F J Dyson[*]

The Man Who Would Make Patterns and Disturb the Universe

Patrick Das Gupta

Freeman J. Dyson, a brilliant theoretical physicist and a gifted mathematician, passed away on 28 February 2020 at the age of 96. A vignette of his outstanding contributions to physical sciences, ranging from the subject of quantum electrodynamics to gravitational waves, is provided in this article. Dyson's futuristic ideas concerning the free will of 'intelligent life' influencing the remote future of the cosmos with 'Eternal Intelligence', Dyson tree, Dyson sphere and so on, have also been discussed briefly.

1 Introduction

Let me begin with some curious observations: A brilliant young person after publishing research papers of significance in pure mathematics, moved over to physics and, within a year or so, published a far-reaching article that would continue to guide future theories. This gem of a research paper synthesized, in a very elegant manner, discordant formalisms propounded independently by three other outstanding theoretical physicists, who later shared the Nobel Prize in Physics for their pioneering work.

With time, our bright and innovative protagonist kept trotting peripatetically in the realm of physical sciences from one field to another. Several distinct scientific ideas are named after him in the area of physical sciences. To top it all, this individual never had an official PhD degree. How often, in the history of science, does one encounter such a sequence of events?

The immensely gifted person, I am referring to, is none other

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than Freeman John Dyson, who sadly left this world on 28 February 2020. Freeman Dyson was born in England to Mildred Lucy Atkey and George Dyson on 15 December 1923. It is generally acknowledged that Dyson's father, Sir George, had been a very talented composer as well as a teacher-musician.

Freeman Dyson excelled in his school studies and at the age of twelve, he moved to Winchester College after winning the first position in a scholarship test in 1936. While he was vigorously pursuing mathematics, concentrating on his favourite subject, number theory, Dyson had already studied Eddington's *The Mathematical Theory of Relativity* by 1939.

In 1941, Dyson was offered a scholarship to study in the Trinity College, Cambridge, where he was taught by famous mathematicians like the legendary G H Hardy. In Cambridge, he also studied physics under one of the greatest theoretical physicists, P A M. Dirac. After World War II, his mentors at Cambridge advised him to take up physics. In particular, the renowned fluid dynamics expert, Sir Geoffrey Ingram Taylor, penned a letter of reference to Hans Bethe, a brilliant theoretical physicist and later a Nobel Laureate, of Cornell University, USA, containing the following lines: "You'll have received an application from Mr. Freeman Dyson to come to work with you as a graduate student. I hope that you will accept him. Although he is only 23 he is in my view the best mathematician in England."

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– Geoffrey Ingram Taylor to Hans Bethe.

Dyson joined Hans Bethe in 1947, and thus began his adventures in physics. Bethe, during that time, inspired by H Kramer's idea of 'renormalization' that the measured energy of a charged particle is a sum of its bare energy and the self-energy acquired by its interactions with the electromagnetic field generated by itself, had published a non-relativistic calculation to explain tiny shifts of the energy levels of a hydrogen atom that was seen in the Lamb-Retherford experiment but was not predicted by Dirac's relativistic electron-positron theory [1–3].

Dyson lost no time in using his mathematical talents to perform a relativistic calculation (while ignoring the intrinsic spin of elec-

trons) to show that the shifts match accurately with experimental results. This work was received by the editors of *Physical Review* on 8 December 1947 and was published in 1948 [4]. Incidentally, around the same time, Julian Schwinger too had published a relativistic calculation to explain the Lamb-Retherford shift of energy levels [5].

Bethe by then had recognized how prodigiously gifted Dyson was, and he went all the way to convince J Robert Oppenheimer of the Institute for Advanced Study, Princeton, that Dyson be accepted in the Institute. Meanwhile, Dyson was discussing with Feynman as well as Schwinger to learn about their distinct formulations concerning the interaction of radiation with charge particles. In 1948, Dyson joined the Institute for Advanced Study, where stalwarts like Einstein, John von Neumann and Kurt Gödel were deeply immersed in research on the fundamental aspects of physical and mathematical sciences.

When Feynman shifted to California Institute of Technology in 1950, Dyson was offered Feynman's professorship at Cornell University, Ithaca. Dyson, after spending about three years in Ithaca, returned to the Institute for Advanced Study as a professor in 1953, and continued his research studies at Princeton, writing prolifically on various topics, till he breathed his last on 28 February 2020.

2 Dyson and QED

Quantum electrodynamics (QED) is a relativistic quantum theory of electromagnetic field, charge particles and their interactions. QED describes, with great precision, myriads of physical processes involving electrons, positrons and photons—creation and annihilation of electron-positron pairs, high energy scattering between electrons, positrons and photons, vacuum polarization that causes the observed Lamb-Retherford shift in hydrogen atoms, etc. Vacuum polarization can be understood intuitively as screening of the electric charge of the atomic nucleus due to spontaneous creation of virtual electron-positron pairs taking place con-

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September 14, 1948:

"... On the third day of the journey a remarkable thing happened; going into a sort of semi-stupor as one does after forty-eight hours of bus riding, I began to think very hard about physics, and particularly about the rival radiation theories of Schwinger and Feynman. Gradually my thoughts grew more coherent, and before I knew where I was, I had solved the problem that had been in the back of my mind all this year, which was to prove the equivalence of the two theories. ..."

The sheer elegance, clarity and sincerity of Dyson's paper stare at one's face. The sequence of names of the physicists appearing in the title is significant. The research paper of Sin-Itiro Tomonaga, indeed was the first one on the subject, followed by the published works of Tomonaga and his co-workers, of Julian Schwinger and then of Richard Feynman [8–17]. Dyson endeavoured to emphasize, in a footnote of his celebrated paper, that Tomonaga and his collaborators had an unequivocal head start in building QED [6].

For a proper understanding of QED, it is, of course, necessary to study the standard literature on the subject that is readily available. Dyson's 1949 paper, which is remarkably pedagogical and crystal clear in its approach, may form an excellent supplement [6]. To appreciate Dyson's paper as well as to motivate young readers to study it, a fleeting glimpse of the basic pre-requisites

has been provided below.

It is well known that fundamental particles not only have intrinsic spin angular momentum, they must also respect the special theory of relativity. In the framework of relativistic quantum mechanics, spin 1/2 free fermions with rest mass m are described by the Dirac equation,

$$\left[i\hbar\gamma^\mu \frac{\partial}{\partial x^\mu} - mc \right] \Psi(x^\alpha) = 0, \tag{1}$$

where the index μ is being summed over $0, 1, 2, 3$ in the product involving the partial derivatives and the Dirac matrices, $\gamma^i \equiv \beta\alpha^i$ for $i = 1, 2, 3$ and $\gamma^0 \equiv \beta$. (The Einstein summation convention wherein repeating indices are assumed to be summed over, has been used throughout this article.)

In eq.(1), $\Psi(x^\alpha)$ represents a column vector with four complex functions $\psi_k(x^\alpha)$, $k = 1, 2, ..., 4$ as entries, describing the spin as well as the anti-particle degrees of freedom,

$$\Psi(x^\alpha) = \begin{pmatrix} \psi_1(x^\alpha) \\ \psi_2(x^\alpha) \\ \psi_3(x^\alpha) \\ \psi_4(x^\alpha) \end{pmatrix} . \tag{2}$$

Dirac equation can be used to study electrons moving with relativistic energies. However, eq.(1) admits both positive energy $E_+ \geq mc^2$ and negative energy $E_- \leq -mc^2$ solutions. Presence of negative energy solutions led Dirac to predict existence of positrons. In the case of plane wave solutions of eq.(1), one may pull out the time-dependent parts and express them as,

$$\Psi_+(x^\alpha) = e^{-\frac{i}{\hbar}E_+ t} \, \mathcal{U}_{+\vec{p}}(\vec{r}) \quad \text{and} \quad \Psi_-(x^\alpha) = e^{-\frac{i}{\hbar}E_- t} \, \mathcal{V}_{-\vec{p}}(\vec{r}). \tag{3}$$

Using the superposition principle, since eq.(1) is linear, general wavepackets for a free spin 1/2 particle can be constructed out of the plane wave solutions of eq.(3). However, relativistic quantum mechanics limiting to just a single spin 1/2 particle is fraught with problems, e.g. Klein paradox.

One can show that combining special relativity with quantum theory necessitates a quantum 'many particle' description . As an

example in 1-dimension [18], if one localizes a particle of rest mass m to a region of Compton wavelength size $\Delta x \sim h/mc$, the uncertainty in the momentum $\Delta p \gtrsim h/\Delta x = mc$. But then, the uncertainty in energy is $\Delta E \sim c\,\Delta p > mc^2$. Hence, the energy uncertainty is large enough to create extra particles.

A natural framework to describe relativistic, 'many particle' quantum systems is the theory of quantum fields. For instance, in a quantum field theory (QFT), $\Psi(x^\alpha)$ appearing in the Dirac equation is elevated to an operator status, $\hat{\Psi}(x^\alpha)$, and the Hamiltonian $\hat{H}_\Psi$, which is the energy operator, is constructed out of the Lagrangian that leads to eq.(1). The particle states corresponding to electrons and positrons are simply the eigenstates of $\hat{H}_\Psi$.

In the classical electromagnetic sector, the time evolution of free electric field $E^i = F^{0i}$ and magnetic field $B_i = \epsilon_{ijk}F^{jk}$ are obtained by solving the Maxwell equations,

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = 0 \,, \tag{4}$$

$F_{\mu\nu}(x^\alpha)$ being the electromagnetic field tensor given by,

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}, \tag{5}$$

where $A_\mu(x^\alpha)$ is the electromagnetic (EM) potential with time-component A_0 being the usual scalar potential $\phi(\vec{r}, t)$ while the space components A^i, $i = 1, 2, 3$ constitute the vector potential $\vec{A}(\vec{r}, t)$.

In the quantum domain, $A_\mu(x^\alpha)$ is turned into a field operator $\hat{A}_\mu(x^\alpha)$. Naturally, the electric and magnetic fields too become space-time dependent operators, and it is their eigenvalues that can be measured at different points as the observable field strengths. Spin 1 photon states emerge out of the QFT of EM radiation as the energy eigenstates of the Hamiltonian $\frac{1}{8\pi} \int (\hat{E}^2 + \hat{B}^2)d^3r$.

So far we have described only free fields. But one must include interaction of charge particles with EM fields in the theory. The mathematical form of the coupling between a charge particle and EM fields emerges very naturally when one demands the full theory to be gauge invariant. In case of the EM fields, it is well

known that given an arbitrary but smooth function $\chi(\vec{r}, t)$, the electric and magnetic fields remain the same under the following gauge transformations,

$$\vec{A}(\vec{r}, t) \rightarrow \vec{A}'(\vec{r}, t) = \vec{A}(\vec{r}, t) + \nabla \chi \tag{6}$$

and,

$$\phi(\vec{r}, t) \rightarrow \phi'(\vec{r}, t) = \phi(\vec{r}, t) - \frac{1}{c} \frac{\partial \chi}{\partial t}, \tag{7}$$

so that the Maxwell's equations are covariant not only under Lorentz transformations but they are also under the above gauge transformations.

Going back to the relativistic quantum mechanics described by eq.(1), if one demands that the Dirac equation be covariant under,

$$\Psi(\vec{r}, t) \rightarrow \Psi'(\vec{r}, t) = e^{iq\frac{\chi(\vec{r}, t)}{\hbar c}} \Psi(\vec{r}, t), \tag{8}$$

as well as under eqs.(6) and (7), it is necessary that the Dirac equation be of the form,

$$\left[i\hbar \gamma^\mu \left(\frac{\partial}{\partial x^\mu} + \frac{iq}{\hbar c} A_\mu(\vec{r}, t) \right) - mc \right] \Psi(x^\alpha) = 0. \tag{9}$$

(For a simple exposition to the details related to gauge invariance in the context of Schrödinger equation, one may read [19].)

The above equation describes a spin 1/2 fermion carrying an electric charge q and interacting with the EM potential A_μ. The wavefuntion $\Psi(\vec{r}, t)$ at any time t can also be obtained using a technique very similar to the Green's function method, given any initial state $\psi(\vec{r}, t_0)$,

$$\psi(\vec{r}, t) = \int K(\vec{r}, \vec{r'}; t, t_0) \psi(\vec{r'}, t_0) d^3 r', \tag{10}$$

if the propagator $K(\vec{r}, \vec{r'}; t, t_0)$ corresponding to eq.(9) has already been determined. In general, one determines the free propagator (i.e. when $A_\mu = 0$ everywhere in the space-time manifold) and then use perturbation theory to take into account interactions.

The propagator method was championed by Feynman in which various interaction processes like vacuum polarization, pair creation, Bhabha scattering, etc., were calculated quite successfully

[15–17]. Feynman used a space-time approach while imposing consistency with the tenets of quantum theory which ordains that one must include superposition of all probability amplitudes corresponding to the intermediate processes that cannot be observed. Feynman's paper on the theory of positron, submitted after Dyson's 1949 publication, of course, acknowledges Hans Bethe and Dyson for the fruitful discussions [16].

To get a gist of the space-time approach, one may consider a very simplified description of an electron getting scattered by a photon. Feynman's method would use the probability amplitude ψ_{AB} for an electron to freely propagate from point A to B by employing the propagator of the free Dirac equation (eq.(1)), and then consider a contact interaction with the EM potential A_μ representing an incoming photon state at point B using a suitable expression determined from the interaction term in eq.(9) and take latter's product with ψ_{AB} to obtain a virtual state that goes from B to C.

Finally, the probability amplitude for the electron as well as an outgoing photon to freely travel from C is calculated by taking the inner product with the amplitude corresponding to the virtual state. Since, points B and C where the interactions have been considered are arbitrary and cannot be observed in experiments, the net probability amplitude for the electron-photon scattering is arrived at by integrating over all possible B and C as demanded by the quantum principle of superposition.

Subsequently, following Stückelberg's interpretation that a positron can be thought of as a electron going backwards in time [20], Feynman could calculate scattering cross-section of electron-positron interactions (i.e. Bhabha scattering) correctly [16]. There is a simple way to see why in Dirac's hole theory, a positron may also be interpreted as an electron for which time is running backwards. In the standard viewpoint, $tE_- < 0$ for negative energy solutions since $E_- < 0$ and $t > 0$ (eq.(3)). But one may also interpret $tE_- < 0$ as due to energy being positive (as in the case of electrons) but time t instead is negative since its direction is reversed. The space-time approach has been more enduring and appealing as it enabled one to quickly visualize the terms that need to be

calculated for a given physical process.

However, unlike the case-by-case technique developed by Feynman, the formalisms developed independently by Tomonaga and Schwinger are mathematically more rigorous, elegant and complete. They are essentially relativistically covariant formulations involving field operators corresponding to spin 1/2 fermions interacting with EM fields [8–14]. To get a flavour of the operator based field theory aspects, one needs to start from the Lagrangian density approach to classical field theory as illustrated below for the spin 1/2 fermions.

Since Ψ is a column vector field, using the adjoint operation, one may define a row vector field,

$$\bar{\Psi}(x^\alpha) \equiv \Psi^\dagger(x^\alpha)\,\gamma_0 = \Psi^\dagger(x^\alpha)\,\beta. \tag{11}$$

Then, eq.(9) can be derived by employing calculus of variation from the following Lagrangian density,

$$\mathcal{L}_D = \bar{\Psi}(x^\alpha)\left[i\hbar\,\gamma^\mu\left(\frac{\partial}{\partial x^\mu} + \frac{iq}{\hbar c}\,A_\mu(\vec{r},t)\right) - mc\right]\Psi(x^\alpha)\,. \tag{12}$$

One may express the above Lagrangian density as a sum of the free Lagrangian density $\mathcal{L}_{D0}$ that only involves the fermionic degrees of freedom and an interaction term containing the coupling between the charged fermions and the EM potential,

$$\mathcal{L}_D \equiv \mathcal{L}_{D0} + \mathcal{L}_{int}, \tag{13}$$

where,

$$\mathcal{L}_{D0} \equiv \bar{\Psi}(x^\alpha)\left[i\hbar\,\gamma^\mu\,\frac{\partial}{\partial x^\mu} - mc\right]\Psi(x^\alpha) \quad \text{and} \quad \mathcal{L}_{int} \equiv -\frac{1}{c}\,j^\mu\,A_\mu\,, \tag{14}$$

$j^\mu(x^\alpha) = q\,\bar{\Psi}(x)\,\gamma^\mu\,\Psi(x)$ being the 4-current density.

Now, in the framework of classical non-relativistic mechanics of point particles, one begins with a Lagrangian $L(q,\dot{q},t)$ corresponding to a particle and obtains the canonical momentum $p \equiv \frac{\partial L}{\partial \dot{q}}$. Using the particle's canonical momentum p, one arrives at the classical Hamiltonian $H(q,p,t) = p\dot{q} - L$.

In the standard canonical quantization scheme, if one wishes to quantize this system, one turns q and p into linear operators $\hat{q}$ and $\hat{p}$, respectively, and imposes the commutation relation,

$$[\hat{q}, \hat{p}] \equiv \hat{q}\hat{p} - \hat{p}\hat{q} = i\,\hbar, \tag{15}$$

so that one possesses a quantum Hamiltonian operator $\hat{H}(\hat{q}, \hat{q}, t)$ (i.e. the energy observable) starting from the classical Lagrangian $L(q, \dot{q}, t)$.

An analogous program is carried out in the case of relativistic quantum fields, except that the infinitely many degrees of freedom of a physical field operator like $\hat{\Psi}(x^{\alpha})$ or $\hat{A}_{\mu}(x^{\alpha})$ and special relativity bring their own subtleties and complications. In this quantization program, firstly the canonical momentum density field and the energy momentum tensor field are obtained from the classical Lagrangian density of a field e.g. $\mathcal{L}_{D0}$ for the fermionic field Ψ. Then, these fields are elevated to the status of field operators by setting up appropriate canonical commutation brackets between the field operators and their canonical momentum operators.

Once the Hamiltonian density operator, $\hat{\mathcal{H}}(\vec{r})$ (e.g. $\hat{\mathcal{H}}_{EM} = \frac{1}{8\pi}(\hat{E}^2 + \hat{B}^2)$, in the case of EM fields), is derived from the Lagrangian density using the canonical prescription, the time evolution of any physical state-vector, $|\psi>$, can be determined from the Schrödinger picture,

$$i\hbar \frac{d|\psi>}{dt} = \hat{H}|\psi(t)> = \int \hat{\mathcal{H}}(\vec{r})d^3r\,|\psi(t)>, \tag{16}$$

where $\hat{H} \equiv \int \hat{\mathcal{H}}(\vec{r})d^3r$ is the Hamiltonian operator.

(In quantum mechanics, the wavefunction $\psi(\vec{r}, t)$ is simply the inner product $(|\vec{r}>, |\psi(t)>) = <\vec{r}\,|\psi(t)>$, with $|\vec{r}>$ being an eigenvector of the position operator $\hat{\vec{r}}$ corresponding to the eigenvalue $\vec{r}$.)

If the Hamiltonian $\hat{H}$ does not depend on time (as in the case of free fields) then for any initial state $|\psi(0)>$, it is straightforward to show from eq.(16) that the state undergoes a unitary evolution generated by the Hamiltonian,

$$|\psi(t)> = e^{-\frac{i}{\hbar}\hat{H}t}\,|\psi(0)> = e^{-\frac{i}{\hbar}t\int \hat{\mathcal{H}}(\vec{r})d^3r}\,|\psi(0)>, \tag{17}$$

In such a situation, one may adopt a Heisenberg picture description wherein the state-vectors remain the same and only the physical observables change with time. But when time-dependent interaction terms are present so that eq.(16) takes the form,

$$i\hbar \frac{d|\psi>}{dt} = (\hat{H} + \hat{H}_{int}(t))|\psi(t)> = \int (\hat{\mathcal{H}}(\vec{r}) + \hat{\mathcal{H}}_{int}(\vec{r}, t))d^3r \, |\psi(t)>,$$

(18)

it is convenient to use the interaction picture in which the state-vector $|\psi(t)>_I$ is define by,

$$|\psi(t)>_I \equiv e^{+\frac{i}{\hbar}\hat{H}t} |\psi(t)>,$$

(19)

where $|\psi(t)>$ is the usual Schrödinger picture state-vector. From eq.(19), it is obvious that the state-vector $|\psi(t)>_I$ satisfies the following equation,

$$i\hbar \frac{d|\psi>_I}{dt} = \hat{H}_{int}(t) |\psi(t)>_I = \int \hat{\mathcal{H}}_{int}(\vec{r}, t) \, d^3r \quad |\psi(t)>_I .$$

(20)

In the case of QED, using eq.(14), one may express the interaction Hamiltonian density operator as, $\hat{\mathcal{H}}_{int}(\vec{r}, t) = \frac{1}{c} \hat{j}^\mu \hat{A}_\mu$.

But eqs.(20) corresponds to a particular inertial frame S for which a time t has been established everywhere in S by synchronizing all the distributed clocks in the frame. If one wishes to express the evolution of a state-vector in a manifestly covariant manner, one needs to express the evolution with respect to space-like 3-dimensional hypersurfaces Σ. Tomonaga-Schwinger formalism had made use of functional derivatives with respect to hypersuface considering an infinitesimal deformation of the space-like hypersurface Σ to obtain an evolution equation in the interaction picture of the form,

$$i\hbar \Big(|\psi(\Sigma + \delta\Sigma)>_I - |\psi(\Sigma)>_I \Big) = \frac{1}{c} \Big(\int_\Sigma^{\Sigma + \delta\Sigma} \hat{\mathcal{H}}_{int} \, d^4x \Big) |\psi(\Sigma)>_I,$$

(21)

which is manifestly covariant as can be deduced from the integral over the space-time proper volume.

Schwinger had presented a sophisticated covariant quantization scheme for fields in great detail [12]. The axiomatic style of

the paper that systematically takes up various subtle issues in a complete and coherent manner is simply awesome. But because of the mathematical rigor, Schwinger's papers on QED appear formidable too.

Schwinger had adopted the interaction picture wherein the state-vector representing a system of photons and charged spin 1/2 fermions evolve from one space-like hypersurface to another only due to the interaction part of the Hamiltonian similar to eq.(21). Both Schwinger and Dyson had emphasized in their papers that all space-time dependent physical observables (i.e. hermitian operators) must mutually commute and thereby be measurable together accurately on any space-like hypersurface. The modern name for this concept is micro-causality. Micro-causality is a consequence of combining both quantum theory and special relativity, as argued below.

It is well established from experiments that when an observable $\hat{O}$ is measured with great precision, the state-vector collapses immediately to one of the former's eigenvectors. Operators that commute have simultaneous eigenvectors, and thus, a set of mutually commuting observables can all be measured simultaneously. This is the reason why, in an ideal measurement, position and momentum cannot be measured simultaneously, as there is no common eigenvector of $\hat{x}$ and $\hat{p}$ since $[\hat{x}, \hat{p}] = i\hbar$.

Now, on a space-like hypersurface Σ, no two events: E_1 at (t_1, x_1, y_1, z_1) and E_2 at (t_2, x_2, y_2, z_2), can be causally connected as $c^2(t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2 < 0$ on Σ. Even a ray of light would not be able to link E_1 and E_2. Hence, one ought to be able to measure any two physical quantities $\hat{O}_1$ at E_1 and $\hat{O}_2$ at E_2 accurately, since the measurements cannot influence each other as they lie outside each other's light cone.

Therefore, simultaneous eigenstates of $\hat{O}_1(E_1)$ and $\hat{O}_2(E_2)$ must exist. This implies that $[\hat{O}_1(E_1), \hat{O}_2(E_2)] = 0$, which precisely is the statement of micro-causality. In particular, even the commutator of a field operator at E_1 and its canonically conjugate momentum field operator at E_2 must vanish if E_1 and E_2 are space-like

separated.

Although Dyson's starting point was Schwinger's approach, he simplified the mathematical apparatus employed by Schwinger considerably without losing track of the physical processes like higher-order radiative reactions and, moreover, he went beyond Schwinger's formulation [6]. In order to solve eq.(21), Dyson introduced perturbation theory and employed a 1-parameter family of space-like hypersurfaces to obtain a unitary operator which in the asymptotic limit reduces to Heisenberg's S-matrix. In a follow-up paper, Dyson obtained integral equations for Green's functions that ensue from field equations [21]. Schwinger, employing alternate methods, arrived independently at the same conclusion [22]. This set consisting of an infinity of hierarchy equations goes by the name of Dyson-Schwinger equations.

In Feynman's formulation of QED, as discussed earlier, every physical process is assigned a probability amplitude in accordance with the principles of quantum theory applied to the Dirac equation (after switching on the EM fields i.e. eq.(9)). It is not surprising that the space-time approach appeared very dissimilar to the Tomonaga-Schwinger formalism, whose focus among other things was to obtain covariant unitary evolution operators based on dynamical fields until Dyson arrived at the scene. He provided a very systematic formulation of Feynman's approach in terms of S-matrix theory [6, 21].

Not only did he prove the equivalence of Feynman and Tomonaga-Schwinger formulations, but he also showed that the renormalization technique gives a finite result at all orders of the perturbation theory. For QED, Dyson was the first person to lay out an exhaustive treatment of renormalization [21]. These were the crucial steps that led the 1965 Nobel Prize in Physics being awarded to Tomonaga, Schwinger, and Feynman.

3 Ferromagnets, Stability of Matter, Gravity and Dyson Bound

Around the 1950s, Dyson was drawn to the subject of ferromagnets and spin waves. Ferromagnets are basically substances in

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which magnetic dipole moments at the lattice sites get aligned in the direction of an applied magnetic field. These magnetic dipole moments associated with atoms (like iron, nickel, or cobalt) arise due to their uncompensated electronic spins. In a seminal study, Felix Bloch had introduced the notion of spin waves in 1930 and had argued that the spin-wave degrees of freedom are important for ferromagnetism at temperatures far below the Curie temperature [23].

As the name suggests, spin waves (the corresponding quanta being called the magnons) are essentially undulating changes in the orientation of magnetic dipole moments propagating along the lattice sites of a ferromagnetic crystal. Following Bloch's insight, calculations were done subsequently by several authors pertaining to the spontaneous magnetization at low temperatures, beyond the leading order term with a temperature dependence of $T^{3/2}$ that Bloch [23] had obtained. But these studies led to conflicting reports on the thermodynamic aspects of ferromagnets until Dyson's two back-to-back papers appeared on this subject in 1956. In the first paper, Dyson addressed the problems posed by the non-orthogonal spin-wave states and provided a suitable formalism to calculate thermodynamic quantities, in terms of these states in the context of a ferromagnet [24].

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Dyson also studied the collision of spin waves and, in particular, gave an exact expression for the free energy that incorporated the effects of spin wave interactions, for he had also calculated the scattering cross-section for two interacting spin waves at low temperatures. In the second paper, Dyson demonstrated the flaws in the earlier papers and calculated the free energy of a Heisenberg model for an ideal ferromagnet, using the techniques developed in his first paper [24] and showed that the effects of interacting spin waves in the spontaneous magnetization begin at order with temperature dependence of T^4 [25].

In the 1960s, inspired by an address of P Ehrenfest to W Pauli that, "...(why) matter should occupy so large a volume... But why are the atoms themselves so big?", Dyson turned his attention to the study of stability of macroscopic systems. Dyson & Lenard

[26], Dyson [27], and Dyson & Lenard [28] were the first to prove rigorously some powerful theorems concerning the importance of Pauli's exclusion principle in the stability of bulk matter made up of non-relativistic fermions interacting with each other through electrostatic forces.

For a quantum system made up of N negatively charged fermions and an arbitrary number of positively charged particles, they proved that there exists a lower bound E_0 to the minimum energy E_{min} so that,

$$E_{min} > E_0 \propto -N\left(\frac{m_e e^4}{2\hbar^2}\right). \tag{22}$$

On the other hand, by considering a non-relativistic quantum system consisting of an equal number N of positively and negatively charged bosons all with the same magnitude of charge $|e|$ as well as identical electronic mass m_e, Dyson proved that if the interactions are purely Coulombic then the ground state energy of the Hamiltonian has an upper bound [27],

$$E_N < -\frac{1}{1944\pi^4}\left(\frac{m_e e^4}{2\hbar^2}\right)N^{7/5} \quad , \tag{23}$$

indicating that the system is likely to be unstable. Hence, the results given by eq.(22) and eq.(23) underscore the significant role that Pauli's exclusion principle plays in ensuring the stability of macroscopic systems and, therefore, of the entire physical world.

According to eq.(23), for bulk matter containing an Avogadro number of such bosons, the energy released while going to the ground state would be at least $\sim 10^{23}$ erg! Interestingly enough, if one considers an astronomically large number of identical, ultra-light bosons interacting only via Newtonian gravity, one can show that such a system is likely to collapse gravitationally into a black hole [29, 30].

Just after publishing his papers on the stability of bulk substances, Dyson shifted his gaze at the topic of gravitational waves (GWs). In any relativistic theory of universal gravitation, if energy and momentum (both being the source of gravity) of or within a body change in an asymmetric manner, the resulting changes in the

gravitational effects would necessarily have to propagate in the form of a wave with speed $\leq c$. Simply put, this is how GWs are generated. In Einstein's general relativity (GR), GWs are produced whenever mass quadrupole moment tensor associated with a source changes with time (see e.g. [30]).

Spurred by J Weber's theoretical work on GWs as well as his stupendous efforts in building a very sensitive resonant bar detector to observe GWs, Dyson calculated the effect of GWs having frequencies in ~ 1 Hz band on bodies like Earth, assuming them to be elastic solids [31]. Dyson incorporated compression, shear-wave, as well as rotation in his analysis and showed that the response to GWs depended on the non-uniformities in the shear-wave modulus. He found that GWs are absorbed by an elastic object when the shear-wave modulus is inhomogeneous. This was the first time that excitations of the normal modes of oscillations of an astrophysical body, like Earth, due to incident GWs was being addressed.

However, in the field of GWs studies, Dyson's name would forever be associated with a conjecture on an upper bound on GW luminosity ([32]; but also see the references provided in [30], on this topic). The quantity $\frac{c^5}{G} = 3.6 \times 10^{59}$ erg s^{-1} $\equiv L_{Dyson}$, which has the dimension of luminosity, is called the Dyson bound on the GW luminosity. It is generally surmised that no GW source can radiate energy at a rate exceeding L_{Dyson}.

Charged with an insight that intelligent species could influence the fate of the evolving universe, Dyson published an exceedingly thought-provoking review paper—'Time without end: Physics and biology in an open universe'

It is somewhat surprising to note that L_{Dyson} emerges also when one considers the well known Planck energy, $E_{Pl} \equiv m_{Pl}c^2 \equiv \sqrt{c^5\hbar/G}$ and the Planck time, $t_{Pl} \equiv \sqrt{\hbar G/c^5}$ to obtain a Planck luminosity [30],

$$\frac{E_{Pl}}{t_{Pl}} = \frac{c^5}{G} \quad , \tag{24}$$

since the Planck constant gets cancelled. Now, quantum fluctuations could have generated many forms of radiation at the time of the initial big bang with a characteristic luminosity $\frac{E_{Pl}}{t_{Pl}} = L_{Dyson}$, but only the classical gravitational signature from GR survives in this expression. There is perhaps an underlying subtlety in the concept of Dyson bound that is yet to be fathomed.

4 Final Leaps of Eternal Intelligence, Trees and Spheres in order to Disturb the Cosmos

After going through Jamal Islam's 'Possible ultimate fate of the universe' [33], Dyson was induced into contemplating the very distant future of the cosmos that follows from the big bang model. Charged with an insight that intelligent species could influence the fate of the evolving universe, Dyson published an exceedingly thought-provoking review paper—'Time without end: Physics and biology in an open universe' [34].

This article is essentially based on the *James Arthur Lectures on Time and its Mysteries* that he had delivered at New York University, in 1978. It begins with reviewing some of the seminal works of Marteen Rees and Steven Weinberg but quickly goes into Dyson's original ideas pertaining to the remote cosmic future.

As the second law of thermodynamics (i.e. monotonic increase in entropy of an isolated system) is invincible, Dyson asked whether it is possible to maintain life and intelligence with the stellar sources of energy (necessary for food production and metabolic activities) fading away eventually in an ever-expanding universe? Dyson explored the possible maneuvers that could be undertaken by a super-advanced civilization to survive as well as to continue thinking, albeit intermittently as the 'intelligent being or automaton' would need to go into long hibernation to ration dying energy resources. This idea is often referred to as Dyson's Eternal Intelligence.

He discussed such futuristic concepts even in his autobiographical book 'Disturbing the Universe' [35]. Dyson's prodigious output is unique and remarkable because not only he proved rigorous theorems in diverse fields of physics based on hard and cold mathematical calculations, he also made imaginative but quantitative speculations concerning the remote future. For instance, taking into account the presence of water in comets, he speculated on the possibility of growing genetically engineered plants on approaching comets (the so-called Dyson Tree).

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Similarly, noting that most of the energy radiated away almost isotropically by stars go wasted, advanced civilization on habitable planets like Earth could increase the efficiency of harnessing the stellar energy by surrounding their host stars in a near isotropic manner by orbiting asteroid-like objects mounted with devices (e.g. 'stellar panels') to absorb the stellar radiation for useful purposes.

One of the consequences of having such a 'Dyson Sphere' (DS) installed by an extraterrestrial civilization around its star would be that the constituent objects of the DS would re-radiate in the infrared part of the visible spectrum, and therefore would be a likely signature of the presence of extraterrestrial intelligence in exoplanets. Dyson's thoughts must be echoing around the frenetic activities currently taking place pertaining to the spate of discoveries of exoplanets.

Acknowledgements

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