Think It Over

This section of Resonance presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to ‘Think It Over’, Resonance, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

Two Simple Partial Fractions*

Problem:

Find the partial fraction expansion of proper rational function of types \( \frac{1}{x^n(x^m+1)} \) and \( \frac{1}{x^n(x^m-1)} \).

Solution:

We may assume without loss of generality that \( m \) divides \( n \) by multiplying with a suitable power of \( x \),

\[
\frac{1}{x^n(x^m+1)} = \frac{1}{x^n} - \frac{1}{x^{n-m}} + \frac{1}{x^{n-2m}} - \frac{1}{x^{n-3m}} + \ldots + (-1)^{\frac{n}{m} - 1} \cdot \frac{1}{x^m} + (-1)^{\frac{n}{m}} \cdot \frac{1}{(x^m + 1)}.
\]

\[
\frac{1}{x^n(x^m-1)} = \frac{1}{x^n} - \frac{1}{x^{n-m}} - \frac{1}{x^{n-2m}} - \frac{1}{x^{n-3m}} - \ldots - \frac{1}{x^m} + \frac{1}{x^m - 1}.
\]

Note that the number of terms is just \( \left( \frac{n}{m} + 1 \right) \) These formulas can be easily checked by mathematical induction. \( x \) could even be like \( y^n \) and this would give partial fraction in terms of \( y^n \).

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