

# Maxwell's Demon, Szilard Engine and Landauer Principle\*

*P S Pal and A M Jayannavar*

The second law of thermodynamics is probabilistic in nature. Its formulation requires that the state of a system be described by a probability distribution. A natural question, thereby, arises as to whether a prior knowledge about the state of the system affects the second law. This question has now been nurtured for over a century, and it was inceptioned by C Maxwell through his famous thought experiment wherein comes the idea of Maxwell's demon. The next important step in this direction was provided by L Szilard, who demonstrated a theoretical model for an information engine incorporating Maxwell's demon. The final step that led to the inter-linkage of information theory and thermodynamics was through Landauer's principle of information erasure that established the fact that *information is physical*. Here, we will present an overview of these three major works that laid the foundations of information thermodynamics.

## Introduction

Almost 150 years ago, Maxwell put forward a thought experiment, thereby threatening the validation of the second law of thermodynamics. He introduced a demon-like creature that controls the dynamics of an isolated system, and while doing so, the entropy of the system decreases [1, 2]. This violates the second law. A classical analysis of Maxwell's demon was conducted by Szilard [3], where he studied an idealized heat engine with one particle gas and directly associated the information acquired by the measurement with the physical entropy and saved the sec-



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## Keywords

Thermodynamics, information, Szilard engine (SZE), measurement.

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ond law. Later, Landauer put forward his famous principle [4, 5], which states that it is information erasure rather than information acquired that is associated with an energetic and hence entropic cost. Now it is a widely known fact that the total system consisting of Szilard engine (SZE), heat reservoir, and demon's memory do not violate the second law.

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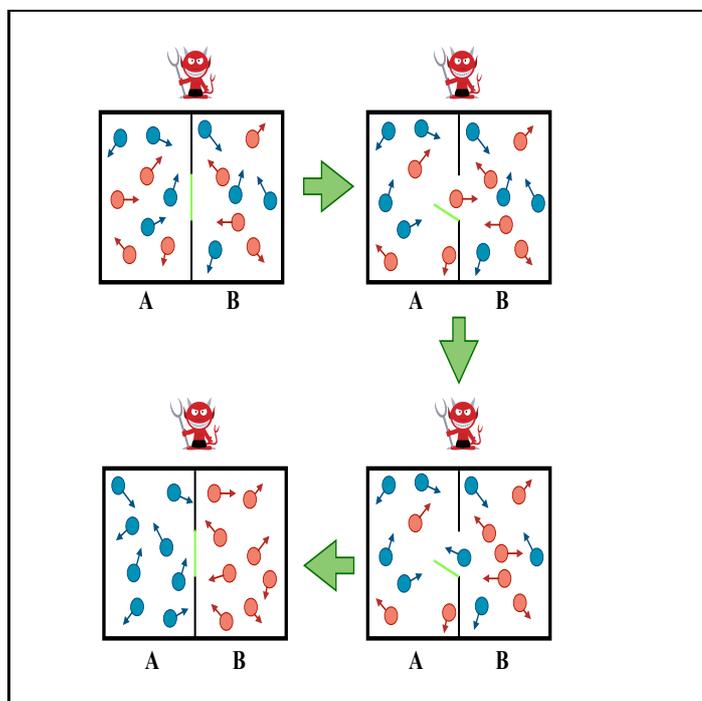
There are intensive interests in quantum thermodynamics of various engines [16–24]. These microscopic engines are the basis of a larger subject of study—quantum biology—for instance, the avian compass [25], and quantum effect in living cells [26].

A generalization of the Szilard engine with a multi-particle system as working substance is discussed here. The work extraction in classical multi-particle Szilard engine will depend upon the initial biasing. Szilard engine is generally studied with the partition being initially put in the middle of the box, i.e., the partition divides the phase space of the working substance into two halves. Here, this division of phase space is made arbitrary by inserting the partition at an arbitrary distance from the boundary. This action changes the probability of particles to stay in a certain volume. In this sense, we are calling it biasing. This affects work extraction. Another important question would be how this biasing can lead to an optimal work extraction in a multi-particle Szilard engine. We find that work extraction can be made larger for  $N$ -particle engine when the partition is inserted near the boundary. Another important factor that affects work extraction is the errors that happen during the measurement. Erroneous measurements, in general, decrease performance as expected.

## 1. Maxwell's Demon

Suppose, we have two objects at different temperatures  $T_1$  and  $T_2$  ( $T_1 < T_2$ ). If they are put in contact with each other, heat will flow from the hot body to the cold one, thereby establishing an





**Figure 1.** Schematic diagram of Maxwell's demon. The blue coloured circles represent the molecules with velocities less than the average velocity while the red circles represent the faster than average molecules.

equilibrium situation with the final temperature being  $T_{\text{fin}}$  ( $T_1 < T_{\text{fin}} < T_2$ ). The fact that the temperature of the cold bath cannot decrease and the temperature of the hot bath cannot increase is prohibited by the second law of thermodynamics.

In 1871, Maxwell proposed a thought experiment that allows this situation —forbidden by the second law—to happen. He considered a box filled with gas at a uniform temperature. Though the mean velocity of any great number of molecules of the gas, arbitrarily selected, is almost exactly uniform, the velocities of each individual molecule are by no means uniform. Now, the box is divided into two parts, *A* and *B* (see *Figure 1*). Maxwell suggested a 'demon', i.e., an intelligent being, that can observe the molecules, is placed near the trapdoor between the two parts. When a faster than average molecule from *A* flies towards the trapdoor, the demon allows the molecule to pass from *A* to *B*. On the other hand, when a slower than average molecule flies from *B* towards the trapdoor, the demon allows it to pass from *B* to *A*. The demon

In 1871, Maxwell proposed a thought experiment that allows this situation —forbidden by the second law—to happen.

doesn't allow slower than average molecules to pass from  $A$  to  $B$  and faster than average molecules to pass from  $B$  to  $A$ . As a result, after some time, the average speed of the molecules in  $B$  would increase while it would decrease in  $A$ . According to the kinetic theory of gases, the average molecular speed is proportional to the temperature. This implies that the temperature decreases in  $A$  and increases in  $B$ , contrary to the second law of thermodynamics. Now, one can use  $A$  and  $B$  as heat baths at different temperatures and can extract work, making it work like a heat engine.

## 2. Szilard Engine (SZE)

A classical demonstration of Maxwell's demon was put forward by Szilard in another thought experiment. Szilard's engine contains a single classical particle, obeying the equation of state of an ideal gas, confined in a box of volume  $V$ .

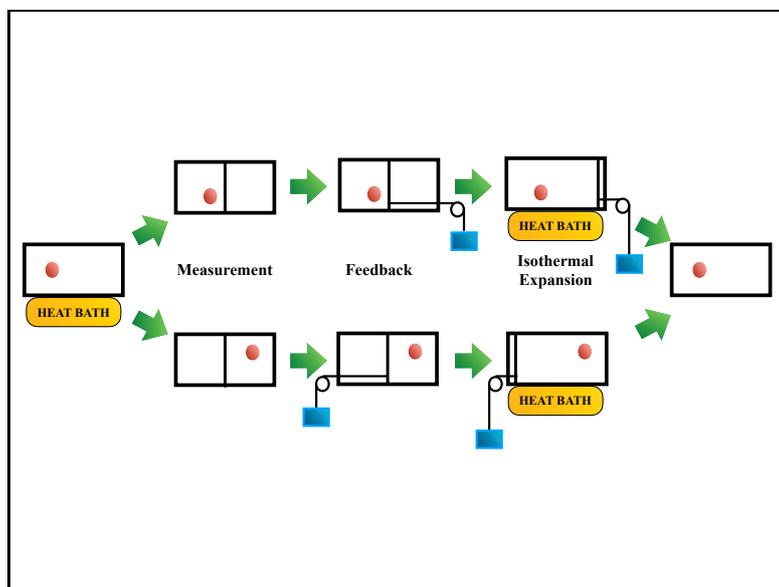
A classical demonstration of Maxwell's demon was put forward by Szilard in another thought experiment [3] (*Figure 2*). Szilard's engine contains a single classical particle, obeying the equation of state of an ideal gas, confined in a box of volume  $V$ . The box is in contact with a thermal reservoir at temperature  $T$ . The particle thermalizes after every collision with the walls of the box. After some time, a demon inserts a moveable wall in the middle of the box and measures the position of the particle. Depending on the measurement outcome, a load is attached to the system. The one-particle gas is then allowed to expand reversibly. In the end, the piston is removed and allowed to thermalize, thereby retaining its original state.

The energetics of the Szilard engine gives insight into the relationship between thermodynamics and information. We can assume that insertion and removal of the piston can be done reversibly, and hence no energy is expended. The only energetic contribution arises from the reversible expansion of the particle against the piston from  $V/2$  to  $V$ . The work done by the piston on the particle is

$$W = - \int_{V/2}^V p dv = -k_B T \int_{V/2}^V \frac{dV}{V} = -k_B T \ln 2. \quad (1)$$

In the next subsections, we will discuss more generalized situations where we incorporate the following facts: (a) the partition





**Figure 2.** Schematic diagram of Szilard engine.

is placed at an arbitrary position inside the box, (b) measurement errors, and (c) multi-particle case.

### 2.1 Single-particle SZE

A single-particle classical gas is confined in a box of length  $L$ , surface area  $A$ , and hence volume  $V = AL$ . The gas is initially at thermal equilibrium with a single thermal bath at inverse temperature  $\beta$ . The cyclic process for the SZE is carried out in the following four steps:

*Step 1. Insertion of partition:* We insert a partition inside the box at position that divides the box in the ratio  $x : 1 - x$  ( $0 \leq x \leq 1$ ). Due to the partition, the particle can be on the left side or the right side. Let's denote the position of the particle by  $r$ . For simplicity, we take  $r = 1$  when the particle is on the left and  $r = 0$  when it is on the right side. It is to be noted that since the initial partitioning of the box is unequal, i.e.,  $x \neq 0.5$ , the probability of the particle to be on either side is not the same. In fact, the probability of the particle to be on the left  $P(1) = x$  whereas the probability of the



particle to be in the right side  $P(0) = 1 - x$ . We do not need to do any work in this step.

*Step 2. Measurement:* Next, we measure the position of the particle. The measurement outcome, denoted by  $q$ , can also take two values, i.e., 1 or 0. After the measurement,  $(r, q)$  represents a particular configuration depicting the actual position of the particle and measured outcome. In the case of error-free measurement,  $r = q$ , and hence we will have only two configurations. Configurations corresponding to erroneous measurement, and its effect on work extraction are discussed later.

*Step 3. Feedback:* We next move the partition quasistatically and isothermally depending on the measurement outcome  $q$ . If  $q = 0$ , i.e., the particle is on the left side, the partition is shifted to the right end of the box. If  $q = 1$ , the partition is shifted to the left end. In this process, we extract work from the engine. Since the process is quasistatic, the work extraction is given by the change in free energy ( $\Delta F$ ) of the particle, i.e.,  $W = \Delta F = (1/\beta) \ln[Z(X_2)/Z(X_1)]$ , where,  $X$  is the position of the partition with  $X_1$  and  $X_2$  being the initial and the final position respectively.  $Z(X)$  is the partition function of the single-particle system with the partition at  $X$ , and it is proportional to  $X$ . Let  $W^{(r,q)}$  denote the work for a particular configuration. Hence,

$$\beta W^{(0,0)} = -\ln x; \beta W^{(1,1)} = -\ln(1-x) \quad (2)$$

*Step 4. Removal of partition:* We remove the partition without any work and the engine returns to its initial state. From the total process, we extract the average work

$$\beta \langle W_{ext} \rangle = -x \ln x - (1-x) \ln(1-x) \quad (3)$$

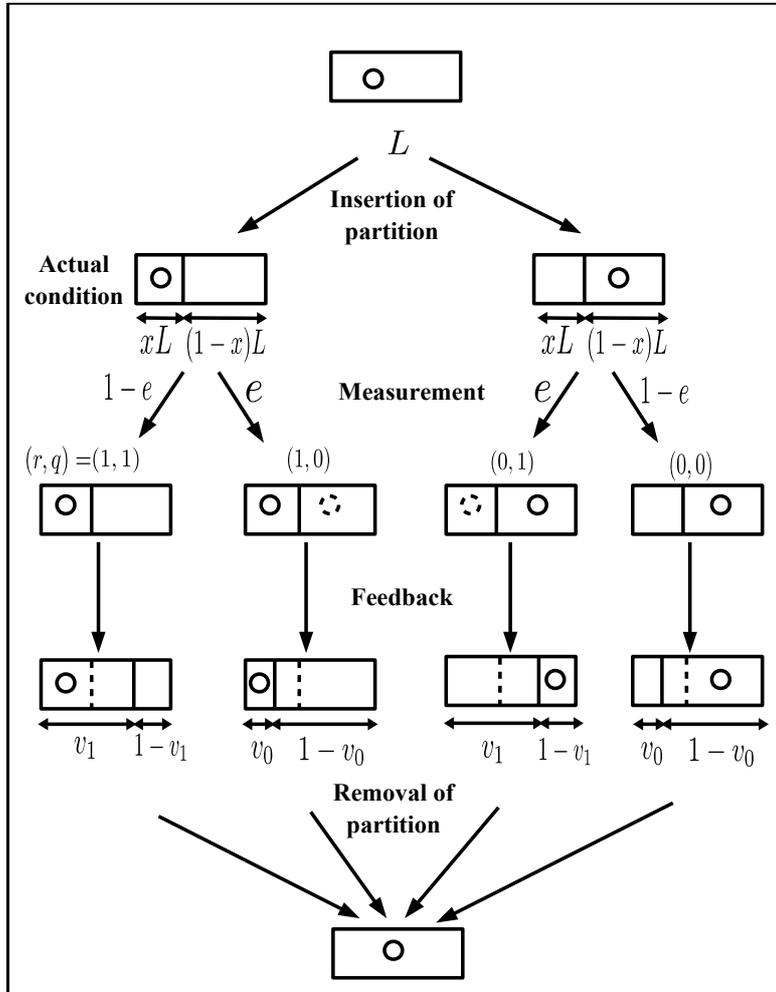
Any measurement process can be accompanied by an error. The measurement error is characterized by the conditional probabilities  $P(q|r)$ .

When the partition is placed in the middle, i.e.,  $x = 1/2$  case, using the above expression we get back (1).

### 2.1.1 Measurement error in single-particle SZE

Any measurement process can be accompanied by an error. The measurement error is characterized by the conditional probab-





**Figure 3.** Single-particle SZE.  $e = 0$  implies error-free measurement, and in that case only two configurations are possible namely  $(r, q) = (1, 1)$  and  $(0, 0)$ .

ities  $P(q|r)$ —the probability of getting an outcome  $q$ , given that the actual particle position is  $r$ . If  $e$  is the error rate, then  $P(0|0) = P(1|1) = 1 - e$ , and  $P(0|1) = P(1|0) = e$ . Now,  $(r, q) = (0, 0)$ ,  $(0, 1)$ ,  $(1, 0)$ ,  $(1, 1)$  will give four possible configurations of the SZE after measurement.

The average information obtained due to the measurement is quan-



tified in the following way:

$$\langle I \rangle = \sum_{r,q=0,1} \tilde{P}(r, q) \ln \left[ \frac{\tilde{P}(r, q)}{P(r)P_m(q)} \right], \quad (4)$$

where  $\tilde{P}(r, q) = P(q|r)P(r)$  is joint probability of actual and measured position of the particle.  $P_m(q) = \sum_{r=0,1} \tilde{P}(r, q) = \sum_{r=0,1} P(q|r)P(r)$  is the probability of a particular measured outcome  $q$ . Using these expressions one can easily calculate that:

$$\begin{aligned} P_m(1) &= x(1 - e) + (1 - x)e, \\ P_m(0) &= xe + (1 - x)(1 - e), \end{aligned}$$

and hence the average information gain is given by:

$$\begin{aligned} \langle I \rangle &= e \ln e + (1 - e) \ln(1 - e) \\ &- [x(1 - e) + (1 - x)e] \ln[x(1 - e) + (1 - x)e] \\ &- [xe + (1 - x)(1 - e)] \ln[xe + (1 - x)(1 - e)]. \end{aligned} \quad (5)$$

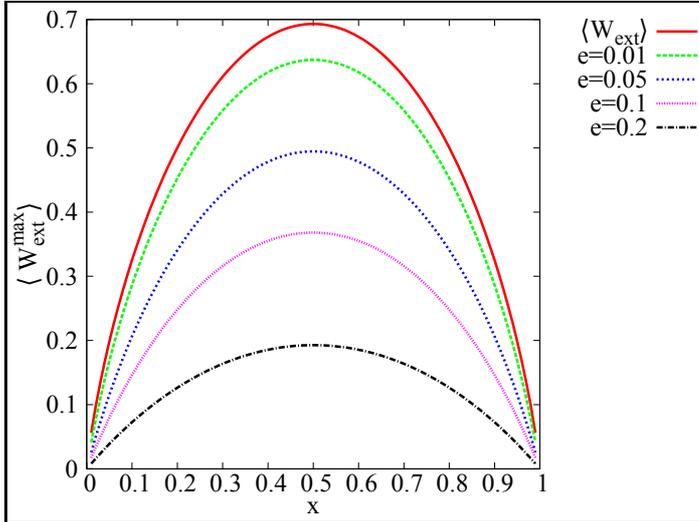
Since the feedback step depends highly on the measurement outcome, it will also be modified due to the presence of error during the measurement step. We assume that after we move the partition, the volume of the box is divided in the ratio as  $v_1 : 1 - v_1$  for  $q = 1$  and  $v_0 : 1 - v_0$  for  $q = 0$  with  $0 \leq v_0, v_1 \leq 1$ . In this process, we extract work from the engine. Let  $W^{(r,q)}$  denote the work for a particular configuration. Hence,

$$\begin{aligned} \beta W^{(1,1)} &= \ln \left( \frac{v_1}{x} \right); \beta W^{(1,0)} = \ln \left( \frac{v_0}{x} \right), \\ \beta W^{(0,1)} &= \ln \left( \frac{1 - v_1}{1 - x} \right); \beta W^{(0,0)} = \ln \left( \frac{1 - v_0}{1 - x} \right). \end{aligned}$$

The average extracted work in the entire process is given by:

$$\begin{aligned} \beta \langle W_{ext} \rangle &= -x \ln x - (1 - x) \ln(1 - x) \\ &+ (1 - e)[x \ln v_1 + (1 - x) \ln(1 - v_0)] \\ &+ e[x \ln v_0 + (1 - x) \ln(1 - v_1)]. \end{aligned} \quad (6)$$





**Figure 4.** Plot of maximum average work extracted from single-particle SZE as a function of the initial position of the partition for different measurement errors.

We then maximize  $\langle W_{ext} \rangle$  under a given measurement error  $e$  and initial position  $x$  of the partition by changing  $v_1$  and  $v_0$ . The maximum value of  $\langle W_{ext} \rangle$  is achieved when,

$$v_1 = \frac{x(1-e)}{(1-e)x + (1-x)e}, \quad (7)$$

$$v_0 = \frac{(1-x)(1-e)}{(1-x)(1-e) + xe}. \quad (8)$$

The maximum work is calculated using the above expressions and is given by the information gained due to measurement:

$$\beta \langle W_{ext}^{max} \rangle = \langle I \rangle \quad (9)$$

which is the modified second law in presence of feedback applied to a cyclic process. In this particular case, it is possible to convert full information into work. *Figure 4* shows the maximum work that can be extracted from the system in the presence of errors during the measurement procedure. Larger the error, lesser the work that is extracted. Work extraction is always degraded due to the presence of errors, as seen from the same figure.

Larger the error, lesser the work that is extracted. Work extraction is always degraded due to the presence of errors.

### 2.2 Multi-particle SZE

In multi-particle SZE, the thermodynamic cyclic consists of the same four steps: insertion of partition, measurement, feedback, and removal of the partition. The multi-particle SZE consists of  $N$  number of particles as the working substance, which is contained in a box of length  $L$ , surface area  $A$ , and hence volume  $V = AL$ .

In multi-particle SZE, the thermodynamic cyclic consists of the same four steps: insertion of partition, measurement, feedback, and removal of the partition. The multi-particle SZE consists of  $N$  number of particles as the working substance, which is contained in a box of length  $L$ , surface area  $A$ , and hence volume  $V = AL$ . The partition is placed initially at a position that divides the box in the ratio  $x : 1 - x$ . Suppose there are  $m$  particles on the left side and  $N - m$  particles on the right. In the next step, the number of particles on the left side is measured. If the measurement procedure is error-free, then the device will exactly measure  $m$  particles on the left. After that, an isothermal expansion is performed in contact with a heat bath at inverse temperature  $\beta$ , where the partition shifts to a position  $l_m$ , determined by the force balance on both sides of the partition. Finally, the wall is removed to complete the cycle. The average work done by the SZE is given by [30]:

$$\langle W_{ext} \rangle = -\frac{1}{\beta} \sum_{m=0}^N p_m \ln \left( \frac{p_m}{f_m} \right) \tag{10}$$

Here  $p_m$  and  $f_m$  are given by:

$$p_m = \frac{Z_{m,N-m}(xL)}{\sum_{m'} Z_{m',N-m'}(xL)}, \tag{11}$$

$$f_m = \frac{Z_{m,N-m}(l_m)}{\sum_{m'} Z_{m',N-m'}(l_m)}, \tag{12}$$

where  $Z_{m,N-m}(X) = Z_m(X)Z_{N-m}(L - X)$  is a partition function that describes the situation of  $m$  particles to the left of the partition and the remaining  $N - m$  to the right, in thermal equilibrium,  $Z_m(X) = cX^m/m!$ , the partition function of a system consisting of  $m$  particles in a box of length  $X$  and  $c$  is some constant. Physically,  $p_m$  denotes the probability that there are  $m$  particles to the left after partition and  $f_m$  represents the probability to choose the case of  $m$  particles on the left side of the wall when the wall is inserted at  $l_m$  in the time backward process. Although (10) is derived for QSZE, surprisingly, it also holds true for work done by



classical SZE derived using classical non-equilibrium thermodynamics. However, for both the cases, the partition function differ, and so does the amount of work. In case of system consisting of  $N$  particles,  $p_m = \binom{N}{m} x^m (1-x)^{N-m}$  and  $l_m = (m/N)L = \alpha L$ .  $f_m$  is calculated in the following way

$$\begin{aligned}
 f_m &= \frac{Z_{m,N-m}(l_m)}{\sum_{m'} Z_{m',N-m'}(l_m)}, \\
 &= \frac{Z_m(\alpha L) Z_{N-m}((1-\alpha)L)}{\sum_{m'} Z_{m'}(\alpha L) Z_{N-m'}((1-\alpha)L)}, \\
 &= \frac{\frac{(\alpha L)^m}{m!} \cdot \frac{[(1-\alpha)L]^{N-m}}{(N-m)!}}{\sum_{m'} \frac{(\alpha L)^{m'}}{m'!} \cdot \frac{[(1-\alpha)L]^{N-m'}}{(N-m')!}}, \\
 &= \binom{N}{m} \frac{(\alpha L)^m [(1-\alpha)L]^{N-m}}{L^N} \\
 &= \binom{N}{m} \alpha^m (1-\alpha)^{N-m}. \tag{13}
 \end{aligned}$$

Inserting these expressions of  $p_m$  and  $f_m$  in (10), we obtain

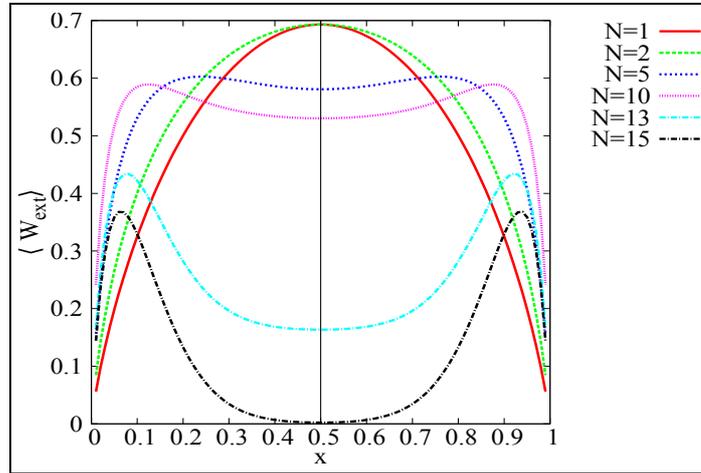
$$\langle W_{ext} \rangle = -T \sum_{m=0}^N \binom{N}{m} x^m (1-x)^{N-m} \ln \left[ \left( \frac{x}{\alpha} \right)^m \left( \frac{1-x}{1-\alpha} \right)^{N-m} \right] \tag{14}$$

Figure 5 depicts the behavior of work done by SZE as a function of biasing for different number of particles. It is seen right away that the work extraction is symmetric about  $x = 0.5$ , which is quite obvious. Another interesting fact to note is that the work done by SZE is the same for  $N = 1$  and  $N = 2$  when the wall is initially kept at  $x = 0.5$ ; it is different otherwise. This is because when there is no biasing, the measurement outcome that there is one particle on both sides does not contribute to the work extraction as the system stays at equilibrium with the wall at  $x = 0.5$ , and it does not move. This is not the case for  $x \neq 0.5$  because in this case, even though there is one particle on each side, the initial position of the partition is not the equilibrium position.

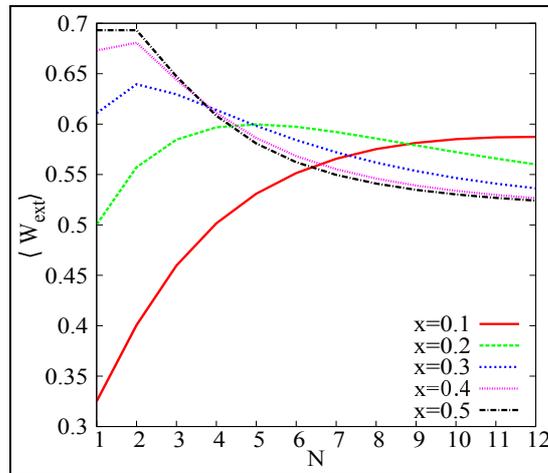
Work extraction has a single peak at  $x = 0.5$  for a single particle, but it splits into two distinct peaks as the number of particles



**Figure 5.** Plot of average work done by multi-particle SZE as a function of the initial position of the partition for different number of particles.



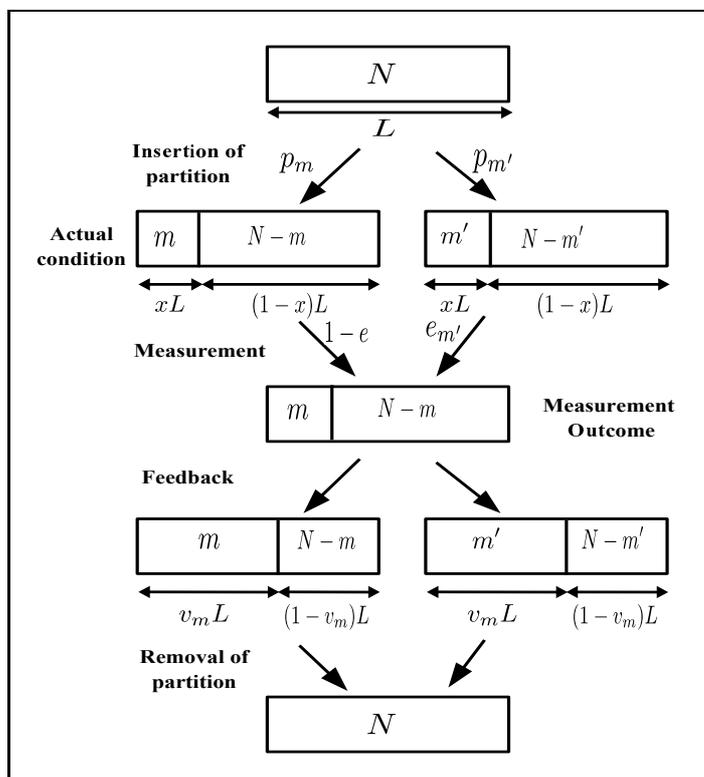
**Figure 6.** Plot of average work extraction as a function of number of particles in multi-particle SZE for different initial positions of the partition.



increases. Due to the emergence of two peaks, local minima in work extraction is visible at  $x = 0.5$ . This minimum value eventually goes to zero as is evident from *Figure 5*. This gives a clear clue that work extraction is more for multi-particle working substance with biasing.

*Figure 6* shows the work extraction as a function of the number of particles for different biasing. The plot depicts the fact that work extraction is small, with a large biasing when the number





**Figure 7.** Multi-particle SZE with errors.

of particles is small. But work done by SZE increases for large biasing when the particle number is large.

### 2.2.1 Measurement error in multi-particle SZE

In multi-particle SZE, the error enters in the measurement step where the measuring device can make an error while counting the number of particles on each side.

In the case of  $N$  identical particle system, there can be  $N + 1$  configurations. Each configuration gives the number of particles on the left and the right side of the box. Consider a measurement outcome with the configuration:  $m$  particles on the left and  $N - m$  particles on the right as shown in *Figure 7*. The actual configurations can be (i)  $m$  particles on the left side and  $N - m$  particles on the right, and (ii)  $m' (\neq m)$  particles on the left and  $N - m'$  particles

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on the right. The probability of observing measured configuration from the former case is  $1 - e$ . The remaining  $N$  actual configurations can be mapped to the measured configuration due to measurement error. This error will depend on the number of particles on the left in the actual configuration. Hence, the probability of observing the measured configuration from the latter case is  $e_{m'}$ . Due to the normalization of probability  $\sum_{m' \neq m} e_{m'} = e$ . These possibilities are clearly shown in *Figure 7*. Depending on the outcome, the partition is shifted to a new position where it divides the box in the ratio  $v_m : 1 - v_m$ . If  $W^{(m,m)}$  represents the work done when the actual and the measured number of particles on the left side are the same (say  $m$ ), and  $W^{(m',m)}$  represents the work done when the actual number of particles is different from the measurement value (say  $m'$  and  $m$  respectively), then

$$\begin{aligned} \beta W^{(m,m)} &= \ln \left[ \frac{Z_{m,N-m}(v_m L)}{Z_{m,N-m}(xL)} \right], \\ \beta W^{(m',m)} &= \ln \left[ \frac{Z_{m',N-m'}(v_m L)}{Z_{m',N-m'}(xL)} \right]. \end{aligned} \quad (15)$$

The average work output given a particular measurement outcome  $m$  is given by:

$$\beta \langle W^{(m)} \rangle = p_m(1 - e)\beta W^{(m,m)} + \sum_{m' \neq m} e_{m'} p_{m'} \beta W^{(m',m)} \quad (16)$$

One can assume the error  $e_{m'}$  to be same for all the  $N$  configurations. In that case,  $e_{m'} = e/N$  should be used in (16). The total average work extraction is given by:

$$\beta \langle W_{ext} \rangle = \sum_{m=0}^N \beta \langle W^{(m)} \rangle \quad (17)$$

Work extraction depends crucially on the nature of the working substance. In case of ideal gas, there is no inter-particle interaction and the partition function of  $n$ -particle system confined in a box of dimension  $X$  is  $Z_n(X) = cX^n/(n!)$ . Using this expression of the partition function, (15) can be easily calculated



$$\begin{aligned}\beta W^{(m,m)} &= \ln \left[ \left( \frac{v_m}{x} \right)^m \left( \frac{1-v_m}{1-x} \right)^{N-m} \right], \\ \beta W^{(m',m)} &= \ln \left[ \left( \frac{v_m}{x} \right)^{m'} \left( \frac{1-v_m}{1-x} \right)^{N-m'} \right].\end{aligned}\tag{18}$$

The above two expressions can be used to calculate the total average extracted work given by (17).

### 3. Landauer's Principle

The thermodynamic process of the Szilard engine is cyclic because the initial and the final states of the particle are identical. Hence the free energy change of the system is zero. This implies that the particle is systematically extracting work cyclically from the thermal reservoir, in contrary to the second law of thermodynamics. The resolution of this apparent violation of the second law was given by Charles Bennett [27]. He investigated the energetics of Szilard's engine using Landauer's principle that relates logical irreversibility with energy consumption during the computing process. The information obtained from measurement is stored in physical systems. As a result, simple logical operations are accompanied by an energetic cost. In the case of Szilard's engine, the particle in the box might have gone through a cyclic process, but the whole system consisting of the particle in the box and memory (which is used to store the information of the position of the particle after the piston is inserted) did not complete a whole cycle. The full cycle will be completed if at the end when the piston is removed, the data in the memory is erased (or reset). This erasure of a bit is a logical irreversible operation since a bit that is initially in one of the two possible states (0 or 1) is set to a reference state irrespective of its initial state. Landauer's principle [4, 5] states that the erasure of a bit requires at least  $k_B T \ln 2$  amount of work dissipated to the thermal environment:

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$$W_{\text{eras}} \geq k_B T \ln 2, \quad (19)$$

Landauer's principle was recently tested experimentally using optically-trapped Brownian particles.

where equality is satisfied in the case of the quasistatic erasing process. Landauer's principle was recently tested experimentally in [28] using optically-trapped Brownian particles. It is to be noted in this context that the error-free measurement process is a logically reversible process because there is a one-to-one correspondence between the state of the system and the measured outcome.

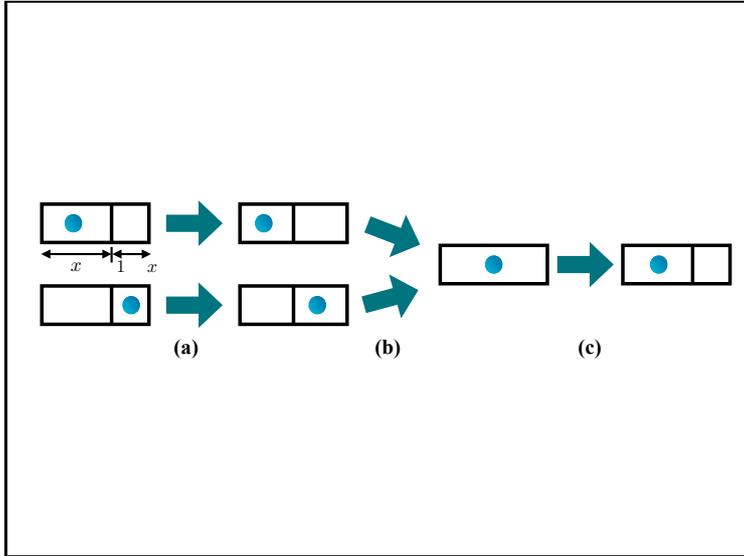
Taking Landauer's principle into account, it is now very clear that in every cycle, there are two contributions to the total work: (i) the work done in the system due to the reversible expansion of the one molecule ideal gas and (ii) work due to the erasure of measured data. Since the measurement has only two outcomes, the measurement of Szilard's engine is equivalent to the measurement of the state of one bit. In every cycle, the state of the demon has to be reset to the reference state in order to carry out the measurement procedure in the next cycle. This resetting process requires a minimum of  $k_B T \ln 2$  amount of work as stated by Landauer. Hence, the work per cycle of the Szilard engine is:

$$W_{\text{tot}} = W + W_{\text{eras}} \geq -k_B T \ln 2 + k_B T \ln 2 = 0, \quad (20)$$

which is in accordance with the second law of thermodynamics.

In recent years, Landauer's principle, which is based on the thermodynamic energy cost of information erasure, has been challenged. Initially, it was thought that the measurement process can be performed without any energy consumption. Now a generalized version of Landauer's principle has been put forward by Sagawa and Ueda [29], that sets a lower bound on the energy cost of measurement and information erasure combined. The analysis by Sagawa and Ueda crucially depends on how one constructs the memory where the information is stored. Instead of giving a





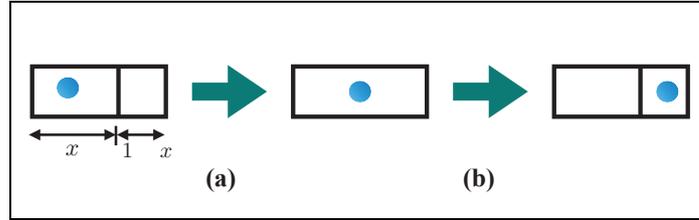
**Figure 8.** Schematic picture of information erasure from an asymmetric memory.

rigorous proof, following [29], here we illustrate this idea using an example of a particle in a box with a partition as a model for a memory (*Figure 8*).

We construct the memory in such a way that the partition divides the box in the volume ratio of  $x : 1 - x$ . When the particle is on the “left” (or on the “right”) side of the box, the memory registers “0” (or “1”). Let’s consider that the initial probability of the outcomes of “0” or “1” is equal, i.e.,  $1/2$ . Now, we will consider the case of quasistatic information erasure from the memory in contact with a heat bath at inverse temperature  $\beta$  and calculate the work required for such a process. Erasure of memory is done in three sub-processes. At first, the information is stored in the memory (*Figure 8*). Then the partition is moved to the center of the box. The average work cost for this process is given by the free energy change, i.e.,

$$\begin{aligned} \beta W_{\text{eras}}^{(1)} &= \frac{1}{2} [\ln x - \ln(1/2)] + \frac{1}{2} [\ln(1 - x) - \ln(1/2)], \\ &= \frac{1}{2} [\ln(2x) + \ln\{2(1 - x)\}]. \end{aligned} \quad (21)$$

**Figure 9.** Schematic picture of the process of information writing to an asymmetric memory.



The partition is then removed, and this process can be regarded as the free expansion of the gas that requires no work. Finally, the box is compressed so that the memory returns to its standard state of “0”. This sub-process occurs at the cost of work given by:

$$\beta W_{\text{eras}}^{(2)} = -\ln x. \quad (22)$$

The total work required for information erasure is

$$\beta W_{\text{eras}}^{\text{M}} = \beta W_{\text{eras}}^{(1)} + \beta W_{\text{eras}}^{(2)} = \ln 2 - \frac{1}{2} \ln \left( \frac{x}{1-x} \right). \quad (23)$$

In the case of symmetric memory, we have  $x = 1/2$ , and the work required for information erasure reduces to  $\ln 2$ , which is the limit set by Landauer’s erasure principle. For any other arbitrary values of  $x$ , Landauer bound is not followed.

We now consider a quasistatic measurement process in presence of a heat bath at temperature  $T$  (Figure 9). Let the memory initially be in the standard state “0”. If the measurement outcome is “0”, the state of the memory does not change. If the measurement outcome is “1”, then the left side of the box expands and pushes the box to the right end, which requires  $(1/\beta) \ln x$  amount of work. The box then compresses from the left side until the partition comes back to its initial position. This compression process requires  $-(1/\beta) \ln(1-x)$  amount of work. Since the probability of measurement outcome “0” or “1” is the same, i.e.,  $1/2$ , the average energy cost for the measurement procedure is given by:

$$\beta W_{\text{meas}}^{\text{M}} = \frac{1}{2} \ln \left[ \frac{x}{1-x} \right]. \quad (24)$$

The total work required for information measurement and erasure process combined is:

$$\beta W^M = \beta W_{\text{meas}}^M + \beta W_{\text{eras}}^M = \ln 2. \quad (25)$$

#### 4. Conclusion

In this article, we have given a flavor of the initial building stages of information thermodynamics by discussing three important works in this field namely Maxwell's demon, Szilard engine, and Landauer's principle. A nice future prospect in this area is to investigate these works in interacting multi-particle systems. It is important to note that all the works have been discussed using the concepts of classical mechanics. In recent years these works have been extended to the quantum regime, and a lot of work is being carried out in quantum thermodynamics. Specifically, Kim *et al.* [30] has given a theoretical analysis of the quantum version of Szilard engine where they studied Szilard engines consisting of Bosons and Fermions. More (less) work (with respect to classical Szilard engine) can be extracted from the Bosonic (Fermionic) Szilard engine due to the indistinguishability of identical particles.

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