Proc. Indian Acad. Sci., Vol. C 2, Part 2, May 1979, pp. 179–195. © Printed in India.

Laser beam propagation in atmospheric turbulence

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MS received 25 August 1978; revised 17 January 1979

Abstract. The optical effects of atmospheric turbulence on the propagation of low power laser beams are reviewed in this paper. The optical effects are produced by the temperature fluctuations which result in fluctuations of the refractive index of air. The commonly-used models of index-of-refraction fluctuations are presented. Laser beams experience fluctuations of beam size, beam position, and intensity distribution within the beam due to refractive turbulence. Some of the observed effects are qualitatively explained by treating the turbulent atmosphere as a collection of moving gaseous lenses of various sizes. Analytical results and experimental verifications of the variance, covariance and probability distribution of intensity fluctuations in weak turbulence are presented. For stronger turbulence, a saturation of the optical scintillations is observed. The saturation of scintillations involves a progressive break-up of the beam into multiple patches; the beam loses some of its lateral coherence. Heterodyne systems operating in a turbulent atmosphere experience a loss of heterodyne signal due to the destruction of coherence.

Keywords. Scintillation; atmospheric optics; inhomogeneous media; turbulence; laser beam propagation.

1. Introduction

The fluctuations in the index of refraction of the earth's atmosphere result in many optical effects long known to astronomers. Some of these effects are twinkling (variation of image brightness), quivering (displacement of image from normal position), tremor disc (smearing of the diffraction image), dancing (continuous movement of a star image about a mean point), wandering (slow oscillatory motions of the image for a period of approximately one minute and angular excursions of a few seconds of arc), pulsation or breathing (fairly rapid change of size of the image), image distortion, and boiling (time-varying non-uniform illumination in a larger spot image). The scattering of electromagnetic waves in the turbulent atmosphere has been studied extensively by radiophysicists also, for example, Wheelon (1959). The development of lasers offered coherent monochromatic sources which opened up for the first time the optical region of the spectrum for communications and various other applications such as imaging, monitoring gaseous pollutants, probing the atmosphere and measuring fluid flow.

The NASA-Marshall Space Flight Centre has been involved in the development of CO_2 Laser Doppler velocimeter systems for atmospheric measurements since the late 1960s. The shorter range continuous wave (CW) system development led to a variety of applications including measurement of the following: aircraft wake vortices (Huffaker *et al* 1975 and Bilbro *et al* 1976a), dust devils (Bilbro *et al* 1976b),

smokestack emissions (Miller & Sonnenschein 1975), and wind profiles. The development of a pulsed system has resulted in measurement programmes for clear air turbulence and wind shear (Weaver *et al* 1976), and severe storm research (Bilbro & Vaughan 1979).

All laser systems which involve propagation of the laser beam over a path of several kilometres through the atmosphere usually experience a degradation of their performance due to a variety of natural causes, such as the gaseous and particulate constituents, fog, snow, rain and atmospheric turbulence. In this paper, only the effects of atmospheric turbulence are considered.

For a laser beam, the turbulence effects are broadly categorised as beam wander, spot dancing, beam spread and scintillations. These effects are due to the temporal and spatial fluctuations of the direction, phase and intensity of the wavefront. A description of the atmospheric inhomogeneities which cause these effects is given in the following section.

2. Relevant parameters of atmospheric turbulence

Optical effects are produced by the variation of the refractive index along the path of the beam. The changes in the refractive index are caused by the fluctuations of temperature which arise from the turbulent mixing of various thermal layers. It is found that the temperature fluctuations obey the same spectral law as the velocity fluctuations. In analogy with the velocity turbulence, the atmosphere may be imagined to consist of a large number of eddies with varying dimensions and refractive indices. For isotropic turbulence, the velocity spectrum $\phi_v\,(K)$ (where the wavenumber $K=2\pi/l$, l being the size of a turbulent eddy) contains the information about the turbulence. $\phi_v(K)$ is intuitively interpreted as the amount of energy in the turbulent eddies of size l. The kinetic energy of turbulence is assumed to be introduced through eddy scale sizes larger than the ' outer scale ' of turbulence, L_0, corresponding to a spatial wavenumber $K_0=2\pi/L_0$. As the wavenumber increases beyond K_0, the turbulence tends to become isotropic and homogeneous. Experiments have confirmed the predictions of Kolmogorov's theory of $K^{-11/3}$ dependence for the three-dimensional spectrum up to wavenumber K_m related to the inner scale of turbulence l_0 by $K_m=5{\cdot}92/l_0$. The spectrum in the dissipation region for which the scale sizes are smaller than l_0 or wavenumbers greater than K_m is steeper than $K^{-11/3}$. The spatial wavenumber region between the inner scale l_0 and the outer scale L_0 obeying the Kolmogorov theory of a spectral slope of $-11/3$ is known as the inertial subrange. l_0 is of the order of several millimetres and L_0 several metres.

At optical wavelengths the refractive index may be related to the temperature through the relation (Valley 1965)

$$n_1 = n-1 = \frac{77{\cdot}6P}{T}\left[1+\frac{0{\cdot}00753}{\lambda^2}\right]\times 10^{-6}, \tag{1}$$

where n is the refractive index, P is pressure in mb, T is temperature in K, and λ is wavelength in μm. The refractive index structure constant C_n^2 is a measure of the strength of atmospheric turbulence and is defined as the mean-squre difference in

the refractive index at two points divided by the separation distance r raised to the 2/3 power, i.e.,

$$C_n^2 = \frac{\langle (n_2 - n_1)^2 \rangle}{r^{2/3}}. \tag{2}$$

The temperature structure constant C_T^2 is similarly defined for temperature fluctuations. Thus C_n may be determined from temperature measurements alone from the relation (Ochs *et al* 1969)

$$C_n = \frac{77{\cdot}6P}{T^2}\left[\frac{1 + 0{\cdot}00753}{\lambda^2}\right] \times 10^{-6}\, C_T. \tag{3}$$

Typical temperature variations of approximately 1 K, resulting in refractive index variations of a few parts in a million, will cause significant effects on optical radiation fields propagating through the atmosphere. C_n^2 is measured in units of $(\text{metres})^{-2/3}$ and varies from 10^{-13} or more for strong turbulence to 10^{-17} or less for weak turbulence.

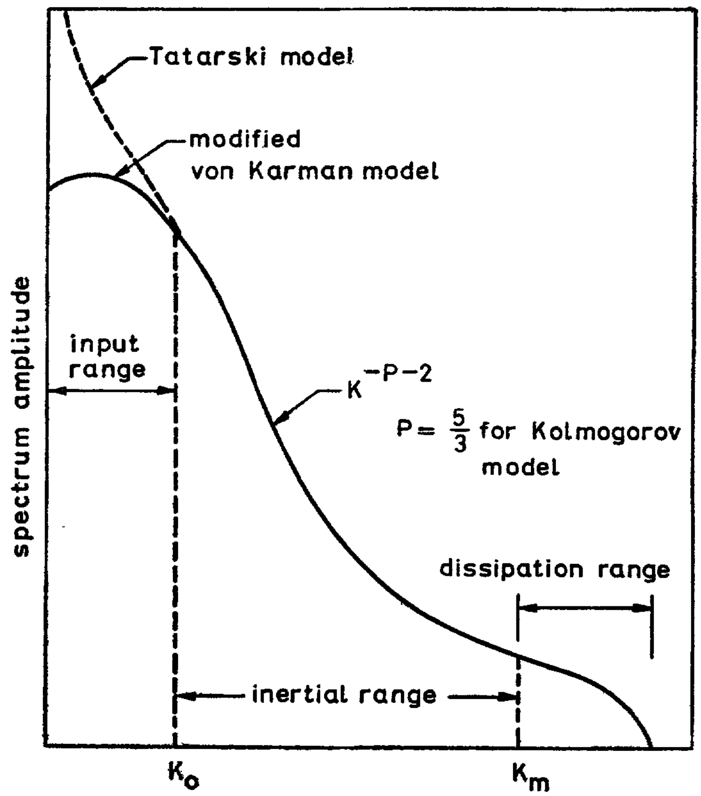

Figure 1. Three-dimensional isotropic turbulence spectrum models for index of refraction fluctuations. $K=2\pi/l$ when l is the size of turbulence body. $K_0=2\pi/L_0$, $L_0=1-100$m. $K_m=2\pi/l_0$, $l_0=1-3$ mm.

Assuming that the form of the temperature spectrum and the refractive index-spectrum is the same as that of the velocity spectrum, Tatarski (1971) gave the following form for the refractive index spectrum:

$$\phi_n(K) = 0{\cdot}033 C_n^2 \, K^{-11/3} \exp(-K^2/K_m^2), \tag{4}$$

where $K_m \, l_0 = 5{\cdot}92$. This spectrum has a singularity at $K = 0$. Physically the singularity implies that the energy per unit volume becomes unbounded as the eddy size increases. This singularity is avoided by using the modified von Karman spectrum

$$\phi_n(K) = \frac{0{\cdot}033 C_n^2 \exp[-K^2(l_0/2\pi)^2]}{[K^2+(l_0/2\pi)^2]^{11/6}}, \tag{5}$$

which gives a bounded variation for $K < 2\pi/L_0$. The three-dimensional refractive index spectrum models according to (4) and (5) are shown in figure 1. Deviations from these models occur if the turbulence is not homogeneous. Deviations have been observed in measurements over paths close to the ground and under conditions of weak turbulence (Kerr 1972). The optical effects of eddies are considered in the next section.

3. Optical properties of turbulent eddies

The optics of the turbulent atmosphere may be described as a collection of weak, moving, three-dimensional, gaseous lenses of varying scale sizes bounded by the inner scale l_0 and the outer scale L_0 of the turbulence. From equation (2), it may be seen that the refractive index fluctuations associated with any scale size l increases as the size of the eddy increases:

$$\langle [\delta n(l)]^2 \rangle = C_n^2 \, l^{2/3}. \tag{6}$$

An eddy of radius l can act as a lens with a focal length $f = R/n_1$, where R is the radius of lens curvature. This is approximated as

$$f \sim l/n_1 \approx l^{2/3} \, C_n^{-1},$$

from equation (6). Turbulence as it naturally occurs can consist of eddies of hot and cold air. Eddies cooler than the ambient air have a higher index of refraction. In this case, n_1 is positive and the lens is convergent. If the eddy consists of warmer air, the index of refraction is lower (giving negative n_1) and the lens is divergent. Typical values in the atmosphere are $C_n \sim 10^{-7}$, $|n_1| \sim 10^{-6}$, and l a few centimetres. Thus the focal length of the atmospheric lenses is $f \sim 10$ km.

The turbulent eddies cause diffraction and refraction of the optical wavefront, and it is instructive to consider these two effects separately. Each eddy of size l bends a ray bundle individually and has a diffraction pattern of approximate width, $\lambda L/l$, where L is the distance between the eddy and the receiver. The diffraction

effects are negligible if $\lambda L/l \ll l$ or if $l \gg (\lambda L)^{\frac{1}{2}}$, and this is the geometric optics regime. But if $\lambda L/l \gg l$, the diffraction pattern entirely determines the intensity distribution of the receiver. Thus the distance L_d beyond which the diffraction effects are important may be defined by $L_d \sim l^2/\lambda$.

The refractive bending of rays by an eddy of size l depends on the refractive index of the eddy and may be calculated by Snell's law to give $\delta\theta \sim \delta n$. δn is zero on the average. The mean squre angular fluctuation for one eddy is $\langle(\delta\theta)^2\rangle \sim \langle(\delta n)^2\rangle$.

Over a path length L, let the beam traverse L/l eddies. Since the deflections are uncorrelated from one eddy to another, they add up as mean squares and the total mean square deflection is

$$\theta_r^2 \sim \frac{L}{l}\langle(\delta n)^2\rangle = C_n^2\, L l^{-1/3}. \tag{7}$$

The effect of refraction is to change the cross-section of the beam from l^2 to $(l \mp L\theta_r)^2$. The amplitudes of the wave before and after refraction are related as

$$A^2 l^2 = (A + \triangle A)^2\,(l \pm L\theta_r)^2.$$

The fractional amplitude change is then given by

$$X = \triangle A/A \sim (L/l)\theta_r.$$

The variance of the amplitude fluctuation is then given by

$$\langle X^2\rangle \sim L^2/l^2\,\langle\theta_r^2\rangle \sim C_n^2\,L^3 l^{-7/3}. \tag{8}$$

The particular scale size which contributes predominantly to the amplitude fluctuation depends on the path length; i.e., different eddies are more effective at different path lengths in producing amplitude fluctuations. This point has been discussed by De Wolf (1969) as follows.

There are no diffraction effects in equation (8). Diffraction effects are negligible if the path length L_r is short or $L_s \ll l/\lambda$ in the geometric optics regime. Since l_0 (inner scale length) is the smallest eddy size, the diffraction by eddies of all sizes is negligible in this case. The smallest eddies then have the largest contribution. Equation (8) then becomes

$$\langle X^2\rangle \sim C_n^2\,L_s^2\,l_0^{-7/3} \text{ for } L_s \ll l_0^2/\lambda. \tag{9}$$

For path lengths L_l greater than L_s, such that $L_l \gg l_0^2/\lambda$, the eddies with sizes greater than $l_l (l > l_l)$ have negligible diffraction, while eddies with sizes smaller than $l_l (l < l_l)$ diffract the rays where

$$l_l = (\lambda L_l)^{1/2}.$$

The diffraction pattern entirely determines the intensity distribution and the refraction is negligible for the small eddies of size $l \ll l_l$; i.e., these eddies simply scatter

the energy, not focussing the rays, and have little effect on the amplitude fluctuations. Geometric optics results may still be applied if the eddies of size $l<l_t$ are excluded. Then the smallest eddy size contributing significantly to the amplitude fluctuation is l_t and $l=l_t$ is substituted in equation (8):

$$\langle X^2 \rangle \sim C_n^2\, L_l^3\, (\lambda L_l)^{-7/6} \sim C_n^2\, k^{-7/6}\, L_l^{-11/6} \text{ for } L_l \gg l_0^2/\lambda, \tag{10}$$

where $k=2\pi/\lambda$. This is also the result obtained by the Rytov approximation. Equation (10) has been interpreted by Tatarski (1971) as the result of the application of geometrical optics to a medium from which all eddies of scale sizes less than $(\lambda L)^{\frac{1}{2}}$ have been removed.

A qualitative description of the effect of the optical inhomogeneities on the wavefront is sometimes helpful and is presented here. As a plane wavefront passes through a region of a relatively lower refractive index, the wavefront will advance as the speed of light is higher. When passing through a region of higher refractive index, the wavefront is slowed down. Thus the refractive inhomogeneities of the atmosphere deform the wavefronts. Portions of the wavefront may be convex or concave in the direction of propagation. The concave portions converge while the convex portions diverge. Thus it is possible for the intensity to build up in some areas. Since the inhomogeneities are random and moving, the bright and the dark areas move randomly. This point of view was developed by the optical astronomers, Meyer-Arendt & Emmanuel (1955).

Young (1970a) obtained equation (10) on considering an aperture of Fresnel-zone size while calculating the normalised variance or the total modulation power obtained with a circular aperture using geometric optics. He observed that ray optics gave the correct parametric dependence because very little power is contributed by the range of spatial frequencies greater than $(\lambda L)^{-\frac{1}{2}}$ which are in the wave optics regime. Thus weak scintillation may be thought of as aperture filtering with a Fresnel-zone size aperture which filters out spatial frequencies higher than $(\lambda L)^{-1/2}$.

A related view developed by the radio astronomers is the phase screen concept. Immediately after passing through a layer of inhomogeneities, only the phase is affected and the intensity remains constant. As the distorted wavefront propagates further, the relative phases will be changed due to different directions of propagation and in this way, the amplitude fluctuations develop. This concept has been applied by Lee and Harp (1969) to wave propagation by dividing the three-dimensional refractive field of the medium into thin slabs perpendicular to the direction of propagation. The effects of the refractive inhomogeneities on the propagation of laser beams in the atmsophere are discussed in the following section.

4. Effect of atmospheric turbulence on laser beams

A qualitative description of the effects, based on the relative sizes of the beam, and the refractive inhomogeneity is given first, followed by the results of Rytov approximation on the wave equation.

If the scale size of the inhomogeneity is much larger than the diameter of the laser beam, the entire beam is bent away from the line of sight and results in beam wander or beam steering. The centre of the beam executes a two-dimensional random

walk in the receiver plane and, for isotropic turbulence, the displacement of the beam from the line of sight will be Rayleigh-distributed. Inhomogeneities of the size of the beam diameter act as weak lenses on the whole or parts of the beam with a small amount of steering and spreading. When the inhomogeneities are much smaller than the beam diameter, small portions of the beam are independently diffracted and refracted and the phase front causes constructive interference over some parts of the receiver and destructive interference over others, leading to alternate bright and dark areas. Since the atmosphere is seldom stationary, the locations of the bright and dark regions change continually and give the pattern of ' boiling '. Thus, the turbulence causes the received field to scintillate in time and space. Since the atmosphere consists of inhomogeneities of all sizes in motion, the laser beam experiences fluctuations of beam size (spreading), beam position (steering or wander), and intensity distribution within the beam (scintillation) simultaneously. The relative importance of these effects depends on the path length, strength of turbulence, and the wavelength of the laser radiation. The variance of irradiance reflects the deviations from the mean due to beam wander, broadening and scintillations. Estimates of these effects for finite beams are discussed below.

Finite transmitter effects have been treated by the Huygens-Fresnel approach and by the transport approximation (Fante 1974). The Huygens-Fresnel principle was extended by Lutomirski & Yura (1971) so that the secondary wavefront is determined by the envelope of spherical wavelets from the primary wavefront as in a vacuum, but each wavelet is determined by the propagation of a spherical wave in the refractive medium for small scattering angles. Hence a knowledge of the spherical wave propagation in the turbulent medium is sufficient to determine the field distribution at an observation point. The mean irradiance distribution at the observation point is characterised by the mutual coherence function (MCF) of a spherical wave and the geometry of the transmitting aperture. The MCF, defined as the cross-correlation function of the complex fields in a direction transverse to the direction of propagation, describes the loss of coherence of an initially coherent wave propagating in a random medium. When the medium is characterised by a refractive index variation that is gaussian with zero mean, the MCF is given by

$$M(\rho, L) = \exp\left[-\tfrac{1}{2} D(\rho)\right], \tag{11}$$

where D is the wave structure function, ρ is the lateral separation between the two points in a plane perpendicular to the direction of propagation, and L is the path length of propagation. For spherical wave propagation and a Kolmogorov spectrum of turbulence, the wave structure function is given by (Fried 1966)

$$D(\rho) = 2{\cdot}91\ k^2\rho^{5/3} \int_0^L ds\ C_n^2(s)\,(s/L)^{5/3}. \tag{12}$$

For constant C_n^2,

$$D(\rho) = 1{\cdot}089\ k^2\ C_n^2\ L\rho^{5/3}. \tag{13}$$

Equation (11) may be written as

$$M(\rho, L) = \exp\left[-(\rho/\rho_0)^{5/3}\right],\ l_0 \ll \rho \ll L_0, \tag{14}$$

where $\rho_0 = \left[1{\cdot}455\, k^2 \int_0^L ds\, C_n^2(s)\, (s/L)^{5/3}\right]^{-3/5}.$

For constant C_n^2

$$\rho_0 = [0{\cdot}545\, k^2\, L C_n^2]^{-3/5}. \tag{15}$$

ρ_0 is the lateral separation such that the MCF becomes equal to $1/e$ and is called the lateral coherence length. If $d > \rho_0$ where d is the diameter of the aperture, the turbulence along the path of the beam reduces the lateral coherence between different elements of the aperture and effectively transforms it into a partially coherent radiator of dimension ρ_0 which decreases with increasing distance from the aperture. But if $d < \rho_0$, the entire aperture behaves like a coherent radiator. Thus the resulting field distribution is characteristic of a coherent aperture with a diameter d_{eff} equal to the smaller of ρ_0 and d. Yura (1972) showed that if $L \gg kd\, d_{\text{eff}}$, the beam properties may be approximated by those of a spherical wave and if $L \ll kd\, d_{\text{eff}}$, the beam properties may be approximated by those of a plane wave.

In the absence of turbulence, a gaussian laser beam will have a farfield angular spread $\theta = 2\lambda/\pi d$. When turbulence is present along the path, the scattering of the beam by the moving turbulent eddies causes additional spreading which is greater than θ under some conditions.

The spreading of the beam consists of the deflection of the beam as a whole by the eddies larger than the beam diameter d and by the broadening of the beam by the eddies smaller than d. The separation of the beam spread into the two components of wander and broadening is usually based on the length of the time of observation. If V is the transverse wind-velocity component, the turbulence eddies interact with the beam at time intervals of the order $\triangle t \sim d/V$. For times much longer than $\triangle t$, the received spot will be fully spread due to wander and broadening. The total spot size, viewed on a time scale $T \gg \triangle t$, is called the long-term spot size and the beam radius of long-term spread is denoted by ρ_L. If a picture with an exposure time much less than $\triangle t$ were taken of the beam, a laser spot of radius ρ_s broadened by the small eddies at some distance ρ_c normal to the line of sight would be observed. ρ_c is the radius of beam-centroidal motion.

The concepts of the beam spread just discussed apply only if the beam is intact. When the beam wander effect is removed, the atmospheric spreading of the beam at the receiver is the short-term beam spread; however, when the turbulence along the path is strong, the beam breaks up into multiple patches and there is not much wander. Kerr & Dunphy (1973) found experimentally at near field that the focussed beam broke up at the receiver into a proliferation of transmitter-diffraction scale patches when there is either transmitter misadjustment or strong turbulence. Raidt & Höhn (1975) observed in recent experiments that the focussed beam at $0{\cdot}63\ \mu$m was broken into several spots of characteristic length $2\lambda L/\pi d$ which is the free-space diffraction limited focussed radius. They verified also the inverse variation of the patch size with the diameter of the beam, that is, the increase in patch size with decrease in beam diameter or vice versa.

The beam quivers while remaining intact as long as the turbulence is relatively weak and the path is not too long or, more precisely, if $L < L_{\text{cr}}$ where L_{cr} is the critical range defined by $L_{\text{cr}} = k\, d_{\text{eff}}^2$ where d_{eff} is the smaller of ρ_0 and d (Yura 1973).

Quantitatively, the spreading of the beam depends on the lateral coherence length ρ_0. If the beam diameter d is much smaller than ρ_0, the effect of turbulence on the beam is negligible and the beam diameter in the farfield will be that due to diffraction, namely, $2\lambda L/\pi d$. However, if the original beam diameter is greater than the lateral coherence length ρ_0, the beam diameter in the farfield is given by $2\lambda L/\pi\rho_0$. That is, the beam behaviour is characteristic of an aperture of diameter ρ_0 instead of d. The turbulence along the path transforms the aperture into a partially coherent radiator. For the initial field distribution of gaussian form given by

$$U(r) = \exp\left[-\frac{2r^2}{d^2} - \frac{ikr^2}{2F}\right],$$

where F is the radius of the curvature, the long term beam radius is obtained as (Yura 1971).

$$\langle \rho_L^2 \rangle \simeq (4L^2/k^2d^2) + (d^2/4)\left(1-\frac{L}{F}\right)^2 + (4L^2/k^2\rho_0). \tag{16}$$

When the turbulence is strong, the beam will be broken up and for this case Klyatskin & Kon (1972) found that the wander of the centre of gravity of the beam may be obtained from

$$\langle \rho_c^2 \rangle \simeq C_n^{8/5}\, k^{-1/15}\, L^{+37/15}, \tag{17}$$

for constant C_n, $L \gg k\,\rho_0^2$ (farfield case), and $d \gg \rho_0$. Taking the ratio of (17) and (16) and observing that the last term on the left-hand side of (16) is dominant for strong turbulence, we obtain

$$(\langle \rho_c^2 \rangle / \langle \rho_L^2 \rangle) \sim (k\rho^2/L)^{1/3}. \tag{18}$$

Since $L \gg k\,\rho_0^2$ then $\rho_c^2 \gg \rho_L^2$, which means that the motion of the beam centroid is negligible compared with the beam spread when the beam breaks up into multiple patches. It is not possible to tell how many patches will form, but the bright patches will be in a zone with mean square radius ρ_L^2 (Fante 1975a).

For ranges less than the critical range, tracking laser transmitters have been proposed and discussed to compensate the beam wander which is a relatively low-frequency phenomenon of the order of 0·5 Hz (Dunphy & Kerr 1974; Sher 1975). For $L < L_{cr}$, the beam centroidal motion is given approximately by Andreev & Gel'fer (1971)

$$\rho_c^2 \simeq 2{\cdot}97\, L^2/k^2\, \rho_0^{5/3}\, d^{1/3}, \tag{19}$$

and appears to satisfy a gaussian distribution on each axis.

One of the basic observations on laser beams is the fluctuations in intensity in addition to fluctuations in beam size and position. Fluctuations in intensity are

called scintillations. Experimental data on scintillations are extensive at present and are usually expressed as the normalised variance of the intensity:

$$(\langle I^2\rangle - \langle I\rangle^2)/\langle I\rangle^2, \tag{20}$$

I is the measured value of the fluctuating intensity and $\langle I\rangle$ is the average. The statistical theories of optical propagation in a randomly inhomogeneous medium usually calculate the variance in the log-amplitude X. The log-amplitude $X(x)$ at any location x is related to the intensity or the irradiance $I(x)$ by

$$X(x) = \ln \frac{A}{\langle A\rangle} = \tfrac{1}{2} \ln \frac{I(x)}{\langle I\rangle}, \tag{21}$$

where A is the amplitude. The variance of the log-amplitude is theoretically given by (Fried 1967c)

$$\sigma_T^2 = 0{\cdot}124\, k^{7/6} L^{11/6} C_n^2, \quad L \gg l_0^2/\lambda, \tag{22}$$

for a spherical wave. Experiments have confirmed the increase of the log-amplitude variance as the 11/6 power of the path length, as the 7/6 power of the optical wavenumber k, and the square of the strength of turbulence for a homogeneous path, i.e., C_n^2 = constant along the path as long as $\sigma_T^2 < 0{\cdot}3$ for the spherical wave (Kleen & Ochs 1970).

Increase of the path length or the strength of turbulence beyond $\sigma_T^2 = 0{\cdot}3$ fails to produce corresponding increase in the measured value of the log-amplitude variance σ_x^2 which slows down, reaches a maximum, and begins to decrease, reaching an asymptotic value of approximately 0·23 independent of wavelength, path length, and parameters of the turbulent medium. Thus the scintillation of an optical beam does not increase indefinitely but tends to saturate for strong turbulence and long paths. This is shown schematically in figure 2. The more precise way of

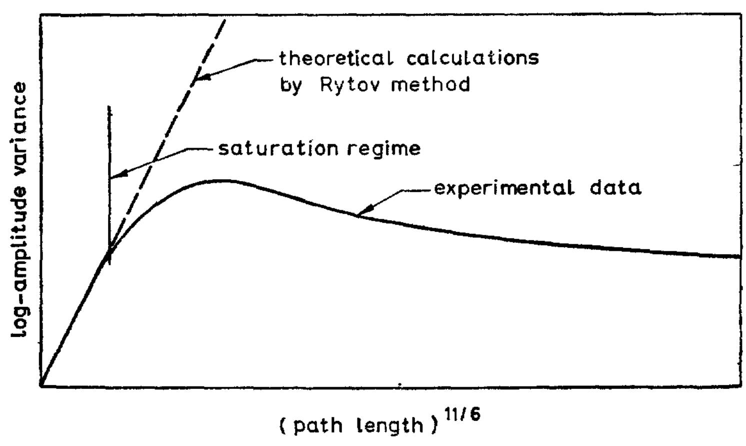

Figure 2. Saturation of the variance of log-amplitude fluctuations of a spherical wave propagating through a homogeneous turbulence path.

showing the saturation effect is to plot the theoretical value of log-amplitude variance along the x-axis and the experimental value of the log-amplitude variance on the y-axis, as is usually done in the literature (Kleen & Ochs 1970).

The probability distribution of the amplitude or intensity fluctuation is an important aspect in optical communications. Theoretically the application of the central limit theorem leads to a normal distribution of the log-amplitude: i.e., the scintillations follow a log-normal distribution. For a unit-amplitude plane wave, the probability density of intensity $p(I)$ satisfies

$$p(I) = 1/[(2\pi)\sigma I] \exp\{-[\ln I+(\sigma^2/2)]^2 (2\sigma^2)^{-1}\},$$

where $\sigma^2 = \ln(1 + \sigma_I^2)$.

For $\sigma_T^2 \leqslant 4$, there is extensive experimental evidence in support of log-normal distribution of intensity. Physically the reason for the log-normal distribution is the multiplicative effect of the refractive-index variations along the path on the intensity; i.e., the field is modulated in each subrange of the path multiplicatively if it is assumed that the effect of atmosphere in each subrange is independent of the degree of coherence. Hence the variation of the logarithm of the amplitude and intensity is the sum of several random variations induced along the path of propagation which leads to a normal distribution due to the central-limit theorem. Experimental measurements by Gracheva *et al* (1975) indicate that the log-normal distribution is a reasonable approximation for $25 < \sigma_T^2 < 100$. For $1 \leqslant \sigma_T^2 \leqslant 25$, there are significant deviations from the log-normal distribution. They did not notice Rayleigh distribution, as predicted by de Wolf (1968) for large values of σ_T^2. Physically, Rayleigh distribution results when the field at the receiver is the sum of a large number of independently scattered fields. The physical model of de Wolf (1969) attributes the Rayleigh distribution to the multiple scattering by off-axis eddies which are larger in strong turbulence.

In addition to temporal variation, the received field exhibits a spatially varying intensity after propagation through turbulence. The spatial structure of scintillations is usually studied by the covariance of the logarithm of the amplitude fluctuations. A correlation length may be defined in several ways. One definition is the value of separation for which the spatial covariance function goes to zero for the first time. The correlation length provides a measure of the separation between two points over which the scintillations are correlated. The concept of the correlation length has been useful for some applications. For example, the theoretical calculation of the log-amplitude variance is usually based on a point detector. In practice, a collector aperture of diameter less than the correlation length is treated as a point detector for comparison of theory and experiment. Another use of the correlation length is in understanding the aperture averaging effect which is discussed later. The intensity covariance function is usually measured and is related to the log-amplitude covariance as, (Fried 1976b)

$$C_I(\rho) = \langle I\rangle \{\exp [4\, C_x(\rho)]-1\}, \tag{23}$$

where the separation is $\rho = | x-x' |$, the covariance of log-amplitude is

$$C_X(\rho) = \langle [X(x)-\langle X\rangle]\, [X(x')-\langle X\rangle]\rangle,$$

and the covariance of intensity is

$$C_I(\rho) = \langle [I(x) - \langle I \rangle]\ [I(x') - \langle I \rangle] \rangle.$$

The intensity and the log-amplitude correlation distances may be seen to be the same from equation (23) but the curves differ greatly. For plane or spherical wave propagation, $C_X(\rho)$ has the form

$$C_X(\rho) = \sigma_X^2 f\ [\rho/(\lambda L)^{1/2}],$$

where f is a function which depends on the mode of propagation. The first zero of f is the correlation length and is approximately $0{\cdot}76\ (\lambda L)^{1/2}$ for a plane wave and $1{\cdot}8\ (\lambda L)^{1/2}$ for a spherical wave based on the perturbation theory of Tatarski. Experiments confirm a correlation length of the order of $(\lambda L)^{1/2}$ when there is no saturation of scintillations.

The classical method of reducing the fluctuations in the received power has been to increase the size of the receiving optics. The reduction of the fluctuations in the received power, when the diameter of the receiving aperture is increased, is called aperture averaging. The physical reasoning is that large apertures tend to average over the statistically independent portions of the scintillation pattern (Fried 1967). The diameter of an independent patch is usually imagined to be of the order of the correlation length. Thus for weak scintillations, the size of the patch is of the order of the Fresnel length $(\lambda L)^{1/2}$. Experiments by Dunphy and Kerr (1973) show that the correlation length decreases as the turbulence gets stronger. Moreover, the covariance curve develops a progressively higher correlation tail and aperture averaging deteriorates as the turbulence strength increases. Figure 3 illustrates the covariance function for several σ_T^2.

To summarise, the phenomenon of saturation of optical scintillation may be characterised by the following experimental observations.

(i) There is a physical break-up of the beam into multiple patches when the turbulence is strong.

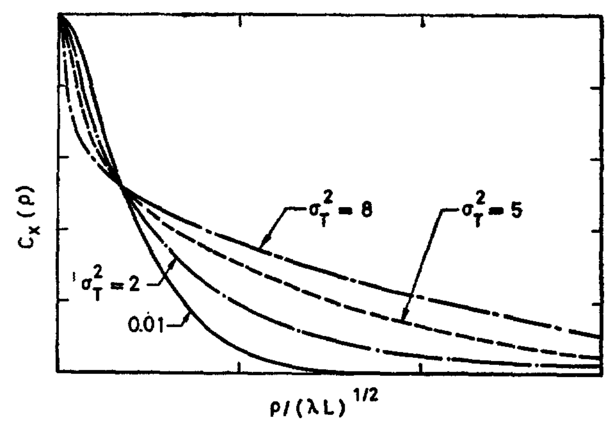

Figure 3. Theoretical curves of the log-amplitude covariance function in weak to strong turbulence from Clifford *et al* (1974).

(ii) The variance of the log-amplitude reaches a maximum instead of increasing indefinitely and then decreases to an asymptotic value of approximately 0·23 independent of the wavelength of the optical radiation and parameters of the turbulent medium as the path length, or the strength of turbulence, or both are increased.
(iii) The spatial covariance function of log-amplitude decays progressively faster but maintains a progressively higher tail as a function of the detector spacing with increasing turbulence strength and path length.
(iv) Aperture averaging becomes progressively less effective for a given size of the aperture as the turbulence strength and the path length are increased.

There were several theoretical attempts to explain the saturation phenomenon in the past but it is only in recent years that the theory is able to account for all of the observed effects. Since the saturation effect is observed under conditions of strong turbulence, a working definition is needed for the conditions specified as weak or strong turbulence. The turbulence is called ' weak ' if the theoretical variance of log-amplitude,

$$\sigma_T^2 = 0{\cdot}124\, k^{7/6}\, L^{11/6}\, C_n^2,$$

satisfies the inequality $\sigma_T^2 < 0{\cdot}3$. The turbulence is called ' strong ' if $\sigma_T^2 > 0{\cdot}3$. Thus the specification of the turbulence conditions involves not just the refractive-index structure constant C_n^2 but a combination of the wavelength of radiation, path length and C_n^2.

One of the earlier attempts to physically explain saturation is that by Young (1970b). When stellar scintillation is observed with small apertures, the total modulation power first increases with zenith angle and then decreases. Young suggested that the smearing or blurring of the details of the wavefront distortions caused by the intervening atmospheric turbulence is responsible for the observed saturation of astronomical scintillation. The blurring effect is produced by the small-scale fluctuations in the wavefront which change rapidly due to the short lifetime of the small eddies. Thus, a shadow pattern produced by a high layer in the atmosphere becomes smeared or washed out by the intervening turbulence as it propagates down through the atmosphere.

Clifford *et al* (1974) give a detailed description of the smearing process in the scintillation pattern in strong turbulence. The wavefront develops small-scale and large-scale distortions while passing through the turbulent medium. When such a distorted wave is incident on a Fresnel-zone-size eddy, the small-scale distortions reduce the resolving power of the eddy while distortions larger than the eddy merely tilt the diffraction pattern of the eddy. The small-scale distortions may be expected to change in detail several times during the lifetime of the larger eddies due to the shorter lifetimes of small eddies. These fluctuations of the small-scale distortions of the wavefront cause the smearing of the diffraction pattern of the Fresnel-size eddy, thus making it less effective as a producer of scintillation. Scintillations remain bounded and do not build up indefinitely due to the smearing by the small-scale eddies.

The smearing effect of strong turbulence is incorporated into the first-order analysis by means of a spectral-filter function which filters out higher spatial frequencies. The spectral-filter function is given by the normalised two-dimensional Fourier transform of the short-term average of the irradiance profile of an optical beam propagating through turbulence. The filter function turns out to be the short-term modulation transfer function (MTF). It is assumed that the log-amplitude covariance of a spherical wave for strong turbulence is given by the first order log-amplitude covariance whose spatial frequency spectrum has been modified by multiplication with the spectral filter function (or the short-term MTF). The log-amplitude covariance thus calculated agrees well with the experiments and is shown in figure 3.

From this covariance function, Clifford & Yura (1974) obtained the following asymptotic behaviour for the log-amplitude variance:

$$\sigma_X^2 = 0{\cdot}36\,(\sigma_T^2)^{-2/5} + 0{\cdot}42\alpha^{5/3}, \tag{24}$$

where α is a constant found from experiments. For $\alpha = 0{\cdot}7$, which is obtained from the strong scintillation data, σ_X^2 tends to 0·23 for large values of σ_T^2.

The perturbation theory of Tatarski (1971) implicitly assumes that the initial coherence is maintained along the propagation path and eddies having scale sizes of the order of the Fresnel length are most effective in producing scintillation. However, as the wave travels through the medium, the transverse spatial coherence decreases continuously and the wave becomes partially coherent. As was previously noted, the spreading of a beam of initial diameter d in a turbulent medium is characteristic of an aperture of diameter ρ_0 (the lateral coherence length) and not of diameter d. The radiator has become partially coherent in the presence of turbulence. Incorporating the loss of coherence of a radiator in the turbulent medium into the analysis, Yura (1974) obtained the saturation of scintillations and the behaviour of the amplitude covariance. He also showed that in the strong turbulence regime, the amplitude correlation length is equal to the lateral coherence length. The turbulent eddies most effective in producing scintillations in the saturation regime are those that have scale sizes of the order of the lateral coherence length and not the Fresnel length.

The asymptotic log-amplitude variance behaviour of $(\sigma_T^2)^{-2/5}$ has been predicted also by Fante (1975b) and Gochelashvily & Shishov (1974) from approximate solutions to the equation satisfied by the fourth moment of the field.

The physical theories and the approximate solutions have been able to predict the observed behaviour of the covariance function, namely, a rapid fall-off followed by a slow decay resulting in a long tail. These curves indicate that there are small-scale and large-scale structures in the scintillation pattern for strong turbulence. The correlation in strong turbulence is over a transverse separation of the order of

$$\rho_0 \sim (\lambda L)^{1/2}/(\sigma_T^2)^{3/5}.$$

As σ_T^2 increases, the lateral coherence length and the amplitude correlation length decrease. The diameter of the detector must be smaller than ρ_0 for point detector performance in strong turbulence.

The aperture averaging which produced a reduction in scintillation noise became less effective for a given aperture of the receiver as the turbulence increased. This effect is due to the presence of large-scale scintillations at the receiver as σ_T^2 increased.

Another quantity of interest is the temporal frequency spectrum of intensity fluctuations observed at a fixed point. The width of the frequency spectrum is of the order of $V/(\lambda L)^{1/2}$ for weak turbulence where V is the velocity component normal to the beam. This is typically 10 to 100 Hz. For strong turbulence, the width of the frequency spectrum is of the order V/ρ_0 which is approximately 100 to 1000 Hz. In fact, the peak of the spectrum occurs at approximately $V/(\lambda L)^{1/2}$ for $\sigma_T^2 < 1$ and at approximately V/ρ_0 for $\sigma_T^2 > 1$.

In addition to the shifting of the peak of the spectrum to higher frequencies, there is a broadening of the spectrum and the maximum value of the spectrum decreases for increasing σ_T^2. These features of the spectrum have been qualitatively observed in the experiments. Finally, to obtain all of the temporal information of the scintillation in the strong turbulence regime, it may be necessary to use an electronic bandwidth of the order of V/ρ_0. Otherwise, the use of a smaller electronic bandwidth may result in an apparent decrease of the log-amplitude variance for increasing values of σ_T^2.

5. Effect of turbulence on heterodyne systems

In a heterodyne receiver the incoming wave which is usually weak is combined with a reference wave on the photodetector surface. Coherent detection results from the square-law response of the photodetector to the incident radiation.

The various effects of atmospheric turbulence on the laser beams are related to image behaviour in a heterodyne system. Comparison of the expressions for the optical power density at a distance L due to a laser transmitter and the expression for the signal-to-noise ratio of a heterodyne receiver detecting a signal from a point source at a distance L shows that they have identical functional dependence on the turbulence parameters. Thus the telescope performance measured in terms of the coherence diameter remains the same whether used as part of a transmitter or as part of a heterodyne receiver in a turbulent atmosphere (Fried & Yura 1972). The reciprocity between the laser transmitter and the heterodyne receiver implies that the average target illumination is related to the power received by the heterodyne receiver. Further, the beam wander and beam spread in the target illumination system are related respectively to image dancing and spread in the heterodyne receiver. Thus the beam-image reciprocity implies the various effects of turbulence on the laser beam in the analysis of the heterodyne detection.

Fried (1967) analysed the optical heterodyne detection of an atmospherically distorted wavefront. A significant finding of this analysis is that there is an upper limit to the achievable signal-to-noise ratio for the heterodyne system operating in atmospheric turbulence no matter how large the detector aperture is. The aperture diameter for the near-peak value of signal-to-noise ratio is of the order of the lateral coherence diameter. This aperture size has the virtue of avoiding large signal power fluctuations which would result in larger collection apertures.

The analysis of the effect of turbulence on the NASA CO_2 laser Doppler System was performed by Murty (1976). From an operational point, the loss of signal due to the turbulence is of interest. The loss of signal arises from the loss of spatial coherence of the beam. The mutual coherence function describes the loss of

coherence and permits the calculation of the loss of signal-to-noise ratio. For a system focussed to infinity, the signal-to-noise ratio is reduced by the factor

$$P = \frac{L^2(1+a^2)+(\pi R^2/\lambda)^2}{L^2+(\pi R^2/\lambda)^2},$$

where $a = R/r_a$ and $r_a = 0{\cdot}1047\ \lambda^{6/5} L^{-3/5} C_n^{-6/5}$,

for a spherical wave. The parameter r_a is proportional to the coherence radius of the laser beam.

In practice the loss of signal due to turbulence is significant in the lower atmosphere for horizontal propagation. For most of the ground-based operations the beam path will point upwards for the detection of atmospheric phenomena and the turbulence effects are less severe.

The loss of signal due to absorption by the molecular gases is considerable for the CO_2 laser systems. Compared to the absorption losses, the loss of signal due to turbulence is in the region of 10–15% up to an altitude of 5 km. Beyond 5 km altitude, the turbulence is usually very weak and the absorption losses dominate.

6. Conclusions

In this paper, we have presented the major effects and some important analytical and experimental results on the propagation of low-power coherent laser beams through isotropic turbulence. A significant parameter which indicates the severity of turbulence effects on practical laser systems is the lateral coherence length ρ_0 which is a measure of the loss of coherence due to propagation in turbulence. The lateral coherence length is given by

$$\rho_0 = 0{\cdot}1586\ \lambda^{6/5} L^{-3/5} C_n^{-6/5}.$$

Thus laser beams having longer wavelengths in the optical region will have larger lateral coherence lengths and will be less affected by atmospheric turbulence. For a numerical example, assume $L=10$ km and $C_n^2=10^{-16}\ cm^{-2/3}$ (medium turbulence condition). The coherence length for these conditions works out to 2·68 m for CO_2 laser at $\lambda=10{\cdot}6\ \mu$m, 1·1 m for CO laser at $\lambda=5{\cdot}07\ \mu$m and 0·57 m for HF laser at $\lambda=2{\cdot}91\ \mu$m. In practice, atmospheric turbulence will limit the maximum useful receiver size of a coherent heterodyne system to approximately the lateral coherence length.

The author acknowledges gratefully the discussions with Mr J W Bilbro and Mr C O Jones, NASA Marshall Space Flight Centre. This work was supported by the NASA Grant NSG-8037.

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