lambda \cdot Z \quad (25)$$

Substituting in (24) we get

$$\bar{F} = (B_A \Phi / \mu_0) \cdot [(P_1 - P_2) / P_1 P_2]. \quad (26)$$

Now, for small values of L_T , as L_T is increased, airgap flux density B_A reaches a limiting value B_{SA} corresponding to saturation flux density B_s of iron. It is then possible to write

$$\Phi = B_{SA} \cdot L_T \cdot \lambda \cdot Z. \quad (27)$$

Substituting in (24) we obtain

$$\bar{F} = \frac{B_{SA}^2 \lambda Z}{\mu_0} \cdot \left[\frac{P_1 - P_2}{P_1 P_2} \right] \cdot L_T \quad (28)$$

This shows that, once B_A has increased to B_{SA} , $\bar{F} \propto L_T$, assuming t/λ and λ/g have already been optimized. Accordingly $\bar{F}$ increases linearly with L_T .

Obviously, there ought to be an optimum value of L_T at which $\bar{F}$ and therefore $\bar{\tau}$ is a maximum. Since the flux Φ crossing the airgap is limited by the constriction in the airgap, the upper and lower limits within which the optimum value of L_T lies may be set as shown by Athani & Khurana (1978b) as follows:

(i) The lower limit is obtained by equating the cross-sectional area of the neck to the active rotor surface area, ie $\pi D L_T = \pi d^2 / 4$. This yields the lower optimum value of L_T as

$$L_{Tl} = d^2 / 4D. \quad (29)$$

(ii) The upper limit of L_T results from equating cross-sectional area of the neck to the rotor iron area, ie $(\pi D L_T)(t/\lambda) = \pi d^2 / 4$. This yields the upper limit of L_T as

$$L_{Tu} = (d^2 / 4D)(\lambda / t) \quad (30)$$

The optimum value of L_T may be determined thus: (26) shows that, once $(P_1 - P_2)/P_1 P_2$ has been optimized with respect to t/λ and λ/g , $\bar{F} \propto (B_A \cdot \Phi)$. Naturally $\bar{F}$ will attain a maximum when the product $(B_A \cdot \Phi)$ reaches a maximum with respect to L_T as shown in figure 22.

6. Optimization of rotor neck

In this section, we consider the optimization of the rotor neck diameter and length.

6.1 Optimization of rotor neck diameter

Bakhuizen & deBoer (1976) advocated a constriction in the rotor neck so as to deeply saturate the magnetic circuit. Our investigation, however, shows that there is no advantage in providing a constriction in the rotor. In fact the constriction limits Φ in the airgap and thereby limits $\bar{F}$. Therefore, constriction should be avoided and d made the same as D . The apprehension that this may give rise to excessive leakage fluxes has been shown to be baseless by our experimental work.

6.2 Rotor neck length

The length of the neck L_N plays an important part in determining the winding inductance L_w . Figure 16 shows variation of L_w against L_N . This enables the designer to select L_N suitably.

7. Experimental verification

Six experimental models have been specially fabricated (Khurana 1978) with a view to verifying experimentally the theoretical predictions of our new theory. The stator plates and rotors of five of these experimental models are shown in figure 17. The fully assembled model containing the sixth rotor is shown in figure 18. The test apparatus used to conduct experimental work is shown in figure 19.

Figure 16. Variation of L_w with L_N

Figure 17. Experimental stator and rotor sets for (a) $t/\lambda = 0.19$, (b) $t/\lambda = 0.27$, (c) $t/\lambda = 0.39$, (d) model no. 5

Figure 18. a) Rotor parts, b) rotor and stator assembly

Figure 19. Experimental set-up for static and dynamic tests

7.1 Effect of t/λ

Three of the models were used to study the effect of t/λ . The major dimensions of these models are given in table 1.

Figure 20 shows a graph of $\bar{\tau}$ vs t/λ ratio for exciting currents of 1A, 3A and 5A. It is clearly seen that the average torque is a maximum for $t/\lambda = 0.27$ in every case. It is interesting to note that this conclusion is corroborated by the τ vs λ curves obtained by Bakhuizen & deBoer (1976) and shown in figure 5.

7.2 Effect of tooth length

Models 4 and 5 were used to study the effect of L_T . The main dimensions of these models are given in table 2.

In model 4 tooth lengths of 12, 17 and 22 mm were obtained by starting with $L_T = 22$ mm and successively cutting down the rotor and stator plates. The same procedure was repeated on model 5 to obtain L_T of 25 mm, 35 mm and 45 mm. Figure 21 shows the graph of $\bar{\tau}$ vs L_T for excitation currents of 1A, 3A and 5A (Athani & Khurana 1978b). It is clearly seen that $\bar{\tau}$ increases with L_T upto a maximum and then

Table 1. Major dimensions of models used to study effect of t/λ

model no	1	2	3
Stator ID (mm)	40.5	40.5	40.5
Rotor dia D (mm)	40.3	40.3	40.3
Air gap g (mm)	0.1	0.1	0.1
Tooth pitch λ (mm)	4.32	4.32	4.32
t/λ ratio	0.19	0.27	0.39
Number of teeth Z	30	30	30
Exciting coil turns N	500	500	500

Figure 20. Average torque vs t/λ curve (experimental)

Figure 21. Graph of torque vs tooth length (experimental)

Table 2. Main dimensions of models used to study the effect of L_T

model no	4	5
Stator inner dia (mm)	40.5	40.5
Rotor dia D (mm)	40.3	40.1
Air gap g (mm)	0.1	0.2
Tooth pitch λ (mm)	4.23	4.23
t/λ ratio	0.27	0.39
Number of teeth Z	30	30
Exciting coil turns N	500	500

falls off as L_T is increased still further. The upper and lower limit of L_T as defined by (29) and (30) are also shown in figure 18. It is seen that the optimum value of L_T is well within these limits.

The dependence of airgap flux Φ_{sA} , airgap flux density of B_A and the product $B_A \Phi_A$, on L_T has also been studied. The results of the study are shown in figure 22. It is seen that

Figure 22. Graph of average torque vs tooth length

$B_A \Phi_A$ reaches a maximum at $L_T \approx 20$ mm which is also the value at which $\bar{\tau}$ also reaches a maximum.

It is interesting to note that these conclusions are corroborated by figure 15, which represents the experimental results of Bakhuizen (1975).

7.3 Effect of airgap (g)

Equation (12) shows that $\bar{F}$ increases as g is decreased. This is because as g decreases, λ/g increases, an increase in λ/g leads to a rise in the value of $g_1 = (P_1 - P_2)/P_1$. To test out this aspect of our theory, an experiment was conducted on model 5. In this model, the airgap was increased from 0.1 mm to 0.2 mm. Maximum torque, neck fluxes, and applied mmf were measured in each case. From these observations were plotted curves of $\bar{\tau}_m$ vs Φ for $g = 0.1$ and 0.2 mm, as shown in figure 23. It is seen that $\bar{\tau}_m$ increases as g decreases under all conditions of excitation. Furthermore, the experimentally obtained increases in $\bar{\tau}_m$ agree well with theoretical values.

Bakhuizen & deBoer (1976) have presented exhaustive data, in graphical form, on the effect of variation of λ/g on the torque, for various values of t/λ . These curves support our premise that $\bar{\tau}_m$ increases as g decreases. Again there is good agreement between the extent of increase in $\bar{\tau}_m$ with reduction in g with values predicted by our general theory rather than with Bakhuizen & deBoers' own theory.

Harris *et al* (1974) also studied the effect of λ/g on specific torque. They observed that, when they varied λ/g from 54 to 18 by increasing the airgap from 0.08 mm to 0.24 mm, the specific torque decreased by 72.2% from 1.38×10^3 to 0.67×10^3 . This agrees well with the reduction of 71.1% predicted by our general theory rather than by their own linear or nonlinear theory.

In this way, our theoretical predictions on the effect of variation in g are supported not only by our own experimental work but also by experimental work published by Bakhuizen & deBoer (1976) and Harris *et al* (1975).

Figure 23. Graph of maximum torque vs Φ for changing airgap g

Figure 24. Graph of maximum torque vs flux Φ for changing neck diameter (d)

7.4 Effect of variation of neck diameter (d)

Bakhuizen & deBoer have recommended the building of a large reluctance in the magnetic circuit by constricting the rotor neck, to ensure deep saturation. Our general theory, on the other hand, does not recommend a constriction in the rotor neck, so that maximum mmf appears across the airgap, thereby maximizing the flux crossing the airgap.

Model 5 was used to test our hypothesis *vis-a-vis* that of Bakhuizen & deBoer. Keeping all other parameters fixed, the rotor diameter was reduced from 35 mm to 30 mm in the first stage and further down to 25 mm in the second stage. Curves of $\bar{\tau}_m$ vs Φ were obtained as shown in figure 24. They clearly show that, for the same Φ , $\bar{\tau}_m$ decreases as d is decreased. Thus our hypothesis is vindicated.

7.5 Effect of rotor neck length (L_N)

The inductance L_w of the phase winding depends upon air gap g , g_2 of (13), and on L_N . It is found that the leakage fluxes and L_w decrease rapidly as L_N is slowly increased from a very low value. However, mmf and therefore L_w tend to increase as L_N increases. As a result of these opposing effects, there is an optimum L_N at which L_w is a minimum. This is shown clearly by the results of sample calculations presented in figure 16.

8. Conclusion

A new general theory of generation of force and torque in variable reluctance stepping motors has been presented. This theory is upwards compatible with the existing linear

Table 3. Results of optimization of rotor parameters

Region	Quantity	Condition	Figure of merit	Optimum				
				t/λ	λ/g	L_T	L_N	d
General	Energy E	Same Φ_m and $\mathcal{F}_m$	$g_1 = (P_1 - P_2)/P_1$	0.27				
	Force F	-do-	$g_2 = g_1/(\lambda/g)$	0.27	12			
Deep saturation	Energy E	Same Φ_m	$g_3 = (P_1 - P_2)/P_1 P_2$	0.15				
		same $\mathcal{F}_m$	$g_7 = P_2(P_1 - P_2)/P_1$	0.5				
	Force F	same Φ_m	$g_4 = g_3/(\lambda/g)$	0.15	12			
		same $\mathcal{F}_m$	$g_8 = g_7/(\lambda/g)$	0.5	24			
Linear region	Energy E	same Φ_m	$g_5 = (P_1 - P_2)/P_1^2$	0.45				
		same $\mathcal{F}_m$	$g_9 = P_1 - P_2$	0.4				
	Force F	same Φ_m	$g_6 = g_5/(\lambda/g)$	0.45	35			
		same $\mathcal{F}_m$	$g_{10} = g_9/(\lambda/g)$	0.4	35			
	Tooth length L_T					Maximum of $B_A \cdot \Phi_A$		
	Rotor neck dia							D
	Rotor neck length L_N						Minimum of L_w	

theory of Mukherji and Neville and the nonlinear theory of Bakhuizen & deBoer and Harris *et al.* It explains successfully all known phenomena in variable reluctance stepping motors, including those for which logical explanations were not available in existing theories. This theory leads systematically to optimization of all parameters determining the rotor geometry as shown in table 3. Our theory is well supported by experimental work carried out on six experimental models as reported in §7. However, the most interesting part of our theory is that its predictions are amply vindicated by experimental work reported by both Bakhuizen & deBoer as well as Harris *et al.* In fact, the predictions of our theory more closely match their experimental work than their own theories. Lastly, our theory leads to a logical step-by-step design procedure (Athani 1979) and computer aided design (Athani 1983) of variable reluctance stepping motors.

Authors are thankful to Electronics Commission, Government of India, for supporting this research.

References

- Athani V V 1979 *Stepping motors and systems* (Leeds: Proc. Third Int. Conf.) p. 60
 Athani V V 1983 *Comput. Electr. Eng.* 10: 41–49
 Athani V V, Khurana S K 1978a *Electr. Machines Electromech.* 2: 293–303
 Athani V V, Khurana S K 1978b *Proc. Inst. Electr. Eng.* 125: 1271–1272
 Athani V V, Khurana S K 1978c New theory of force and torque production in variable reluctance machines, Res. Rep. No. IIT-B/EE-CS/RR/780501, Indian Inst. Technol. Bombay
 Bakhuizen A J C 1975 *Incremental motion control systems and devices* (Proc. fourth Annual Symp: Illinois Univ.)
 Bakhuizen A J C, deBoer R 1976 On the calculation of permeances and forces between doubly slotted structures, TH Rep. 76-E-65. Eindhoven University of Technology
 Harris M R 1974 *Incremental motion control systems and devices*, (Proc. third Annual Symp:)
 Harris M R, Hughes A, Lawrenson P J 1975 *Proc. Inst. Electr. Eng.* 122: 1121–1127
 Harris M R, Andjargholi V, Lawrenson P J, Hughes A 1977 *Proc. Inst. Electr. Eng.* 124: 1215–1224
 Khurana S K 1978 *Mechanism of torque production in variable reluctance stepper motors*, Ph.D. Thesis, Indian Inst. Technol., Bombay
 Mukherji K C, Neville S 1971 *Proc. Inst. Electr. Eng.* 118: 1258–1268
 Mukherji K C, Tustin A 1974 *Proc. Inst. Electr. Eng.* 121: 965–974
 Ward P A, Lawrenson P J, 1977 *Proc. Inst. Electr. Eng.* 124: 542–544

List of symbols

a	equivalent ratio of width to length of gap for a slot pitch where permeance is a minimum
b	equivalent ratio of width to length of gap for a slot pitch where permeance is a maximum
B_A	airgap flux density
B_M	peak airgap flux density
B_S	saturation flux density
B_{SA}	saturation airgap flux density
d	neck diameter

D	rotor diameter
E	energy
$\bar{F}$	average force
F_m	maximum average force
f_1	figure of merit of Mukherji & Neville (1971)
f_2	figure of merit of Bakhuizen & deBoer (1976)
f_3	figure of merit of Harris <i>et al</i> (1977)
G	figure of merit of Mukherji & Tustin
g	airgap
$g_1, \dots, g_6$	figures of merit as per our new theory
I	phase current
L_N	neck length
L_T	tooth length
L_w	inductance of stator winding
$P(0)$	airgap permeance for rotor position no. 1
$P(-x)$	airgap permeance for rotor position no. 2
P_1	permeance factor for rotor position no. 1
P_2	permeance factor for rotor position no. 2
p_r	rotor slot pitch
p_s	stator slot pitch
R	permeance ratio = $p_r/(p_r - p_s)$
s	slot width
t	tooth width
Z	number of teeth
$\mathcal{F}$	mmf applied
$\mathcal{F}_m$	peak mmf across the airgap
μ_0	permeability of free space
Φ_A	airgap flux density
Φ_m	maximum airgap flux density
λ	tooth pitch = $s + t$