A Revisit to the Double-prism Experiment of J. C. Bose *

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Optical tunneling is a fascinating phenomenon. Some of the early optical tunneling experiments were conducted in the microwave range. The double-prism experiment of Sir J. C. Bose is the earliest among them. The theory of optical tunneling, based on Maxwell equations in electrodynamics, was developed a few years after Bose’s study. As far as our knowledge goes, there is no paper studying the conformity of Bose’s findings with the theory. In the present work, we have made a qualitative and quantitative investigation of the experimental measurements of Bose by comparing his results with the theoretical predictions and find a remarkably good agreement.

1. Introduction

A young professor of physics (Figure 1) at the Presidency College, Kolkata, was studying the optical properties of millimeter waves with his indigenous apparatus in 1894. In his experiment, a beam of millimeter waves was incident on a right-angled prism, and it was totally internally reflected by the hypotenuse of the prism. When another prism was placed at a small distance from the previous one, keeping the hypotenuse faces parallel to each other (Figure 2), it was observed that a fraction of the incident beam passed through the air-space (enclosed between the plane parallel surfaces of the two prisms) into the second prism. By gradually reducing the thickness of the air-space, he found that the intensity of the transmitted wave increased. It was the first observation of optical tunneling using microwaves (Figure 3).

Keywords
Evanescent wave, optical tunneling, frustrated total internal reflection.

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*Vol.27, No.3, DOI: https://doi.org/10.1007/s12045-022-1326-1
The young physicist was Sir Jagadis Chandra Bose, one of the pioneers of microwave research. His findings were published in the *Proceedings of the Royal Society* in 1897 [1]. The paper was communicated by his physics professor Lord Rayleigh. This penetration of waves through the air film is now known as ‘frustrated total internal reflection’ (FTIR) and has opened up a new branch of physics called the near field optics. FTIR has many useful applications such as touch sensing devices, fingerprint imaging used in biometrics, tactile sensors for robot grippers, etc.

FTIR is also regarded as the electromagnetic analog of quantum tunneling where the air gap serves as the potential barrier, and this analogy was first mentioned by Sommerfeld. Because of this analogy, FTIR is popular by the name optical tunneling.

At the time of Bose’s observation, the theory of optical tunneling was not well developed. In 1902, Elmer E. Hall performed an extensive study [3], both theoretical and experimental, on the penetration of light from a denser medium into a rarer medium when the condition of total internal reflection is maintained. His theory was based on the old notion of the existence of ether. He
had an idea of verifying the theory of optical tunneling using millimeter waves. Perhaps, Hall did not notice the similar studies of Bose, which he had performed nearly a decade ago.

2. Optical Tunneling

When a monochromatic beam of light undergoes total internal reflection at an interface, say between a glass prism and air, a small exponentially decaying disturbance, called evanescent wave, penetrates the less dense medium. Evanescent wave travels a small distance parallel to the interface and its intensity rapidly falls off to zero. It carries no time-averaged energy through the interface into the rarer medium (air). But when another prism is placed against the first one such that the air-gap is less than a wavelength, the evanescent wave can penetrate the second prism and excite a propagating wave that carries energy through the air gap. This phenomenon is called frustrated total internal reflection or optical tunneling. The electrodynamic theory of evanescent wave and optical tunneling based on Maxwell equations is presented in the appendix section.

Optical tunneling occurs when the evanescent wave extends with sufficient amplitude across the rare medium into another medium of higher refractive index. From the Fresnel relations, which are
Figure 3. Microwave apparatus (Photo courtesy: Bose Institute archives.
http://www.jcbose.ac.in/museum-gallery).

Figure 4. Tunneling of light through an air film of thickness \( d \) enclosed between the plane parallel hypotenuses of two prisms. \( OP \) indicates the lateral shift of the reflected wave known as the Goos-Hänchen shift.

A direct consequence of the Maxwell equations and the appropriate boundary conditions, it is possible to connect the incident, reflected, and transmitted amplitudes of the electric fields of light in both sides of the air-gap [4]. If \( d \) is the thickness of the air film and \( \theta \) be the angle of incidence (Figure 4) then the transmission coefficient \( T \) is given by

\[
T = \frac{1}{\alpha \sinh^2(\beta x) + 1},
\]

(1)
where the dimensionless quantities $\beta$ and $x$ are given by

$$\beta = 2\pi(n^2 \sin^2 \theta - 1)^{\frac{1}{2}},$$

(2)

and

$$x = \frac{d}{\lambda}.$$

(3)

The parameter $\alpha$ depends on the state of polarization of the incident wave. For the perpendicular component of the incident field

$$\alpha_{\perp} = \frac{(n^2 - 1)^2}{4n^2 \cos^2 \theta(n^2 \sin^2 \theta - 1)},$$

(4)

For the parallel component we have

$$\alpha_{\parallel} = \alpha_{\perp}(n^2 + 1) \sin^2 \theta (n^2 - 1)^2.$$  

(5)

From (1) it is found that for a given value of $d/\lambda$ and for a particular polarization, the transmission coefficient $T$ depends on the angle of incidence and the refractive index of the prism.

Figure 5 illustrates the variation of $T$ with $d/\lambda$ for four different values of refractive indices and for both states of polarizations. Here the angle of incidence is kept at $\theta = 45^\circ$. We see that the transmitted amplitude increases with the decrease in refractive index.

This can easily be seen from (1) that for observing the phenomenon of tunneling with visible light, the air gap must be at least $10^{-4}$ mm, which is of the order of the wavelength of visible light. For $d \approx \lambda$, the transmitted intensity reduces to 5 to 10% of the intensity of
the incident light depending on the values of the refractive indices of the prisms. Therefore, to perform experimental studies on optical tunneling with visible light, high precision instruments are required. It is not an easy task to measure such a small ($\sim 10^{-4}$ mm) thickness of the air gap and vary it with equal precision. The use of microwaves could simplify the experimental arrangements since microwaves have wavelengths of a few millimeters, and tunneling can easily be demonstrated with an air gap of several millimeters. Bose exactly did that. He used microwave radiations of wavelength of about 20 mm. In the next section, an outline of his double-prism experiment is presented.

3. The Pioneering Experiment

Bose used a spark-gap radiator for generating microwaves and a spiral-spring receiver for detecting the transmitted or reflected waves. These were the earliest versions of microwave emitters and receivers. During the initial stage of the development of microwave technology, electromagnetic waves were generated by spark gaps, and the waves were called Hertzian waves. These waves had wide frequency bands. By inserting a metal sphere in between the sparking elements, Oliver Lodge, an eminent British scientist, filtered out some of the unwanted frequencies. But after a few strikes of the sparks on the metal sphere, its surface became rough, which introduced spurious radiations. Bose modified the emitter by covering the metal sphere with platinum and placing it between two hollow metal hemispheres. Platinum has a very high melting point $\sim 1772{^\circ}$C. This modified version was able to generate radiations in the desired frequency range with higher intensity. Bose succeeded in generating microwaves with wavelengths ranging from 2.5 cm to 5.0 mm.

During Bose’s time, receivers for the Hertzian waves were called coherers. Bose’s spiral-spring coherer (Figure 6) was a great advance over the then-existing receivers [5]. Metal filling coherers were prevalent at that time. Bose replaced the irregular metal filings with a steel spring. With its numerous point-contacts, the
sensitivity of Bose’s coherer was very high. Unlike metal filling coherer, in his spring coherer, the loss of sensitivity due to fatigue after long-continued radiation, could be restored by slightly changing the applied voltage. A galvanometer was used for detecting the current produced in the receiver during the reception of microwaves.

Bose’s study consisted of three parts. In the first part of his study, he tried to observe the influence of the angle of incidence on the reflected and the transmitted components of the incident wave in a double prism. In the second part, he examined how the transmitted intensity varied with the wavelength of the incident wave, where the angle of incidence was kept constant. The effect of varying the thickness of the air-space between two identical prisms on total internal reflection was studied in the last part.

In the first part, he used two hemicylindrical prisms of glass, each with a radius of 12.5 cm and measured the reflected and transmitted amplitudes in terms of galvanometer currents for three different angles of incidence \( \theta \), 30°, 45° and 60°.

It was one of the important findings in his earlier experiments with microwaves that the critical angle of glass for the millimeter waves is 29°. The thickness of the parallel air space between the

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**Figure 6.** Bose’s diagram of his spiral spring detector [5].
Figure 7. Variation of transmission coefficient with $d/\lambda$ for three different angles of incidence for a fixed value of refractive index (2.063). For a particular value of the wavelength and of the thickness of the air film, $T$ increases with a decrease in angle of incidence.

plane faces of the hemicylinders was gradually diminished, starting from a greater value of 20 mm. It was observed that the transmission begins when the air-gap was reduced to 14 mm for an angle of incidence $\theta = 30^\circ$. It was 10.3 mm for $\theta = 45^\circ$ and 7.6 mm for $\theta = 60^\circ$. For the case of total reflection, he concluded, “The minimum effective thickness is thus seen to undergo a diminution with the increase of the angle of incidence.” An equivalent limiting value of the thickness of the air-space for the transmitted part could be the maximum effective thickness where the transmitted intensity was just detectable. Figure 7 illustrates the theoretical variation of the transmission coefficient $T$ with $d/\lambda$ for three different angles, and these are the angles used by Bose in his study. Here the refractive index is kept at a constant value of 2.063 (corresponding to the critical angle of $29^\circ$). From Figure 7 one may observe that the maximum effective thickness (for a given $\lambda$) decreases with the increasing value of $\theta$, and this is in agreement with the observation of Bose. It is, therefore, needless to mention that the findings of Bose are correct, at least at the qualitative level.

Bose also varied the wavelength by changing the distance between the sparking surfaces. In this part of his investigation, he used two right-angled prisms, which were obtained by cutting a cube of glass (of side 4.5 cm) across a diagonal. Keeping the angle of incidence fixed at $45^\circ$, he measured the minimum thickness required for the total reflection for three different distances between the sparking surfaces. At the time of his double-prism experiment, he was not sure about the exact wavelengths of the
microwave radiations generated in his radiators. He considered that the wavelengths were proportional to the distance between the sparking surfaces. Later, he determined the wavelength of the microwaves using a cylindrical grating arrangement. This experiment was another marvel of the great experimentalist. He obtained a PhD degree from the University of London for this work. He found that the wavelength was approximately equal to twice the distance between the sparking surfaces.

In the last part of his study, Bose measured the reflected and transmitted portions by gradually decreasing the thickness ($d$) of the air-space. The distance between the sparking surfaces of the radiator was kept constant at 10.1 mm. About the results obtained in this part of his study, he commented, “The results are to be taken more as qualitative, as no reliance can be placed on the sensibility of the receiver being absolutely uniform.” The qualitative agreement of his findings with the theoretical predictions has already been discussed. Here we would try to fit the experimental data (tabulated in his paper) with the mathematical expression of the transmission coefficient $T$ as given in equation (1). $T$ is calculated by dividing the galvanometer deflections due to the transmitted portion by the deflection recorded when the whole radiation is transmitted. We set this maximum reading as 240 divisions since Bose wrote: “Half the total radiation gave a deflection of 120 divisions.” Also, the wavelength is chosen to be $\lambda \sim 20.2$ mm. For the $T$ vs. $d/\lambda$ plot (Figure 8) the required data have been extracted from his paper and are given in Table-1. In the fitting process, we exclude the point (in $T$ vs. $d/\lambda$ curve) obtained for

<table>
<thead>
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<th>$d$ (mm)</th>
<th>$d/\lambda$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.015</td>
<td>1.0</td>
</tr>
<tr>
<td>1.8</td>
<td>0.089</td>
<td>0.667</td>
</tr>
<tr>
<td>3.6</td>
<td>0.178</td>
<td>0.625</td>
</tr>
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<td>4.5</td>
<td>0.223</td>
<td>0.500</td>
</tr>
<tr>
<td>5.4</td>
<td>0.267</td>
<td>0.417</td>
</tr>
<tr>
<td>7.2</td>
<td>0.356</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Bose measured the reflected and transmitted portions by gradually decreasing the thickness ($d$) of the air-space. The distance between the sparking surfaces of the radiator was kept constant at 10.1 mm.
Figure 8. Dependence of the transmission coefficient $T$ versus $d/\lambda$. Points are the experimental data of Bose and the continuous curve is the best fit obtained with the relation $T = 1/(\alpha \sinh^2(\beta d) + 1)$. The point represented by the open square is not taken into account into the fitting process.

$d = 1.8$ mm, since this point appears to be much erroneous compared to the other data points. The fitting parameters for the best fit have values $\alpha = 0.4$ and $\beta = 5.6$.

These are almost of the same order of magnitude of their theoretical estimates as obtained from equations (2), (4) and (5): $\alpha_\perp = 1.125$, $\alpha_\parallel = 2.530$ and $\beta = 6.67$ (we use $\theta = 45^\circ$, $n = 2.063$ and $\lambda = 20.2$ cm). It is, therefore, really an interesting observation that the experimental measurements of Bose are in agreement with the theoretical predictions. Here we must keep in mind the difficulties he encountered in recording the data using the microwave instruments of the earliest version. The best fit curve (Figure 8) is found to be good as well.

4. Conclusion

The double prism apparatus of Bose with variable air-space has been used as an adjustable attenuator (of millimeter-waves) in the 1.3 mm multibeam receiver on the 12 m telescope in the National Radio Astronomy Observatory at Kitt Peak [6]. We all know that Sir J. C. Bose was not interested in patenting his inventions of millimeter-wave instruments, as he strongly believed that any scientific invention or knowledge should be kept open for all. His double-prism experiment has been used as an illustration of optical tunneling in many standard textbooks of optics [7]. The exper-
iment is also demonstrated in *The Feynman Lectures on Physics* [8]. Unfortunately, none of these books has cited the name of Sir J. C. Bose. He was a pioneer in the field of experimental studies on optical tunneling using microwaves and the experiment, in particular, should be named: *the double prism experiment of J. C. Bose.*

The double-prism experiment can be incorporated in our undergraduate physics course so that the students at this level would more readily appreciate the analogous tunneling phenomenon in quantum mechanics. The concept of optical tunneling is not beyond the scope of undergraduate level knowledge in physics. The double-prism experiment with microwaves can easily be realized in a physics laboratory and for this, we only require two identical right-angled prisms made of wax (or, perspex), a transmitter with a microwave generator (klystron), and a suitable detector (may be a horn antenna).

5. Appendix

*Electrodynamic theory of evanescent waves*

We assume the total internal reflection of a monochromatic plane wave at the inner surface (glass-air interface) of a prism (see *Figure 9*). The solution for the transmitted electric field can be expressed as

\[ \hat{E}_t(\vec{r}) = \hat{E}_{0t} e^{i(\vec{k}_t \cdot \vec{r} - \omega t)} , \]

where

\[ \vec{k}_t \cdot \vec{r} = k_{tx} x + k_{ty} y . \]

Here the plane of incidence of the wave is assumed to be the x-y plane and hence the wave vector \( \vec{k} \) is confined in the x-y plane. Again

\[ k_{tx} = k_t \sin \theta_t , \]

and

\[ k_{ty} = k_t \cos \theta_t . \]

From Snell’s law, we have

\[ n = \frac{\sin \theta_t}{\sin \theta_i} . \]
where \( n \) is the refractive index of the prism.

Hence

\[ k_{ix} = k_i n \sin \theta_i, \]

and

\[ k_{iy} = \pm k_i (1 - n^2 \sin^2 \theta_i)^{\frac{1}{2}}. \]

In the case of total internal reflection

\[ \sin \theta_i > \frac{1}{n}. \]

Hence

\[ k_{iy} = \pm ik_i (n^2 \sin^2 \theta_i - 1)^{\frac{1}{2}} \quad (i = \sqrt{-1}). \]

Therefore (6) reduces to the following form

\[ E_i(x, y) = E_{0i} e^{-\frac{y}{2}} e^{i(k_{ix} x - \omega t)}, \quad (7) \]

where we have neglected the positive exponential because for this the amplitude grows exponentially with \( y \) which is physically unacceptable. Here

\[ \delta = \frac{1}{k_i(n^2 \sin^2 \theta_i - 1)^{\frac{1}{2}}} = \frac{\lambda}{2\pi(n^2 \sin^2 \theta_i - 1)^{\frac{1}{2}}}, \]

\( \lambda \) being the wavelength of the transmitted light in air. Hence the transmitted wave advances along the surface in the \( x \)-direction. Its amplitude drops off exponentially as it penetrates the air medium; it is, therefore, called the evanescent wave (the wave that is disappearing quickly). Energy of such a surface wave circulates back and forth across the interface but on the average (over time) it gives no net flow of energy in the direction of its propagation, and therefore, the reflection becomes total.
The transmission coefficient in optical tunneling

Here, we present only an outline of the theory of optical tunneling. For a detailed discussion the reader is referred to [4]. Although the evanescent wave decays exponentially in the rare medium, in presence of another dielectric medium, we have to account for multiple reflections on the surfaces involved (Figure 10). The reflection coefficient of the thin dielectric layer can be obtained by adding all the multiple reflection contributions as

$$ r = \frac{r_{12} + r_{23} e^{-i\delta}}{1 + r_{23} r_{12} e^{-i\delta}} $$  (8)

where the phase shift between consecutive reflected rays is defined as

$$ \delta = \frac{4\pi d}{\lambda} \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1}. $$  (9)

The reflection coefficients are obtained from the Fresnel relations for both polarizations of the incident wave as

$$ (r_{12})_\| = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}, $$  (10)

and

$$ (r_{23})_\| = \frac{n_3 \cos \theta_2 - n_2 \cos \theta_3}{n_3 \cos \theta_2 + n_2 \cos \theta_3}. $$  (11)

One can write down similar expressions for the reflection coefficients for perpendicular component. Here, we consider specifically the case of optical tunneling.
Then, \( \delta = \frac{4 \pi d}{\lambda} \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1} \) becomes imaginary, since \( n_1 \sin \theta_1 > n_2 \) (the incident angle being greater than the critical angle). This results in a damped real field. The transmission coefficient can be obtained from the relation \( T = 1 - |r|^2 \). Substituting (9), (10) and (11) into (8) we find

\[
T = \frac{1}{\alpha \sinh^2 (\beta d/\lambda) + \gamma},
\]

where

\[
\beta = 2\pi (n_1^2 \sin^2 \theta_1 - n_2^2)^{\frac{1}{2}}.
\]

The parameters \( \alpha \) and \( \gamma \) depend on the state of polarization of the incident wave. For the perpendicular component of the incident field

\[
\alpha_\perp = \frac{(N_2^2 - 1)(N_2^2 N_1^2 - 1)}{4N_2^2 \cos \theta_1(N_2^2 \sin^2 \theta_1 - 1)(N_1^2 - \sin^2 \theta_1)^{\frac{3}{2}}},
\]

\[
\gamma_\perp = \frac{[(N_2^2 - \sin^2 \theta_1)^{\frac{1}{2}} + \cos \theta_1]^2}{4 \cos \theta_1(N_1^2 - \sin^2 \theta_1)^{\frac{3}{2}}}. \tag{15}
\]

For the parallel component we have

\[
\alpha_\parallel = (\alpha_\perp / N_1^2)[(N_2^2 + 1) \sin^2 \theta_1 - 1][(N_1^2 N_2^2 + 1) \sin^2 \theta_1 - N_1^2], \tag{16}
\]

\[
\gamma_\parallel = \frac{[(N_2^2 - \sin^2 \theta_1)^{\frac{1}{2}} + N_1^2 \cos \theta_1]^2}{4N_1^2 \cos \theta_1(N_1^2 - \sin^2 \theta_1)^{\frac{3}{2}}}. \tag{17}
\]

Here, \( N_1 = n_3 / n_1 \) and \( N_2 = n_1 / n_2 \). For the special case \( n_1 = n_3 = n \), and \( n_2 = 1 \), \( \alpha \) and \( \gamma \) reduce to

\[
\alpha_\perp = \frac{(n^2 - 1)^2}{4n^2 \cos^2 \theta (n^2 \sin^2 \theta - 1)},
\]

\[
\alpha_\parallel = \alpha_\perp [(n^2 + 1) \sin^2 \theta - 1]^2,
\]

and

\[
\gamma_\perp = \gamma_\parallel = 1.
\]

Acknowledgment

I am grateful to Professor (retd) Soumen Kumar Roy of the Department of Physics, Jadavpur University, Kolkata, India, for helpful discussions.
Suggested Reading