The Knight’s Tour Problem and Rudrata’s Verse*
A View of the Indian Facet of the Knight’s Tour

G S S Murthy

If a chess-knight is moved on a vacant chess-board $[8 \times 8$ square] such that it visits each one of the 64 squares once and once only, the knight is said to execute a Knight’s Tour. Solution to the knight’s tour problem was known in India as early as the 9th century AD as a demonstration of wizardry in composing 32-syllable verses in Sanskrit. A pair of meaningful verses is composed in such a manner that when one verse is written serially (left to right and top to bottom) one syllable a square to fill up $8 \times 4$ cells — half of a chess board – the other verse appears as the Knight’s Tour. The earliest example of this skill in poetry-composition is given in a Sanskrit treatise on poetics, kāvyālaṅkāra written by Rudraṭa who lived around the ninth century A.D. Knight’s Tour as a mathematical problem was first noticed and discussed in the West by Leonard Euler in the eighteenth century. After providing the back ground to the subject as a puzzle on the chess-board, a problem in mathematics and as a challenge in verse-composition, the article discusses the special characteristic of Rudrata’s example where the pair of verses reduces to a single verse.

1. Introduction

If a chess-knight is moved on a vacant chess-board $[8 \times 8$ square] such that it visits each one of the 64 squares once and once only, the knight is said to execute a Knight’s Tour. The particular sequence of squares which the knight takes is the path of the knight’s tour. [In this article, we shall use the word “cell” to denote “square”.] Working out a path is actually a mathematical problem and it is not at all easy to come up with a solution, by trial and error.

Keywords
Knight’s Tour Problem, Rudraṭa, cit-rakāvyam, turaṅga-padam, Sanskrit literature, cyclic permutation.

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Solution to the knight’s tour problem was known in India as early as the 9th century AD in the guise of exemplary skill in composing 32-syllable verses in Sanskrit. A pair of meaningful verses is composed in such a manner that when a specified verse of the pair is written serially (left to right and top to bottom) one syllable a cell to fill up $8 \times 4$ cells – half of a chess board – the other verse follows the path of the Knight’s Tour. The verses have to not only meet the requirements of prosody but also have to be contextually meaningful. The earliest example of this skill in poetry-composition is given in a Sanskrit treatise on poetics, *kāvyālaṅkāra* written by Rudraṭa who lived around the ninth century A.D. This type of literary wizardry in Sanskrit comes under the general category, *citrakāvyam*, which could be translated as “figure-poetry” or “amusing poetry”. *Citrakāvyam* is again a sub-category of Śabda alaṅkāra, which deals with “word-based ornamentation” of poetry.

The knight’s tour problem which would have originated as a puzzle on the chess-board was later taken up as a problem in Mathematics. However in India it showed itself up as a way of demonstrating exemplary skill in composing verses in Sanskrit. The following paragraphs give a brief introduction to the knight’s tour problem, as a puzzle on the chess-board, as a problem in mathematics and as a challenge in verse-composition in Sanskrit.

### 2. Knight’s tour as a chess puzzle

It is believed that Chess had its origin in India, where it is called catu-ranga(having four divisions). A 12th century work Mānasollāsaḥ of king Someśvara, an encyclopaedic compendium meant for a king describes inter alia the game of caturanga. From a perusal of this work, one could surmise the following equivalence in the names of the pieces: rook = ratha (chariot), knight= turaga (horse), bishop= gaja (elephant), king = rājā (king), queen= mantrī (minister) and pawn = padāti (foot-soldier). The game seems to have spread from

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1. Movements ascribed to the chariot (ratha) and elephant (gaja) in Mānasollāsaḥ are at variance with the movements prescribed to rook and bishop in modern chess.
India via Persia to Arab countries where it was called shatranj, an obvious distortion of caturanga. The Sanskrit name for the manner a knight moves on the chess board is turaga-pada.

A 10th century Arab, Abu Bakr Al-suli, who was a king’s companion, known for his skill in playing chess, wrote a book in Arabic on chess strategy, where he discussed the knight’s tour problem as a chess-puzzle [https://en.wikipedia.org/wiki/Abu_Bakr_bin_Yahya_al-Suli]. Mānasollāsaḥ, referred to above, which was written two centuries later also describes a solution of Knight’s tour as a puzzle. It is interesting to see how Someśvara, the author of Mānasollāsaḥ explains the solution. First he provides a co-ordinate system for the 8×8 chess board. For reasons of his own, he chooses the consonants c, g, n, ṭ, r, s, p to denote the 8 columns (or rows) and vowels a ā i ī u ū e ai to denote the 8 rows (or columns). The resulting co-ordinate system for the 8 × 8 chess board will be as shown in Figure 1. Any cell is uniquely defined by the combination of the chosen consonant and vowel.

![Figure 1. Co-ordinate system of Someśvara](image-url)
Now he gives the knight’s tour as a sequence of 64 syllables broken into 8 sets of 8 syllables for convenience in memorizing. The 8 sets are as follows:

\[
\begin{align*}
\text{pa} & \text{ si pu se \ taş ne cai gă } \\
\text{nī} & \text{ cu gi ca nā \ ūa sā pē } \\
\text{sū} & \text{ pai re dai ge dā gu ci } \\
\text{ga} & \text{ dā ra pā \ sī pū sai \ ūe } \\
\text{nai} & \text{ ce nū gai cū \ gi cā \ na } \\
\text{tā} & \text{ sa pi su pe rai de nu } \\
\text{tū} & \text{ rī di ūr rī rū ĭ } \\
\text{du nī ci gā da rā ūf rū } \\
\text{[प सि पु से टे ने चे गू ]} \\
\text{नी चु गिया टा सा पी } \\
\text{मू पे रे दे गे दू गू चिया } \\
\text{ग दा र पा सी पु से टे } \\
\text{ने चे नू गे चू गी चा न } \\
\text{टा सी गु पे रे दे नु } \\
\text{दू नि ची गा द रा टी रु } \\
\end{align*}
\]

These sequences are devoid of any meaning or prosodic constraints. The resulting knight’s tour is shown in Figure 2. The knight’s tour can be traced by following the numbers in the cells 1, 2, 3 ---64.

Someśvara also gives the possible knight moves from various cells as per the following:

\[
\begin{align*}
\text{koṇapārśvasthitasīya turagasya padatrayam} & \ 573 \\
\text{koṇasthasya padadvandvarṇ prānte padacatusṭayam} & \\
\text{dvitiyavalaye kone haye padacatusṭayam} & \\
\text{dvitiyavalaye‘nyatra padaśṭṭkam nigadyate} & \\
\end{align*}
\]
Arough translation of this is given below:

“When the knight is in the corner, it has 2 moves; when it is in the cell beside the corner it has 3 moves. If it is in the side (other than

Figure 2. Knight’s tour of Someśvara.
3. Knight’s Tour as a Problem in Mathematics

It is only in the 18th century that the famous mathematician and physicist Leonard Euler heard of the chess puzzle and looked at it as a problem in mathematics. One of the solutions to the problem given by Euler is shown in Figure 4. [https://www.researchgate.net/publication/265424016_Knight’s_tours_for_cubes_and_boxes]
His solution fills up the bottom-half of a $8 \times 8$ chess-board. But the solution is such that the $33^{rd}$ move of the knight can take it to the bottom right corner cell of the top-half. A little reflection will show that the top-half can be easily filled up following the route of the bottom-half. The $64^{th}$ cell that the knight occupies is such that it can jump to the $1^{st}$ cell. The solution is therefore re-entrant and the knight can indefinitely keep moving in the endless circuit. Euler showed how new solutions can be generated from known solutions and how incomplete solutions could be repaired to lead to a knight’s tour. Euler also showed how solutions for larger chess-boards, say having $10 \times 10$ cells, could be obtained by suitably patching up solutions for smaller boards.

The next important work in arriving at a solution for the knight’s tour problem was by Warnsdorff in the 19th century who gave a heuristic rule for choosing the next move for the knight.[https://en.wikipedia.org/wiki/Knight%27s_tour#Warnsdorff%27s_rule] As per Warnsdorff, the knight should be moved to the cell from which the knight will have the fewest onward moves. Let us take an example for understanding this rule:

*Figure 5* shows a partially filled knight’s tour where the knight has reached the $7^{th}$ cell of the tour. For the next step there are 2 choices either cell C2 or cell D3. We see that from cell C2, three choices are available (A3, B4, E3) and from cell D3, 5 choices (C1, B2, B4, F4, F2) are available. As per Warnsdorff Rule, one should choose cell C2 which has less number of choices (3) and not D3 which has 5 choices. It is, of course, possible to have two or more choices for which the number of onward moves is equal; there are various methods for breaking such ties and we shall not go into the details.

If we now have a re-look at *Figure 3* which gives the number of possible moves for a knight as per Someśvara, a thought occurs if around the 12th century a rule similar to Warnsdorff rule was known among experts in the field in India.

Research work on the mathematics of the knight’s tour problem proliferated in the 20th century in several directions, such as extending the problem to squares and rectangles of different sizes and
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Figure 5. Warnsdorff Rule.

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3-D cuboids, calculating the number of all possible solutions etc. Treating the knight’s tour problem as a simpler instance of the more general Hamiltonian path problem in graph theory, attempts have been made for finding out clever heuristics and efficient algorithms for arriving at a solution.

In recent times Prof. D E Knuth, well known mathematician and computer theorist, has studied the problem and its history extensively and credit is due to him for having discovered and publicized the fact that the earliest solution to the problem originated in India not later than 9th Century AD. There are strips of his lectures on You Tube [https://www.youtube.com/watch?v=C95hHI7cVNs] which give a glimpse of his deep interest in the subject.

4. Śabda alaṅkāra (word-based ornamentation) and Citrakāvyam (Figure-poetry)

Before we get into the details of knight’s tour as a display of literary wizardry in Sanskrit, it is worthwhile knowing the background to such skills and their place in Sanskrit literature. We shall therefore provide a brief introduction to Śabda alaṅkāra and Citrakāvyam in the foregoing.

Alliteration and rhyming are well recognized as techniques in composing verses and competent poets have judiciously employed them
to advantage. Corresponding words in Sanskrit are anuprāsa (alliteration) and “yamaka” (rhyming). While the word, “rhyming” is often associated with the end of lines in a verse, “yamaka” has a broader meaning in Sanskrit.

When a consonant is repeatedly used in a line of verse, it is called “anuprāsa”. If the repeated consonant is soft it could accentuate a mood of love or melancholy and if it is hard it could enhance a mood of valour and war. However, when a sequence of syllables is repeatedly used in a verse it is called “yamaka”. The sequence by itself need not be meaningful and even when meaningful, more often than not, the meaning will be different at each instance of repetition. “Anuprāsa” and “yamaka” were the earliest forms of “Śabdaalaṅkāra”, namely “word-based ornamentation”. Vālmīki’s Rāmāyaṇam, which holds an exalted position in world literature and which perhaps was written in 2nd century BC, has a display of yamaka (repetition of syllabic sequences) in the 5th canto of the 5th book, Sundarākāṇḍa as the following verse exemplifies:

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tataḥ sa madhyamgatamanṣumantam jyotsṇavitānaṃ muhurudvam-
mantam
dadarśadhīmānbhuvibhānumantaṃgoṣṭhevṛṣam mattamivabhra-
mantam ॥
```

“Then intelligent Hanuman looked at the brilliant Moon in the zenith of the sky spewing frequently a spread of moon-light, and wandering like a proud bull in its enclosure.”

One cannot miss the pleasing repetition of the sequence "man-
tam”[मन्तम] and the fact that at each instance of repetition it occurs as a part of a different word.

Kālidāsa who could not have been later than 5th century AD indulges in yamaka in his work Raghuvaṃśam at places. By the time poet Bhāravi composed his magnum opus, “Kirātārjunīyam” an epic poem sometime earlier to 7th century AD, Śabda alaṅkāra had widened its base beyond “anuprāsa” and “yamaka”.

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In the 15th canto of Kirātārjunīyam, Bhāravi demonstrates “Gomūtrikābandha” one of the earliest forms of citrakāvyam in the following verse. The verse is written in a figure [Figure 6] which represents the zig-zag path traced on the ground by the urine of a cow as it urinates while walking. (go=cow, mūtra =urine, bandha=constraint)

नासुरो'याṃ न वा नागो धरसंख्या न राक्षसः।
नासुक्षम याया वा भोगो धरणवस्था हि राजसः॥

(“He is not a daitya; he is not a nāga(serpent); he is not a demon of mountainous proportions; He is just a pleasant enthusiastic energetic human being walking on this earth.”)

Similarly diagrams may represent a sword, a wheel, a lotus of 4, 8, 12, 16 petals and such figure-verses are named after the thing the figure represents, khadga-bandha (sword), cakra-bandha (wheel), aśṭa-dala-padma-bandha (8-petalled lotus) etc. The main feature of these figure-verses (often called “citrabandha”s) is that some syllables are repeatedly used as pivots.

While Bhāravi demonstrated only three types of figure-verses, Rājānaka Ratnākara who is believed to have lived in the 9th century devotes the 48th canto of his Haravijayam to several types of citrabandhas, where he has attempted the knight’s tour also. Haravijayam is the earliest of the poetic works where we come across the knight’s tour. Rudraṭa, who wrote, “Kāvyālaṅkāra”, a trea-
tise on Literary Theory in the 9th century was the earliest among literary theorists who discussed śabda alāṅkāras and especially citrakaṅvyam including knight’s tour in detail. We could thus say that citrakaṅvyams had reached a stage of maturity by the time of Rudraṭa and that they came into vogue during the 8th-9th centuries.

There are other types of constraints, employed in citrakaṅvyam, such as the following:

1. The verse does not contain certain types of consonants say labials (pa-varga) or gutturals (ka-varga).
2. The verse contains 1, 2 or 3 consonants only.
3. The verse uses only one syllable. An example is
yāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyāyायायायायायायायायायायायायायायायायायायायायायायायायायāyायायायायायायायायायात्।
पादुकासहम्-30-26 II
["यायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायायात्।
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पादुकासहम्-30-26 II]

4. The verse when read in reverse yields another meaningful, contextually relevant verse. There are works of this type where the verses, when read normally, narrate one story and when read in reverse narrate another story.

5. The verse written row-wise in a matrix yields another verse or same verse when read as per a specified manner. Knight’s tour comes in this category.

Two factors associated with Sanskrit facilitate such literary acrobatics. 1. Possibility of splitting a compound word in different ways as per the rules of sandhi\(^2\) and samāsa\(^3\). 2. Existence of a large number of words which have multiple meanings.

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**Notes:**

2 euphonic coalition causing modifications of the final letters of a word and the beginning letters of the next word in a sentence.

3 Construction of compound words.
Later literary theorists who were not just compilers of knowledge but were also original thinkers did not speak highly of word-based ornamentations (śabda alaṅkāra) as they could only be understood by the very learned among connoisseurs and as they often resulted in semantic banality. However, citrakāvyam continued to be popular among aspiring poets as a means of demonstrating their wizardry in versification. We thus see competent poets indulging in these literary acrobatics right up to the 20\textsuperscript{th} and 21\textsuperscript{st} centuries.

5. Verse-pairs for Knight's Tour

We shall now have a look at two verse-pairs composed as knight’s tour (turagabandha) the first by Vedāntadeśika in Pādukāsahasram and the second by Rājānaka Ratnākara in Haravijayam.

Vedāntadeśika who lived in the later part of the 13\textsuperscript{th} century was a pious devout person who led a simple life, an erudite scholar who wrote philosophical treatises and a gifted poet whose literary works have earned him a place among the pantheon of great Sanskrit poets. His work Pādukāsahasram consisting of 1000 verses is a hymn to the sandals of Śrīrāma which were worshipped by Bharata when the former lived in the forests. The work has 32 cantos and the 30\textsuperscript{th} canto called citrapaddhati consists of 40 verses where Vedāntadeśika demonstrates his skill in citrabandhas and in every possible type of literary jugglery. Needless to say that it is not easy to understand the meaning of these verses without the help of a commentary written by an equally competent and erudite scholar.

In this work of his, the pair of verses which demonstrates knight’s tour is as follows [\textit{Figure 7}]:

\begin{verbatim}
sthirāgasāṁ sadārādhya vihatākatatāmatā \\
 satpāduke sarā sāmā raŋgarājapadannaya
\end{verbatim}

\begin{verbatim}
स्थिरागसां सदाराध्यानि विहताकततामताः \\
सत्पादुके सरामा रञ्जराजपदनया
\end{verbatim}

4 Dr S. Rajaraman has been presented with Vyas samman (2019) by The Government of India for his accomplishments in the genre of Citrakāvyam.
![Vedāntadeśika’s knight tour verses.](image)

### Oh Sandal-pair of the Creator! You are to be worshipped all the time by the ordinary folk whose sins are ever present. You demolish the sorrows and unpleasant happenings. Lead me jingling to the feet of God]


<table>
<thead>
<tr>
<th>सिंह1</th>
<th>रा30</th>
<th>ग9</th>
<th>रा20</th>
<th>च3</th>
<th>रा24</th>
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<td>चे14</td>
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<td>रा15</td>
<td>रा32</td>
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<td>प्ल8</td>
<td>रा13</td>
<td>ध्व22</td>
<td>च5</td>
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![Figure 7](image)

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The three verses above form one sentence. The second verse is the main verse and the third verse is the knight’s tour verse. The meaning of the verses is roughly as follows:

Suciroma’s lustrous angry army, which had a fear-less roar, which was served by horsemen and magicians, which had gained advantage by virtue of having swords in hand, which comprised of soldiers protected by the large army, whose sounds indicated the
arrival of final annihilation and which attacked Kali, the divine force was defeated by her.

It needs to be mentioned that Dr. Knuth was so much fascinated by the knight’s tour verses in Sanskrit that he composed a similar pair of verses in English! Whereas in Sanskrit each cell contains a syllable, in Knuth’s verses each cell contains a word. [Figure 9 & 10] [https://www.iiitb.ac.in/CSL/projects/Chitrukavya/visuals.html]
6. Rudraṭa’s Verse

Having had a look at knight’s tour verse-pairs, let us have a look at Rudraṭa’s composition which has a unique feature. The 15th verse of the 5th Chapter of his Kāvyālāṅkāra which reads as follows is a knight’s tour verse.

\[ \text{senā līlīnā nālī līnānā nānālīlī} \]
\[ \text{nālīnālīile nālīnī lānānānālīlī} \]
\[ \text{नालीलीलीलीलीलीनालीलीनालीलीलीलीली} \]

It can be translated as follows:

“I, a truthful well-read man, a leader of a group, helpful to servants, praise the army which has as its leader a man who praises playful persons”.

Namisādhu, the commentator, after explaining the meaning of the verse, gives a cryptic mnemonic verse in his commentary which reads as follows:

\[ \text{कशझेनागभटाय तथखेवेञराघबे} \]
\[ \text{षजेथाढे पचेमेथे दाेणसछलडे फङे} \]
\[ \text{कशझेनागभटाय तथखेवेञराघबे} \]
\[ \text{षजेथाढे पचेमेथे दाेणसछलडे फङे} \]

He further says, “Write this verse over ‘\text{senā lī}’” \[\text{सेनाली}---\] verse serially each letter over its corresponding letter. Then following the alphabetical order of the letters in the mnemonic verse you can get the knight’s move verse.”

The cryptic verse of Namisādhu has no meaning and contains each of the Sanskrit consonants from \text{क(ka)} to \text{स(sa)} only once. (There is apparently an error in the mnemonic verse, which has occurred while printing or copying manuscript. It reads \text{थ} instead of \text{ध} and that leads to double occurrence of \text{थ} and missing of \text{ध}.) Vowels attached to the consonants are just there to meet the constraints of prosody and have no value or meaning.
Following Namisādhu’s explanation, the corrected mnemonic verse has been placed over Rudraṭa’s verse in Figure 11. For clarity, Rudraṭa’s verse is in Roman script, while Namisādhu’s mnemonic is in Devanāgarī script. Alphabetical serial number of the letter is also given.

If we follow the alphabetical order it clearly gives the knight’s tour, which turns out to be the same as what Vedāntadesīka has used for his verse-pair. [It is very likely that the poets knew the solution through an earlier knight’s tour verse-pair or a mnemonic verse, same as or similar to, the one given by Namisādhu.]

Rudraṭa has simplified the complexity of composition by adopting only 4 syllables. His composition is unique as his knight’s tour verse is the same as the main verse.

It is natural to investigate if his use of only 4 distinct syllables is connected with the knight’s tour sequence yielding the same verse.
It indeed turns out that in order that the knight’s tour sequence yield the same verse, there cannot be more than 4 distinct syllables in the verse.

To prove this, we set up a table of equality (Figure 12) where the 2 serial cell-numbers in a column have to be the same syllable:

This table of equality leads to the interesting result that the following cells have to have the same syllable:

- 2, 11, 7, 28, 29, 12, 24, 6, 22, 31, 17, 19, 10, 13, 30.
- The following cells have to have the same syllable:
  - 3, 5, 32, 27, 14, 20, 4, 15, 26, 8, 18, 25, 23, 16, 9,
  - These two sets of cells turn out to be the two disjoint cycles of the knight’s tour permutation. Cell No. 1 and Cell No. 21 can have distinct syllables.

Rudraṭa’s verse adheres to the above criteria where cell No. 1 contains syllable से ‘se’, cell No. 2, 11 etc contain syllable ना ‘nā’, cell No. 3, 5 etc contain syllable ली ‘li’ and cell No. 21 contains syllable ले ‘le’. It may be noted that the syllable in cell 21 of the main verse is also the syllable in cell 21 of the knight’s tour verse.

Now a question arises. Let us suppose that instead of Rudraṭa’s knight’s tour we are interested in composing a verse based on any of the other knight’s tours we have had a look at. Then, what are the conditions that will need to be satisfied, if we want to ensure that

<table>
<thead>
<tr>
<th>Main verse cell No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knight's tour cell No.</td>
<td>1</td>
<td>11</td>
<td>5</td>
<td>15</td>
<td>32</td>
<td>22</td>
<td>28</td>
<td>16</td>
<td>3</td>
<td>13</td>
<td>7</td>
<td>24</td>
<td>30</td>
<td>20</td>
<td>26</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Main verse cell No.</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
<th>31</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knight's tour cell No.</td>
<td>19</td>
<td>25</td>
<td>10</td>
<td>4</td>
<td>21</td>
<td>31</td>
<td>16</td>
<td>6</td>
<td>23</td>
<td>8</td>
<td>14</td>
<td>29</td>
<td>12</td>
<td>2</td>
<td>17</td>
<td>27</td>
</tr>
</tbody>
</table>

**Figure 12.** Table of equality of cells to ensure that Rudrata’s main verse and knight’s tour verse are same.
the verse read along knight’s tour is the same as the main verse? The answer can be worked out in the same manner as was worked out in the case of Rudraṭa’s verse.

It would however be appropriate to have a look at this procedure at an elementary level as follows. Supposing we permute a set of elements (a, b, c, d, e, f) as follows:

\[ a \rightarrow b \rightarrow c \rightarrow a; \quad d \rightarrow e \rightarrow d; \quad f \rightarrow f. \]

This permutation can be written as \((abc)(de)(f)\). The various alternatives can be listed as

\[ \text{abc de f, bca ed f, cab de f, abc ed f, bca de f, cab ed f,} \]

The number of distinct alternatives is 6, LCM of the periods of the two cyclic permutations. If we say that the elements of the set are such that this permutation leaves the order of the elements of the set unchanged, it implies \(a = b = c; \quad d = e\). That means there can only be 3 independant elements, one for each disjoint cycle.

This logic is clearly extendable to the case of Rudraṭa’s knight’s tour verse. In the case of the latter, the knight’s tour verse coincides with the main verse. When we set up the set of equations to enforce this coincidence, the disjoint cycles separate out automatically and we are led to the result that all the elements of a disjoint cycle will have to be a common syllable. In case of Rudraṭa’s knight’s tour as there are 2 disjoint cycles each having 15 elements and 2 fixed points there can be a maximum of 4 syllables only. Similarly for knight’s tours of Ratniśara, Someśvara and Euler, the maximum number of syllables that the verse can have in order that the knight’s tour verse be the same as main verse can be worked out and the details are as per Figure 13.

Figure 13 shows that among three \(4 \times 8\) knight’s tours studied, Ratnakara’s and Euler’s require 3 syllables. Someśvara’s \(8 \times 8\) knight’s tour will need a pair of main verses, each verse having 32 syllables and it will require 4 syllables.

Figure 13 (Column D) also indicates the structure of permutation in each of the four knight’s tours. Rudraṭa’s knight’s tour is a permutation of the elements of set 1, 2, 3, ----32 which maps the 15
### Figure 13. Required number of syllables and their cell no.s for the knight’s tour verse to be the same as main verse (4 different tours).

<table>
<thead>
<tr>
<th>A Verse-pair of</th>
<th>B No.of syllables</th>
<th>C Syllable No.</th>
<th>D Cell No.s where the syllable has to appear (Also indicates the permutation structure: Fixed points and cyclic permutations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rudraṭa (4x8)</td>
<td>4</td>
<td>1</td>
<td>Cell No. 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>Cell No. 21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>Cells (2, 11, 7, 28, 29, 12, 24, 6, 22, 31, 17, 19, 10, 13, 30.) 15 cells</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>Cells No. (3, 5, 32, 27, 14, 20, 4, 15, 26, 8, 18, 25, 23, 16, 9) 15 cells</td>
</tr>
<tr>
<td>Ratnakara</td>
<td>3</td>
<td>1</td>
<td>Cell No. 16</td>
</tr>
<tr>
<td>(4x8)</td>
<td></td>
<td>2</td>
<td>Cell No. 28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>Cells No. (1, 29, 13, 27, 11, 2, 19, 15, 31, 24, 3, 25, 18, 32, 30, 7, 8, 23, 9, 6, 14, 21, 20, 5, 4, 10, 12, 17, 22, 26) 30 cells</td>
</tr>
<tr>
<td>Somēśvara</td>
<td>4</td>
<td>1</td>
<td>Cell No. 11</td>
</tr>
<tr>
<td>(8x8)</td>
<td></td>
<td>2</td>
<td>Cell No. 26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>Cells No. (57, 29, 52, 37, 6, 23, 13, 18, 64, 46, 48, 21, 15, 50, 44, 53, 45, 4, 5, 15, 45, 63, 36, 16, 60, 10, 5, 40, 17, 54, 28, 58, 19, 47, 31, 58, 35, 22, 30, 62, 42, 49, 38, 12, 1) 45 cells</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>Cells No. (3, 61, 25, 9, 20, 32, 39, 2, 51, 27, 41, 34, 7, 8, 14, 33, 24) 17 cells</td>
</tr>
<tr>
<td>Euler 4x8</td>
<td>3</td>
<td>1</td>
<td>Cell No. 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>Cell No. 23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>Cells No. (2, 18, 25, 12, 24, 29, 21, 14, 13, 7, 5, 32, 6, 15, 3, 28, 27, 17, 19, 10, 20, 4, 22, 6, 13, 11, 7, 30, 31, 16, 9, 26) 30 cells</td>
</tr>
</tbody>
</table>

Elements of subset (2, 11, 7, 28, 29, 12, 24, 6, 22, 31, 17, 19, 10, 13, 30) to each other in a cyclic fashion and 15 elements of subset (3, 5, 32, 27, 14, 20, 4, 15, 26, 8, 18, 25, 23, 16, 9) to each other in a cyclic fashion, while elements 1 and 21 remain fixed. Similar statements can be made in respect of the other three knight’s tours.
It is interesting and intriguing too, to note that there are two and only two fixed points in each of the four knight’s tours studied.

7. Conclusion

Solution to the knight’s tour problem was known in India as early as the 9th century AD in the form of a literary skill, where a 32-syllable verse written on a $8 \times 4$ chess-board yields another meaningful verse if read as per the movement of a knight visiting all the 32-cells one and once only. Rudraṭa’s Kāvyālāṅkāra provides a verse employing only 4 distinct syllables where the knight’s tour yields the same verse. This article, first gives an introduction to knight’s tour as a chess puzzle, as a mathematical problem and as demonstrator of wizardry in Sanskrit versification and then works out the maximum number of distinct syllables that the verses could contain in order that the knight’s tour verse be the same as the main verse, in respect of four different knight’s tours. It is seen that only in respect of Rudraṭa’s knight’s tour a maximum of 4 distinct syllables could be accommodated. The article also indicates the cyclic permutation associated with each of the four knight’s tours.

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