Classroom

In this section of *Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

Beyond $\pi^e < e^\pi$: Proof Without Words of $b^a < a^b (b > a \geq e)$ *

Rajib Mukherjee
Department of Mathematics
Krishnath College
Berhampore
West Bengal 742 101, India
Email:
rajibmukherjee786@gmail.com

Manishita Chakraborty
Panihati Mahavidyalaya for Girls'
West Bengal 700 110, India
Email: manisitach@gmail.com

Using elementary properties of calculus there are at least two [1, 2] visual proofs of $\pi^e < e^\pi$. Also, there is a nice visual proof [3] of $A^B > B^A$ for $e \leq A < B$. It seems to be an interesting problem to visually compare $\boldsymbol{a^b}$ and $\boldsymbol{b^a}$ where the argument does not depend on the shape of a graph more complicated than $\boldsymbol{y = \ln x}$.

In this note, using elementary function and an elementary property of calculus we give a visual proof of:

$$\boxed{a^b > b^a, b > a \geq e}$$

$$(x \ln x - x)_a^b = \int_a^b \ln x \, dx < \ln b(b - a)$$

$$\Rightarrow \; a \ln b < (b - a) + a \ln a$$

$$\Rightarrow \; a \ln b < (b - a) \ln a + a \ln a \; (a \geq e)$$

$$\Rightarrow \; a \ln b < b \ln a$$

$$\Rightarrow \; \boxed{b^a < a^b}$$

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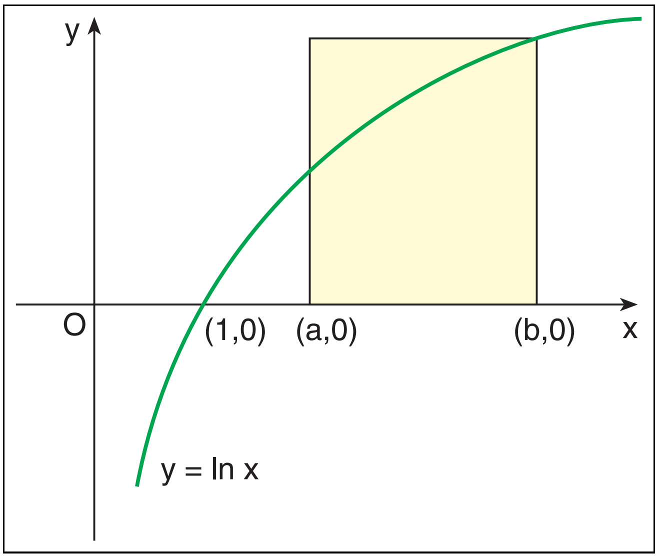

Note : $a = e, b = \pi$ $\Rightarrow \pi^e < e^\pi$	**Example :** $a = 9, b = \sqrt{83}$ $\Rightarrow 9^{\sqrt{83}} > (\sqrt{83})^9$

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Suggested Reading

[1] Fouad Nakhli, $e^\pi > \pi^e$, *Mathematics Magazine*, Vol.60, No.3, p.165, 1987.

[2] Bikash Chakraborty, A visual proof that $\pi^e < e^\pi$, *The Mathematical Intelligencer*, 41, 56, 00I 10.007/ s00283-018–9816-4, 2019.

[3] Charles D Gallant, Proof without words: Comparing B^A and A^B for $A < B$", *Mathematics Magazine*, Vol.64, No.1, p.31, 1991 doi: 10.2307/2690451.

[4] Roger B Nelsen, Proofs without words: Exercise in visual thinking, *The Mathematical Association of America*, 1993.

[5] Roger B Nelsen, Proofs without words II: More exercise in visual thinking, *The Mathematical Association of America*, 2000.