Freeman Dyson, the Mathematician*

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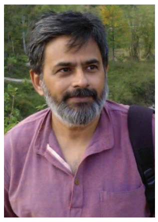
A mathematician of the highest rank
with ideas ever-brimming with a full tank.
He designed special lenses
to 'see' Ramanujan's congruences
partitioned through the yet-invisible crank!



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The one person who comes closest to the legacy of Hermann Weyl was Freeman Dyson, who contributed enormously to both Physics and Mathematics. His books and talks are testaments to his prolificity in writing widely on the world at large and on science and mathematics in particular. To name a few of his books, he has (talked and) written on 'Bombs and Poetry', 'Imagined Worlds', 'Origins of Life' and 'Birds and Frogs'. Remarkably, Dyson contributed handsomely to what is termed 'pure mathematics'. One would expect a physicist-mathematician to interest himself mainly in problems of an 'applied' nature. Here, we take a necessarily brief peek into some of his 'purely mathematical' work. The suggested reading at the end can be referred to for more details.

1. Statistical Mechanics – Dyson's Conjecture

Keywords
Congruences for partitions, rank and crank, random matrix theory, Riemann zeta function, Dyson transform, quasi-crystals.

In an enormously influential paper, 'Statistical theory of energy levels of complex systems', *I. J. Math. Phys.*, 3:140–156, 1962, Dyson proposed a new type of ensemble transforming the whole subject into purely group-theoretic language. In the paper, Dyson studied new types of statistical ensembles, demonstrating a mathematical idealisation of the concept of considering "all physical

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systems with equal probability". He studied three such ensembles based on the orthogonal, unitary and symplectic groups. He establishes an exact mathematical correspondence between the distribution of eigenvalues of a random matrix and the distribution of positions of charges in a finite Coulomb gas at a finite temperature T. The corresponding temperature T is $1, 1/2$ or $1/4$ according to as to whether the matrix is taken from an orthogonal, unitary or symplectic ensemble.

In this paper, Dyson arrives at some purely mathematical conjectures one of which is the so-called 'Dyson conjecture' (Conjecture C in the paper), asserting that for positive integers $a_1, \cdots, a_N$, the constant term of the rational function $\prod_{i \neq j}(1 - z_i/z_j)^{a_i}$ is the multinomial coefficient $\frac{(\sum_i a_i)!}{\prod_i (a_i)!}$. The conjecture was proved soon afterwards but the richness of the idea can be divined from the fact that the conjecture was generalized by Macdonald to all root systems (Dyson's conjecture corresponding to the type A_n) and proved later by Cherednik using double affine Hecke algebras. As a matter of fact, the conjecture is a mathematical reformulation of another conjecture (Conjecture A) which specifies precisely the statistical properties of a Coulomb gas of N particles.

1.1 *A Simple Proof of Dyson's Conjecture*

Here is a 'good' proof of the conjecture. The proof by I J Good uses just Lagrange's interpolation formula.
Write $f(\mathbf{x}, a_1, \cdots, a_n) = \prod_{i \neq j}(1 - x_i/x_j)^{a_i}$.
Now, interpolation formula gives the function that equals 1 at all x_i's to be

$$\sum_j \prod_{i:i \neq j} \frac{t - x_i}{x_j - x_i}.$$

Putting $t = 0$, we have

$$1 = \sum_j \prod_{i:i \neq j} (1 - x_j/x_i)^{-1}.$$

Multiplying $f(\mathbf{x}, a_1, \cdots, a_n) = \prod_{i \neq j}(1 - x_i/x_j)^{a_i}$ by the above expression for 1, we get

$$f(\mathbf{x}, a_1, \cdots, a_n) = \sum_j f(\mathbf{x}, a_1, \cdots, a_j - 1, \cdots, a_n)$$

where the right hand side expression has terms where the variable a_j is reduced by 1 and other variables the same. If the sought for constant term of $f(\mathbf{x}, \mathbf{a})$ is denoted by $g(\mathbf{a})$, then

$$g(\mathbf{a}) = \sum_j g(a_1, \cdots, a_j - 1, \cdots, a_n) \cdots (\spadesuit)$$

Now, clearly if $a_j = 0$, the term x_j occurs with a strictly negative power in $f(\mathbf{x}, \mathbf{a})$, and would, therefore, not contribute to the constant terms. In other words,

$$g(a_1, \cdots, a_n) = g(a_1, \cdots, a_{j-1}, a_{j+1}, \cdots, a_n) \;\; if \;\; a_j = 0.$$

Clearly $g(0, \cdots, 0) = 1$. Thus, ($\spadesuit$) along with the last two equalities determine the general value $g(\mathbf{a})$ recursively. The multinomial coefficient

$$M(a_1, \cdots, a_n) = \frac{(\sum_i a_i)!}{\prod_i (a_i)!}$$

satisfies the same recursion and initial value. Thus, $g = M$.

2. Random Matrices and Zeta Zeroes

The story is supposed to have started in 1972 when the analytic number theorist Hugh Montgomery met Dyson at the Institute for Advanced Study—this conversation is sometimes called ”one of the most celebrated denouements in mathematics in recent years.” In Montgomery’s words,

“I [Montgomery] took afternoon tea that day in Fuld Hall with [Sarvadaman] Chowla. Freeman Dyson was standing across the room. I had spent the previous year at the Institute and I knew him perfectly well by sight, but I had never spoken to him. Chowla said: ”Have you met Dyson?” I said no, I hadn’t. He said: ”I’ll introduce you.” I said no, I didn’t feel that I had to meet Dyson.

Chowla insisted, and so I was dragged reluctantly across the room to meet Dyson. He was very polite, and asked me what I was working on. I told him I was working on the differences between the non-trivial zeros of Riemann's zeta-function, and that I had developed a conjecture that the distribution function for those differences had integrand $1 - \sin^2(\pi u)/\pi^2 u^2$. He got very excited. He said: "That's the form factor for the pair correlation of eigenvalues of random Hermitian matrices!"

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3. Birds and Frogs

In a very incisive article titled 'Birds and Frogs', Dyson talks about two types of mathematicians. He says, " Birds fly high in the air and survey broad vistas of mathematics out to the far horizon. They delight in concepts that unify our thinking and bring together diverse problems from different parts of the landscape. Frogs live in the mud below and see only the flowers that grow nearby. They delight in the details of particular objects, and they solve problems one at a time. I happen to be a frog."
"The world of mathematics is both broad and deep, and we need birds and frogs working together to explore it."
"Rene Descartes was a bird, and Francis Bacon is a frog."

In 'Birds and Frogs', Dyson writes of a viable method to attack the Riemann hypothesis:

[1] As a mathematician, I use the word 'attack' in the sense of trying to prove it or rather than attack its proposed veracity!

"A quasi-crystal is a distribution of discrete point masses whose Fourier transform is a distribution of discrete point frequencies. Or to say it more briefly, a quasi-crystal is a pure point distribution that has a pure point spectrum.... Here comes the connection of the one-dimensional quasi-crystals with the Riemann hypothesis. If the Riemann hypothesis is true, then the zeros of the zeta-function form a one-dimensional quasi-crystal according to the definition. They constitute a distribution of point masses on a straight line, and their Fourier transform is likewise a distribution of point masses, one at each of the logarithms of ordinary prime numbers and prime-power numbers.... My suggestion is the following. Let us pretend that we do not know that the Riemann Hypothesis is true. Let us tackle the problem from the other end. Let us try to obtain a complete enumeration and classification of one-dimensional quasicrystals. That is to say, we enumerate and classify all point distributions that have a discrete point spectrum...We shall then find the well-known quasi-crystals associated with PV[2] numbers, and also a whole universe of other quasicrystals, known and unknown. Among the multitude of other quasi-crystals we search for one corresponding to the Riemann zeta-function and one corresponding to each of the other zeta-functions that resemble the Riemann zeta-function. Suppose that we find one of the quasi-crystals in our enumeration with properties that identify it with the zeros of the Riemann zeta-function. Then we have proved the Riemann Hypothesis and we can wait for the telephone call announcing the award of the Fields Medal. These are of course idle dreams. The problem of classifying one-dimensional quasi-crystals is horrendously difficult, probably at least as difficult as the problems that Andrew Wiles took seven years to explore. But if we take a Baconian point of view, the history of mathematics is a history of horrendously difficult problems being solved by young people too ignorant to know that they were impossible. The classification of quasi-crystals is a worthy goal, and might even turn out to be achievable."

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[2]Here PV stands for Pisot and Vijayaraghavan.

4. Partitions and Crank

One of the most riveting short papers by Dyson, titled 'Some guesses in the theory of partitions', was published in the *Eureka Magazine* in 1944. He begins the article thus:

"Professor Littlewood, when he makes use of an algebraic identity, always saves himself the trouble of proving it; he maintains that an identity, if true, can be verified by anybody obtuse enough to feel the need of verification. My object in this following note is to confute this assertion."

"After a few preliminaries, I state certain properties of partitions which I am unable to prove; these guesses are then transformed into algebraic identities which are also unproved, although there is conclusive numerical evidence in their support; finally, I indulge in some even vaguer guesses concerning the existence of identities which I am not only unable to prove but also unable to state. I think this should be enough to disillusion anyone who takes Professor Littlewood's innocent view of the difficulties of algebra. Needless to say, I strongly recommend my readers to supply the missing proofs, or even better, the missing identities."

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For the partition function $p(n)$ that counts the number of ways to partition n into (unordered) sums of positive integers, Ramanujan's famous congruences assert the divisibility properties:

$$5|p(5n+4), 7|p(7n+5), 11|p(11n+6) \ \forall \ n.$$

Although they were proved analytically, one would like natural counting functions that would count $p(5n+4)/5$, $p(7n+5)/7$, and $P(11n+6)/11$. In the *Eureka* paper, Dyson defined the notion of the rank of a partition to be the largest part minus the number of parts. Dyson conjectured that the number of partitions of $5n+4$ with rank equal to each given congruence class mod 5 is equally numerous (that is, independent of the class), thereby explaining the congruence $5|p(5n+4)$. He made a similar conjecture about the rank belonging to the 7 congruence classes mod 7 for partitions of $7n+5$. Both these were proved later by Atkin and Swinnerton-Dyer. Dyson observed that the rank does not separate the classes mod 6 while considering the partitions of $11n+6$.

Here, Dyson came up with a brilliant idea. He looked at various tables of partition functions and "guessed" the existence of a more esoteric hypothetical function he christens the 'crank' of a partition, which would explain the congruences for $p(11n + 6)$. In 1988, George Andrews and Frank Garvan defined the crank of a partition as follows. It is defined to be the largest part if the partition has no 1's. If there are $\omega > 0$ one's in the partition, then the difference "(the number of its parts bigger than ω)—ω is defined to be the crank. They showed that with this notion of the crank, we have the correct analogue for $p(11n + 6)/11$ of the above results with rank for $p(5n + 4)/5$ and $p(7n + 5)/7$.

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5. Missed Opportunities

In a wonderful talk (the Gibbs lecture published in the *Bulletin of the American Mathematical Society*, 1972) titled 'Missed Opportunities', Dyson says wistfully:

"I shall examine in detail some examples of missed opportunities, occasions on which mathematicians and physicists lost chances of making discoveries by neglecting to talk to each other. My purpose in calling attention to such incidents is not to blame the mathematicians or to excuse the physicists for our failure in the last twenty years to equal the great achievements of the past. My purpose is not to lament the past but to mould the future. It is obviously absurd for me to imagine that I can mould the future with a one-hour lecture. The fact that Hilbert in 1900 and Minkowski in 1908 succeeded in doing it does not give me any confidence that I can do it too. But at least I have learned from Hilbert and Minkowski that one does not influence people by talking in generalities. Hilbert and Minkowski gave specific suggestions of things that mathematicians and physicists could profitably think about; I shall try to follow their style. I shall try to convince you by examining actual cases that the progress of both mathematics and physics has in the past been seriously retarded by our unwillingness to listen to one another. And I will end with an attempt to identify some areas in which opportunities for future discoveries are now being missed." We will describe one of the examples

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Dyson gives to demonstrate the kind of things these 'missed opportunities' entail.

Quoting him, "Long after I became a physicist, I retained a sentimental attachment to the (Ramanujan) τ-function, and as a relief from the serious business of physics I would from time to time go back to Ramanujan's papers and meditate on the many intriguing problems that he left unsolved. Four years ago, during one of these holidays from physics, I found a new formula for the τ-function, so elegant that it is rather surprising that Ramanujan did not think of it himself. The formula is:"

$$\tau(n) = \sum \frac{\prod_{1\leq i<j\leq 5}(a_i - a_j)}{1!2!3!4!}$$

where the sum is over all $a_i \equiv i \bmod 5$ with

$$a_1 + a_2 + a_3 + a_4 + a_5 = 0;$$

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 = 10n.$$

As the Dedekind eta function $\eta(x) = x^{1/24} \prod_{n=1}^{\infty}(1 - x^n)$ satisfies

$$\eta(x)^{24} = \sum_{n\geq 1} \tau(n)x^n$$

the above expression of Dyson for τ gives a formula for the 24-th power of the eta function. Dyson discovered that there were analogous formulae for $\eta(x)^d$ for

$$d = 3, 8, 10, 14, 15, 21, 24, 26, 28, 35, 36, \cdots$$

Then Dyson goes on to say:

"I stared for a little while at this queer list of numbers (3). As I was, for the time being, a number theorist, they made no sense to me. My mind was so well compartmentalized that I did not remember that I had met these same numbers many times in my life as a physicist. If the numbers had appeared in the context of a problem in physics, I would certainly have recognized them as the dimensions of the finite-dimensional simple Lie algebras. Except for 26. Why 26 is there I still do not know. The others all correspond to simple algebras: $A_l, A_2, B_2, G_2, A_3, B_3, A_4, D_4, A_5, B4$

and so on. For example, $d = 24$ (above) corresponds to the algebra A_4 and in the structure of the formula (above) you can see the root system of A_4. So I missed the opportunity of discovering a deeper connection between modular forms and Lie algebras, just because the number theorist Dyson and the physicist Dyson were not speaking to each other. This story has a happy ending. Unknown to me the English geometer, Ian MacDonald, had discovered these same formulae as special cases of a much more general theory. In his theory, the Lie algebras were incorporated from the beginning, and it was the connection with modular forms which came as a surprise. Anyhow, MacDonald established the connection and so picked up the opportunity which I missed."

6. Tidbits

6.1 *Counterfeit Coin*

The famous problem asks for identification of one 'defective' coin among $(3^n - 3)/2$ coins of which all others have equal weights if we are allowed n weighings. The weighing is done on a two-sided pan without any weights provided, and we may place coins on either side. We are asked also to determine if the defective coin is heavier or lighter; if we are asked only to determine which coin is defective among $(3^n - 1)/2$ coins, it turns out we can do so in n weighings. The solutions are usually given by a case-by-case branching method where the weighing at the ith stage depends on the outcomes of the weighing in the previous step. Dyson gave a delightful new solution where a pre-determined algorithm right at the beginning could be given which, when followed, determines the bad coin and also whether it is heavier or lighter! His proof also shows that n weighings will not suffice if we have more than $(3^n - 3)/2$ coins. The algorithm exists for any number of coins not exceeding $(3^n - 3)/2$ but the extreme case of $(3^n - 3)/2$ coins is the most interesting. As the solution is easy to describe, we do so here.

Dyson gave a delightful new solution where a pre-determined algorithm right at the beginning could be given which, when followed, determines the bad coin and also whether it is heavier or lighter!

The algorithm only requires labelling the $(3^n - 3)/2$ coins in the base 3 as follows.

Write $N = \frac{3^n-3}{2}$ for simplicity of notation and we will give two labels for each coin as follows.

The N coins are numbered $1, 2 \cdots , N$ and the first label of coin i is its base 3 expansion in n-digits (if the number of digits is less than n, one puts 0's, in the beginning, to make it genuinely n digits). The second label of the coin i is obtained by subtracting the first label from the number $3^n - 1$ in base 3 (which consists of n two's). Thus, whichever digit in the first label is 0 becomes 2 in the second label and vice versa; the digits 1 remain the same. In this manner, each coin gets two labels and any n-digit base 3 number (other than the same digit repeated n times) can occur just once as a label. Among the two labels of any coin, one calls 'clockwise', if the first change of digit starting from the left-most digit takes either 0 to 1 or 1 to 2 or 2 to 0. Otherwise, it is called anticlockwise. Thus, for any coin, one label is clockwise and the other is anticlockwise. For each $i \leq n$, we divide the set of coins into 3 sets $C(i, 0), C(i, 1), C(i, 2)$ where $C(i, d)$ is made up of the set of all those coins whose clockwise label has digit d int he i-th place form the left. Note that since the cyclic permutation $0 \mapsto 1, 1 \mapsto 2, 2 \mapsto 0$ maps $C(i, 0)$ to $C(i, 1)$, $C(i, 1)$ to $C(i, 2)$ and $C(i, 2)$ to $C(i, 0)$, these sets are equinumerous; that is,

$$|C(i, d)| = N/3 \;\; \forall \;\; i \leq n, d = 0, 1, 2.$$

The algorithm is described as follows.

At the i-th weighing, the coins for $C(i, 0)$ and $C(i, 2)$ are placed on the left side and the right side of the pan respectively. The set $C(i, 1)$ is kept aside.

We define the number a_i to be $0, 1$ or 2 according as to whether the left side is heavier, both are equal or the right is heavier in the i-th weighing.

Consider the number $A = 3^{n-1}a_1 + 3^{n-2}a_2 + \cdots + 3a_{n-1} + a_n$.

From the i-th weighing, it is clear then that the defective coin is heavier and its clockwise label has a_i as its i-th digit, or lighter and its anticlockwise label has a_i as its i-th digit.

Therefore, after n weighings, we know all the a_i's and the coin one of whose labels is the number A is the defective coin. Also, it is heavier or lighter according to whether this label is clockwise or anticlockwise!

6.2 *Stable Table*

In 1951, Dyson came up with a very clever argument to prove that corresponding to any continuous function f defined from the 2-dimensional unit sphere S^2 to the real numbers, one can find two orthogonal diameters so that f takes the same value at all the four endpoints (which, of course, form the vertices of a square). The result turns out actually to be equivalent to the general case when orthogonality of diameters can be replaced with the condition that they meet at any given angle. More precisely, Dyson's theorem is equivalent to:

For ant continuous function $f : S^2 \to \mathbb{R}$*, and any fixed positive real number* r *with* $0 < r < 2$*, there are points* $x, y \in S^2$ *at distance* r *so that* f *takes the same value at* $x, -x, y, -y$.

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In 1955, C. Yang generalized Dyson's theorem to general dimensions; he showed:

For any positive integers n, d *and any continuous function from* S^{dn} *to* $\mathbb{R}^d$*, there exist* n *mutually orthogonal diameters whose* $2n$ *endpoints are mapped to the same point.*

The special case $n = 1$ is the well known Borsuk–Ulam theorem.

The theorem of Dyson on four points on a sphere is directly related to the 'stable table' problem.

A rectangular table placed on a slightly inclined floor (but one which is continuous) that wobbles as not all legs are resting, can be brought to a stable position simply by a rotation.

This is a form of the intermediate value theorem but more precisely follows from a 1954 theorem of Livesay that (generalizes Dyson's theorem from the square case and) asserts: *For any continuous function* f *defined on the unit sphere, we can position a given rectangular table with all its vertices on the sphere such that* f *takes on the same value at all four vertices.*

The above theorem of Dyson on spheres is not related to the concept of 'Dyson sphere' which is a thought experiment by him popularized in 1960. His idea was that if the energy demands of human civilization increased for a long enough period, a time would come when it would require all the energy produced by the Sun. He thought of a system of orbiting "shells" that would intercept and collect all of the Sun's energy.

6.3 *Algebraic Topology*

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This topological result, which addresses simplicial homology groups is too technical to recall here in its full generality, but it is a generalization of an easier to state theorem due to Lebesgue. Lebesgue's result asserts that if an open subset of $\mathbb{R}^n$ is covered by a finite number of closed sets of sufficiently small diameter, then some $n + 1$ of them must have a common point.

6.4 *Sixth Fermat Number*

One knows that the largest prime known of the form $2^k + 1$ is $2^{16} + 1$. The fifth Fermat number $2^{2^5} + 1$ was shown in an elementary manner to be divisible by 641 by Euler. A similar elementary proof was given by Dyson to prove that $2^{2^6} + 1$ is divisible by 274177, and therefore, cannot be prime. Basically, Dyson's observation amounts to the following:

If f is odd and $2^n \equiv 1 \bmod f$, and if the number $q = 1 + 2^r f$ divides $1 + 2^{2m}\left(\frac{2^n-1}{f} - 2^r\right)^2$, the q also divides $1 + 4^{m+n+r}$.

This enables him to not only obtain Euler's result by checking that $q = 641 = 1 + 2^7 \times 5 = 2^4 + 5^4$ divides $1 + 2^{10}5^6$ (corresponds to $m = 5, n = 4$ in Dyson's observation) but also shows that since $q = 274177 = 1 + 2^8 \times 3^2 \times 7 \times 17$ divides $1 + 15409^2$ (corresponds to $m = 0, n = 24$), it divides also $1 + 4^{32} = 1 + 2^{2^6}$.

6.5 *Dyson Transform for Ramare*

In 1995, Olivier Ramare proved that every even integer is a sum of at the most 6 prime numbers. A crucial lemma that went into his proof was due to Dyson, and is now known as the Dyson transform; it permits us to 'transform' elements from one summand to another. More precisely, this is the following Lemma:

Let $A = \{a_1 < a_2 < \cdots\}, B = \{0 = b_1 < b_2 < \cdots\}$ *be sequences of non-negative integers. For any* $e \in A$*, define*

$$A' = A \cup (B + e), B' = B \cap (A - e).$$

Then, we have $A' + B' \subset A + B, e + B' \subset A', 0 \in B'$ *and, for each* m*,*

$$(A \cap [1, m]) + (B \cap [1, m - e]) = (A' \cap [1, m]) + (B' \cap [1, m - e]).$$

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Suggested Reading

[1] **F J Dyson, Some guesses in the theory of partitions, *Eureka*, (Cambridge) Vol.8, pp.10–15, 1944.**

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