From Complexity to Simplicity and Back*

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The Nobel Prize of 2021 highlighted the importance of understanding the complex dynamical processes that govern the evolution of Earth’s climate. Two of the Nobel laureates, Syukuro Manabe and Klaus Hasselmann pioneered the creation of a robust theoretical and mathematical framework for using a hierarchy of models of varying complexity to study a variety of questions, the most important of which may be: how do we quantify the effects of human activities on the Earth’s climate? Three main contributions of Manabe and Hasselmann on which this article focuses are: (i) simple radiative-convective models that study, among other factors, the effect of changes of CO₂ concentration; (ii) a methodology to derive simpler, stochastic climate models from more complex, coupled models for the weather; and (iii) mathematical techniques called fingerprinting that quantify the human impact on the climate.

“The climate system is undoubtedly one of the most complex systems scientists have tried to come to grips with...even the most powerful supercomputers fall far short of capturing all of the processes known to be important for the system dynamics...understanding the system becomes mandatory. Understanding is not achieved through complex numerical models, but with simpler conceptual models.”

– Klaus Hasselmann, in the Preface of Stochastic Climate Models, P. Imkeller, J-S. von Storch (eds.)

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1. Diverse Approaches to Study the Complex Climate System

Imagine standing on a sea-shore in Kerala during the monsoon onset, surrounded by clouds and thunder, the roaring ocean, and the lush green vegetation soaking in the much-awaited rain. There may not be a more beautiful moment to think about the complexity of the Earth system; a complexity brought about by the intricate web of interactions among everything surrounding you at that instance. So, imagine that instead of humming a melodious song or reciting a poem, you are tempted to think of mathematical models of the spectacle, and more broadly, about the climate and the changes that are occurring. You will soon realize that the Earth’s climate is a complex dynamical system for many distinct reasons, some of which are as follows.

1. The climate is dependent on the interactions between a variety of coupled subsystems, each of which is complex on its own. Some examples of such subsystems, illustrated in Figure 1, that are commonly included in climate models are: atmosphere, land surface, ocean and sea ice, aerosols, carbon cycle, vegetation, and many others [7, Figures 1-13].

2. The dynamics of most of these subsystems themselves contain multiple spatial and temporal scales. Spatial variability ranges from sub-millimeter scale cloud processes, which are not ‘resolved’ or explicitly simulated but represented through parameterisations to ‘continent-scale’ ocean currents and cloud systems, which emerge from the dynamics. Temporal variations are equally vast—from hourly variations of storms and rain to multi-decadal oscillations of the ocean. One of the essential features is the lack of a clear separation between the various scales involved, and this leads to many of the difficulties in studying the climate system.

3. Since the coupling between the subsystems may be quite strong, an understanding of one of them in isolation may not suffice to understand the dynamics of the climate. There are emergent phenomena due to interactions between different subsystems. Of course, this is a key property of many, if not most, complex systems and is not restricted to climate.
4. Biological systems are also complex in a similar fashion, with most of the above characteristics, as expressed so beautifully by Lewis Thomas\(^1\). But there is a major difference—there is only one Earth! Trivial as it may sound, the implication is that unlike the multiple realisations of the biological systems that can be studied in the laboratory, we do not have the luxury of performing ‘controlled double-blind trials’ or laboratory experiments by varying important parameters, e.g., CO\(_2\) concentration that controls the climate system as a whole. (It may be argued that we are performing an uncontrolled experiment with it!)

Faced with this complexity, at least two distinct approaches have been used to make progress in building up our understanding of the climate, and the 2021 Nobel Prize is essentially a recognition of the importance of both of these approaches since the work of Syukuro Manabe and Klaus Hasselmann beautifully illustrates their power [13, 24, 33]. These two approaches may be summarised as follows.

1. On one hand, building simple models to capture essential fea-

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**Figure 1.** Different components of the climate system that are included in the current climate models. Figure from https://www.gfdl.noaa.gov/earth-system-model/. Reproduced with permission from John P. Dunne.
1. I have been trying to think of the Earth as a kind of organism, but it is no go. I cannot think of it this way. It is too big, too complex, with too many working parts lacking visible connections. The other night, driving through a hilly, wooded part of southern New England, I wondered about this. If not like an organism, what is it like, what is it most like? Then, satisfactorily for that moment, it came to me: it is most like a single cell.” From *The Lives of a Cell: Notes of a Biology Watcher* by Lewis Thomas

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The discussion up to this point focused on the conceptual and scientific difficulties in studying a complex system, albeit one of immense importance since we all live in and depend on the intricate processes and balances that keep this system in a state that is conducive to life as we know it. Hence understanding changes in the climate is not just a problem of purely academic interest but has direct consequences for the fate of humanity as a whole. One of the important contributions of Hasselmann has been in developing systematic methods, now collectively called fingerprinting, to identify the human-caused global warming signal. This aspect will be described in detail in section 3.

Before describing the scientific contributions of Manabe and Hasselmann, it is vital to note one crucial feature of research in a field such as the earth sciences—a feature that is seldom emphasised and often overlooked: significant and substantial progress is almost always achieved through (i) several small contributions of a large number of researchers, (ii) technological advances that
steadily improve the computational and observational capabili-
ties, and (iii) very rarely through ‘fundamental breakthroughs’
by one or a few scientists [2, 3]. Thus the short account of the
radiative-convective equilibrium, fingerprinting, and stochastic mod-
els focusing on the contributions of Manabe and Hasselmann does
not provide a complete picture of the progress achieved in these
areas, nor does it do justice to the immensely crucial contributions
of a large number of other scientists and mathematicians. Such a
review is clearly beyond the scope of this article, and arguably be-
yond the scope of the present author. The interested reader may
refer to some of the existing reviews [6, 9, 10, 18, 36, 39 and re-
ferences therein], though this is a woefully short and biased list of
reviews.

2. Hierarchy of Models in the Work of Syukuro Manabe

Physical modelling of the global climate change, and in particu-
lar, the question of sensitivity of the climate to changes in the
concentration of greenhouse gases such as CO₂ is the main fo-
cus of the work of Syukuro Manabe over the last 5–6 decades.
He and his coworkers were the first researchers to develop, in the
1960s, the so-called radiative-convective models (RCM). These
models are one-dimensional in the sense that they model the ver-
tical profile of the atmospheric temperature \( T(z) \) as a function
of the height, or equivalently, of pressure, assuming hydrostatic
balance. The RCM do not take into account the variation of the
temperature with latitude, longitude, or seasons. Other examples
of one-dimensional models are the so-called energy balance mod-
els, the most well-known being those by Budyko [5] and Sellers
[44].

The basic idea of an RCM is to capture the temperature change
in a column of the atmosphere, and the models that Manabe stud-
iied included the radiative processes (absorption, emission, and
scattering of light). The early models did not explicitly include
the energy exchange due to condensation or evaporation of wa-
ter vapour (clouds and rain) but are being studied recently [12,
The main insight of Manabe was to realize that instead of capturing these complex processes, the model could enforce the observed lapse rate, which is the rate of change of temperature with height, in the troposphere. This is called the convective adjustment and allows the models to be used reliably without explicit simulation of the much more complicated convective processes (ascent of hot air, descent of cold air, and condensation or evaporation of water droplets in clouds). The observed lapse rate used by Manabe was 6.5 degrees per kilometer in the troposphere.

The basic equation used in the RCM [16, 30] is quite simply

\[ \rho c_p \frac{\partial T}{\partial t} = -\frac{\partial R}{\partial z}, \]

where \( R \) is the radiative flux (both shortwave and longwave). The model includes the temperature at a fixed number of heights (or pressure levels). The right-hand side is calculated by specifying a profile of the important gases such as carbon dioxide, water vapor, and ozone, and the laboratory observed absorption and emission properties. The details of calculating the radiative flux are elaborated in [27] and in the appendices of [30, 31]. The temperature difference between adjacent layers is continually adjusted to achieve a pre-specified lapse rate. Thus for every model time step, there is an ‘inner loop’ (that mimics an enthalpy conserving mixing process in the unstable layers, as described in Appendix 1 of [31]) for attaining this lapse rate.

The other insight of Manabe was to compare the results of two different numerical experiments: one where the relative humidity is specified, and another where the absolute humidity is specified. In the first case, the change in surface temperature due to the doubling of CO\(_2\) concentration was 2.3\(^\circ\) C, whereas it was only 1.3\(^\circ\)C in the latter case. This difference indicated that the water vapor exerts strong positive feedback on the Earth’s climate [26]. Another major discovery of these studies was that with increasing carbon dioxide, the temperature of the surface and the troposphere of the Earth increase but the temperature of the stratosphere decreases (e.g., left panel of Figure 2).
The results from these studies did not remain just an academic curiosity from studying a single model because of a couple of reasons: (i) the extreme care with which this work was performed, and (ii) it was followed up by a long series of equally influential and seminal studies by Manabe’s group at Princeton. Some of these achievements are listed below.

1. The papers [30, 31] contain extensive studies of the response of their model to systematic variations of many different parameters involved, including relative humidity, water vapor mixing ratio, ozone, cloudiness, etc. Thus, in addition to the now-well-known and celebrated figure from [31] showing changes in vertical profile of temperature due to changes in CO₂ concentration (left panel of Figure 2 here), the paper contains about 15 other figures and 8 tables (mostly not discussed in the recent flurry of expositions of their work) documenting the results of these variations. A representative result, showing the effect of variation of surface albedo, is reproduced here in the right panel of Figure 2.

2. Over the subsequent decades, Manabe’s group continued to de-
velop models of increasing complexity, in particular, general circulation models and coupled atmosphere-ocean models, that repeatedly reproduced the results from the simple RCM used by them in the mid-60s, while at the same time giving insights on many other aspects of climate dynamics.

3. Papers [28, 32] used a general climate model developed by Manabe’s group to reproduce the earlier results about the heating of the troposphere and cooling of the stratosphere due to the increased concentration of carbon dioxide. These papers also contained new results indicating an increase in the intensity of the hydrological cycle (rain and evaporation), a decrease in the snow cover and albedo at higher latitudes, and a host of other features. These papers also started identifying the limitations of GCM, particularly in simulating the precipitation (which shows a significant bias that continues to be the case even for modern climate models).

4. Reference [25] studies the effect of the hydrology of the Earth’s surface, by representing the soil as boxes with limited water-holding capacity, while [4] studied the effects of orography by comparing model simulations with and without mountains, identifying the essential role of mountains in the hydrological cycle.

Even though the above list is by no means an exhaustive summary of all the contributions of Syukuro Manabe, it gives a flavor of the style of his scientific research.

3. Integration of Mathematical Theory and Observations in the Work of Klaus Hasselmann

The two main aspects of the work of Hasselmann that will be emphasised here are (i) the development of stochastic climate models, and (ii) the development of methods to isolate the anthropogenic climate change signals. Though the beginnings of both these research programs go back to the 1970s, their real impact was felt only in the early part of this century because the implementation of his ambitious program needed appropriate mathematical theory, extensive observations mainly from satellites, and
computational capabilities for simulation of complex models and processing the observed and simulated data-sets. The two subsections describe the basic ideas behind these research programs.

3.1 Need for Stochastic Climate Models

The main aim of stochastic models in the context of climate science is best described in the introduction of Hasselmann’s highly influential paper [14]: “An understanding of the origin of climatic variability, in the entire spectral range from extreme ice age changes to seasonal anomalies is a primary goal of climate research. Yet...there exists today [in 1976] no generally accepted, simple explanation for the observed structure of climate variance spectra.” Hasselmann’s approach was inspired by Einstein’s model for the motion of a Brownian particle. Just as the seemingly random motion of a particle suspended in a fluid is the result of its collisions with the fluid molecules, the effect of the relatively fast dynamics of the atmospheric processes such as the winds, clouds, etc., can be modelled by a stochastic or random forcing of the slowly varying components of the climate, particularly the ocean. The puzzle was that the spectrum (the Fourier transform of the fluctuations) of the climatic variables (e.g., sea surface temperature) was observed to be ‘red noise’.

One of the illustrations of the above phenomena may be found in the work of Manabe and Stouffer [29]. It is quite well-known


2Different interpretations of the term ‘red noise’ exist depending on the context. In this article, I will use ‘red noise’ to mean the spectrum of the Ornstein-Uhlenbeck process.
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Such 'red noise spectra' are now essentially textbook material and illustrations are included even with standard software such as NCL, e.g., https://www.ncl.ucar.edu/Applications/spec.shtml. I am using this reference to provide continuity with discussion in the previous section.

[37] that the Ornstein-Uhlenbeck process, satisfying the following stochastic differential equation (using the example of [29])

$$C \frac{dT}{dt} = -\gamma T + f_w,$$

for the sea surface temperature $T$ has spectral density

$$G(\omega) = \frac{(F_0/C)^2}{(\gamma/C)^2 + \omega^2},$$

where $f_w$ is white noise from the 'weather'. (See next paragraph for more details.) Using the output of a 1000-year integration of two different models, the authors of [29] obtained the power spectrum of monthly mean sea-surface temperature anomaly which fitted very well to the above 'red noise spectrum' as shown in the right panel of Figure 3 reproduced here from [29]. This readily confirms the proposal made by Hasselmann in [14]. Earlier attempts at detecting the red noise were made by Hasselmann and his collaborators [8, 22], with one representative result shown in the left panel of Figure 3 reproduced here from [8].

Hasselmann's idea was that (2) arises from a set of coupled equations of the following form.

$$\frac{dx}{dt} = u(x, y) \text{ (weather)}, \quad \frac{dy}{dt} = v(x, y) \text{ (climate)},$$

where the 'fast' components $x$ (weather variables such as the wind, humidity, pressure, etc.) have a timescale $\tau_x$ of a few hours to days that is significantly smaller than timescale $\tau_y$ for the 'slow' components $y$ (climate variables such as ice coverage, land foliage, etc.) that vary over several months, years, or longer.

Hasselmann proposed a sketch of how an appropriate averaging procedure may be used so that the 'averaged' equation for the fluctuations of the slow $y$-variable contains the effects of the fast dynamics of the $x$-variables only in a 'noise' term, thus resulting in equations of the form (2). This broad idea—the so-called 'Hasselmann's program' [18]—is intricately connected to the dynamical systems theory of deterministic chaotic systems on the
one hand and the large deviation theory for stochastic and random dynamical systems on the other.

But non-trivial, challenging, and exciting mathematical difficulties have prevented substantial progress beyond a few specialized or idealized situations, as may be gathered from the following representative statements: “Hasselmann’s program has never really been implemented... [though] the program has not lost its importance and significance” [18, p.142, Ludwig Arnold] and “Hasselmann’s stochastic climate model is a mesoscopic description. It ignores the details of the weather fluctuations. In its original version, it is a cognitive model, aimed at understanding a specific physical process. It has not yet been advanced to a comprehensive and realistic model of the climate system on the mesoscopic level” [18, p.285, Peter Müller]. Recent work on rigorously deriving stochastic models for a slowly varying system by averaging of fast dynamics in a slow-fast system can be found in [11, 35, 38] (and to reiterate the earlier warning, this is a woefully short and biased list).

3.2 Fingerprints of Human Impact on the Climate

In addition to stochastic climate models, another important contribution of Hasselmann is in the development of a reliable methodology for detecting climate change and understanding its causes, a research area known as the detection and attribution of climate change [46]. It is now widely acknowledged [40] to have been initiated in Hasselmann’s paper entitled ‘On the signal-to-noise problem in atmospheric response studies’ [15]. The main idea presented in that paper could not be implemented until the advent of two crucial inputs: (i) sufficiently long time series of climate observations, mainly from satellites, and (ii) sufficient computational power to perform long climate simulations. With the availability of these two ingredients, a series of papers over the last two decades [1, 23, 41–43, 45 and references therein] have shown ‘unequivocally’ and ‘virtually certainly’ the human influence on the warming of the atmosphere, the global retreat of glaciers, acidification of the surface open ocean, and related changes in the

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climate [19, 34].

4. Looking Forward by Learning from the Past

The physical basis of earth sciences has been firmly established for more than a hundred years since the pioneering work of Vilhelm Bjerknes who suggested the use of hydrodynamics and thermodynamics to study the atmosphere. It took several more decades until the work of Manabe to provide a reliable methodology to study the long term changes in the Earth’s climate. The mathematical developments needed to unravel the role of stochasticity, pioneered in the work of Hasselmann, are still underway and will continue to grow only through fruitful collaborations between physicists, earth scientists, and mathematicians. With the recognition of the work of Manabe and Hasselmann by the Nobel Committee for Physics, the artificial distinction between the first two is now erased, as was expressed recently by Manabe: “With a Nobel prize in Physics under our discipline’s belt, it gives me and climate modelling colleagues the credibility and recognition we have yearned for: climate science is real science.”

Suggested Reading


[39] V. Ramanathan and J. A. Coakley Jr, Climate modeling through radiative-


