Classroom

In this section of *Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

Arbitrary vs. Random*

Aniruddha V Deshmukh
Applied Mathematics and Humanities Department
Sardar Vallabhbhai National Institute of Technology
Surat, Gujarat 395 007
India.
Email:
aniruddha480@gmail.com

In this article, we try to see the difference between the two words—'arbitrary' and 'random', which often confuse students. The article is a product of discussions with my batchmates and juniors who have raised the question, "What is the difference between arbitrary and random?" and "When should we use the word arbitrary and what is its significance?" The article tries to address these questions in general.

Introduction

During our school days, we usually deal with Mathematics, which is not too rigorous as to include proofs of theorems that could be difficult to understand. Even in the 10 + 2 level, we accept a lot of Mathematics intuitively without proofs. All we care to do at the time is to deduce new expressions from known ones and manipulate them to 'solve' problems. This is what is expected of us at that level and is in fact required.

However, things start to change when a student pursues Mathematics for his/her higher studies (which are the undergraduate

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and postgraduate courses in Mathematics). A student, especially at the undergraduate level, when exposed to mathematical rigour (and proofs) for the first time, often gets intimidated by the language used. Sometimes, the language also confuses. One such example is the use of the word ‘arbitrary’. It is quite common to look at statements like “Let $x \in X$ be arbitrary” or “Let n be an arbitrarily chosen natural number”.

The first thing that any (new) student would do, is ask “what is the meaning of this?” and not surprisingly, the student is often told, “choose randomly.” In fact, we have even witnessed Mathematics teachers telling their students that the meaning of ‘arbitrary choice’ is a ‘random choice’. The result of this answer is a permanent impression in the student’s brain that ‘arbitrary’ is related to ‘probability’, and this is clearly because students learn probability and ‘randomness’ from their lower standard up until the students pursue higher studies.

Such an interpretation of words leads to confusion while reading (and writing) proofs and ultimately a failure in understanding the concept altogether. This article is designed to make a distinction between these words.

The Words: Arbitrary and Random

If one tries to look at the actual meaning of the words: ‘arbitrary’ and ‘random’, one may find the following definitions:

The words ’arbitrary’ and ’random’ are indeed, interchangeable when it comes to the English language, and we have been using them in the same sense for long. But, when it comes to Mathematics, things start to change.

Arbitrary: Subject to individual will or judgment without restriction; contingent solely upon one’s discretion.

Random: Proceeding, made, or occurring without definite aim, reason, or pattern.

In fact, the dictionaries and thesaurus often give them as synonyms of each other. These words are, indeed, interchangeable when it comes to the English language, and we have been using them in the same sense for long. But, when it comes to Mathematics, things start to change.

It is not that their meaning is different. This would cause quite a trouble while communicating in English! While keeping its meaning same, Mathematicians have devised a rule for their usage. Hence, in Mathematics, these words are not synonyms or interchangeable and should be used with proper care. A wrong use once can cause the failure of all the statements that follow. Let us look at their difference now.

Mathematicians have devised a rule for the usage of the words 'arbitrary' and 'random'. In Mathematics, these words are not synonyms or interchangeable and should be used with proper care. A wrong use once can cause the failure of all the statements that follow. Let us look at their difference now.

The Difference

The first time any student comes across the word 'random' is when the student is studying probability. So, if we are to say, "Let x be a random integer", it leads us to ask (naturally), "What is the probability distribution?" For suppose, there is a statement "If x is a random integer, then $x \neq 0$". Then, under the circumstances, it is completely possible to have a probability distribution wherein the probability of choosing 0 from all the integers is 0. So, the statement can indeed be true, given the special probability distribution.

On the other hand, a student comes across the word 'arbitrary', when the student is dealing with quantified statements in proofs. In many Mathematics books, the quantified logical statement $\forall x \in X, P(x)$ is translated into English as "For any arbitrarily chosen x in the set X, the proposition $P(x)$ holds true". Here, not as above, one should not (and will not) ask about the "probability distribution". If we look at the quantified statement carefully, it specifies "If we take any x in the set X (without any bias or judgement), then $P(x)$ will hold true". So, let us look at the statement considered above with the word 'random' replaced by the word 'arbitrary'. Now, the statement becomes, "If x is an arbitrary integer, then $x \neq 0$". And this statement is completely false. Why is it false? Well, because the use of 'arbitrary' has allowed us to take any integer x from the set of integers and still $x \neq 0$ should hold. But this would imply that the statement should also hold for 0, which is an integer, which tells us that the statement made is not true.

Conclusion: The Use

The word 'random' is to be used only when probabilities are taken into account.

While it is made clear that the word 'random' is to be used only when probabilities are taken into account, we conclude the article by looking at how to use the word 'arbitrary'. As discussed in the above section, we can use the word 'arbitrary' when we have statements involving the universal quantifier $\forall$. So, when we say $\forall x \in X$ or "Let $x \in X$ be arbitrary", we are giving somewhat a 'label' to the element that will be taken into consideration. The actual and intuitively most sensible way to prove a statement involving the universal quantifier is to check the statement's truth value for every element in the considered set. However, practically this is not possible, especially when the sets are infinite and/or abstract in a sense we do not actually have elements but rather only their idea. So, now we move towards our weapon of 'arbitrary choice'. We choose any element x 'arbitrarily', meaning without any bias or judgement. Now, if the statement we want to prove holds true for this choice, it must hold for all other choices (since we do not have a bias in the choice), and we say that the theorem is proved.

Again, this is a place where students get confused a lot. When we say 'choice', even as mentioned here, students start thinking ways of actually 'picking up' an element from the set. It is to be noted by the students and also by the teachers that here, 'choice' is not made in a sense of 'picking up'. As said earlier, when we say "Let x be arbitrary", we are actually providing a label with which we can work. Now, the arbitrary choice enables us to keep any element behind this label x and prove our statement. Thus, we are saving a lot of trouble by working with the label we gave rather than actually working with elements.

In this manner, the two words 'arbitrary' and 'random' differ from each other, especially in terms of Mathematics. This difference is an important aspect of Mathematical rigour and should be incorporated in classes by students and teachers. Students should try to keep themselves clear in the understanding of the meaning and use of these words, and at the same time, teachers should not encourage the interchangeable use of these words by correcting the students wherever necessary. This will make understanding Mathematical statements (and hence proofs) easy rather than intimidating.

Suggested Reading

[1] A Kumar, S Kumaresan, B K Sarma, *A Foundation Course in Mathematics*, Narosa publication, 2018.

[2] S Kumaresan, *Problems-Set Theory and Foundations*, MTTS Trust.

[3] Kaj (https://math.stackexchange.com/users/139618/kaj), Arbitrary vs. Random, URL (version: 2017-11-21): https://math.stackexchange.com/q/2529446